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RMO
2016

REGIONAL MATHEMATICAL OLYMPIAD 2016

TEST PAPER WITH SOLUTION & ANSWER KEY

REGION: RAJASTHAN, CHHATTISGARH, JHARKHAND, ORRISA, MADHYA PRADESH

CENTRE : JAIPUR, RAIPUR, RANCHI, BHUVNESWAR, INDORE

Date: 16th October, 2016 | Duration: 3 Hours | Max. Marks: 102

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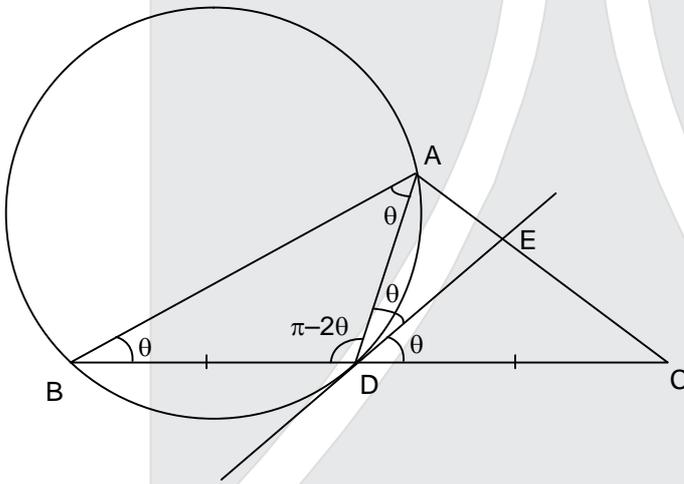
:: IMPORTANT INSTRUCTIONS ::

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- Answer all the questions.
- All questions carry equal marks. Maximum marks: 102.

Answer to each question should start on a new page. Clearly indicate the question number.

1. Let ABC be a triangle and D be the mid-point of BC. Suppose the angle bisector of $\angle ADC$ is tangent to the circumcircle of triangle ABD at D. Prove that $\angle A = 90^\circ$.

Sol.



Given $BD = DC$, let the angle bisector of $\angle ADC$ meet AC at E ,
Further assume $\angle CDE = \angle ADE = \theta$.

Since the angle bisector is tangent at D ,
 $\angle ABC = \theta$ (angle in alternate segment are equal)

Now $\angle ABD = \pi - 2\theta \Rightarrow \angle BAD = \theta \Rightarrow ABD$ is an isosceles triangle

So $AD = BD = CD \Rightarrow D$ is equidistant from vertices A, B, C

$\Rightarrow \Delta$ is circumcentre lies on triangle and is mid-point of $BC \Rightarrow \angle A = 90^\circ$

2. Let a, b, c be three distinct positive real number such that $abc = 1$.

Prove that $\frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)} \geq 3$

Sol.

$$\begin{aligned} & - \left(\frac{a^3(b-c) + b^3(c-a) + c^3(a-b)}{(a-b)(b-c)(c-a)} \right) \\ &= - \left(\frac{a^3b - a^3c + b^3c - b^3a + c^3a - c^3b}{(a-b)(b-c)(c-a)} \right) \\ &= - \left(\frac{ab(a-b)(a+b) + c(b^3 - a^3) + c^3(a-b)}{(a-b)(b-c)(c-a)} \right) \\ &= - \left(\frac{ab(a+b) - c(a^2 + b^2 + ab) + c^3}{(b-c)(c-a)} \right) \\ &= - \left(\frac{a^2b + ab^2 - a^2c - b^2c - abc + c^3}{(b-c)(c-a)} \right) \\ &= - \left(\frac{a^2(b-c) + ab(b-c) + c(c-b)(c+b)}{(b-c)(c-a)} \right) \\ &= - \left(\frac{a^2 + ab - c(c+b)}{(c-a)} \right) \\ &= - \left(\frac{a^2 + ab - c^2 - bc}{(c-a)} \right) \\ &= - \left(\frac{(a-c)(a+c) + b(a-c)}{(c-a)} \right) = a + b + c \end{aligned}$$

AM \geq GM

$$\frac{a+b+c}{3} \geq (abc)^{1/3}$$

$$a + b + c \geq 3$$

3. Let a,b,c,d,e,f be positive integers such that

$$\frac{a}{b} < \frac{c}{d} < \frac{e}{f}$$

Suppose $af - be = -1$. show that $d \geq b + f$.

Sol. $\frac{a}{b} < \frac{c}{d} < \frac{e}{f}$

$af - be = -1$

Now to show that $d \geq b + f$

$ad + \lambda_1 = bc$ (1) λ_1 and $\lambda_2 \in I^+$

$cf + \lambda_2 = de$ (2)

$af + 1 = be$ (3)

multiply the (1) equation by f

$bcf = afd + \lambda_1 f$

$b(de - \lambda_2) = d(be - 1) + \lambda_1 f$

$bde - \lambda_2 b = bde - d + \lambda_1 f$

$d = \lambda_2 b + \lambda_1 f$

$d \geq b + f$

4. There are 100 countries participating in an olympiad. Suppose n is a positive integer such that each of the 100 countries is willing to communicate in exactly n languages. If each set of 20 countries can communicate in at least one common language, and no language is common to all 100 countries, what is the minimum possible value of n ?

Sol. Let there be 20 languages everybody speaks.

$P_1 = \{L_1, L_2, \dots, L_{20}\}$

$P_2 = \{L_1, L_2, \dots, L_{20}\}$

⋮

⋮

$P_{80} = \{L_1, L_2, \dots, L_{20}\}$

$P_{81} = \{L_2, L_3, \dots, L_{20}, L_{21}\}$

$P_{82} = \{L_1, L_3, \dots, L_{20}, L_{21}\}$

⋮

$P_{100} = \{L_1, L_2, \dots, L_{19}, L_{21}\}$

Now a group of 20 selected from $P_1 - P_{80}$ will be able to communicate, while a group of 20 from $P_{81} - P_{100}$ will have common L_{21} . If some are chosen from $P_1 - P_{80}$ and some from $P_{81} - P_{100}$, then at maximum 19 persons will be chosen from $P_{81} - P_{100}$, \therefore at maximum 19 of L_1, \dots, L_{20} languages will be lost and one will still remain common with $P_1 - P_{80}$ in set L_1, L_2, \dots, L_{20} .

Now to understand why $N < 20$ is not possible.

Consider $N = 19$.

Assume $P_1 - P_{99}$ speaks L_1, L_2, \dots, L_{19}

So P_{100} speaks (L_{20}, \dots, L_{38})

Obviously in a group of 20 when P_{100} is selected they don't have common language.

P_1, \dots, P_{98} speaks (L_1, \dots, L_{19})

P_{99} & P_{100} have 9 and 10 languages

$P_{99} \equiv \{L_1, \dots, L_9, L_{20}, \dots, L_{29}\}$

$P_{100} \equiv \{L_{10}, \dots, L_{19}, L_{20}, \dots, L_{28}\}$

whenever P_{99} & P_{100} are chosen in group of 20 no common language will be there.

P_1, \dots, P_{97} speak (L_1, \dots, L_{19})



P_{98}, P_{99}, P_{100} will have these 19 languages : No language is common among them in these 19 languages.

$$P_{98} \equiv \{L_1, L_2, L_{20}, \dots, L_{36}\}$$

$P_{99} \equiv \{L_3, L_4, L_5, L_{20}, \dots, L_{35}\}$ In set of 20, when P_{98}, P_{99}, P_{100} are selected common language

$$P_{100} \equiv \{L_6, L_7, L_{20}, \dots, L_{36}\}$$

Likewise P_1, \dots, P_{81} speaks (L_1, \dots, L_{19})

$$P_{82} = \{L_1, L_{20}, \dots, L_{37}\}$$

$$P_{83} = \{L_2, L_{20}, \dots, L_{37}\}$$

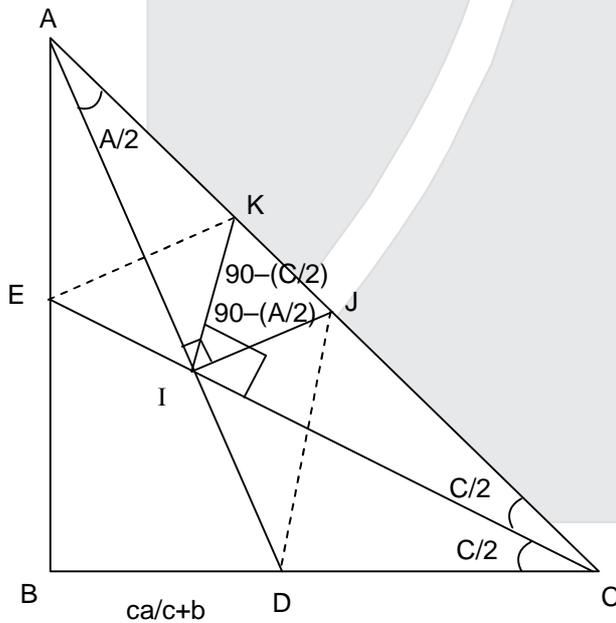
⋮

$P_{100} = \{L_{19}, L_{20}, \dots, L_{37}\}$ Now when these 19 persons are chosen i -group of 20, common language will exist.

Hence Answer is 20

5. Let ABC be a right-angled triangle with $\angle B = 90^\circ$. Let I be the incentre of ABC. Extend AI and CI, let them intersect BC in D and AB in E respectively. Draw a line perpendicular to AI at I to meet AC in J, draw a line perpendicular to CI at I to meet AC in K. Suppose $DJ = EK$. Prove that $BA = BC$.

Sol.



Now given $DJ = EK$

$$DJ^2 = EK^2$$

$$ID^2 + IJ^2 = EI^2 + IK^2$$

$$IE^2 + IC^2 \tan^2 \frac{C}{2} = (ID)^2 + (IA)^2 \tan^2 \frac{A}{2} \Rightarrow \tan \frac{A}{2} = \frac{IJ}{IA} \quad (\text{since in } \triangle AIJ)$$

$$\text{Now } IE = \frac{c}{a+b+c} EC \quad (\text{I divide CA in the ratio } a+b : c)$$

$$IC = \frac{a+b}{a+b+c} EC$$

$$\text{and } ID = \frac{a}{a+b+c} AD \quad \text{and } AI = \frac{b+c}{a+b+c} AD$$



$$\text{Now } c^2 (EC)^2 + (a+b)^2 EC^2 \tan^2 \frac{C}{2} = a^2 (AD)^2 + (b+c)^2 AD^2 \tan^2 \frac{A}{2}$$

$$\text{Now in } \triangle EBC \quad \tan \frac{C}{2} = \frac{EB}{a} = \frac{ac}{a+b} \cdot \frac{1}{a} \Rightarrow (a+b) \tan \frac{C}{2} = c$$

$$\text{In } \triangle ABD \quad \tan \frac{A}{2} = \frac{BD}{c} = \frac{ac}{(b+c)c} \Rightarrow (b+c) \tan \frac{A}{2} = a$$

$$\Rightarrow c^2 (EC)^2 + c^2 (EC)^2 = a^2 (AD)^2 + a^2 (AD)^2$$

$$cEC = aAD \Rightarrow \frac{ac}{\cos \frac{C}{2}} = \frac{ac}{\cos \frac{A}{2}} \quad \left(\cos \frac{C}{2} = \frac{a}{EC} \right)$$

$$\Rightarrow \cos \frac{C}{2} = \cos \frac{A}{2} \Rightarrow \frac{C}{2} = \frac{A}{2} \Rightarrow C = A$$

6. (a) Given any natural number N, prove that there exists a strictly increasing sequence of N positive integers in harmonic progression.

Sol. Consider the sequence

$$\frac{N!}{N}, \frac{N!}{N-1}, \frac{N!}{N-2}, \frac{N!}{N-3}, \dots, \frac{N!}{N-(N-1)}$$

$$\Rightarrow \frac{N!}{N}, \frac{N!}{N-1}, \frac{N!}{N-2}, \dots, \frac{N!}{1} \text{ are in H.P.}$$

Hence \forall natural numbers N, we get a strictly increasing H.P. of N positive integers.

- (b) Prove that there cannot exist a strictly increasing infinite sequence of positive integers which is in harmonic progression.

Sol. Consider a harmonic progression whose first term, $T_1 = p$

second term $T_2 = q$ where $q > p$, $p, q \in \mathbb{N}$ and all terms are positive integers.

$$\text{Now for this H.P. } T_r = \frac{1}{\frac{1}{p} + (r-1) \left(\frac{1}{q} - \frac{1}{p} \right)}$$

$$\Rightarrow T_r = \frac{pq}{2q - p + r(p - q)}$$

Now for $r > \frac{2q-p}{q-p}$, term of H.P. are negative,

Which is a contradiction

Hence the proof.

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1787

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