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DIVISION: JEE (MAIN+ADVANCED)
ACADEMIC SESSION: 2023-24

PERIODIC ASSESSMENT TEST (PAT)

STUDENT SUPPORT BOOKLET (SSB)

Answer Key (AK) | Standard Hints (SH) | Text Solutions (TS) | Weightage Sheet (WS)

CLASS	XII	COURSE NAME	VIJETA	COURSE CODE	JP
PHASE CODE(S)	05JP	TOTAL PAGES	1	BATCH CODE(S)	05JP

Target Examination & Year:

JEE (MAIN+ADVANCED) 2024

TEST PATTERN	TEST TYPE	TEST CODE & SEQUENCE
JEE (ADVANCED)	PART TEST (PT)	APT 02

DATE & DAY:
20th August 2023 | Sunday

Duration & Time:
Paper-1 : 3 Hrs | 11:30 AM to 02:30 PM
Paper-2 : 3 Hrs | 03:00 PM to 06:00 PM

Contents:

- Weightage Sheet (WS) ☐
- Answer Key (AK) ☑
- Standard Hints (SH) ☐
- Text Solutions (TS) ☑
- Resonance Student's Critical Analysis of Learning for Excellence (ResoSCALE) ☐
- Student Self Assessment Sheet (SAS) ☐
- Video Solutions (VS) ☐

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ANSWER KEY (AK)

PAPER-1											
PART-I : MATHEMATICS	Q.No.	1	2	3	4	5	6	7	8	9	10
	Ans.	BCD	ABCD	BD	AB	AD	BC	AC	BCD	BCD	ABC
	Q.No.	11	12	13	14	15	16	17	18		
	Ans.	BD	ABCD	ABCD	ABC	B	C	B	C		
PART-II : PHYSICS	Q.No.	19	20	21	22	23	24	25	26	27	28
	Ans.	ABD	BC	AD	B	ABD	C	BD	ABC	D	CD
	Q.No.	29	30	31	32	33	34	35	36		
	Ans.	BC	A	AB	C	B	B	D	A		
PART-III : CHEMISTRY	Q.No.	37	38	39	40	41	42	43	44	45	46
	Ans.	AC	ACD	ABCD	BCD	ABCD	ABCD	ACD	AD	BC	ACD
	Q.No.	47	48	49	50	51	52	53	54		
	Ans.	ABCD	BCD	ABD	AD	D	B	B	C		
PAPER-2											
PART-I : MATHEMATICS	Q.No.	1	2	3	4	5	6	7	8	9	10
	Ans.	AC	BC	AC	CD	BC	ABC	ABCD	AD	ABCD	ABCD
	Q.No.	11	12	13	14	15	16	17	18		
	Ans.	01.00	11.00	06.00	10.00	01.00	02.00	02.00	05.00		
PART-II : PHYSICS	Q.No.	19	20	21	22	23	24	25	26	27	28
	Ans.	AD	B	B	BCD	D	AC	AC	B	AD	BC
	Q.No.	29	30	31	32	33	34	35	36		
	Ans.	05.00	08.00	00.45	16.00	06.00	05.00	05.00	03.00		
PART-III : CHEMISTRY	Q.No.	37	38	39	40	41	42	43	44	45	46
	Ans.	BC	ACD	BCD	ACD	AB	AB	CD	CD	ACD	ABC
	Q.No.	47	48	49	50	51	52	53	54		
	Ans.	04.00	27.00	05.00	06.00	05.00	05.00	04.00	06.00		

STUDENT'S SPACE

PAPER-1

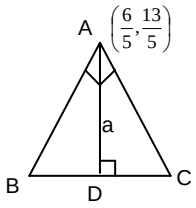
PART-I: MATHEMATICS

1. ∵ Given lines are perpendicular and intersect at $A\left(\frac{6}{5},\frac{13}{5}\right)$

∵ दी गई रेखाएं लम्बवत् होगी तथा $A\left(\frac{6}{5},\frac{13}{5}\right)$ पर प्रतिच्छेद करती है।

Equations of angle bisectors of the given line are $x+3y=9$ and $3x-y=1$

दी गई रेखा के कोण अर्द्धकों के समीकरण $x+3y=9$ और $3x-y=1$ है।



Third side (let BC) will be parallel to these bisectors.

तीसरी भुजा BC इन अर्द्धकों के समान्तर होगें।

Let माना AD = a $\Rightarrow$ AB = $a\sqrt{2}$

∵ Area of $\Delta ABC = 10$

$\Rightarrow \frac{1}{2}(a\sqrt{2})^2 = 10 \Rightarrow a = \sqrt{10}$

Let the equation of BC is $x+3y=k$

$\therefore \left|\frac{\frac{6}{5}+\frac{39}{5}-k}{\sqrt{1+9}}\right| = \sqrt{10} \Rightarrow k = -1, 19$

Thus equation of BC is $x+3y+1=0$ or $x+3y-19=0$

If the equation of BC is $3x-y=k_1$

$\therefore \left|\frac{\frac{18}{5}-\frac{13}{5}-k_1}{\sqrt{10}}\right| = \sqrt{10} \Rightarrow k_1 = -9, 11$

Hence equation of BC is $3x-y+9 = 0$ or $3x-y-11=0$

2. $x = -\sqrt{3} - \frac{r}{2}$, $y = 0 + r\frac{\sqrt{3}}{2}$

$\frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2$

$\Rightarrow 3r^2 - 2r - (4\sqrt{3} + 8) = 0 \begin{matrix} r_1 \\ r_2 \end{matrix}$

$r_1 = PA$, $r_2 = -PB$

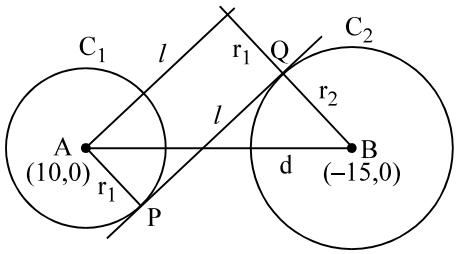
$r_1 + r_2 = PA - PB = \frac{2}{3}$, $r_1 r_2 = PA(-PB)$

$= -\frac{(4\sqrt{3}+8)}{3}$

$r_1 - r_2 = PA + PB = \sqrt{(r_1+r_2)^2 - 4r_1r_2}$

$= \frac{2}{3}\sqrt{25+12\sqrt{3}}$

3.



Centres are (10, 0) and (– 15, 0)

केन्द्र (10, 0) और (– 15, 0) है।

$r_1 = 6$; $r_2 = 9$

$d = 25$

$r_1 + r_2 < d$

$\Rightarrow$ circles are separated वृत्त अलग अलग है।

$PQ_{(min)} = \sqrt{d^2-(r_1+r_2)^2} = \sqrt{625-225} = 20$

$PQ_{(max)} = \sqrt{d^2-(r_1-r_2)^2} = \sqrt{625-9} = \sqrt{616}$

4. Family of circle $S + \lambda L = 0$

वृत्तों का निकाय $S + \lambda L = 0$

$S_1 \equiv (x-3)(x-6) + (y-7)(y-5) + \lambda(2x+3y-27) = 0$

$x^2 + y^2 + x(2\lambda - 9) + y(3\lambda - 12) + (53 - 27\lambda) = 0$

common chord of family of circle and given circle is $S_1 - S_2 = 0$ (S_2 given circle)

दिए गए वृत्तों के निकाय की उभयनिष्ठ जीवा $S_1 - S_2 = 0$ (S_2 दिया गया वृत्त)

$(-5x - 6y + 56) + \lambda(2x + 3y - 27) = 0$

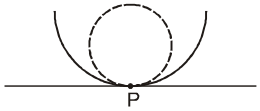
which represents family of lines passing through the point $\left(2, \frac{23}{3}\right)$

$\Rightarrow a = 2, b = \frac{23}{3}$

बिन्दु $\left(2, \frac{23}{3}\right)$ से गुजरने वाले रेखा निकाय को व्यक्त करता है $\Rightarrow a = 2, b = \frac{23}{3}$

5. $at^2 = 2at$ point

$t = 0, t = 2$ (0, 0), (4, 4)



(I) when $P \equiv (0, 0)$

equation of tangent of parabola $y^2 = 4x$ at (0, 0) is

$y \cdot y_1 = 2(x + x_1)$

$x = 0$

Now equation of circle is $x^2 + y^2 + \lambda(x) = 0$

pass through the point (1, 0)

$\lambda = -1$

So equation is $x^2 + y^2 - x = 0$

(II) when point (4, 4)

equation of tangent at point (4, 4) is $2x - 4y + 8 = 0$

now circle, $(x - 4)^2 + (y - 4)^2 + \mu(2x - 4y + 8) = 0$

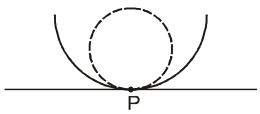
pass through point (1, 0)

so equation of circle is

$x^2 + y^2 - 13x + 2y + 12 = 0$

हल. $at^2 = 2at$ बिन्दु

$t = 0, \quad t = 2$ (0, 0), (4, 4)



(I) जब $P \equiv (0, 0)$

$x^2 + y^2 + \lambda(x) = 0$

बिन्दु (1, 0) से गुजरता है।

$\lambda = -1$

$y^2 = 4x$ के बिन्दु (0, 0) पर स्पर्श रेखा का समीकरण

समीकरण

$x^2 + y^2 - x = 0 \qquad y \cdot y_1 = 2(x + x_1)$

$x = 0$

(II) जब बिन्दु (4, 4) है। $2x - 4y + 8 = 0$

$(x - 4)^2 + (y - 4)^2 + \mu(2x - 4y + 8) = 0$

बिन्दु (1, 0) से गुजरता है।

समीकरण

$x^2 + y^2 - 13x + 2y + 12 = 0$

6. Equation of tangent to $y^2 = 4x$, whose slope is m, is $y = mx + \frac{1}{m}$

i.e. $m^2 x - my + 1 = 0$

it is tangent to the circle $(x - 3)^2 + y^2 = 9$

$\therefore \quad \left|\frac{3m^2 + 1}{\sqrt{m^4 + m^2}}\right| = 3$

$\therefore \quad 9m^4 + 6m^2 + 1 = 9m^4 + 9m^2$

i.e. $3m^2 = 1$

$\Rightarrow \quad m = \pm \frac{1}{\sqrt{3}}$

$\therefore$ Equation of the tangent is

$y = \pm\left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)$ i.e. $\sqrt{3}\,y = \pm(x + 3)$

परवलय $y^2 = 4x$ की स्पर्श रेखा का समीकरण, जिसकी प्रवणता m है, $y = mx + \frac{1}{m}$ है।

अर्थात् $m^2 x - my + 1 = 0$

यह वृत्त $(x - 3)^2 + y^2 = 9$ की स्पर्श रेखा है।

$\therefore \quad \left|\frac{3m^2 + 1}{\sqrt{m^4 + m^2}}\right| = 3$

$\therefore \quad 9m^4 + 6m^2 + 1 = 9m^4 + 9m^2$

अर्थात् $3m^2 = 1$

$\Rightarrow \quad m = \pm \frac{1}{\sqrt{3}}$

$\therefore$ स्पर्श रेखा की समीकरण

$y = \pm\left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)$ अर्थात् $\sqrt{3}\,y = \pm (x + 3)$ है।

7. $\lim\limits_{x\to 0} \frac{ae^{2x} - b\cos 2x + ce^{-2x} - x\sin x}{x\sin x} = 1$

$$\lim_{x\to 0} \frac{a\left(1+2x+\frac{(2x)^2}{2!}+....\right) - b\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-.....\right) + c\left(1-2x+\frac{(2x)^2}{2!}-....\right)}{x^2} = 2$$

$\Rightarrow a - b + c = 0$.....(1)

$2a - 2c = 0$ .....(2)

$2a + 2b + 2c = 2$ .....(3)

$\Rightarrow$ a = c and और b = 2a

$\Rightarrow a + 2a + a = 1 \quad \Rightarrow \quad a = 1/4 = c$

$b = \frac{1}{2}$

$f(t) = \frac{3}{4}t^2 - \frac{1}{4}t + \frac{1}{4} = \frac{3t^2 - t + 1}{4}$

8. $f(x) = ([x] + [-x])([\sin x] + [-\sin x]) + x$

$$= \begin{cases} x\ , & x \in I \\ x\ , & x \in \frac{n\pi}{2}, n \in I \\ x+1\ , & \text{otherwise अन्यथा} \end{cases}$$

points of discontinuity are x = 1,2,3, $\frac{\pi}{2}$

असंततता के बिन्दु x = 1,2,3, $\frac{\pi}{2}$

9. Given f(x+y) = f(x) + f(y) + xy

.....(1)

and $\lim\limits_{h\to 0} \frac{f(h)}{h} = 3$

$f'(x) = \lim\limits_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \frac{f(h) + xh}{h} = 3 + x$

$\Rightarrow f(x) = 3x + \frac{x^2}{2} + c$

in equation (1) put x=0=y

$\Rightarrow f(0) = 0$

Therefor $f(x) = 3x + \frac{x^2}{2}$

हल. दिया है f(x+y) = f(x) + f(y) + xy

.....(1)

और $\lim\limits_{h\to 0} \frac{f(h)}{h} = 3$

$f'(x) = \lim\limits_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \frac{f(h) + xh}{h} = 3 + x$

$\Rightarrow f(x) = 3x + \frac{x^2}{2} + c$

समीकरण (1) में x=0=y $\Rightarrow f(0) = 0$

अतः $f(x) = 3x + \frac{x^2}{2}$

10. $\lim\limits_{x\to \frac{\pi}{2}^+} \cot x = 0^-$

$\lim\limits_{x\to \frac{\pi}{2}^+} f(x) = \pi$

now $\lim\limits_{x\to \frac{\pi}{2}^+} g(f(x)) = g(\pi) = \text{sgn}(0) = 0$

$\lim\limits_{x\to \frac{\pi}{2}^+} g(x) = 1$

$f\left(g(x) + \frac{\pi}{4} - 1\right) = \cos^{-1} 1 = 0$

$\lim\limits_{x\to \frac{\pi}{2}^-} \cot x = 0^+$

$f(x) = \cos^{-1} 0 = \frac{\pi}{2}$

g(f(x)) = 1

$\lim\limits_{x\to \frac{\pi}{2}^-} g(f(x)) = 1$

Soइसलिए $\lim\limits_{x\to \frac{\pi}{2}^-} f\left(g(x) + \frac{x}{4} - 1\right) = 0$

11. We have यहाँ $g = \frac{1}{f}$

$\therefore g' = -\frac{1}{f^2}.f' \quad \Rightarrow \quad \frac{f'}{f} = -g'f = -\frac{g'}{g}$

or या $g'' = \frac{2}{f^3}(f')^2 - \frac{f''}{f^2}$

$$\Rightarrow \frac{g''}{g'} = -\frac{2f'}{f} + \frac{f''}{f'}$$

$$\Rightarrow \frac{f''}{f'} - \frac{g''}{g'} = \frac{2f'}{f} = \frac{-2g'}{g}$$

12. $2^x + 2^y = 2^{x+y}$

... (i)

diff. both sides w.r.t.x

$$2^x \cdot \ell n2 + 2^y \cdot \ell n2 \frac{dy}{dx} = 2^{x+y} \cdot \ell n2\left(1 + \frac{dy}{dx}\right)$$

$$2^x - 2^{x+y} = (2^{x+y} - 2^y)\ \frac{dy}{dx} \text{ .... (ii)}$$

$$\frac{2^x(1-2^y)}{2^y(2^x-1)} = \frac{dy}{dx}$$

from (i) & (ii)

$$2^x - 2^x - 2^y = (2^x + 2^y - 2^y)\ \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2^y}{2^x}$$

$$\frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y} - 2^y} = \frac{-2^y}{2^y(2^x-1)}$$

$$= \frac{1}{1-2^x} = 1 - 2^y$$

हल. $2^x + 2^y = 2^{x+y}$

... (i)

समीकरण (i) के दोनों पक्षो का x के सापेक्ष अवकलन करने पर

$$2^x \cdot \ell n2 + 2^y \cdot \ell n2\ \frac{dy}{dx} = 2^{x+y} \cdot \ell n2\left(1 + \frac{dy}{dx}\right)$$

$$2^x - 2^{x+y} = (2^{x+y} - 2^y)\ \frac{dy}{dx} \text{ .... (ii)}$$

$$\frac{2^x(1-2^y)}{2^y(2^x-1)} = \frac{dy}{dx}$$

समी. (i) तथा (ii) से

$$2^x - 2^x - 2^y = (2^x + 2^y - 2^y)\ \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2^y}{2^x}$$

$$\frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y} - 2^y} = \frac{-2^y}{2^y(2^x-1)}$$

$$= \frac{1}{1-2^x} = 1 - 2^y$$

13. $\cos(|x|) = \cos x$,

$(x^2-1)|x^2-3x+2| = (x-1)\,|x-1|\,.(x+1)\,|x-2|$

The only point where f is not differentiable is x = 2

हल. $\cos(|x|) = \cos x$,

$(x^2-1)|x^2-3x+2| = (x-1)\,|x-1|\,.(x+1)\,|x-2|$

f केवल x = 2 पर अवकलनीय नही है।

14. Let माना $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

and और $z = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \sin\theta$ रखने पर $\Rightarrow \theta = \sin^{-1}x$

$\therefore\ y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right) = \sin^{-1}$

$(\sin 2\theta)$

$y = 2\theta$

$= 2\sin^{-1}x$

$$\frac{dy}{dx} = 2 \times \frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$z = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$$

$$\frac{dz}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\frac{dz}{dy} = \frac{dz/dx}{dy/dx} = \frac{\sqrt{1-x^2}}{1+x^2}$$

Sol. (15-16)

LT-3, 4

Let θ, ϕ, α are the roots of $x^3 + 2x^2 + px + q = 0$, then

माना θ, ϕ, α समीकरण $x^3 + 2x^2 + px + q = 0$ के मूल है, तब

$\theta + \phi + \alpha = -2$ .....(i)

$\theta\phi + \phi\alpha + \theta\alpha = p$ .....(ii)

$\theta\phi\alpha = -q$ .....(iii)

and let θ, ϕ, β are the roots of $x^3 + x^2 + px + r = 0$

तथा माना θ, ϕ, β समीकरण $x^3 + x^2 + px + r = 0$ के मूल है

$\therefore \quad \theta + \phi + \beta = -1$ .....(iv)

$\theta\phi + \phi\beta + \theta\beta = p$ .....(v)

$\theta\phi\beta = -r$ .....(vi)

From eqs. (i) and (iv), we get

(i) तथा (iv) से

$\alpha - \beta = -1$ .....(vii)

From eqs. (ii) and (v), we get

(ii) तथा (v) से

$(\theta + \phi)(\alpha - \beta) = 0$

$\Rightarrow \quad \theta + \phi = 0$

Now from eqs. (i) and (iv), we get

अब समीकरण (i) तथा (iv) से

$\alpha = -2$ and $\beta = -1$

Now $f(x) = \begin{cases} a^{x\log_{(1+x)}3} & ; -1 < x < 0 \\ a & ; \quad x = 0 \\ b\dfrac{\ln\left(e^{x^2} + 2\sqrt{x}\right)}{\tan\sqrt{x}} & ; \ 0 < x < 1 \end{cases}$

Now $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} e^{-h\log_{(1-h)}3}$

$= \lim\limits_{h\to 0} e^{-\frac{h\ell n3}{\ell n(1-h)}} = \lim\limits_{h\to 0} e^{\ell n3} = 3$

and $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} \dfrac{b\ell n\left(e^{h^2} + 2\sqrt{h}\right)}{\tan\sqrt{h}}$

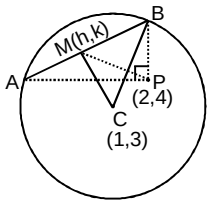
$= \lim\limits_{h\to 0} \dfrac{b\ell n\left(e^{h^2} + 2\sqrt{h}\right)}{\sqrt{h}}$

$= \lim\limits_{h\to 0} \dfrac{b\left(2he^{h^2} + \dfrac{1}{\sqrt{h}}\right)}{\left(\dfrac{e^{h^2} + 2\sqrt{h}}{2\sqrt{h}}\right)}$

$= \lim\limits_{h\to 0} \dfrac{2b\left(2h^{\frac{3}{2}}e^{h^2} + 1\right)}{\left(e^{h^2} + 2\sqrt{h}\right)} = 2b$

So $a = 3$, $b = \dfrac{3}{2}$

Sol. (17-18)



Let M be (h, k) माना M(h, k) है। r

$= \sqrt{1+9+6}$

AM = BM = PM = 4

$= \sqrt{(h-2)^2 + (k-4)^2}$

$CB^2 = CM^2 + MB^2$

$16 = (h-1)^2 + (k-3)^2 + (h-2)^2 + (k-4)^2$

$\cancel{16} = 2h^2 + 2k^2 - 6h - 14k + 1 + 9 + 4 + \cancel{16}$

$2h^2 + 2k^2 - 6h - 14k + 14 = 0$

$\Rightarrow \quad x^2 + y^2 - 3x - 7y + 7 = 0$

PART-II: PHYSICS

19. If they are connected by Cu wire, their potential will be come same.

$\dfrac{kq}{r}$ = same $\Rightarrow q \propto r$

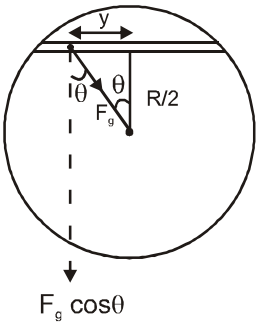
$E = \dfrac{kq}{r^2} \propto \dfrac{r}{r^2} \propto \dfrac{1}{r}$

so $\dfrac{E_1}{E_2} = \dfrac{r_2}{r_1}$

and $\sigma = \dfrac{q}{4\pi r^2} \propto \dfrac{r}{r^2} \propto \dfrac{1}{r}$

so $\dfrac{\sigma_1}{\sigma_2} = \dfrac{r_2}{r_1}$

20.



$F_g = \frac{GMmr}{R^3}$

pressing force दबाव बल $= F_g \cos\theta$

$= \frac{GMmr\cos\theta}{R^3}$

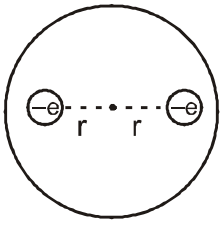
$= \frac{GMm}{2R^2}$ = constant . नियत

$a = \frac{Fg\sin\theta}{m} = \frac{GMr\sin\theta}{R^3}$

$a = \frac{GMy}{R^3}$

21. Refer to answer key

22.



Lets find electric field at radial distance (r) using gauss's theorem

माना त्रिज्य दूरी (r) वैद्युत क्षेत्र गाउस प्रमेय के उपयोग से ज्ञात करते है।

$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$

$E(4\pi r^2) = \int_{r=0}^{r=r} (\rho_0 r) 4\pi r^2 \, dr$

$E = \frac{\rho_0 r^2}{4\varepsilon_0}$

For equilibrium of the electrons

इलैक्ट्रॉनों की साम्यावस्था के लिए

(e) $\left(\frac{\rho_0 r^2}{4\varepsilon_0}\right) = \frac{1}{4\pi\varepsilon_\circ}\frac{e^2}{(2r)^2}$, $r = \left(\frac{e}{4\pi\rho_0}\right)^{1/4}$

distance between the electrons इलैक्ट्रॉनों की

मध्य दूरी $= 2r = 2\left(\frac{e}{4\pi\rho_0}\right)^{1/4}$

23. (A) Potential at the centre due to +Q charge is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} = \frac{Q}{8\pi\varepsilon_0 R}$, and that due to the induced charges = 0

So $V_{centre} = \frac{Q}{8\pi\varepsilon_0 R} = V_{surface}$

(A) +Q आवेश के कारण केन्द्र पर विभव $\frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} = \frac{Q}{8\pi\varepsilon_0 R}$, और प्रेरित आवेश के कारण = 0

अतः $V_{केन्द्र} = \frac{Q}{8\pi\varepsilon_0 R} = V_{सतह}$

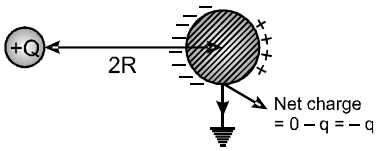
(B) In step II, two objects of different potential are tauched, so heat will be produced in the form of sparking

(B) II पद में भिन्न–भिन्न विभव की दो वस्तु एक दूसरे के सम्पर्क में है। अतः ऊष्मा चिन्गारी के रूप में उत्पन्न होगी।

(C) No heat is produced

(C) कोई ऊष्मा उत्पन्न नहीं होगी

(D)



After grounding, net potential at the centre of sphere

भूसम्पर्कित करने के पश्चात् गोले के केन्द्र पर कुल विभव

$$V = \frac{KQ}{2R} + \frac{K(-q)}{R} = 0$$

$\Rightarrow \quad q = \frac{Q}{2}$

$\Rightarrow$ Charge on the sphere is $-\frac{Q}{2}$.

गोले पर आवेश $-\frac{Q}{2}$ होगा

24. $V_C = V_D = 0 \qquad V_A = \frac{kp}{r^2} \qquad V_B = -\frac{kp}{r^2}$

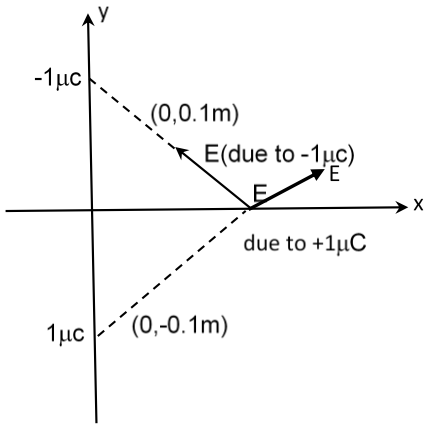
25. $V_P = \frac{kq_0}{2a} - \frac{kq_0}{a} + \frac{kq_0}{a/2} = \frac{3kq_0}{2a}$.

26. The potential of the two surface will be equal when the whole charge Q flows from inner to other shell.

दो सतह के विभव समान होगें जब सम्पूर्ण आवेश Q आन्तरिक से बाह्य कोश को प्रवाहित होगा।

27. Refer to answer key

28.



$V_O = 0$

$V_\infty = 0$ $\quad \Delta U = q\Delta V = 0$

$\therefore$ C is correct सही है।

From symmetry of the problem components of field by both charge cancel in x-axis and add up in y-axis.

प्रश्न की ज्यामिती से, दोनों आवेशों की वजह से विद्युत क्षेत्र का x-अक्ष का घटक आपस में रद्द हो जायेगा तथा y-अक्ष का घटक आपस में जुड़ जायेगा।

29.

(After collision टक्कर के बाद)

$v = v_A + v_B$ .....(1)

$\frac{1}{2} = \frac{v_B - v_A}{v}$

$v_B - v_A = \frac{v}{2}$ .....(2)

$v_B = \frac{3v}{4}$

$v_A = \frac{v}{4}$

$I_B = m\left(\frac{3v}{4} - 0\right) = \frac{3mv}{4}$

loss in K.E. गतिज ऊर्जा में हानि $= \frac{1}{2}mv^2 = \frac{1}{2}m\left[\left(\frac{v}{4}\right)^2 + \left(\frac{3v}{4}\right)^2\right] = \frac{3}{16}mv^2.$

30. Refer to answer key

31. Apply momentum conservation in horizontal direction

क्षैतिज दिशा में संवेग संरक्षण से

$m_1v_1 = mv + \frac{m_1v_1}{3}$

$v = \frac{2m_1v_1}{3m} = \sqrt{5g\ell}$ (for just complete circular motion) (वृत्तीय गति पूर्ण करने के लिए)

$v_1 = \frac{3m}{2m_1}\sqrt{5g\ell}$.

32. Rate of heat produced ऊष्मा उत्पादन की दर

$$\frac{d\theta}{dt} = \frac{v^2}{R} = \frac{v^2}{R_0(1+\alpha(T-0))} =$$

$\frac{v^2}{R_0(1+\alpha T)}$ and तथा $\frac{d\theta}{dt} = \frac{dT}{dt}$ ms

$$\Rightarrow \quad ms\frac{dT}{dt} = \frac{v^2}{R_0(1+\alpha T)}$$

$$\int_{T=0}^{T=T}(1+\alpha T)dT = \frac{v^2}{R_0 ms}\int_{t=0}^{t=t}dt$$

$$T + \frac{\alpha T^2}{2} = \frac{v^2}{R_0 ms}t$$

$$t = \frac{R_0 ms}{v^2}\left(T + \frac{\alpha T^2}{2}p\right)$$

33. $F_e\left(\frac{\ell}{2}\right) = C\theta$

$\Rightarrow \frac{kQ_1Q_2}{r^2}\left(\frac{\ell}{2}\right) = C\theta$

$\theta = 15°$

34. $\frac{kQQ}{r^2}\left(\frac{\ell}{2}\right) = C\theta$

$\frac{kQ^2\ell}{2\ C} = \theta r^2$

= same for all the observation

$= 10\ (8\ cm)^2$

$= 10\ (8 \times 10^{-2}\ m)^2$

get $Q = 16\ \mu C$

35. Refer to answer key

36. Refer to answer key

PART-III: CHEMISTRY

37. Refer to answer key

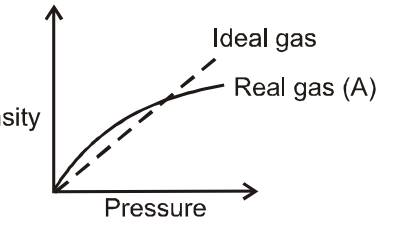
38. $\frac{r_X}{r_y} = \sqrt{\frac{M_Y}{M_X}} = \frac{1}{5}$

$$\frac{r_y}{r_z} = \sqrt{\frac{M_z}{M_y}} = \frac{1}{6}$$

$$\frac{r_z}{r_x} = \sqrt{\frac{M_x}{M_z}} = \sqrt{\frac{M_x}{M_y}} \times \sqrt{\frac{M_y}{M_z}} = 5 \times 6 = 30.$$

39. Thory based

40. **(C)**



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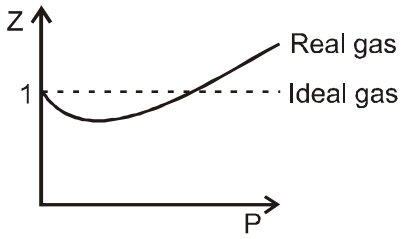

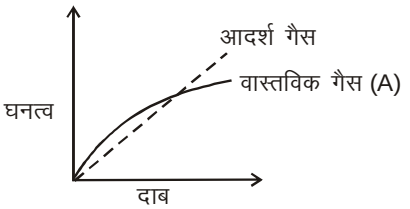

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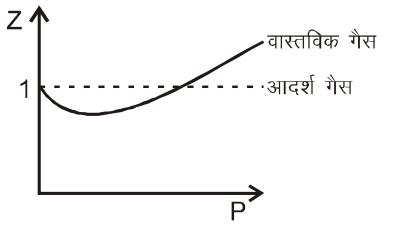


41. This radius ratio suggests exact fitting in rock salt type structure ($\because \frac{r}{R} = 0.414$).
यह त्रिज्या अनुपात रॉक साल्ट प्रकार की संरचना में पूर्णतः व्यवस्थित होता है ($\because \frac{r}{R} = 0.414$)।

42. Fact based
तथ्यात्मक

43. Constituent particles are polar molecule in ice, SO_2 & HCl.
CO_2 is a molecular solid

हल. बर्फ, SO_2 तथा HCl में घटक कण ध्रुवीय अणु होते हैं।
CO_2 एक आण्विक ठोस है।

44. Formula will be = X_3Y_4
सूत्र होगा = X_3Y_4

45. Frenkel defect is created when an ion leaves its correct lattice site and occupies an interstitial site, i.e., dislocation.
फ्रेंकल त्रुटि उत्पन्न होती है जब एक आयन इसके सही जालक स्थल से मुक्त होता है तथा अन्तराकाशी स्थलो को घेरता है अर्थात् अव्यवस्था। (विस्थापन)

47. Refer to answer key

48. (B) $Ni(HSO_3)_2$
nickel (II) hydrogen sulphite
(निकल (II) हाइड्रोजन सल्फाइट
or nickel (II) bisulphite
या निकल (II) बाईसल्फाइट)
(C) $Sr(PO_3)_2$
Stronsium metaphosphate
(स्ट्रॉन्शियम मेटाफॉस्फेट)
(D) CsOBr
Cesium hypobromite
(सिजियम हाइपोब्रोमाइट)

49. (A) H_2SO_{4-} Sulphuric acid (सलफ्यूरिक अम्ल)
(B) H_3BO_{3-} Boric acid (बोरिक अम्ल)
(C) HNO_{2-} Nitrous acid (नाइट्रस अम्ल)
(D) $HClO_{3-}$ Chloric acid (क्लोरिक अम्ल)

51. Number of Ca^{+2} ions = $\frac{1}{8} \times 8 = 1$
Number of Ti^{4+} = 1
Number of O^{-2} ions = $6 \times \frac{1}{2} = 3$
Total no. of atoms = 5

Sol. Ca^{+2} आयनों की संख्या = $\frac{1}{8} \times 8 = 1$
Ti^{4+} की संख्या = 1
O^{-2} आयनों की संख्या = $6 \times \frac{1}{2} = 3$
परमाणुओं की कुल संख्या = 5

52. Refer to answer key

PAPER-2

PART-I: MATHEMATICS

1. Any point on line through A is
$(-2 + r\cos\theta\ ,\ -3 + r\sin\theta)$
$\therefore$ $(-2 + AB\cos\theta\ ,\ -3 + AB\sin\theta)$ lies on $x + 3y = 9$

$\therefore AB = \dfrac{20}{(\cos\theta + 3\sin\theta)}$,

similarly $AC = \dfrac{4}{(\cos\theta + \sin\theta)}$

$\therefore$ $4 = \cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta$
$4 + 4\tan^2\theta = 1 + 4\tan\theta + 3\tan^2\theta$
$\tan^2\theta - 4\tan\theta + 3 = 0$
$\tan\theta = 1$ or $\tan\theta = 3$

$\Rightarrow$ required lines are
$y + 3 = x + 2$ or $y + 3 = 3(x + 2)$

हल. रेखा पर कोई बिन्दु जो A से गुजरता है
$(-2 + r\cos\theta\ ,\ -3 + r\sin\theta)$

$\therefore$ $(-2 + AB\cos\theta\ ,\ -3 + AB\sin\theta)$ रेखा $x + 3y = 9$ पर स्थित है।

$\therefore$ $AB = \dfrac{20}{(\cos\theta + 3\sin\theta)}$, इसी प्रकार

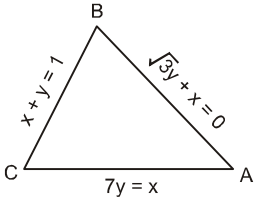
$AC = \dfrac{4}{(\cos\theta + \sin\theta)}$

$\therefore$ $4 = \cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta$
$4 + 4\tan^2\theta = 1 + 4\tan\theta + 3\tan^2\theta$
$\tan^2\theta - 4\tan\theta + 3 = 0$
$\tan\theta = 1$ or $\tan\theta = 3$

$\Rightarrow$ अभीष्ट रेखाएँ
$y + 3 = x + 2$ or $y + 3 = 3(x + 2)$

2. Let slope of given lines $m_1 = \dfrac{1}{7}$, $m_2 = \dfrac{-1}{\sqrt{3}}$,
$m_3 = -1$
Hence interior angle of triangle

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3} + 7}{7\sqrt{3} - 1} > 0$$



$$\tan B = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} > 0$$

$$\Rightarrow \tan C = \frac{m_3 - m_1}{1 + m_2 m_1} = \frac{-1 - \frac{1}{7}}{1 - \frac{1}{7}} = \frac{-8}{6} < 0$$

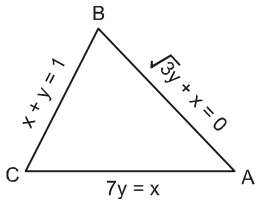
Hence angle C is obtuse therefore circumcentre and orthocentre lies outside the triangle.

हल. दी गई रेखाओं की प्रवणताऐं क्रमशः है $m_1 = \dfrac{1}{7}$,

$m_2 = \dfrac{-1}{\sqrt{3}}$, $m_3 = -1$

अतः त्रिभुज के अन्तःकोणों के लिये

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3} + 7}{7\sqrt{3} - 1} > 0$$



$$\tan B = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} > 0$$

$$\Rightarrow \tan C = \frac{m_3 - m_1}{1 + m_2 m_1} = \frac{-1 - \frac{1}{7}}{1 - \frac{1}{7}} = \frac{-8}{6} < 0$$

अतः कोण C अधिक कोण है अतः परिकेन्द्र एवं लम्बकेन्द्र त्रिभुज के बाहर होगें।

3. Equation of radical axis of the given circle is $x = 0$. If one circle lies completely inside the other, center of both circles should lie on the same of radical axis and radical axis should not intersect the circles.
दिये समीकरण की मूलाक्ष का समीकरण $x = 0$.
यदि एक वृत्त दूसरे वृत्त के अन्दर पूर्णतयाः है। तब दोनों के केन्द्र समान मूलाक्ष पर स्थित होगें। तथा मूलाक्ष वृत्तों को प्रतिच्छेद नहीं करती है।

$\Rightarrow (-a_1)(-a_2) > 0$

$\Rightarrow a_1 a_2 > 0$ and $y^2 + c = 0$ should have imaginary roots

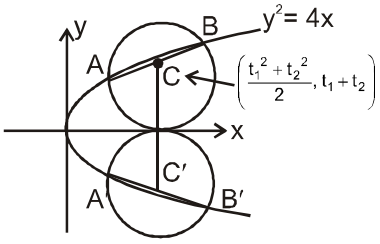
$\Rightarrow a_1 a_2 > 0$ और $y^2 + c = 0$ काल्पनिक मूल रखता है।

$\Rightarrow c > 0$

4. $A(t_1^2, 2t_1)$, $B(t^2_2, 2t_2)$

centre of circle वृत्त का केन्द्र $\left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$

$\Rightarrow |t_1 + t_2| = r$, slope of AB की प्रवणता $= \frac{2}{t_1 + t_2} = \pm \frac{2}{r}$



5. (b,c) We have,

$x^2 + y^2 - 8x - 16y + 60 = 0$ ..... (i)

Equation of chord of contact from (-2,0) is -

$2x - 4(x - 2) - 8y + 60 = 0$

$\Rightarrow$ $3x + 4y - 34 = 0$ ..... (ii)

Solving eqs. (i) and (ii), we get

$$x^2 + \left(\frac{34 - 3x}{4}\right)^2 - 8x - 16\left(\frac{34 - 3x}{4}\right) + 60 = 0$$

$\Rightarrow 16x^2 + 1156 - 204x + 9x^2 - 128x - 2176 + 192x + 960 = 0$

$\Rightarrow 5x^2 - 28x - 12 = 0$

$\Rightarrow (x - 6)(5x + 2) = 0$

$\Rightarrow x = 6$ & $-2/5$

$\Rightarrow$ Points are (6, 4), $\left(-\frac{2}{4}, \frac{44}{5}\right)$

6. $x^2 + (ax^2 - b)^2 = 1$

$\Rightarrow a^2x^4 + (1 - 2ab)x^2 + (b^2 - 1) = 0$

$\Rightarrow a^2t^2 + (1 - 2ab)t + (b^2 - 1) = 0$

$\Rightarrow f(t) = 0$

$D = 4a^2 - 4ab + 1$

$a > b > 1$

$\Rightarrow D > 0$, $f(0) > 0$ and $\frac{2ab - 1}{2a^2} > 0$

$\Rightarrow$ $t_1 > 0, t_2 > 0$

$\Rightarrow$ four distinct real values of x

$b < -1 \Rightarrow D > 0, f(0) > 0$ and $\frac{2ab - 1}{2a^2} < 0$

$\Rightarrow$ $t_1 < 0, t_2 < 0 \Rightarrow$ no real value of $x - 1 < b < 1 \Rightarrow f(0) < 0$

$\Rightarrow t_1 > 0, t_2 < 0$

$\Rightarrow$ two distinct real values of x

हल. $x^2 + (ax^2 - b)^2 = 1$

$\Rightarrow a^2x^4 + (1 - 2ab)x^2 + (b^2 - 1) = 0$

$\Rightarrow$ $a^2t^2 + (1 - 2ab)t + (b^2 - 1) = 0$

$\Rightarrow f(t) = 0$

$D = 4a^2 - 4ab + 1$

$a > b > 1$

$\Rightarrow D > 0, f(0) > 0$ और $\frac{2ab - 1}{2a^2} > 0$

$\Rightarrow$ $t_1 > 0, t_2 > 0$

$\Rightarrow$ x की चार भिन्न-भिन्न वास्तविक संख्याएँ

$b < -1 \Rightarrow D > 0, f(0) > 0$ और $\frac{2ab - 1}{2a^2} < 0$

$\Rightarrow$ $t_1 < 0, t_2 < 0 \Rightarrow$ x के किसी भी मान के लिये नहीं

$-1 < b < 1 \Rightarrow f(0) < 0$ $\Rightarrow t_1 > 0, t_2 < 0$

$\Rightarrow$ x के दो वास्तविक मानों के लिये

7. Put x = 1, 2, 3, 4 in f(x), f ′(x), f″(x) f‴(x) respectively

We get,

$f'(2) + f''(3) + f'''(4) = 0$

$12 f(1) + 3f'(2) + f''(3) = 0$

$18 f(1) + 2f'(2) - f''(3) = 0$

$6f(1) - f'''(4) = 0$

We get $f(1) = f'(2) = f''(3) = f'''(4) = 0$

So f(x) = 0

हलः f(x) में x = 1, 2, 3, 4 रखने पर f(x), f ′(x), f″(x) f‴(x) क्रमशः

$f'(2) + f''(3) + f'''(4) = 0$

$12 f(1) + 3f'(2) + f''(3) = 0$

$18 f(1) + 2f'(2) - f''(3) = 0$

$6f(1) - f'''(4) = 0$

$f(1) = f'(2) = f''(3) = f'''(4) = 0$ पर

इसलिए $f(x) = 0$

8. $\lim\limits_{x\to\infty}\dfrac{2^{\left(\frac{x^n}{e^x}\right)} - 3^{\left(\frac{x^n}{e^x}\right)}}{x^n}$ is 0

and तथा $\lim\limits_{x\to\infty}\dfrac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left(\left(\frac{2x+1}{x-1}\right)^x\right)}$

$\lim\limits_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$

$= \lim\limits_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)\left(\dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\right)$

$= \lim\limits_{x\to\infty}\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}} = \dfrac{1}{\infty} = 0$

$\ln \sec^{-1}\left(\left(\dfrac{2x+1}{x-1}\right)^x\right)$

Let माना $y = \left(\dfrac{2x+1}{x-1}\right)^x$

$\log y = x\log\left(\dfrac{2x+1}{x-1}\right)$

$\lim\limits_{x\to\infty}\log y = \lim\limits_{x\to\infty} x\log\left(\dfrac{2x+1}{x-1}\right)$

$= \infty \times \ln 2$

$\Rightarrow \quad \lim\limits_{x\to\infty}\log y = \infty$

$\Rightarrow \quad \lim\limits_{x\to\infty} y = \infty$

$\therefore \quad \sec^{-1}(\infty) = \dfrac{\pi}{2}$

$x_2 = \dfrac{\pi/2}{\pi/2} = 1$

9. $\because \Delta = 4 - 4.4^{\text{cosec}^2\alpha}\left(\beta^2 - \beta + \dfrac{1}{2}\right) \geq 0$

$\Rightarrow 1 - 4^{\text{cosec}^2\alpha}\left(\left(\beta - \dfrac{1}{2}\right)^2 + \dfrac{1}{4}\right) \geq 0$

$\Rightarrow 4^{\text{cosec}^2\alpha}\left(\left(\beta - \dfrac{1}{2}\right)^2 + \dfrac{1}{4}\right) \leq 1$

$\Rightarrow \text{cosec}^2\alpha = 1$ and $\beta = \dfrac{1}{2}$

$\therefore$ $4\sin^2\alpha + 1 = 5$ and और $2(\sin^2\alpha + \beta) = 3$

$\because f(0^-) = \lim\limits_{x\to 0^-}\dfrac{a - ax\left(x - \frac{x^3}{3!} + \ldots\ldots\right) + b\left(1 - \frac{x^2}{2!} + \ldots\ldots\right) + 5}{x^2}$

$= \lim\limits_{x\to 0^-}\dfrac{(a+b+5) + x^2\left(-a - \frac{b}{2}\right) + x^3 + \ldots\ldots}{x^2}$

$\therefore$ $a + b + 5 = 0$ and और $f(0^-) = -a - \dfrac{b}{2}$

$\because f(0^+) = \lim\limits_{x\to 0^+}\left(1 + \dfrac{x(c+dx^2)}{x^2}\right)^{\frac{1}{x}}$

$= \lim\limits_{x\to 0^+}\left(1 + \dfrac{c+dx^2}{x}\right)^{\frac{1}{x}}$ to exist $c = 0$

$= \lim\limits_{x\to 0^+}\left(1 + \dfrac{c+dx^2}{x}\right)^{\frac{1}{x}}$ विद्यमान होने के लिए

$c = 0$

$= \lim\limits_{x\to 0^+}(1 + dx)^{1/x}$

$= e^d$

$\therefore$ $f(x)$ is continuous at $x = 0$

$\therefore$ $x = 0$ पर $f(x)$ सतत है।

$\therefore$ $f(0^-) = f(x) = f(0^+)$

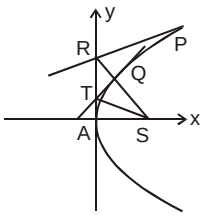
$-a - \dfrac{b}{2} = 3 = e^d$ and और $a + b + 5 = 0$

$a = -1$; $b = -4$; $c = 0$, $d = \ln 3$.

10. If equation of tangent at P, Q and vertex A of a parabola are $3x + 4y - 7 = 0$, $2x + 3y - 10 = 0$ and

$R \equiv (1, 1)$

$T \equiv (2, 2)$



Equation of RS is $4x - 3y - 1 = 0$

RS का समीकरण $4x - 3y - 1 = 0$

Equation of TS is $3x - 2y - 2 = 0$

TS का समीकरण $3x - 2y - 2 = 0$

$\therefore$ focus $S \equiv (4, 5)$

length of latus rectum $= 4 \times \dfrac{1}{\sqrt{2}} = 2\sqrt{2}$

नाभिलम्ब की लम्बाई $= 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$

axis is $x + y - 9 = 0$

अक्ष $x + y - 9 = 0$ है।

vertex $\equiv \left(\frac{9}{2}, \frac{9}{2}\right)$

शीर्ष $\left(\frac{9}{2}, \frac{9}{2}\right)$ है।

11. $\because \lim\limits_{x\to 0} \frac{3^{\sin^{-1}x} - ax + b}{2x + 1 - 1} = \ln\sqrt{3}$

$\therefore \lim\limits_{x\to 0} [\cos^{-1}x] = 1$

$\Rightarrow \lim\limits_{x\to 0} \frac{3^{\sin^{-1}x} - ax + b}{2x} = \ln\sqrt{3}$

$\therefore$ limit to exist Nr $\to 0 \quad \Rightarrow 1 + b = 0$

$\Rightarrow b = -1$

$\therefore \lim\limits_{x\to 0} \frac{3^{\sin^{-1}x} - ax - 1}{2x} = \ln\sqrt{3}$

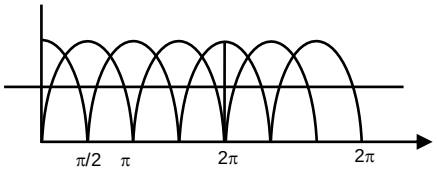
$\Rightarrow \lim\limits_{x\to 0} \left(\frac{3^{\sin^{-1}x} - 1}{2x} - \frac{a}{2}\right) = \ln\sqrt{3}$

$\Rightarrow \frac{1}{2}\ln 3 - \frac{a}{2} = \ln\sqrt{3} \quad \Rightarrow a = 0$

$\therefore a - b = 1$

12. From the graph total number of non-differentiable points = 11

आरेख से अवकलनीय नहीं होने के बिन्दुओं की संख्या = 11



13. $y = |x - 1|^{\sin x}$

$y = (1 - x)^{\sin x}$ at $x = -\frac{\pi}{2}$

$\log y = \sin x . \log (1 - x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \log (1 - x) - \frac{\sin x}{1 - x}$

$\frac{dy}{dx} = |x - 1|^{\sin x}\left[\cos x . \log(1 - x) - \frac{\sin x}{1 - x}\right]$

$= \left(1 + \frac{\pi}{2}\right)^{-1}\left[0 + \frac{1}{1 + \frac{\pi}{2}}\right] = \left(1 + \frac{\pi}{2}\right)^{-2}$

$\Rightarrow \quad a = \frac{1}{2}, b = -2$

So इसलिए $\frac{1}{a} + 4b = 2 - 8 = -6$

14. $f(x) = \begin{cases} P(x), & x \le 0 \\ x^{\left(1+\frac{1}{x}\right)} & x > 0 \end{cases}$

$f(x) = \begin{cases} ax + b & ; \quad x \le 0 \\ x^{\left(1+\frac{1}{x}\right)} & ; \quad x > 0 \end{cases}$

$f'(x)\big|_{\text{at } x=1} = x^{\left(1+\frac{1}{x}\right)}\left[-\frac{1}{x^2}\ln x + \left(1 + \frac{1}{x}\right).\frac{1}{x}\right]$

$f'(x)\big|_{\text{at } x=1} = 2 = f(-1)$

$2 = -a + b$

Continuity at x = 0

x = 0 सतत्

$\lim\limits_{x\to 0^+} f(x) = b$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^{\left(1+\frac{1}{x}\right)} = 0$

so, $b = 0$; $a = -2$

$f(x) = \begin{cases} -2x & ; \quad x \le 0 \\ x^{\left(1+\frac{1}{x}\right)} & ; \quad x > 0 \end{cases}$

From the definition of greatest integer, in $x \in [0,5]$

$x \in [0,5]$ में महत्तम पूर्णांक फलन की परिभाषा से

$[P(x)] = [-2x]$ is non derivable at 10 points

$[P(x)] = [-2x]$, 10 बिन्दुओं पर अवकलनीय नहीं है।

15. $ax^3 + x - 1 - a = 0 \begin{cases} 1 \\ \alpha \\ \beta \end{cases}$

$(a \neq -1\ 0)$

$\Rightarrow (x-1)(ax^2 + ax + (a+1)) = 0$ (using theory of equation)

$1 + \alpha + \beta = 0$ and $\alpha\beta = \dfrac{(a+1)}{a}$

Now, $\lim\limits_{x\to\frac{1}{\alpha}} \dfrac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x} - 1)(x-1)}$

Substituting $x = \dfrac{1}{t}$

$\lim\limits_{t\to\alpha} \dfrac{(1+a)\frac{1}{t^3} - \frac{1}{t^2} - a}{(e^{1-\frac{\alpha}{t}} - 1)\left(\frac{1}{t} - 1\right)}$;

$\lim\limits_{t\to\alpha} \dfrac{at^3 + t - (1+a)}{(e^{1-\frac{\alpha}{t}} - 1)(t-1)}$;

$\lim\limits_{t\to\alpha} \dfrac{(t-1)\left((at^2 + at - (a+1)\right)}{t^2 \dfrac{(e^{1-\frac{\alpha}{t}} - 1)}{\left(1-\frac{\alpha}{t}\right)(t-1).\left(1-\frac{\alpha}{t}\right)}}$

$\lim\limits_{t\to\alpha} \dfrac{at^2 + at - (a+1)}{t(t-\alpha)}$

$\lim\limits_{t\to\alpha} \dfrac{a(t-\beta)}{t} = \dfrac{a(\alpha-\beta)}{\alpha} \equiv \dfrac{a\ell(k\alpha-\beta)}{\alpha}$

$\Rightarrow \ell = 1$ and $k = 1$ so $k\ell = 1$

16. $t^2 + 1 = x - t$ and और $t^2 + 1 = y + t$

$\Rightarrow \quad t = \dfrac{x-y}{2}$

$\therefore (x-y)^2 + 4 = 2(x+y)$

$(x-y)^2 = 2(x+y-2)$

$\left(\dfrac{x-y}{\sqrt{2}}\right)^2 = \sqrt{2}\left(\dfrac{x+y-2}{\sqrt{2}}\right)$

$\therefore$ length of latus rectum नाभिलम्ब की लम्बाई $= \sqrt{2}$

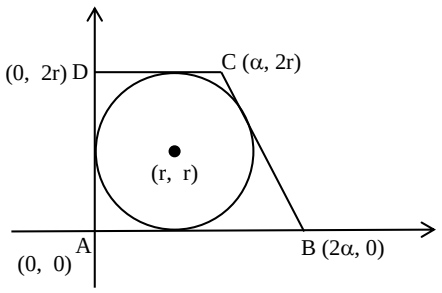
17. $18 = \dfrac{1}{2}(3\alpha)(2r) \Rightarrow \alpha r = 6$

Line $y = -\dfrac{2r}{\alpha}(x - 2\alpha)$ is tangent to

$(x-r)^2 + (y-r)^2 = r^2$

$2\alpha = 3r$ and $\alpha r = 6$

$r = 2$



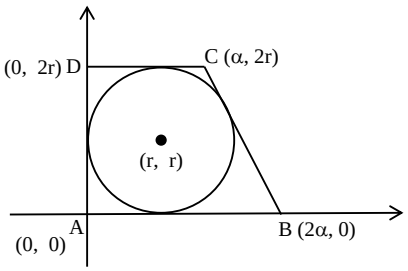
हल. $18 = \dfrac{1}{2}(3\alpha)(2r) \Rightarrow \alpha r = 6$

रेखा $y = -\dfrac{2r}{\alpha}(x - 2\alpha)$ वृत्त

$(x-r)^2 + (y-r)^2 = r^2$ की स्पर्श रेखा है।

$2\alpha = 3r$ और $\alpha r = 6$

$r = 2$



18. $\dfrac{dx}{dy} = \dfrac{\frac{dx}{dt}}{\frac{dy}{dt}} = \dfrac{12t^2}{12t^3} = \dfrac{1}{t}$

$\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dt}\left(\dfrac{1}{t}\right).\dfrac{dt}{dy} = \dfrac{-1}{t^2}.\dfrac{1}{12t^3} = \dfrac{-1}{12t^5}$

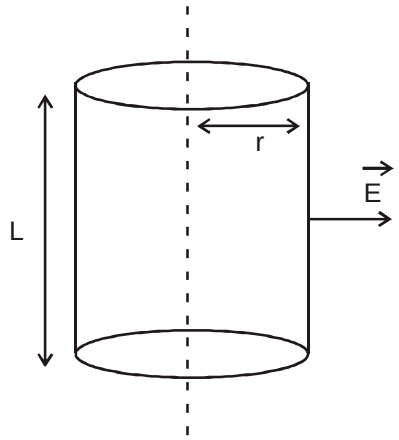
So $\dfrac{\frac{-1}{12t^5}}{\left(\frac{1}{t}\right)^n}$ is constant $\Rightarrow n = 5$

इसलिए $\dfrac{\frac{-1}{12t^5}}{\left(\frac{1}{t}\right)^n}$ अचर है। $\Rightarrow \quad n = 5$

PART-II: PHYSICS

19. Case (i) $x < R$

Let a Gaussian surface is a cylinder of radius r and length equal to given cylinder



$\because \vec{E}$ and $\vec{ds}$ are parallel to each other, so :

$$\frac{q_{in}}{\epsilon_o} = \int \vec{E}.\vec{ds} = \int E\, ds$$

$$\Rightarrow \frac{\rho(\pi r^2)L}{\epsilon_o} = E\int ds = E.\, 2\pi r\, L$$

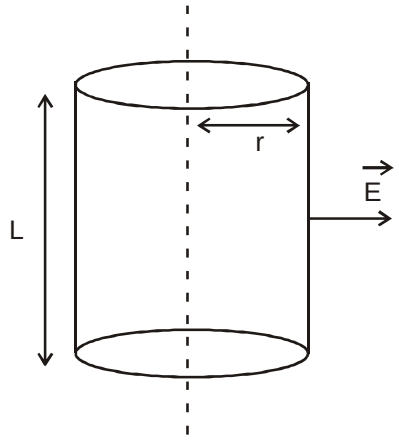
$$\Rightarrow E = \frac{\rho r}{2\,\epsilon_o}$$

(ii) $x \geq R$: Again by $\frac{q_{in}}{\epsilon_o} = \int \vec{E}.\,d\vec{s}$ z $E \int ds$

$$\Rightarrow \frac{\rho(\pi R^2)L}{\epsilon_o} = E\,(2\pi\, r)L \;\Rightarrow\; E = \frac{\rho R^2}{2\,\epsilon_o\, x}$$

हल. स्थिति (i) $x < R$

दिये गये बेलन (सिलेण्डर) के समान एक r त्रिज्या और L लम्बाई की गॉउसीयन सतह (बेलनाकार) मानते है।



$\vec{E}$ और $\vec{ds}$ दोनो एक दूसरे के समान्तर है अतः

$$\frac{q_{in}}{\epsilon_o} = \int \vec{E}.\vec{ds} = \int E\, ds$$

$$\Rightarrow \frac{\rho(\pi r^2)L}{\epsilon_o} = E\int ds = E.\, 2\pi r\, L$$

$$\Rightarrow E = \frac{\rho r}{2\,\epsilon_o}$$

(ii) $x \geq R$

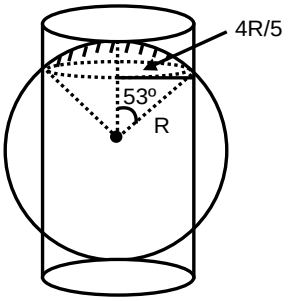
पुनः $\frac{q_{in}}{\epsilon_o} = \int \vec{E}.\,d\vec{s} = E\int ds$

$$\Rightarrow \frac{\rho(\pi R^2)L}{2\,\epsilon_o} = E\,(2\pi\, r)L$$

$$\Rightarrow E = \frac{\rho R^2}{2\,\epsilon_o\, x}$$

20. Refer to answer key

22. (1) for h = 2R लिये $r = \frac{4R}{5}$



Shaded charge छायांकित आवेश = 2π (1 – cos53º) × $\frac{Q}{4\pi} = \frac{Q}{5}$

$\therefore \quad q_{enclosed} = \frac{2Q}{5}$

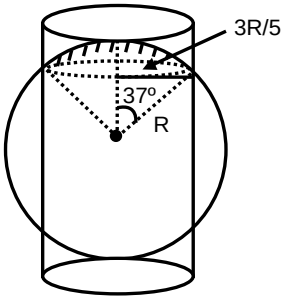
$\therefore \quad \phi = \frac{2Q}{5\varepsilon_0}$

$\therefore$ for h > 2R r = $\frac{4R}{5}$

$\therefore \quad \phi = \frac{2Q}{5\varepsilon_0}$

h > 2R के लिये $r = \frac{4R}{5}$ $\quad \therefore \phi = \frac{2Q}{5\varepsilon_0}$

(2) for h = 2R लिये $r = \frac{3R}{5}$



Shaded charge छायांकित आवेश = 2π (1 – cos37°) × $\frac{Q}{4\pi} = \frac{Q}{10}$

$\therefore \quad q_{enclosed} = \frac{Q}{5}$

$\therefore \quad \phi = \frac{Q}{5\varepsilon_0}$

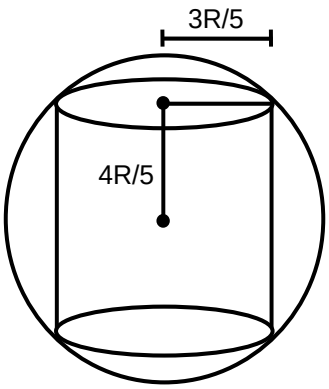
$\therefore \quad$ for h > 2R r = $\frac{3R}{5}$

$\therefore \quad \phi = \frac{Q}{5\varepsilon_0}$

h > 2R के लिये r = $\frac{3R}{5}$

$\therefore \quad \phi = \frac{Q}{5\varepsilon_0}$

(3) suppose माना h = $\frac{8R}{5}$ r = $\frac{3R}{5}$

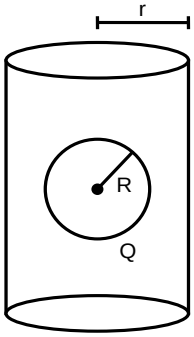


$\phi = 0$

so for $h < \frac{8R}{5}$ $\phi = 0$

इसलिये $h < \frac{8R}{5}$ के लिये $\phi = 0$

(4) h > 2R r > R



$\phi = \frac{Q}{\varepsilon_0}$ clearly from Gauss' Law गाऊस के नियम से स्पष्ट है।

24. f = mg sinθ

and तथा

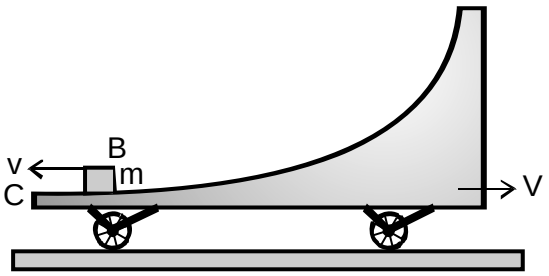
PE sinθ = mg sinθ. R

q 2R E sinθ = mg sinθ R

$$E = \frac{mg}{2q}$$

$\therefore$ **Ans.** (A) & (C)

25.



mv = MV …(i)

$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = mgH$ …(ii)

Solving हल करने पर

$$v = \sqrt{\frac{2gHM}{M+m}} \;\&\; V = \sqrt{\frac{2gH}{M+m}.\frac{m^2}{M}}$$

Maximum relative velocity of cart w.r.t. block गुटके के संदर्भ में कार्ट का अधिकतम सापेक्ष वेग

$$= V + v = \sqrt{\frac{2gH(M+m)}{M}}$$

$\therefore$ time taken by block to travel from B to C गुटके के द्वारा B से C जाने में लिया गया समय

$$t = \frac{L}{V+v} = L\sqrt{\frac{M}{2gH(M+m)}}$$

26. $V = a + br^3$

$E = -\frac{dv}{dr} = -3br^2$

flux through a Gaussian spherical surface of radius r is

$\phi = E(4\pi r^2) = -12\pi br^4$

$\phi = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad q_{in} = \phi\,\varepsilon_0$

$q_{in} = -12\pi\varepsilon_0 br^4$

$\rho = \frac{dq}{d(vol)} = \frac{-48\pi br^3 dr}{4\pi r^2 dr} = -12\varepsilon_0 br$

28. Velocity of the block just after the impulse is given by :

$P_f - P_i = Jext = mV - 0 = \frac{1}{2}\times(1\times10^{-3})\times(4\times10^4)$

$V = 5$ m/sec.

For maximum compression $V_f = 0$ for an instant

$W_{nc} = KE\uparrow + PE\uparrow$

$-(\mu_K mg)\,x_{max} = (0 - \frac{1}{2}m(5)^2) + \frac{1}{2}Kx^2_{max}$

Solving we get

$x_{max} = 1$ m

At maximum compression, applied force on the block is $kx = (40)(1) = 40$ N

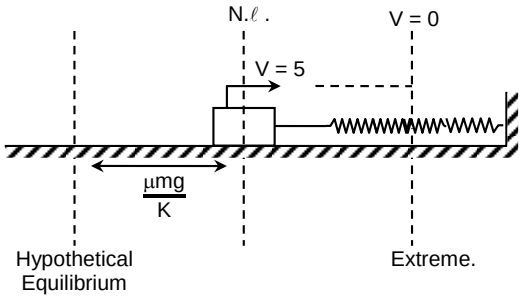
And the shear strength $= \mu_s N = \left(\frac{3}{4}\right)(40) = 30$N

So the block will turn back.

We can also estimate the time taken by the block to reach the maximum displaced position

The hypothetical equilibrium position will be behind the natural position by distance of $\frac{\mu mg}{K}$

(from $Kx = \mu mg$)



$\therefore$ Time taken by the block to go from hypothetical equilibrium to the extreme position $= \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m}{K}}$

$\Rightarrow$ Time taken by the block from initial position (Natural length) to the extreme position will be less than $\frac{T}{4}$

$t < \frac{\pi}{2}\sqrt{\frac{m}{K}}$

हल : आवेग देने के ठीक बाद ब्लॉक का वेग:

$P_f - P_i = Jext = mV - 0 = \frac{1}{2}\times(1\times10^{-3})\times(4\times10^4)$

$V = 5$ m/sec.

अधिकतम सम्पीडन के लिए, $V_f = 0$ किसी क्षण पर

$W_{nc} = KE\uparrow + PE\uparrow$

$-(\mu_K mg)\,x_{max} = (0 - \frac{1}{2}m(5)^2) + \frac{1}{2}Kx^2_{max}$

हल करने पर

$x_{max} = 1$ m

अधिकतम सम्पिडन पर, ब्लॉक पर कार्यरत बल kx $= (40)(1) = 40$ N

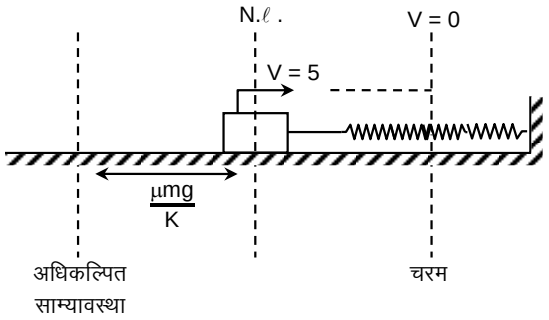
अपरूपण बल $= \mu_s N = \left(\frac{3}{4}\right)(40) = 30$N

अतः ब्लॉक आपस आयेगा।

हम ब्लॉक द्वारा अधिकतम विस्थापन तक पहुँचने के समय की गणना भी कर सकते है।

अधिकल्पित साम्यावस्था की स्थिति प्राकृतिक स्थिति से $\frac{\mu mg}{K}$ दूरी पर होगी।

($Kx = \mu mg$ से)



∴ ब्लॉक के अधिकल्पित साम्यावस्था की स्थिति से चरम स्थिति तक जाने में लिया गया समय = $\frac{T}{4}=\frac{\pi}{2}\sqrt{\frac{m}{K}}$

$\Rightarrow$ ब्लॉक के प्रारम्भिक स्थिति (प्राकृतिक लम्बाई) से चरम स्थिति तक जाने में समय $\frac{T}{4}$ से कम होगा।

$t<\frac{\pi}{2}\sqrt{\frac{m}{K}}$

30. Heat produced in the wire = 1^2 Rt

heat = $(2)^2 \times 5 \times 84 = 20 \times 84$ j

heat = $\frac{20\times 84}{4.2}$ cal = 400 cal

$\theta = ms\Delta T$

$400 = (100)\left(\frac{1}{2}\right)\Delta T$

$\Rightarrow \quad \Delta T = 8^oC$

31. $m_1 = (d_1)(A_1\,\ell)\ m_2 = (d_2)(A_2\,\ell)$

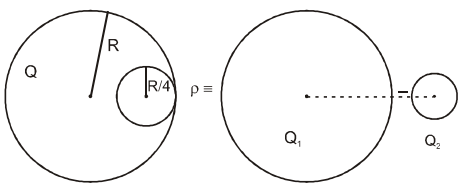
$R_{A1}=\frac{\rho_1\ell}{A_1}, R_{cu}=\frac{\rho_2\ell}{A_2}$

$m_1R = (\rho_1 d_1)\,\ell^2\ m_2R = (\rho_2 d_2)\,\ell^2$

$\frac{m_1}{m_2}=\frac{\rho_1 d_1}{\rho_2 d_2}=\frac{2.4\times10^{-8}\times2.7\times10^3}{1.6\times10^{-8}\times9\times10^3}$

$\frac{m_1}{m_2}=\frac{9}{20}$

32.



Let E_1 is the electric field at point P due to Q_1 and E_2 is the electric field at point P due to Q_2

माना E_1, Q_1 के कारण बिन्दु P पर विद्युत क्षेत्र है, तथा E_2, Q_2 के कारण बिन्दु P पर विद्युत क्षेत्र है।

$E = E_1 - E_2$

$= \frac{kQ_1}{R^2}-\frac{kQ_2}{(R/4)^2}$

$= \frac{k}{R^2}\frac{4}{3}\pi R^3\rho-\frac{16k}{R^2}\frac{4}{3}\pi\left(\frac{R}{4}\right)^3\rho$

$= kR\pi\rho\left[\frac{4}{3}-\frac{1}{3}\right] \qquad = k\pi R\rho$

$= kR\pi\frac{Q}{\frac{4}{3}\pi\left(R^3-\frac{R^3}{64}\right)} \quad = \frac{kQR}{R^3}\frac{3}{4}\frac{64}{63}$

$= \frac{16}{21}\frac{kQ}{R^2}.$

34. When the sphere leaves the surface, its speed will be v

$= \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}=\sqrt{\frac{2\times10\times7}{1+\frac{2}{5}}} \Rightarrow v = 10$ m/s

Then the ball will perform a projectile motion.

Range = $u\sqrt{\frac{2h}{g}}=10\sqrt{\frac{2\times1.25}{10}}=5m$

जब गोला सतह को छोड़ता है, तब इसकी चाल v

$= \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}=\sqrt{\frac{2\times10\times7}{1+\frac{2}{5}}} \Rightarrow v = 10$ m/s

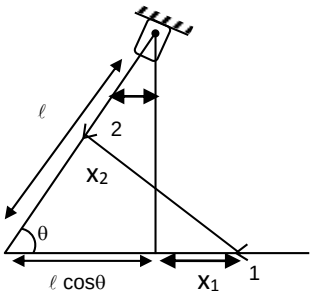
तब गेंद प्रक्षेप्य गति करेगी।

परास = $u\sqrt{\frac{2h}{g}}=10\sqrt{\frac{2\times1.25}{10}}=5m$

35. Refer to answer key

36. For equilibrium, the combined centre of mass should be vertically below the point of suspension.

For this



$m_1x_1 = m_2x_2$

$$m\left(\frac{\ell}{2} - \ell\cos\theta\right) = (m)\frac{\ell\cos\theta}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

PART-III: CHEMISTRY

38. $T_B = 350$ K

$$T_c = \frac{8a}{27Rb} = \frac{8}{27} \times 350 \text{ K}$$

So the gas can't be liquefied above 103.7 K
इसलिए गैस 103.7 K के ऊपर द्रवीकृत नहीं हो सकती है।

39. (A) Packing fraction संकुलन प्रभाज

$$= \frac{4 \times \frac{4}{3}\pi\left(R_+^3 + R_-^3\right)}{[2(R_+ + R_-)]^3} \times 100$$

= 69.77%

(C) In CsCl, chloride ion forms crystal and Cs^+ ion occupy cubical void.
CsCl में, क्लोराइड आयन क्रिस्टल बनाता है तथा Cs^+ आयन घनीय रिक्तिका में भरे होते हैं।

40. In bcc coordination number is 8.
bcc संरचना मे समन्वय संख्या 8 है।

41. (C) for monoclinic system
$a \neq b \neq c$ & $\alpha = \gamma = 90°,\ \beta \neq 120°$
(D) packing fraction of B.C.C. is 68%

हल. (C) एकनताक्ष निकाय के लिए $a \neq b \neq c$ तथा $\alpha = \gamma = 90°,\ \beta \neq 120°$
(D) B.C.C. का संकुलन प्रभाज 68% है।

42. Refer to answer key

44. Refer notes.
नोट्स देखें।

46. Perchloric acid is $HClO_4$
परक्लोरिक अम्ल $HClO_4$ है।

47. Number of A ions per unit cell = $\frac{1}{8} \times 8 = 1$

Number of B ions per unit cell = $\frac{1}{2} \times 6 = 3$

Empirical formula = AB_3.

हल. प्रति एकक कोष्ठिका A आयनों की संख्या = $\frac{1}{8} \times 8$ = 1
प्रति एकक कोष्ठिका B आयनों की संख्या = $\frac{1}{2} \times 6 = 3$ मूलानुपाती सूत्र = AB_3.

48. $$d = \frac{ZM}{N_A a^3}$$

$$2.72 = \frac{4 \times M}{6.02 \times 10^{23} \times (404 \times 10^{-10})^3}$$

$$M = \frac{2.72 \times 6.02 \times (404)^3}{4 \times 10^7} = 26.99 = 27$$

49. A, B, D, E & F

50. Except Be and Mg
Be व Mg के अलावा।

51. (A) $Be(OH)_2 < Mg(OH)_2 < Ca(OH)_2 < Ba(OH)_2$ Basic character
(B) $BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3$.
Decomposition temperature
(C) $Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3 < Cs_2CO_3$ Water solubility
(D) $Na_2O_2 < K_2O_2 < Rb_2O_2 < Cs_2O_2$ Stability
(E) $LiHCO_3 < NaHCO_3 < KHCO_3 < RbHCO_3 < CsHCO_3$ Stability
(F) NaF > NaCl > NaBr > NaI
Melting point
(G) $Li^+_{(aq)} < Na^+_{(aq)} < K^+_{(aq)}$
Ionic mobility in aqueous medium
(A) $Be(OH)_2 < Mg(OH)_2 < Ca(OH)_2 < Ba(OH)_2$ क्षारीय अभिलक्षण

(B) $BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3$.
विघटन ताप

(C) $Li_2CO_3 < Na_2CO_3 < K_2CO_3$
$< Rb_2CO_3 < Cs_2CO_3$ जल विलेयता

(D) $Na_2O_2 < K_2O_2 < Rb_2O_2 < Cs_2O_2$
स्थायित्व

(E) $LiHCO_3 < NaHCO_3 < KHCO_3$
$< RbHCO_3 < CsHCO_3$ स्थायित्व

(F) NaF > NaCl > NaBr > NaI गलनांक

(G) $Li^+_{(aq)} < Na^+_{(aq)} < K^+_{(aq)}$
जलीय विलयन में आयनिक चालकता

54. [(iv), (v), (vi), (viii), (xi) (xii)]

---- TEXT SOLUTIONS (TS) END ----

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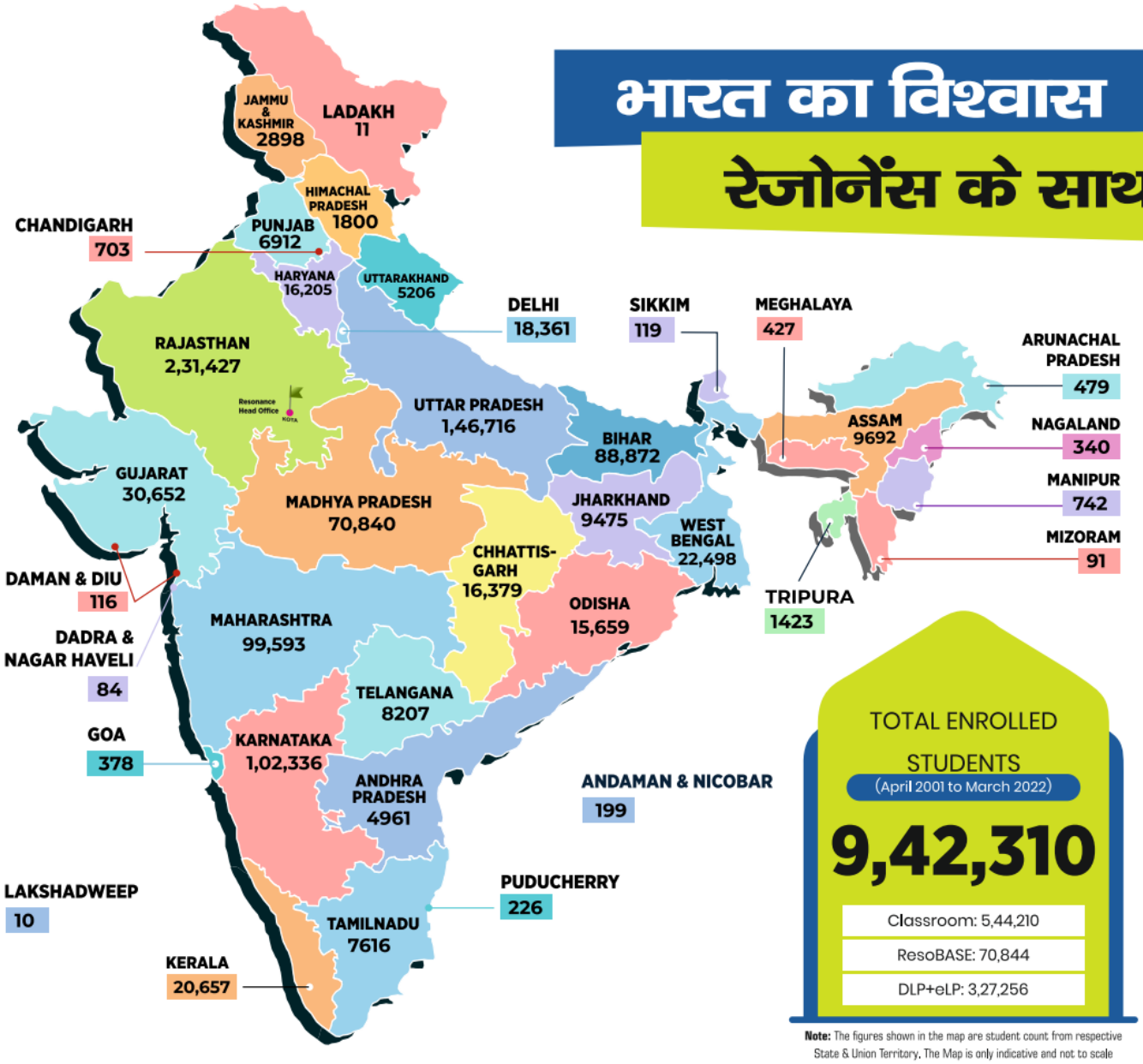
रेजोनेंस के साथ



TOTAL ENROLLED STUDENTS
(April 2001 to March 2022)

9,42,310

- Classroom: 5,44,210
- ResoBASE: 70,844
- DLP+eLP: 3,27,256

Note: The figures shown in the map are student count from respective State & Union Territory. The Map is only indicative and not to scale

Resonance : The Legacy of 21 Years (2001-2022) of Academic Excellence

- **JEE (Adv.) / IIT-JEE** ▸ **50 हजार +** SELECTIONS SINCE 2002
  229 AIRs in TOP-100 (Classroom + DLP)
- **JEE (Main) / AIEEE** ▸ **2.40 लाख+** SELECTIONS SINCE 2009
  136 AIRs in TOP-100 (Classroom + DLP)
- **NEET (UG) / AIPMT** ▸ **19 हजार+** SELECTIONS SINCE 2012
  19 AIRs in TOP-100 (Classroom + DLP)

- **NTSE** SINCE 2006 ▸ **2440** Scholars
- **KVPY** SINCE 2006 ▸ **2859** Fellowship Winners
- **OLYMPIADS** SINCE 2006 ▸ **52** Medalists (Gold/Silver/ Bronze) in International Olympiads
- **CA & CS** SINCE 2013 ▸ **4179** Selections — 5 Times AIR-1 in CA & CS Exams
- **CLAT, SET & GPTU** SINCE 2014 ▸ **77** Selections — AIR-1 in GPTU