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BHUBANESWAR STUDY CENTRE

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Toppers in JEE (Advanced) 2018

ADMISSION OPEN for Session 2019-20

CLASSES:

7 8 9 10 11 12

TARGET: JEE (Main + Advanced)

JEE (Main) | AIIMS/ NEET

Pre-foundation

SCHOLARSHIP UPTO 90%

Entrance Test Dates:

17th, 31st March & 7th April 2019

Resonance®
Educating for better tomorrow
Pre-foundation Career Care Programmes (PCCP) Division

PRE-BOARD_SESSION-2018-19

SUBJECT : MATHEMATICS

CLASS-X (CBSE PATTERN)

Time: 3 Hours **Max. Marks : 80**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

a) All questions are compulsory.

b) The question paper consists of 30 questions divided into four sections – A, B, C and D.

c) Section A contains 6 questions of 1 mark each which are subjective questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each, Section D contains 8 questions of 4 marks.

d) Use of calculator is not permitted.

NAME OF THE CANDIDATE : .....ROLL NO. : .....

I have read all the instructions and shall abide by them

I have verified the identity, name and roll number of the candidate.

- -
Signature of the Candidate

- -
Signature of the Invigilator

PCCP Head Office :

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Behind City Mall, Jhalawar Road, Kota (Rajasthan)-324005

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Section-A

1. The coordinates of one end point of a diameter of a circle are (4, – 1) and the coordinates of the centre of the circle are (1, – 3). Find the coordinates of the other end of the diameter.

2. Find the roots of the quadratic equation $x^2 + 2\sqrt{2}x - 6 = 0$ by using the quadratic formula.

OR

Show that the roots of the equation $x^2 + ax - 4 = 0$ (where $a \in R$) are real and distinct

3. Find the value of expression $\dfrac{\sin 30° + \tan 45° - \sec 60°}{\text{cosec } 30° - \cot 45° - \cos 60°}$.

OR

Find the value of the expression $(\cos\theta - 1)(1 + \cos\theta)(1 + \cot^2\theta)$.

4. Find the 11th term of the AP : $-5, \dfrac{-5}{2}, 0, \dfrac{5}{2}, \ldots$

5. ΔABC and ΔPQR are similar triangles such that $\angle A = 32°$ and $\angle R = 65°$, then find $\angle B$.

6. Write the statement of fundamental theorem of arithmetics.

Section-B

7. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

OR

Use Euclid's division algorithm to find the HCF of 10224 and 9648.

8. In an A. P. 5 times the 5th term is equal to 8 times the 8th term, then find its 13th term .

OR

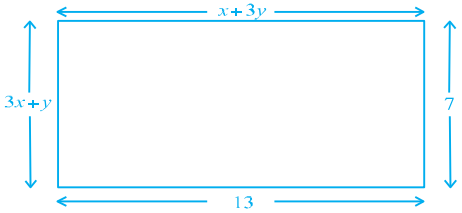
Which term of the AP: 53, 48, 43,... is the first negative term ?

9. Show that the points (1, 1), (– 2, 7) and (3, – 3) are collinear.

10. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. Then find the probability that the selected student is not from A, B and C

11. An integer is chosen between 0 and 100. What is the probability that it is
(i) divisible by 7 ? **(ii)** not divisible by 7 ?

12. Find the values of x and y in the following rectangle **[see Figure].**



Section-C

13. Prove that $5 - 2\sqrt{3}$ is an irrational number

14. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, then find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

15. Solve : $\frac{25}{x+y} - \frac{3}{x-y} = 1, \frac{40}{x+y} + \frac{2}{x-y} = 5$

16. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, – 4). How many such points are there ?

OR

Let P and Q be the points of trisection of the line segment joining the points A(2, – 2) and B(– 7, 4) such that P is nearer to A. Find the coordinates of P and Q.

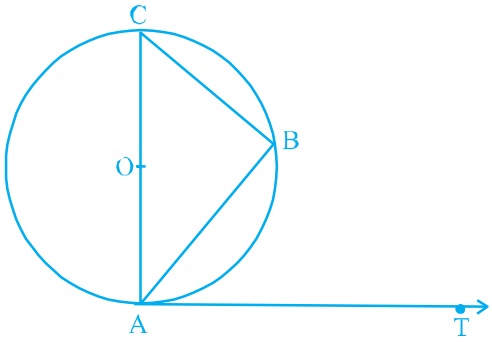
17. Evaluate :

$\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sec^2 50^\circ - \cot^2 40^\circ} + 2\,\text{cosec}^2\, 58^\circ - 2\cot 58^\circ \tan 32^\circ - 4 \tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ$

OR

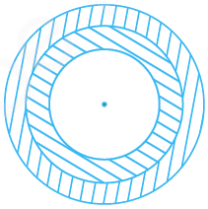
Given $2y \cos\theta = x \sin\theta$ and $2x \sec\theta - y \,\text{cosec}\,\theta = 3$, then find the value of $x^2 + 4y^2$.

18. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that $\angle BAT = \angle ACB$.



19. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height *h*. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β, respectively. Prove that the height of the tower is $\left(\frac{h\tan\alpha}{\tan\beta - \tan\alpha}\right)$.

20. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1: 2:3, then find the ratio of the areas of three regions.



OR

All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm^2 . (Use $\pi = 3.14$).

21. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then find the number of marbles that the cube can accomodate.

OR

A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8cm. Find the height of the cone.

22. The mean of the following frequency distribution is 25.2. Find the missing frequency x

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	8	X	10	11	9

Section-D

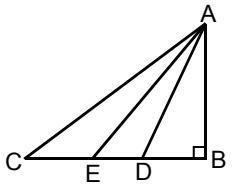
23. Solve for x : $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$, $x \neq 1, 2, 3$

OR

A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

24. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.

25. In figure, D and E trisect BC , then prove that $8AE^2 = 3AC^2 + 5AD^2$.



26. Construct a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the Δ ABC.

27. A man on a cliff observes a boat at an angle of depression of 30º which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60º. Find the total time taken by the boat from the initial point to reach the shore.

OR

At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$. On walking 240 m nearer the tower, the tangent of the angle of elevation becomes $\frac{3}{4}$. Find the height of the tower.

28. The mean of the following frequency distribution is 50, but the frequencies f_1 and f_2 in classes 20-40 and 60-80, respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	f_1	32	f_2	19

OR

The median of the following data is 50. Find the values of p and q, if the sum of all the frequencies is 90.

Marks	**Frequency**
20-30	p
30-40	15
40-50	25
50-60	20
60-70	q
70-80	8
80-90	10

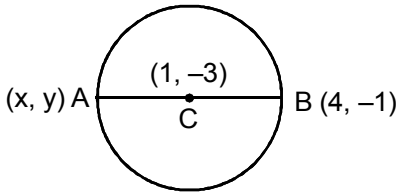
29. Water flows at the rate of 10m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24m ?

30. Prove that $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$ = $\sec\theta\operatorname{cosec}\theta + 1$.

HINTS & SOLUTIONS

SECTION-A

1. Let A (x, y) be the other end of the diameter and centre C (1, – 3) is mid point of end points of a diameter.



So, $1 = \frac{x+4}{2}$, $-3 = \frac{y-1}{2}$

$x = -2, y = -5$

$\therefore$ Required point is (– 2, – 5).

2. $x^2 + 2\sqrt{2}x - 6 = 0$

Here $a = 1, b = 2\sqrt{2}, c = -6$

$b^2 - 4ac = (2\sqrt{2})^2 - 4 \times 1 \times -6$

$= 8 + 24 = 32$

The roots are given by

$= x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\sqrt{2} \pm \sqrt{32}}{2 \times 1}$

$= \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2} = x = \frac{-2\sqrt{2} + 4\sqrt{2}}{2}$ or $x = \frac{-2\sqrt{2} - 4\sqrt{2}}{2}$

$x = \sqrt{2}$ or $x = -3\sqrt{2}$

Hence, roots are $\sqrt{2}$ and $-3\sqrt{2}$.

OR

$D = a^2 - 4(-4)$

$= a^2 + 16$

Here D > 0, therefore roots are real and distinct

3. $\frac{\sin 30^\circ + \tan 45^\circ - \sec 60^\circ}{\text{cosec}\, 30^\circ - \cot 45^\circ - \cos 60^\circ} = \frac{\frac{1}{2} + 1 - 2}{2 - 1 - \frac{1}{2}} = \frac{-\frac{1}{2}}{\frac{1}{2}} = -1.$

OR

$(\cos\theta - 1)(1 + \cos\theta)(1 + \cot^2\theta)$

$= (\cos^2\theta - 1)(\text{cosec}^2\theta)$

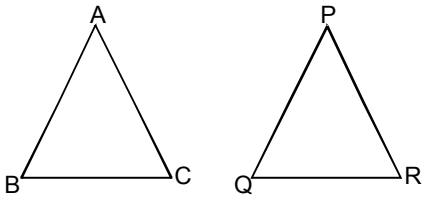
$= -\sin^2\theta \cdot \text{cosec}^2\theta = -1.$

4. Here, n = 11, $a = -5$ $d = \frac{-5}{2} - (-5) = \frac{5}{2}$

$a_n = a + (n - 1)d$

$\therefore \quad a_{11} = -5 + (11 - 1)\left(\frac{5}{2}\right) = -5 + 10 \times \frac{5}{2} = -5 + 25 = 20.$

5.



$\Delta ABC \sim \Delta PQR$

$\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

$\therefore \angle P = 32°$, $\angle C = 65°$

In ΔABC

$\Rightarrow \quad \angle A + \angle B + \angle C = 180° \quad \Rightarrow \quad 32° + \angle B + 65° = 18°$

$\Rightarrow \quad \angle B = 180° - 32° - 65° \quad \Rightarrow \quad \angle B = 180° - 97° = 83°.$

SECTION-B

7. Number ending at 0 or 5 is divisible by 5. Now, $(12)^n = (2 \times 2 \times 3)^n = 2^{2n} \times 3^n$

It does not have 5 in prime factorization. So, 12^n can never end with 5 and zero.

OR

$10224 = 9648 \times 1 + 576$

$9648 = 576 \times 16 + 432$

$576 = 432 \times 1 + 144$

$432 = 144 \times 3 + 0$

HCF = 144

8. $5T_5 = 8T_8 \quad \Rightarrow \quad 5(a + 4d) = 8(a + 7d)$

$\Rightarrow 5a + 20d = 8a + 56d \quad \Rightarrow \quad 3a + 36d = 0$

$\Rightarrow a + 12d = 0$

now $T_{13} = a + 12d = 0$.

OR

Given A.P. is 53, 48, 43, …

$\therefore \quad a = 53, d = 48 - 53 = -5$

Let the nth term of A.P. is the first negative term.

Then, $a_n < 0 \quad \Rightarrow \quad a + (n - 1)d < 0$

$\Rightarrow \quad 53 + (n - 1)(-5) < 0 \quad \Rightarrow \quad -5(n - 1) < -53$

$\Rightarrow \quad 5(n - 1) > 53 \quad \Rightarrow \quad 5n - 5 > 53$

$\Rightarrow \quad 5n > 53 + 5 \quad \Rightarrow \quad n > 11.6$

$\therefore \quad n = 12$

Hence, the first negative term of A.P. is 12th term, i.e.,

9. A (1, 1), B (– 2, 7) and C (3, – 3)

AB $= \sqrt{(-2-1)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

BC $= \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$

CA $= \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

$\because \quad AB + CA = BC$

$3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

$5\sqrt{5} = 5\sqrt{5}$

So, the points A, B and C are collinear

10. T(E) = 23
F(E) = not form A, B, C i.e., = 23 – (4 + 8 + 5)
F(E) = 23 – 17 = 6
$\therefore$ P(F) = $\frac{6}{23}$.

11. (i) Numbers between 0 and 100 divisible by 7 are 7, 14, 21,...., 98 = (AP)
Here, a_n = 98, a = 7, d = 7.
$\Rightarrow$ $a_n = a + (n-1)d$
$\Rightarrow$ $98 = 7 + (n-1)7$
$\Rightarrow$ $98 - 7 = (n-1)7$
$\Rightarrow$ $\frac{91}{7} = (n-1)$
$\Rightarrow$ $(n-1) = 13$
$\Rightarrow$ $n = 13 + 1 = 14$.
$\therefore$ F(E) = 14 and T(E) = 99
$\therefore$ P(E) = $\frac{14}{99}$.

(ii) Number of favourable outcomes of getting a number which is not divisible by 7 = 99 – 14 = 85
$\therefore$ P(E) = 85
$\therefore$ P(E) = $\frac{F(E)}{T(E)} = \frac{85}{99}$.

12. As the opposite sides of a rectangle are equal so by figure, we conclude that
$3x + y = 7$ ... (1)
$x + 3y = 13$ ... (2)
$9x + 3y = 21$ [From (1)]
$x + 3y = 13$
– – –
$8x = 8$ [Substracting (ii) from (i)]
$x = 1$
Now, $3(1) + y = 7$ [From (i)]
$\Rightarrow$ $y = 7 - 3$ $\Rightarrow$ $y = 4$ and $x = 1$
Hence the required values of x and y are 1 and 4 respectively.

SECTION-C

13. If possible, suppose $5 - 2\sqrt{3}$ is rational. Therefore, we can find two integers a, b ($b \neq 0$) such that

$5 - 2\sqrt{3} = \frac{a}{b}$ $\Rightarrow$ $2\sqrt{3} = 5 - \frac{a}{b}$ $\Rightarrow$ $\sqrt{3} = \frac{1}{2}\left(5 - \frac{a}{b}\right)$

Since a, b, 2, and 5 are integers, $\frac{1}{2}\left(5 - \frac{a}{b}\right)$ is a rational number. Hence, $\sqrt{3}$ should be rational.

But we know that $\sqrt{3}$ is an irrational number. Therefore, our assumption is wrong.

Hence, $5 - 2\sqrt{3}$ is irrational.

14. $f(x) = x^2 - 1$ $\alpha\beta = -1$ $\alpha + \beta = 0$
Required polynomial:

$$x^2 - \left[\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}\right]x + \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = x^2 - \left[\frac{2\left(\alpha^2 + \beta^2\right)}{\alpha\beta}\right]x + 4$$

$= x^2 - \left[\frac{2\left[(\alpha+\beta)^2 - 2\alpha\beta\right]}{\alpha\beta}\right]x + 4 \quad = x^2 - \frac{-4(-1)}{-1}x + 4 \; = x^2 + 4x + 4.$

15. $\frac{25}{x+y} - \frac{3}{x-y} = 1$ .....(i) $\frac{40}{x+y} + \frac{2}{x-y} = 5$ .....(ii)

multiplying (i) by 2 & (ii) by 3

$\frac{50}{x+y} - \frac{6}{x-y} = 2$ .....(iii) $\frac{120}{x+y} + \frac{6}{x-y} = 15$ .....(iv)

Adding (iii) & (iv) we get

$\frac{120}{x+y} + \frac{50}{x+y} = 17$

$\frac{170}{x+y} = 17$

$x + y = 10$ .....(vii)

Multiplying (i) by 8 & (ii) by 5 , we get

$\frac{200}{x+y} + \frac{24}{x-y} = 8$ .....(v) $\frac{200}{x+y} - \frac{10}{x-y} = 25$ ....(vi)

(vi) – (v)

$\frac{24+10}{x-y} = 17$

$x - y = \frac{34}{17} = 2$

$x - y = 2$ .....(viii)

By adding (vii) & (viii) we get

$2x = 12$

$x = 6$ & $y = 4$ Answer.

16. Let point P(x, 0) be a point on x-axis, and A be the pont (7, – 4)

So, AP = $2\sqrt{5}$ [Given]

$\Rightarrow$ $AP^2 = 4 \times 5 = 20$

$\Rightarrow$ $(x - 7)^2 + [0 - (-4)]^2 = 20$

$\Rightarrow$ $x^2 + 49 - 14x + 16 = 20$

$\Rightarrow$ $x^2 - 14x - 20 + 65 = 0$

$\Rightarrow$ $x^2 - 14x + 45 = 0$

$\Rightarrow$ $x^2 - 9x - 5x + 45 = 0$

$\Rightarrow$ $x(x - 9) - 5(x - 9) = 0$

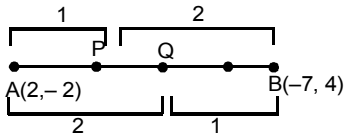
$\Rightarrow$ $(x - 9)(x - 5) = 0$

$\Rightarrow$ $x - 9 = 0$ or $x - 5 = 0$

$\Rightarrow$ $x = 9$ or $x = 5$

Hence, there are two such points on x-axis whose distance from (7, – 4) is $2\sqrt{5}$. Hence, required points are (9, 0), (5, 0).

OR



For the dividing ratio is 1 : 2

$P(x_1, y_1)$, $Q(x_2, y_2)$ for point Q the dividing ratio is 2 : 1.

$x_1 = \frac{4-7}{3} = \frac{-3}{3} = -1$ $\qquad$ $x_2 = \frac{-14+2}{3} = -4$

$y_1 = \frac{-\ 4+4}{3} = 0$ $\qquad$ $y_2 = \frac{8-2}{3} = 2$

P (-1 , 0) & Q (-4 , 2)

17. $\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sec^2 50 - \cot^2 40^\circ}$ + 2 cosec^2 58º – 2cot 58º

tan 32º – 4 tan 13º tan 37º tan 45º tan 53º tan 77º

$= \frac{\cos^2(90^\circ - 70^\circ) + \cos^2 70^\circ}{\sec^2(90^\circ - 40^\circ) - \cot^2 40^\circ}$ + 2 cosec^2 58º – 2cot 58º tan (90º – 58º)

– 4 (tan 13º tan 77º) (tan 37º tan 53º) tan 45º

$= \frac{\sin^2 70^\circ + \cos^2 70^\circ}{\text{cosec}^2 40^\circ - \cot^2 40^\circ}$ + 2$\text{cosec}^2$58º – 2$\cot^2$ 58º

– 4 [tan (90º – 77º) tan 77º] [tan (90º – 53º) tan 53º] [1]

$= \frac{1}{1} + 2 - 4 = -1.$

OR

$\frac{2x}{\cos\theta} - \frac{y}{\sin\theta} = 3$

$\frac{2x\sin\theta - y\cos\theta}{\cos\theta\sin\theta} = 3$

$2x\sin\theta - y\cos\theta = 3\cos\theta\sin\theta$ ...(i)

$x\sin\theta - 2y\cos\theta = 0$ ...(ii)

(ii) × 2, then (i) – (ii)

$2x\sin\theta - y\cos\theta = 3\cos\theta\sin\theta$

$2x\sin\theta - 4y\cos\theta = 0$

$- \quad + \quad -$

$3y\cos\theta = 3\cos\theta\sin\theta$

$y = \sin\theta$

put $y = \sin\theta$ in (ii)

$2y\cos\theta = x\sin\theta$

$2\times\sin\theta\cos\theta = x\sin\theta$

$x = 2\cos\theta$

$x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta = 4$

18. **Given :** Chord AB, diameter AOC and tangents at A of a circle with centre O.

To prove : $\angle BAT = \angle ACB$

Proof : Radius OA and tangent AT at A are perpendicular.

$\therefore$ $\angle OAT = 90^\circ$ (radius at the point of contact of tangent is perpendicular)

$\Rightarrow$ $\angle BAT = 90^\circ - \angle BAC$ .....(i)

AOC is diameter

$\therefore$ $\angle B = 90^\circ$

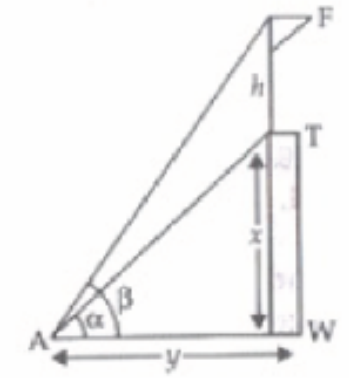
$\Rightarrow$ $\angle C + \angle BAC = 90^\circ$

$\Rightarrow$ $\angle C = 90^\circ - \angle BAC$ .....(ii)

From (i) and (ii), we get

$\angle BAT = \angle ACB$. Hence, proved.

19. Let the height of vertical tower (TW) = x.
And, the height of flag staff (TF) = h (Given)
The angle of elevation at A on ground from the base and top of flag staff are α, β respectively.



Let AW = y
In ΔTWA, we have

$$\tan\alpha = \frac{x}{y}$$

$$\Rightarrow \quad y = \frac{x}{\tan\alpha} \qquad (I)$$

Now, in ΔFWA, we have

$$\tan\beta = \frac{x+h}{y}$$

$$\Rightarrow \quad y\tan\beta = x + h$$

$$\Rightarrow \quad \frac{x\tan\beta}{\tan\alpha} = x + h \qquad [\text{From (I)}]$$

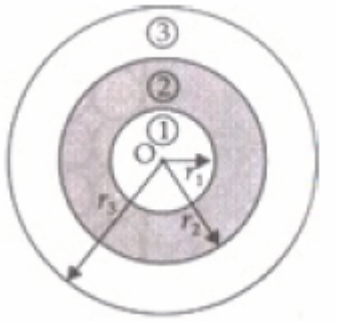
$$\Rightarrow \quad x\tan\beta = x\tan\alpha + h\tan\alpha$$

$$\Rightarrow \quad x(\tan\beta - \tan\alpha) = h\tan\alpha$$

$$\Rightarrow \quad x = \frac{h\ \tan\ \alpha}{\tan\beta - \tan\alpha}$$

Hence, proved

20. $d_1 : d_2 : d_3 = 1 : 2 : 3$ [Given]
$= 1d : 2d : 3d$ [× by d]



$$d_1 : d_2 : d_3 = \frac{d}{2} : \frac{2d}{2} : \frac{3d}{2} \qquad \left[\times \text{ by } \frac{1}{2}\right]$$

$$\Rightarrow \quad r_1 : r_2 : r_3 = r, 2r, 3r$$

$$\therefore \quad r_1 = r, r_2 = 2r, r_3 = 3r.$$

Now, $A_1 = \pi r^2$

$A_2 = \pi(2r)^2 = 4\pi r^2$

$A_3 = \pi(3r)^2 = 9\pi r^2$

Area of region between first and second circles.

$= A_2 = A_2 = 4\pi r^2 = \pi r^2 = 3\pi r^2$

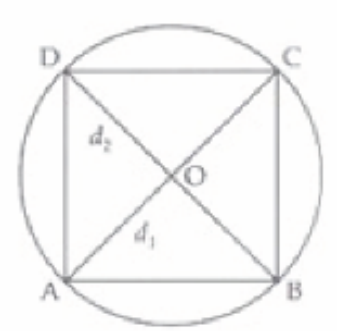
Area of region between second and third circles
$= A_3 - A_2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2$.

$\therefore$ Required ratio $= \pi r^2 : 3\pi r^2 : 5\pi r^2$.

On dividing all the three ratios by πr^2, we get required ratio of areas of three regions as 1 : 3 : 5.

OR

All the vertices of a rhombus lie on a circle so rhombus is a square and its diagonals are of length 2r cm.



Area of circle = 1256 cm^2

$\Rightarrow$ $\pi r^2 = 1256\ cm^2$

$\Rightarrow$ $r^2 = \dfrac{1256}{\pi}$

$\Rightarrow$ $r^2 = \dfrac{1256 \times 100}{314} = 400$

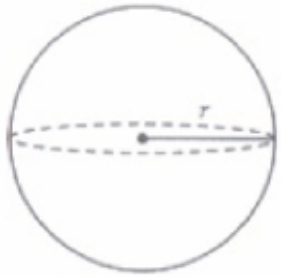
$\Rightarrow$ $r = \sqrt{400} = 20$ cm.

$\therefore$ Area of rhombus $= \dfrac{1}{2} d_1 d_2 = \dfrac{1}{2} \times 2r \times 2r$

$= 2r^2 = 2 \times 20 \times 20$

$\Rightarrow$ Area of rhombus = 800 cm^2.

21. Let the spherical marble has radius r.

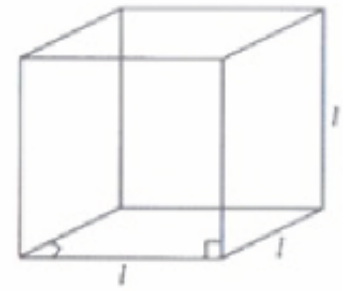


Diameter of the marble = 0.5 cm

$\Rightarrow$ $r = \dfrac{0.5}{2}$ cm

$\Rightarrow$ r =0.25 cm

Cube ℓ = 22 cm



Let n marbles can fill the cube.

$\therefore$ Volume of n marbles = $\left(1-\frac{1}{8}\right)$ part of volume of cube

$\Rightarrow$ $n \cdot \frac{4}{3}\pi r^3 = \frac{7}{8} \times \ell^3$.

$\Rightarrow$ $n = \frac{7\ell^3}{8} \times \frac{3}{4\pi r^3} = \frac{7\times3\times22\times22\times22\times7}{8\times4\times22\times0.25\times0.25\times0.25}$

$\Rightarrow$ $n = \frac{7\times3\times22\times22\times22\times100\times100\times100\times7}{8\times4\times22\times25\times25\times25}$

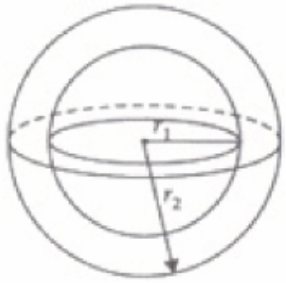
$= 7 \times 3 \times 22 \times 22 \times 2 \times 7 = 42 \times 484 \times 7$

$n = 142296$

So, cube can accomodate upto 142296 marbles so right answer is 142296

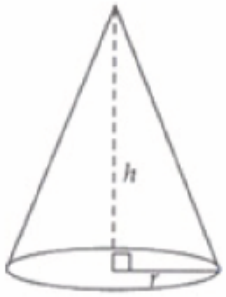
OR

Main concept : During recasting a shape into another its' volume does not change. Spherical shell.



$r_1 = \frac{4}{2} = 2$ cm

$r_2 = \frac{8}{2} = 4$ cm.



Cone

$r = \frac{8}{2} = 4$ cm

h = ?

During recasting volume remains same so

Volume of cone = Volume of hollow spherical shell.

$\Rightarrow$ $\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$

$\Rightarrow$ $\frac{\pi}{3}r^2h = \frac{\pi}{3} \times 4\,[r_2^3 - r_1^3]$

$\Rightarrow$ $r^2h = 4\,[r_2^3 - r_1^3]$

$\Rightarrow$ $4 \times 4h = 4[(4)^3 - (2)^3]$

$\Rightarrow$ $4h = 64 - 8$

$\Rightarrow$ $h = \frac{56}{4}$

$\Rightarrow$ $h = 14$ cm.

22.

C.I.	Frequency	x_i	f_ix_i
$0-10$	8	5	40
$10-20$	x	15	15x
$20-30$	10	25	250
$30-40$	11	35	385
$40-50$	9	45	405
	$\Sigma f_i = 38 + x$		$\Sigma f_i x_i = 1080 + 15x$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\frac{252}{10} = \frac{1080 + 15x}{38 + x}$$

$126\,(38 + x) = 5(1080 + 15x)$

$4788 + 126x = 5400 + 75x$

$51x = 612$

$$x = \frac{612}{51}$$

$x = 12.$

SECTION-D

23.

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{(x-1)(x-2)}{(x-1)(x-2)} + \frac{(x-2)-(x-3)}{(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{1}{x-2} - \frac{1}{x-1} + \frac{1}{x-3} - \frac{1}{x-2} = \frac{2}{3}$$

$$-\frac{(x-3)+(x-1)}{(x-3)(x-1)} = \frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$

$\Rightarrow \quad (x-1)(x-3) = 3$

$x^2 - 4x + 3 = 3$

$x(x-4) = 0$

$x = 0, 4$

OR

Let speed of stream = x km/hr

$$\frac{32}{24-x} - \frac{32}{24+x} = 1$$

$$32\left\{\frac{24+x-24+x}{(24+x)(24-x)}\right\} = 1$$

$64x = 24^2 - x^2$

$x^2 + 64x - 576 = 0$

$x^2 + 72x - 8x - 576 = 0$

$x(x+72) - 8(x+72) = 0$

$x = 8$ km/hr .

24. $a - d, a, a + d$

$a - d + a + a + d = 12$

$\Rightarrow 3a = 12$

$\Rightarrow a = 4$

$(a - d)^3 + a^3 + (a + d)^3 = 288$

$\Rightarrow a^3 - d^3 - 3ad(a - d) + a^3 + a^3 + d^3 + 3ad(a + d) = 288$

$\Rightarrow 3a^3 - 3a^2d + 3ad^2 + 3a^2d + 3ad^2 = 288$

$\Rightarrow 3a^3 + 6ad^2 = 288$

$\Rightarrow 3(4)^3 + 6 \times 4\, d^2 = 288$

$\Rightarrow 24d^2 = 288 - 192$

$\Rightarrow d^2 = \frac{96}{24} = 4$

$\Rightarrow d = 2$

So, numbers are 2, 4, 6.

25. Let $CE = ED = BD = x$

To prove : $8AE^2 = 3AC^2 + 5AD^2$

In ΔACB

$AC^2 = AB^2 + BC^2$

$AC^2 = AB^2 + (3x)^2$

$AC^2 = AB^2 + 9x^2$

In ΔADB

$AD^2 = AB^2 + DB^2$

$AD^2 = AB^2 + x^2$

In ΔAEB

$AE^2 = AB^2 + BE^2$

$AE^2 = AB^2 + 4x^2$

R.H.S. $3AC^2 + 5AD^2$

$= 3(AB^2 + 9x^2) + 5(AB^2 + x^2)$

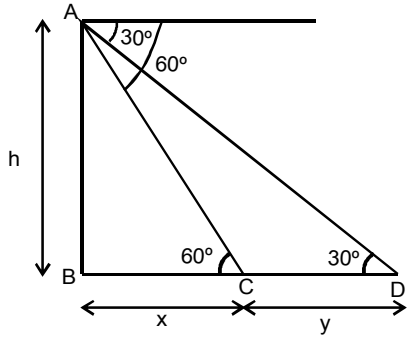
$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$

$= 8AB^2 + 32x^2$

$= 8(AB^2 + 4x^2)$

$= 8AE^2$ **LHS.**

27.



Let the distance covered be y m in 6 min.

$\therefore$ Speed of boat = $\frac{y}{6}$ m/min

In ΔABC

$\tan 60^\circ = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$

In ΔABD

$\tan 30^\circ = \frac{h}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{h}{\frac{h}{\sqrt{3}}+y}$

$\frac{h}{\sqrt{3}} + y = h\sqrt{3}$

$y = \frac{2h}{\sqrt{3}}$

So, $\frac{x}{y} = \frac{1}{2}$

$x = \frac{y}{2}$

$\therefore$ Total distance to be covered $x + y = \frac{3y}{2}$

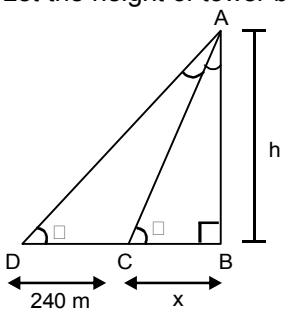
$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ Total time taken $= \frac{3y}{2\times y} \times 6 = 9$ min.

OR

$\tan\theta = \frac{5}{12}$, $\tan\phi = \frac{3}{4}$

Let the height of tower be h.



In ΔABC

$\tan\phi = \frac{h}{x}$

$x = \frac{h}{\tan\phi}$

In ΔABD

$\tan\theta = \frac{h}{x+240}$

$x + 240 = \frac{h}{\tan\theta}$

$$\frac{h}{\tan\phi} + 240 = \frac{h}{\tan\theta}$$

$$240 = h\left(\frac{1}{\tan\theta} - \frac{1}{\tan\phi}\right).$$

$$h = \frac{240 \times \tan\phi\tan\theta}{\tan\phi - \tan\theta}$$

$$h = \frac{240 \times \frac{3}{4} \times \frac{5}{12}}{\frac{3}{4} - \frac{5}{12}}$$

[Substituting the value of tan θ and tan ϕ]

$$h = \frac{75}{4/12} = 225 \text{ m}$$

So, the height of tower = 225 m.

28.

$C.I.$	$Mid\ \ values(x_i)$	$Frequency\ \ (f_i)$	$x_i f_i$
$0-20$	10	17	170
$20-40$	30	f_1	$30f_1$
$40-60$	50	32	1600
$60-80$	70	f_2	$70f_2$
$80-100$	90	19	1710
		$\Sigma f_i = 68 + f_1 + f_2$	$\Sigma x_i f_i = 3480 + 30f_1 + 70f_2$

Given: $\Sigma f_i = 120$

$\Rightarrow$ $68 + f_1 + f_2 = 120$

$\Rightarrow$ $f_1 + f_2 = 52$ .....(i)

Also, $\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i}$

$\Rightarrow$ $50 = \frac{3480 + 30f_1 + 70f_2}{68 + f_1 + f_2}$

$\Rightarrow$ $3400 + 50f_1 + 50f_2 = 3480 + 30f_1 + 70f_2$

$\Rightarrow$ $20f_1 - 20f_2 = 3480 - 3400$

$\Rightarrow$ $20f_1 - 20f_2 = 80$

$\Rightarrow$ $f_1 - f_2 = 4$ .....(ii)

Solving (i) and (ii), we get

$f_1 + f_2 = 52$

$f_1 - f_2 = 4$

———————

$2f_1 = 56$

———————

$\Rightarrow$ $f_1 = 28$ and $f_2 = 24$.

OR

Marks	Frequency	C.F.
$20-30$	p	p
$30-40$	15	$p+15$
$40-50$	25	$p+40$
$50-60$	20	$p+60$
$60-70$	q	$p+60+q$
$70-80$	8	$p+68+q$
$80-90$	10	$p+78+q$

We know, sum of all frequencies = 90

$\Rightarrow$ $p + 78 + q = 90$

$\Rightarrow$ $p + q = 12$ .....(i)

Also,

$$\text{Median} = \ell + \left(\frac{\frac{n}{2}-c}{f}\right) \times h$$

Where, ℓ = lower limit of the median class

$n = 90$

$\Rightarrow$ $\frac{n}{2} = 45$

C = C.F. preceding of the median class

f = frequency of the median class

h = class size.

$\Rightarrow$ $50 = 50 + \left(\frac{45-(p+40)}{20}\right) \times 10$

$\Rightarrow$ $0 = \left(\frac{5-p}{20}\right) \times 10$

$\Rightarrow$ $p = 5$

Putting in equation (i), we get

$q = 7$.

$P = 5$ & $q = 7$

29. When a fluid (water) flows through a pipe of a area of cross-section A with velocity v, then volume of water coming from pipe in time t.

= Area of cross-section × Length

= A × v.t [$\because$ V = Area of base × Height]

Cylinder	Cone
$A = \pi r^2$	$R = \frac{40}{2}$ cm $= 0.2$ m
$r = \frac{5 \text{ mm}}{2} = \frac{5}{2000}$ m	$H = 24$ cm $= 0.24$ m

$\frac{1}{400}$

$v = 10$ m/min

$\Rightarrow$ $v = \frac{10}{60}$ m/s $= \frac{1}{6}$ m/s

$\therefore$ Volume of flowing water = Volume of cone.

$\Rightarrow$ Area of base × height (dist.) $= \frac{1}{3}\pi R^2 H$

$\Rightarrow$ $A \times v.t. = \frac{1}{3}\pi R^2 H$

$\Rightarrow$ $\pi r^2 . v. t = \frac{1}{3}\pi R^2 H$

$\Rightarrow$ $r^2.vt = \frac{1}{3} R^2 H.$

$\Rightarrow$ $\frac{1}{400}\times\frac{1}{400}\times\frac{1}{6}t = \frac{1}{3} \times 0.2 \times 0.2 \times 0.24$

$\Rightarrow$ $t = \frac{2\times 2\times 24\times 400\times 400\times 6}{3\times 10000}$

$\Rightarrow$ $t = 4 \times 24 \times 4 \times 4 \times 2$ sec.

$= \frac{4\times 24\times 4\times 4\times 2}{60}$ min $= \frac{512}{10} = 51.2$ min. $= 51$ min. $+ 0.2$ min $= 51$ min $+ 0.2 \times 60$ sec.

$\Rightarrow$ t = 51 min and 12 sec.

Hence, conical tank will fill in 51 min and 12 sec.

30. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$

$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$

$= \frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$

$= \frac{(\tan^2\theta+1+\tan\theta)}{\tan\theta}$

$= \tan\theta + \cot\theta + 1$

$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$

$= \frac{1}{\sin\theta\cos\theta}+1$

$= \sec\theta\operatorname{cosec}\theta + 1.$

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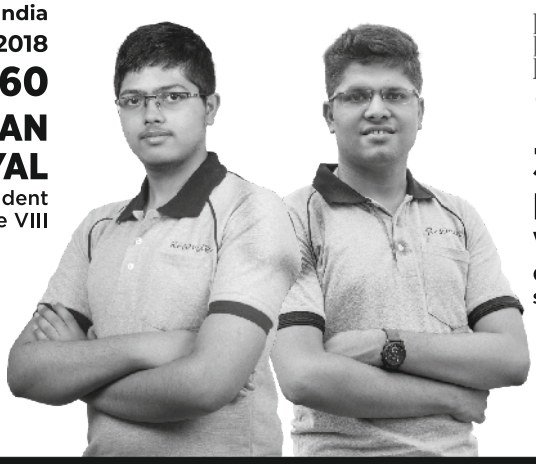

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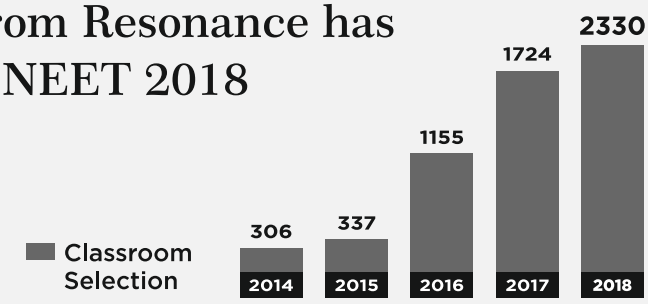

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