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**RMO
2016**

REGIONAL MATHEMATICAL OLYMPIAD 2016

TEST PAPER WITH SOLUTION & ANSWER KEY

REGION : GUJARAT | CENTRE : SURAT

Date: 09th October, 2016 | Duration: 3 Hours | Max. Marks: 102

Resonance's Forward Admission & Scholarship Test (ResoFAST)

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Test Dates

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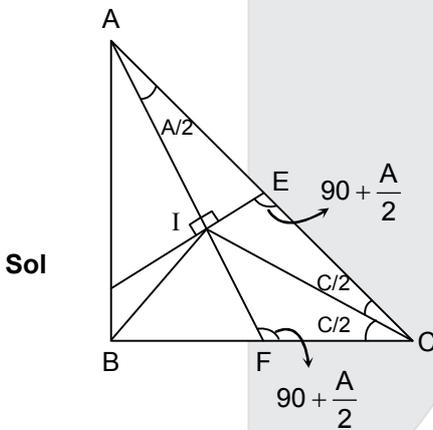


:: IMPORTANT INSTRUCTIONS ::

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- Answer all the questions.
- All questions carry equal marks. Maximum marks: 102.

Answer to each question should start on a new page. Clearly indicate the question number.

1. Let ABC be a right-angled triangle with $\angle B = 90^\circ$. Let I be the incentre of ABC. Let AI extended intersect BC at F. Draw a line perpendicular to AI at I. Let it intersect AC at E. Prove that $IE = IF$.



Given : In $\triangle ABC$, $\angle ABC = 90^\circ$, angle bisector of $\angle BAC$ cut BC at F, I is incentre of $\triangle ABC$,

Line perpendicular to I through I cuts AC at E

To prove : $IE = IF$

Construction : Join IC

Prove : $\angle IAE = \frac{\angle A}{2}$ (AI is the angle bisector of $\angle BAE$)

$\angle IEC = 90 + \frac{A}{2}$ { $\angle IEC$ is exterior angle of $\angle AEI$ for $\triangle AEI$ }

$\angle IFC = 90 + \frac{A}{2}$ { $\angle IFC$ is exterior angle of $\angle AFB$ in $\triangle AFB$ }

Now in $\triangle IEC$ and $\triangle IFC$

$\angle IEC = \angle IFC = 90 + \frac{A}{2}$ {from 1 and 2}

$$\angle ECI = \angle FCI = \frac{C}{2} \quad \{CI \text{ is angle bisector of } \angle ECF\}$$

$$IC = IC \text{ (common)}$$

$$\text{so } \triangle IEC \cong \triangle IFC \quad \{\text{AAS congruency criterion}\}$$

$$\Rightarrow IE = IF \quad \{\text{corresponding sides of congruent triangles}\}$$

2. Let a, b, c be positive real number such that

$$\frac{a}{1+b} + \frac{b}{1+c} + \frac{c}{1+a} = 1$$

$$\text{Prove that } abc \leq \frac{1}{8}.$$

Sol. $a(1+c)(1+a) + b(1+b)(1+a) + c(1+b)(1+c) = (1+a)(1+b)(1+c)$

$$\Rightarrow (a^2 + b^2 + c^2) + (a^2c + b^2a + c^2b) = 1 + abc \quad \dots\dots\dots(1)$$

$$\text{Now } \frac{a^2 + b^2 + c^2}{3} \geq (abc)^{2/3} \quad (\text{AM} \geq \text{GM})$$

$$\Rightarrow a^2 + b^2 + c^2 \geq 3(abc)^{2/3} \quad \dots\dots\dots(2)$$

$$\frac{a^2c + b^2a + c^2b}{3} \geq (abc) \quad (\text{AM} \geq \text{GM})$$

$$\Rightarrow a^2c + b^2a + c^2b \geq 3abc \quad \dots\dots\dots(3)$$

add (2) and (3) we get

$$(a^2 + b^2 + c^2) + (a^2c + b^2a + c^2b) \geq 3(abc)^{2/3} + 3abc \quad \dots\dots\dots(4)$$

$$\Rightarrow 1 + abc \geq 3(abc)^{2/3} + 3(abc) \quad (\text{using (1) and (4)}) \quad \dots\dots\dots(5)$$

$$\text{Let } (abc)^{1/3} = t$$

$$\text{Now } 1 + t^3 \geq 3t^2 + 3t^3$$

$$\Rightarrow (1+t)(1+t^2-t) \geq 3t^2(1+t) \Rightarrow (1+t)(3t^2-t^2+t-1) \leq 0 \Rightarrow (1+t)^2(2t-1) \leq 0$$

$$\Rightarrow t \leq \frac{1}{2} \Rightarrow (abc)^{1/3} \leq \frac{1}{2} \Rightarrow abc \leq \frac{1}{8}$$

3. For any natural number n, expressed in base 10, let S(n) denote the sum of all digits of n. Find all natural numbers n such that $n^3 = 8S(n)^3 + 6nS(n) + 1$.

Ans. 17

Sol. $n^3 = 8(S(n))^3 + 6nS(n) + 1$

$$\Rightarrow (2S(n))^3 + (-n)^3 + 1^3 - 3(2S(n))(-n)(1) = 0$$

$$\Rightarrow (2S(n) + 1 - n) \left[\frac{(2S(n)+n)^2 + (1+n)^2 + (2S(n)-1)^2}{2} \right] = 0$$

Because $(n + 1)^2$ is always positive so second factor is positive

$$\Rightarrow 2S(n) + 1 - n = 0$$

$$\Rightarrow n = 2s(n) + 1 \quad \dots\dots\dots(1)$$

Let $n = a_k a_{k-1}, \dots\dots\dots a_2 a_1 a_0$ where $a_k, a_{k-1}, \dots\dots, a_2, a_1, a_0$ represent digits

$$\text{Now } (10^k a_k + 10^{k-1} a_{k-1} + \dots + 10a_1 + a_0) = 2(a_k + a_{k-1} + \dots + a_1 + a_0) + 1$$

$$\text{Because } 2(a_k + a_{k-1} + \dots + a_1 + a_0) + 1 \geq 2(9(k+1)) + 1$$

{Equality holds when all digit are equal to 9}

$$\text{so, } (10^k a_k + 10^{k-1} a_{k-1} + \dots + 10a_1 + a_0) \geq 2(9(k+1)) + 1$$

Which can holds only for $k = 0, 1$

Case-I $K = 0$

It means n is single digit number

$$\text{Here } S(n) = n \Rightarrow n = 2n + 1 \quad \{\text{using (1)}\}$$

$$\Rightarrow n = -1 \Rightarrow \text{no natural number is possible}$$

Case-II $K = 1$

It means n is two digit number.

$$\text{Here } 10a_1 + a_0 = 2(a_1 + a_0) + 1 \quad \{\text{using (1)}\}$$

$$\Rightarrow 8a_1 = a_0 + 1 \Rightarrow a_0 = 7 \text{ and } a_1 = 1 \Rightarrow n \text{ is } 17$$

4. How many 6-digit natural numbers containing only the digits 1,2,3 are there in which 3 occurs exactly twice and the number is divisible by 9 ?

Ans. 0

Sol. Let digits of six digit number are $a_5, a_4, a_3, a_2, a_1, a_0$ where $a_0, a_1, \dots\dots a_5 \in \{1, 2, 3\}$ in which two of them must be equal 3 and other four equals to either 1 or 2

$$\text{Let } a_5 = a_4 = 3 \text{ \& } a_3, a_2, a_1, a_0 \in \{1, 2\}$$

$$\text{Now } a_5 + a_4 + a_3 + a_2 + a_1 + a_0 = 9k$$

{If number is divisible by 9 then sum of digits is multiple of 9}

$$\Rightarrow a_3 + a_2 + a_1 + a_0 + 6 = 9k \Rightarrow a_3 + a_2 + a_1 + a_0 = 3, 12, 21, \dots$$

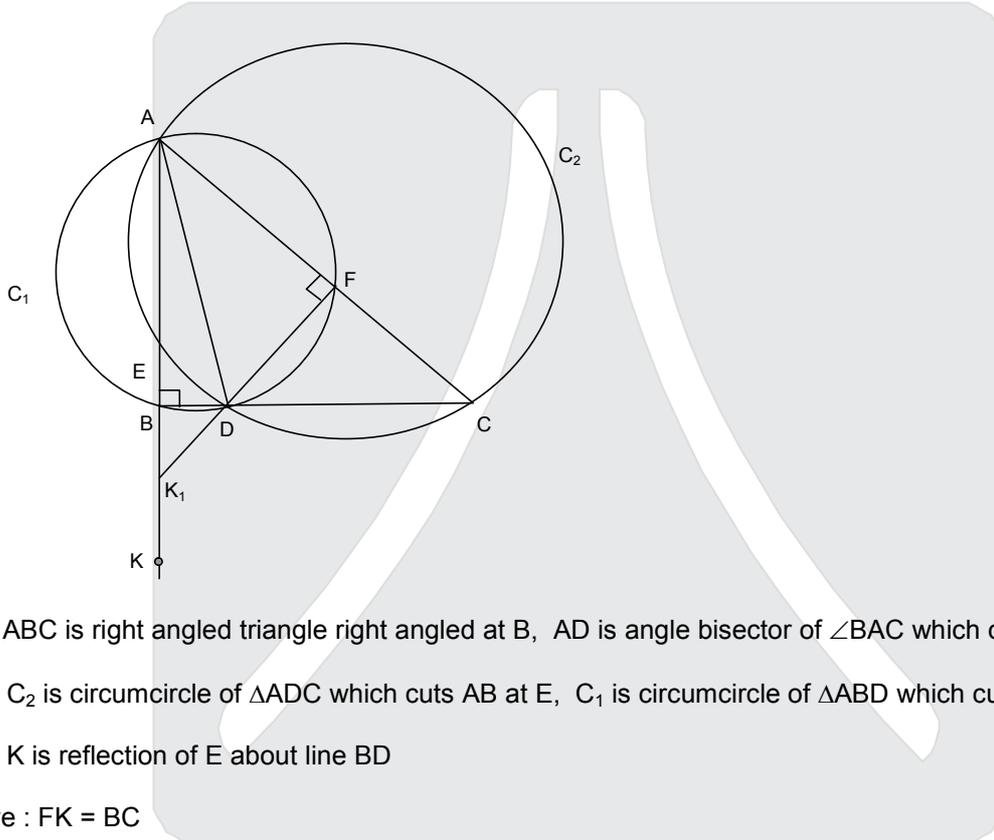
But minimum value of $a_0 + a_1 + a_2 + a_3 = 4$ (which is not possible) and maximum value of

$a_0 + a_1 + a_2 + a_3 = 12$ (Which occur when $a_0 = a_1 = a_2 = a_3 = 3$)

⇒ no number is possible

5. Let ABC be a right-angled triangle with $\angle B = 90^\circ$. Let AD be the bisector of $\angle A$ with D on BC. Let the circumcircle of triangle ACD intersect AB again in E. and let the circumcircle of triangle ABD intersect AC again in F. Let K be the reflection of E in the line BC. Prove that $FK = BC$.

Sol.



Given : ABC is right angled triangle right angled at B, AD is angle bisector of $\angle BAC$ which cuts BC at D.

C_2 is circumcircle of $\triangle ADC$ which cuts AB at E, C_1 is circumcircle of $\triangle ABD$ which cuts AC at F.

K is reflection of E about line BD

To prove : $FK = BC$

Construction : Join FD and extend it so that its cuts extension of AB at K_1 .

Proof : Points ABDF are concyclic (given)

AD is diameter of circumcircle (C_1) of $\triangle ABD$ ($\angle ABD$ is 90°)

$\angle AFD = 90^\circ$ (because AD is diameter)

In $\triangle ABD$ and $\triangle AFD$

$\angle B = \angle F = 90^\circ$, $\angle BAD = \angle FAD = \frac{\angle A}{2}$ (AD is angle bisector), AD is common

so $\triangle ABD \cong \triangle AFD$ (AAS congruency theorem)

⇒ $AB = AF$ and $BD = FD$ (1)

$$\angle AED = 180^\circ - \angle ACD \quad (\text{sum of opposite angles of cyclic quadrilateral AEDC is } 180^\circ)$$

$$\angle BED = \angle ACD \quad (\text{Linear pair axiom})$$

$$\angle EDB = 90^\circ - \angle ACD \quad (\Delta EBD \text{ is right triangle) } \dots\dots(2)$$

$$\angle FDC = 90^\circ - \angle ACD = \angle BDK_1 \quad (\angle FDC \text{ and } \angle BDK_1 \text{ are verticals opposite angle) } \dots\dots(3)$$

Now In ΔEBD and ΔK_1BD

$$\angle EDB = \angle K_1DB \text{ (from (2) and (3)), } \angle EBD = \angle K_1BD \text{ (both are } 90^\circ), \text{ BD is common}$$

$$\text{so, } \Delta EBD \cong \Delta K_1BD \quad (\text{AAS congruency theorem})$$

$$\Rightarrow EB = BK_1 \quad (\text{corresponding sides of congruent triangle})$$

$$\Rightarrow K_1 \text{ is reflection of E about line BD}$$

$$\Rightarrow K_1 \text{ is same as K}$$

Now In ΔBDK and ΔFDC

$$\angle BDK = \angle FDC, \angle KBD = \angle CFD = 90^\circ, \text{ BD} = \text{DF (from (1))}$$

$$\Rightarrow \Delta BDK \cong \Delta FDC$$

$$\text{BD} = \text{DF} \dots\dots\dots(4)$$

$$\text{DC} = \text{DK} \dots\dots\dots(5)$$

Add (4) and (5) we get $BC = FK$ hence proved.

6. Show that the infinite arithmetic progression $(1, 4, 7, 10, \dots)$ has infinitely many 3-term subsequences in harmonic progression such that for any two such triples $\langle a_1, a_2, a_3 \rangle$ and $\langle b_1, b_2, b_3 \rangle$ in harmonic progression one has $\frac{a_1}{b_1} \neq \frac{a_2}{b_2}$

Sol. Consider three terms of this sequence $a = 3p + 1, b = 3q + 1, c = 3r + 1$ where p, q, r are in A.P.

So a, b, c will also be in A.P.

Now $\frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c} \Rightarrow bc, ca, ab$ are in H.P. and all three of bc, ca, ab are of the form $3n + 1$ for some $n \in \mathbb{N}$, hence there will be infinite triplets of $(bc, ca, ab) \equiv (a_1, a_2, a_3)$ which are in H.P.

If (a_1, a_2, a_3) and (b_1, b_2, b_3) are two such triplets.

$$\text{Let } a_1 = (3q + 1)(3r + 1), a_2 = (3r + 1)(3p + 1), a_3 = (3p + 1)(3q + 1)$$

where $p + r = 2q$

$$\text{and } b_1 = (3m + 1)(3n + 1), (3n + 1)(3\ell + 1), (3\ell + 1)(3m + 1)$$

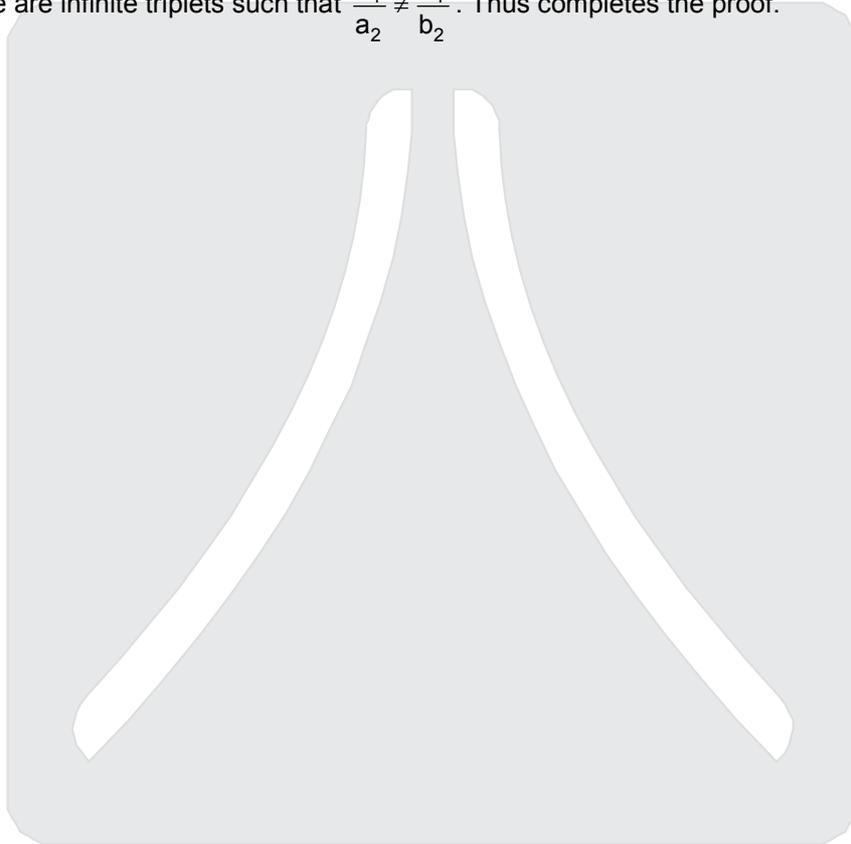
where $\ell + n = 2m$

$$\text{If } \frac{a_1}{b_1} = \frac{a_2}{b_2} \Rightarrow \frac{(3q+1)(3r+1)}{(3m+1)(3n+1)} = \frac{(3r+1)(3p+1)}{(3n+1)(3\ell+1)}$$

$$\Rightarrow \frac{(3q+1)}{(3m+1)} = \frac{(3p+1)}{(3\ell+1)}$$

Now for a choice of q, m, p , the value of ℓ is fixed, but ℓ can be chosen arbitrary for a given m .

Hence there are infinite triplets such that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Thus completes the proof.



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Study Center Cities (29): Rajasthan: Kota, Jaipur, Jodhpur, Udaipur, Ajmer, Sikar; **Bihar:** Patna; **Chattisgarh:** Raipur; **Delhi; Gujarat:** Ahmedabad, Surat, Rajkot, Vadodara; **Jharkhand:** Ranchi; **Madhya Pradesh:** Bhopal, Gwalior, Indore, Jabalpur; **Maharashtra:** Aurangabad, Mumbai, Nagpur, Nanded, Nashik, Chandrapur; **Odisha:** Bhubaneswar; **Uttar Pradesh:** Agra, Allahabad, Lucknow; **West Bengal:** Kolkata;

Test Dates: 20.11.2016, 25.12.2016, 15.01.2017

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Other Test Cities (74): Rajasthan: Ajmer, Sikar, Sri Ganganagar, Alwar, Bhilwara, Bikaner, Bharatpur, Churu, Abu Road, Barmer; **Bihar:** Arah, Bhagalpur, Purnia, Samastipur, Gaya, Sitamari, Nalanda, Begu Sarai, Madhubani, Muzzafarpur; **Delhi NCR:** Noida, Gurgaon, Faridabad, Ghaziabad; **Haryana:** Bhiwani, Rewari, Hisar, Kaithal, Mahendargarh; **Jharkhand:** Jamshedpur, Bokaro, Dhanbad; **J&K:** Jammu; **Madhya Pradesh:** Satna, Singhroli, Guna, Sahdol, Chattarpur; **Maharashtra:** Pune, Latur, Akola, Jalgaon, Sangli; **North East:** Guwahati, Jorhat; **Odisha:** Rourkela, Sambalpur; **Punjab:** Amritsar, Jhalandhar, Bhatinda; **Uttarakhand:** Dehradun, Haridwar; **Uttar Pradesh:** Kanpur, Varanasi, Jhansi, Jaunpur, Bareilly, Rai Bareilly, Sultanpur, Saharanpur, Aligarh, Gorakhpur, Mathura, Rampur; **West Bengal:** Durgapure; **Gujrat:** Gandhinagar, Anand, Jamnagar, Vapi, Mehsana; **Chattisgarh:** Bilaspur, Bhillai; **Himachal Pradesh:** Mandi, Chandigarh;

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5111

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JEE (Main) 2016

28090

CCP: 20429 | DLP/ e-LP: 7661

AIIMS 2016

213

CCP: 32 | DLP/ e-LP: 181

NEET 2016

1787

CCP: 1155 | DLP/ e-LP: 632

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