Resonance
Educating for better tomorrow

IIT-JEE 2012

PAPER - 1

PART - III : MATHEMATICS

Section I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1, –1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T(2, 1,4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Sol. **Ans. (A)**

Equation of QR is

$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$$

Let P ≡ $(2 + \lambda, 3 + 4\lambda, 5 + \lambda)$

$10 + 5\lambda - 12 - 16\lambda - 5 - \lambda = 1$

$-7 - 12\lambda = 1$

$\Rightarrow \quad \lambda = \frac{-2}{3}$

then P ≡ $\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Let S = $(2 + \mu, 3 + 4\mu, 5 + \mu)$

$$\vec{TS} = (\mu)\hat{i} + (4\mu + 2)\hat{j} + (\mu + 1)\hat{k}$$

$$\vec{TS} \cdot (\hat{i} + 4\hat{j} + \hat{k}) = 0$$

$\mu + 16\mu + 8 + \mu + 1 = 0$

$\mu = -\frac{1}{2}$

$$S = \left(\frac{3}{2}, 1, \frac{9}{2}\right)$$

$$PS = \sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2+\frac{4}{9}+\left(\frac{13}{3}-\frac{9}{2}\right)^2} = \sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}} = \sqrt{\frac{1}{18}+\frac{4}{9}} = \sqrt{\frac{9}{18}} = \frac{1}{\sqrt{2}}$$

42. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}$ + K

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}$ + K

(C) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}$ + K

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}$ + K

Sol. **Ans (C)**

Put secx + tanx = t

(secx tanx + $\sec^2$x) dx = dt

secx . t dx = dt

secx – tanx = $\frac{1}{t}$

secx = $\frac{t+\frac{1}{t}}{2}$

$$\int \frac{\sec x.dt}{t^{9/2}.t} = \int \frac{1}{2}\frac{\left(t+\frac{1}{t}\right)}{t.t^{9/2}}dt$$

$$= \frac{1}{2}\int\left(\frac{1}{t^{9/2}}+\frac{1}{t^{13/2}}\right)dt$$

$$= -\frac{1}{2}\left[\frac{2}{7t^{7/2}}+\frac{2}{11t^{11/2}}\right]+k$$

$$= -\frac{1}{t^{11/2}}\left[\frac{t^2}{7}+\frac{1}{11}\right]+k$$

43. Let z be a complex number such that the imaginary part of z is non zero and a = $z^2 + z + 1$ is real. Then a cannot take the value

(A) –1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

43. Ans (D)

Here $z^2 + z + 1 - a = 0$

$$\Rightarrow \quad z = \frac{-1 \pm \sqrt{4a-3}}{2}$$

Here $a \neq \frac{3}{4}$ otherwise z will be purely real.

44. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathrm{IR}$, then f is

(A) differentiable both at x = 0 and at x = 2

(B) differentiable at x = 0 but not differentiable at x = 2

(C) not differentiable at x = 0 but differentiable at x = 2

(D) differentiable neither at x = 0 nor at x = 2

Sol. Ans (B)

(I) for derivability at x = 0

$$\text{L.H.D.} = f'(0^-) = \lim_{h\to 0^+} \frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to 0^+} \frac{h^2.\left|\cos\left(-\frac{\pi}{h}\right)\right| - 0}{-h}$$

$$= \lim_{h\to 0^+} -h\,.\left|\cos\frac{\pi}{h}\right| = 0$$

$$\text{RHD} \quad f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{h^2.\left|\cos\left(\frac{\pi}{h}\right)\right| - 0}{h} = 0$$

So f(x) is derivable at x = 0

(ii) check for derivability at x = 2

$$\text{RHD} = f'(2^+) = \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0^+} \frac{(2+h)^2.\left|\cos\left(\frac{\pi}{2+h}\right)\right| - 0}{h}$$

$$= \lim_{h\to0^+} \frac{(2+h)^2.\cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to0^+} \frac{(2+h)^2.\sin\left(\frac{\pi}{2}-\frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to0^+} \frac{(2+h)^2.\sin\left(\frac{\pi h}{2(2+h)}\right)}{\left(\frac{\pi}{2(2+h)}\right)h} . \frac{\pi}{2(2+h)}$$

$$= (2)^2 . \frac{\pi}{2(2)} = \pi$$

LHD $= \lim_{h\to0^+} \frac{f(2-h)-f(2)}{-h}$

$$= \lim_{h\to0^+} \frac{(2-h)^2.\left|\cos\left(\frac{\pi}{2-h}\right)\right|-0}{-h}$$

$$= \lim_{h\to0^+} \frac{(2-h)^2.\left(-\cos\left(\frac{\pi}{2-h}\right)\right)-0}{-h}$$

$$= \lim_{h\to0^+} \frac{(2-h)^2\cos\left(\frac{\pi}{2-h}\right)}{h}$$

$$= \lim_{h\to0^+} \frac{(2-h)^2.\sin\left(\frac{\pi}{2}-\frac{\pi}{2-h}\right)}{h}$$

$$= \lim_{h\to0^+} \frac{(2-h)^2.\sin\left(-\frac{\pi h}{2(2-h)}\right)}{\left(-\frac{\pi h}{2(2-h)}\right)} . \frac{-\pi}{2(2-h)}$$

$$= -\pi$$

So f(x) is not derivable at x = 2

45. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150 (C) 210 (D) 243

Sol. Ans (B)

	B_1	B_2	B_3
Case-1:	1	1	3
Case-2:	2	2	1

Ways of distribution $= \dfrac{5!}{1!\,1!\,3!\,2!}\cdot 3! + \dfrac{5!}{2!\,2!\,1!\,2!}.3!$

$= 150$

46. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right) = 4$, then

(A) a = 1, b = 4 (B) a = 1, b = –4

(C) a = 2, b = –3 (D) a = 2, b = 3

Sol. Ans (B)

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right) = 4$$

$$\lim_{x\to\infty}\left(\frac{x^2(1-a)+x(1-a-b)+(1-b)}{x+1}\right) = 4$$

Limit is finite

It exists when $1 - a = 0 \quad \Rightarrow a = 1$

then $$\lim_{x\to\infty}\left(\frac{1-a-b+\dfrac{1-b}{x}}{1+\dfrac{1}{x}}\right) = 4$$

$\therefore \quad 1 - a - b = 4 \quad \Rightarrow \quad b = -4$

47. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto (B) onto but not one-one

(C) one-one but not onto (D) neither one-one nor onto

Sol. Ans (B)

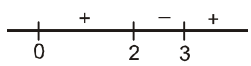
$F : [0, 3] \to [1, 29]$

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$



in given domain function has local maxima, it is many-one

Now at $x = 0 \quad f(0) = 1$

$x = 2 \quad f(2) = 16 - 60 + 72 + 1 = 29$

$x = 3 \quad f(3) = 54 - 135 + 108 + 1$

$= 163 - 135 = 28$

Has range = [1, 29]

Hence given function is onto

48. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Sol. Ans (A)

Circle $x^2 + y^2 = 9$

line $4x - 5y = 20$

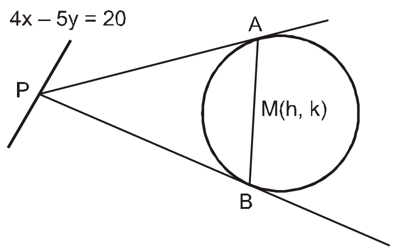
$P\left(t, \frac{4t-20}{5}\right)$

equation of chord AB whose mid point is M (h, k)

$T = S_1$

$\therefore \quad hx + ky = h^2 + k^2$ ....(1)

equation of chord of contact AB with respect to P.



$T = 0$

$tx + \left(\frac{4t-20}{5}\right)y = 9$ ....(2)

comparing equation (1) and (2)

$$\frac{h}{t} = \frac{5k}{4t-20} = \frac{h^2+k^2}{9}$$

on solving

$45k = 36h - 20h^2 - 20k^2$

$\Rightarrow$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

49. Let $P = [a_{ij}]$ be a 3 × 3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Sol. Ans (D)

Given $P = [a_{ij}]_{3\times3}$ $b_{ij} = 2^{i+j}$ aij

$Q = [b_{ij}]_{3\times3}$

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad |P| = 2$$

$$Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{bmatrix}$$

Determinant of Q = $\begin{vmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{vmatrix}$

$= 4 \times 8 \times 16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{vmatrix}$

$= 4 \times 8 \times 16 \times 2 \times 4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= 2^2 . 2^3 . 2^4 . 2^1 . 2^2 . 2^1$
$= 2^{13}$

50. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes.

Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R.. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

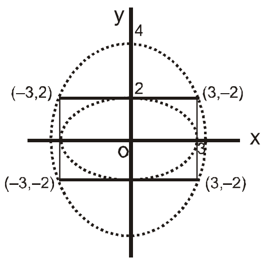
Sol. **Ans (C)**

Let required ellipse is

$E_2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes thorugh (0, 4)

$0 + \frac{16}{b^2} = 1 \quad \Rightarrow \quad b^2 = 16$



It also passes through (±3, ±2)

$\frac{9}{a^2} + \frac{4}{b^2} = 1$

$\frac{9}{a^2} + \frac{1}{4} = 1$

$\frac{9}{a^2} = \frac{3}{4} \quad \Rightarrow \quad a^2 = b^2 (1 – e^2)$

$\frac{12}{16} = 1 – e^2$

$e^2 = 1 – \frac{12}{16} = \frac{4}{16} = \frac{1}{16}$

$e = \frac{1}{2}$

Section II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

51. If y(x) satisfies the differential equation $y' - y \tan x = 2x \sec x$ and y(0), then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

(B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$

(D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Sol. **Ans (AD)**

$$\frac{dy}{dx} - y \tan x = 2x \sec x$$

$y(0) = 0$

$$\text{I.F.} = e^{-\int \tan x\, dx} = e^{-\log \sec x}$$

I.F. = cos x

$$\cos x \,.\, y = \int 2x \sec x . \cos x\, dx$$

$\cos x.\ y = x^2 + c$

$c = 0$

$y = x^2 \sec x$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}.\sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2}.\sqrt{2} + \frac{\pi^2}{16}.\sqrt{2}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}.2 = \frac{2\pi^2}{9}$$

$$y'\left(\frac{\pi}{3}\right) = 2\frac{\pi}{2}.2 + \frac{\pi^2}{9}.2.\sqrt{3}$$

$$\frac{4\pi}{3} + \frac{2\pi^2\sqrt{3}}{9}$$

52. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denotes respectively the events that the engines E_1 E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P\left[X_1^c \mid x\right] = \frac{3}{16}$

(B) P[Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$

(D) $P[X \mid X_1] = \frac{7}{16}$

Sol. **Ans** **(BD)**

$P(x_1) = \frac{1}{2}$

$P(x_2) = \frac{1}{4}$

$P(x_3) = \frac{1}{4}$

$P(x) = P(E_1 E_2 E_3) + P(\overline{E}_1 E_2 E_3) + P(E_1 \overline{E}_2 E_3) + P(E_1 E_2 \overline{E}_3)$

$= \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}$

$P(x) = \frac{1}{4}$

(A) $P\left(\frac{x_1^c}{x}\right) = \frac{P(x_1^c \cap x)}{P(x)}$

$$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

(B) $P(\text{exactly two} / x) = \frac{P(\text{exactly two} \cap x)}{P(x)} = \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$

(C) $P(x / x_2) = \dfrac{P(x \cap x_2)}{P(x_2)} = \dfrac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}{\frac{1}{4}} = \dfrac{5}{8}$

(D) $P(x / x_1) = \dfrac{P(x \cap x_1)}{P(x_1)} = \dfrac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}{\frac{1}{2}} = \dfrac{7}{16}$

53. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy

(A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

Sol. Ans (ACD)

As $\tan(2\pi - \theta) > 0$, $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, $\theta \in [0, 2\pi]$

$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow 1 < 2\sin(\theta + \phi) < 2$

$\Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1$

As $\theta + \phi \in [0, 4\pi]$

$\Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $\theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

$\Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta$ or $\frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$

$\Rightarrow \phi \in \left(-\frac{3\pi}{2}, \frac{-2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$ $\left(\because \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)\right)$

54. If S be the area of the region enclosed by y = e^{-x^2}, y = 0, x = 0, and x = 1. Then

(A) $S \ge \frac{1}{e}$ (B) $S \ge 1 - \frac{1}{e}$ (C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Sol. **Ans (ABD)**

$$I = \int_0^1 e^{-x^2} dx$$

$-x^2 \le 0$

$e^{-x^2} \le 1$

$$\int_0^1 e^{-x^2} dx \le 1$$

$x^2 \le x \Rightarrow -x^2 \ge -x \Rightarrow e^{-x^2} \ge e^{-x}$

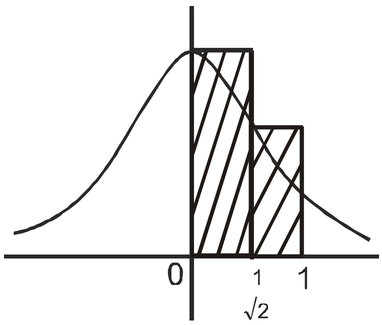
$$\Rightarrow I \ge \int_0^1 e^{-x} dx$$

$$\ge -\left(e^{-x}\right)_0^1$$

$$\ge -\left(\frac{1}{e} - 1\right)$$

$I \ge 1 - \frac{1}{e} \Rightarrow$ (B) is correct

Since If $I \ge 1 - \frac{1}{e} \Rightarrow I > \frac{1}{e}$ $\Rightarrow$ (A) is correct



$I < \frac{1}{\sqrt{2}} \times 1 + \frac{1}{\sqrt{e}} \times \left(1 - \frac{1}{\sqrt{2}}\right)$

So Ans. D

55. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line 2x – y = 1. The points of contacts of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

Sol. **Ans (AB)**

Slope of tangents = 2

Equation of tangents y = 2x ± $\sqrt{9.4 - 4}$

$\Rightarrow$ y = 2x ± $\sqrt{32}$

$\Rightarrow$ 2x – y ± $4\sqrt{2}$ = 0 ....(i)

Let point of contact be (x_1, y_1)
then equation (i) will be identical to the equation

$$\frac{xx_1}{9} - \frac{yy_1}{4} - 1 = 0$$

$$\therefore \frac{x_1/9}{2} = \frac{y_1/4}{1} = \frac{-1}{\pm 4\sqrt{2}}$$

$$\Rightarrow (x_1, y_1) \equiv \left(-\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \text{ and } \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Section III : Integer Answer Type

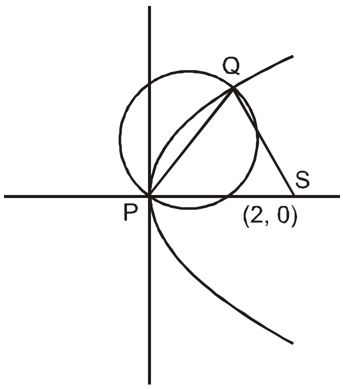
This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

56. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is.

Sol. **Ans (4)**

Focus is (a, 0) ≡ (2, 0)

P ≡ (0, 0)



$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ \frac{\alpha^2}{8} & \alpha & 1 \end{vmatrix}$$

$= \frac{1}{2}(2\alpha) = \alpha$ we need y coordinate of Q.

$(2t^2, 4t)$ satisfies circle

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$t^3 + 3t - 4 = 0$

$(t - 1)(t^2 + t + 4) = 0$

t = 1

so Ans. 4

57. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 p(3) = 2 , then $p'(0)$ is

Sol. **Ans (9)**

$p' = \lambda(x-1)(x-3) = \lambda(x^2 - 4x + 3)$

$p(x) = \lambda(x^3/3 - 2x^2 + 3x) + \mu$

$p(1) = 6$

$6 = \lambda(1/3 - 2 + 3) + \mu$

$6 = \lambda(1/3 + 1) + \mu$

$18 = 4\lambda + 3\mu$ ...(i)

$p(3) = 2$

$2 = \lambda(27/3 - 2 \times 9 + 9) + \mu$

$2 = \mu$

$\mu = 2 \Rightarrow \lambda = 3$

$p'(x) = 3(x-1)(x-3)$

$p'(0) = 3(-1)(-3)$

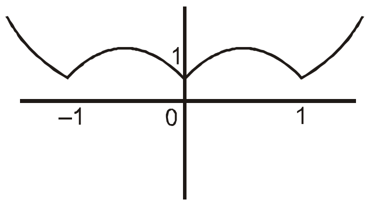
$= 9$

58. Let f : IR → IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. **Ans (5)**

$f(x) = |x| + |x^2 - 1|$

$$f(x) = \begin{cases} -x + x^2 - 1 & x < -1 \\ -x - x^2 + 1 & -1 \le x \le 0 \\ x - x^2 + 1 & 0 < x < 1 \\ x + x^2 - 1 & x \ge 1 \end{cases}$$



$$f(x) = \begin{cases} x^2 - x - 1 & x < -1 \\ -x^2 - x + 1 & -1 \le x \le 0 \\ -x^2 + x + 1 & 0 < x < 1 \\ x^2 + x - 1 & x \ge 1 \end{cases}$$

59. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}}\right)$ is

Sol. **Ans (4)**

Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}.......}} = t$

$\sqrt{4 - \frac{1}{3\sqrt{2}}t} = t$

$4 - \frac{1}{3\sqrt{2}} t = t^2 \Rightarrow$

$t^2 + \frac{1}{3\sqrt{2}} t - 4 = 0 \Rightarrow 3\sqrt{2}\, t^2 + t - 12\sqrt{2} = 0$

$t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}}$

$t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

60. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

Sol. **Ans (3)**

$6 - 2\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{c} - 2\vec{c}\cdot\vec{a} = 9$

$\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right) = \frac{-3}{2}$

$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 \geq 0$

$3 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right) \geq 0$

$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} \geq \frac{-3}{2}$

Since $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-3}{2}$

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = 0 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \left|2\vec{a} + 5(-\vec{a})\right| = |3\vec{a}| \Rightarrow 3$