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RMO
2016

REGIONAL MATHEMATICAL OLYMPIAD
2016

TEST PAPER WITH SOLUTION & ANSWER KEY

REGION: DELHI CENTRE

Date: 09th October, 2016 | Duration: 3 Hours

Resonance's Forward Admission & Scholarship Test (ResoFAST)

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Test Dates
20th Nov 16 | 27th Nov 16 | 11th Dec 16 | 25th Dec 16 | 15th Jan 17

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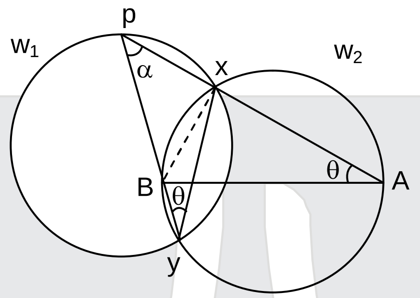
Important Note : Read the instructions provided on the answer seripts carefully before attempting the paper.

Problem 1. Given are two circles ω_1, ω_2 which intersect at points X, Y. Let P be an arbitrary point on ω_1. Suppose that the lines PX, PY meet ω_2 again at points A, B respectively, Prove that the circumcircles of all triangles PAB have the same radius

Sol.



$\Delta PXY \sim \Delta PBA$

$\Rightarrow \frac{AB}{XY} = \frac{PB}{PX} = \frac{PA}{PY}$

As $r_1 = \frac{XY}{2\sin\alpha}$ $\Rightarrow \sin\alpha$ is constant

Now $r_2 = \frac{AB}{2\sin\alpha}$

As length of chord AB is independent of position of P so AB is constant $\Rightarrow r_2$ is constant

Problem 2. Consider a sequence $(a_k)_k \geq 1$ of natural numbers defined as follows : $a_1 = a$ and $a_2 = b$ with a, b > 1 and gcd (a, b) = 1 and for all k > 0, $a_{k+2} = a_{k+1} + a_k$. Prove that for all natural numbers n and k, $\gcd(a_n, a_{n+k}) < \frac{a_k}{2}$.

Sol. Sequence $<a_n>$ is a, b, a + b, a + 2b, 2a + 3b, 3a + 5b, 5a + 8b, ...

Fibonacci sequence F_n is 1, 1, 2, 3, 5, 8, 13............

$\Rightarrow an = F_{n-2}\, a + F_{n-1}\, b$

As gcd(a, b) = 1 $\Rightarrow$ all terms of sequence $< a_n >$ are pairwise coprime

Now $a_{n+k} = a_{n+k-1} + a_{n+k-2}$ (coefficient are F_2, F_1)

$= 2(a_{n+k-2}) + a_{n+k-3}$ (coefficient are F_3, F_2)

$= 3(a_{n+k-3}) + 2a_{n+k-4}$ (coefficient are F_4, F_3)

$= 5a_{n+k-4} + 3a_{n+k-5}$ (coefficient are F_5, F_4)

After 'k' iterations, we get

$a_{n+k} = F_{k+1}a_n + F_k\, a_{n-1}$

Now $(a_n, a_{n+k}) = (a_n, F_{k+1}a_n + F_k a_{n-1})$

$= (a_n, F_k\, a_{n-1})$

$= (a_n, F_k)$ ($\because$ a_n and a_{n-1} are coprime)

$\le F_k$

As $a_k = F_{k-2}\, a + F_{k-1}\, b$

and $a \ge 2$, $b \ge 2$ (both connot be 2 as (a, b) = 1)

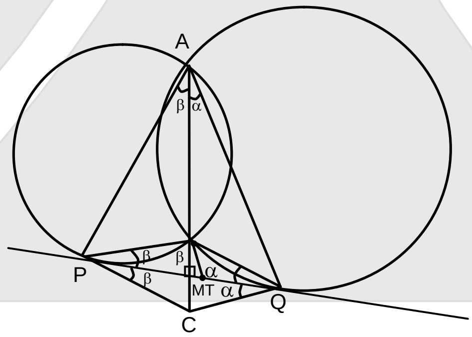
$\Rightarrow a_k > 2(F_{k-2} + F_{k-1})$

$\Rightarrow a_k > 2F_k$

$\Rightarrow (a_n, a_{n+k}) \le F_k < \dfrac{a_k}{2}$

Problem 3. Two circles C_1 and C_2 intersect each other at points A and B. Their external common tangent (closer to B) touches C_1 at P and C_2 at Q. Let C be the reflection of B in line PQ. Prove that $\angle CAP = \angle BAQ$

Sol.



Let $\angle PQC = \alpha$

$\Delta CMQ \cong \Delta BMQ$ $\therefore$ $\angle BQM = \alpha$

$TQ^2 = TB.\ TA$

$\Rightarrow \dfrac{TQ}{TB} = \dfrac{TA}{TQ}$ $\Rightarrow$ $\Delta BTQ \sim \Delta QTA$ $\Rightarrow$ $\angle QAT = \angle BQT = \alpha$

$\therefore$ $\angle A + \angle C = (\beta + \alpha) + (180° - (\beta + \alpha)) = 180°$

$\Rightarrow$ APCQ is cyclic quadrilateral $\therefore$ $\angle CAP = \angle CQP = \alpha = \angle BAQ$

Problem 4. Let a, b, c be positive real numbers such that a + b + c = 3. Determine, with certainty, the largest possible value of the expression

$$\frac{a}{a^3+b^2+c} + \frac{b}{b^3+c^2+a} + \frac{c}{c^3+a^2+b}$$

Sol. AM $\geq$ HM

$$\Rightarrow \frac{a(a^2)+b(b)+c(1)}{a+b+c} \geq \frac{a+b+c}{\frac{a}{a^2}+\frac{b}{b}+\frac{c}{1}}$$

$$\Rightarrow \frac{a^3+b^2+c}{9} \geq \frac{a}{1+a+ac}$$

$$\Rightarrow \frac{a}{a^3+b^2+c} \leq \frac{1+a+ac}{9}$$

Given expression $E \leq \frac{3+a+b+c+ab+bc+ca}{9}$

$$\Rightarrow E \leq \frac{6+ab+bc+ca}{9}$$

As $a^2 + b^2 + c^2 \geq ab + bc + ca$

$\Rightarrow (a + b + c)^2 \geq 3(ab+bc+ca)$ $\Rightarrow 3 \geq ab+bc+ca$ $\Rightarrow E \leq \frac{6+3}{9} \Rightarrow E \leq 1$

Problem 5. (a) A 7-tuple $(a_1,a_2,a_3,a_4,b_1,b_2,b_3)$ of pairwise distinct positive integers with no common factor is called a shy tuple If

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 = b_1^2 + b_2^2 + b_3^2$$

and for all $1 \leq i < j \leq 4$ and $1 \leq k \leq 3$, $a_i^2 + a_j^2 \neq b_k^2$. Prove that there exists infinitely many shy tuples.

(b) Show that 2016 can be written as a sum of squares of four distinct natural numbers.

Sol. (a) We can observe that

$(a + b + c)^2 + (– a + b + c)^2 + (a – b + c)^2 + (a + b – c)^2$

$= (2a)^2 + (2b)^2 + (2c)^2$

$$\Rightarrow a_1^2 + a_2^2 + a_3^2 + a_4^2 = b_1^2 + b_2^2 + b_3^2$$

If 2a, 2b, 2c have a common factor α then it will be a factor of a + b + c, – a + b + c, a – b + c and a + b – c so the given equality can be divided by α^2 such that the 7-uple has no common factor. There are infinite possibilities to choose 2a, 2b, 2c so there exist infinitely many shy tuples.

Sol. (b) $2016 = 16 \times 126 = 16\,(1^2 + 3^2 + 4^2 + 10^2)$

Problem 6. A deck of 52 cards is given. There are four suites each having cards numbered 1,2,.........13. The audience chooses some five cards with distinct numbers written on them. The assistant of the magician comes by, looks at the five cards and turns exactly one of them face down and arranges all five cards in some order. Then the magician enters and with an agreement made beforehand with the assistant, he has to determine the face down card (both suite and number). Explain how the trick can be completed.

Sol. At least two cards have same suit let P_1, P_2 has same suit then the assistant will turn down one of P_1, P_2 (say P_1) and arrange P_2, P_3, P_4, P_5 in such a way that P_2 takes first position so the magician know that the card turned down and the card having first position have same suit. This will help him to identify suit of P_1 now there are 6 ways to arrange P_3 P_4 P_5 as LMH, LHM, MLH, MHL, HLM, HML

(low number, middle number, high number)

If difference of numbers obtained on P_1 and P_2 is ≤ 6 then turn down the higher one and give first position to lower one. Let us assume P_1 has number k and P_2 has no k + 1, k + 2, k + 3, k + 4, k + 5 or k + 6. If P_2 has k + 1 then show LMH.

If P_2 has k + 2 then show LHM and soon. If difference is > 6 then turn down lower one and give first position to higher one and add 1,2,3,4,5,6 for arrangements LMH, LHM, MLH, MHL, HLM, HML in the higher number for example adding 4 to 11 indicates 2.

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Test Cities for ResoFAST - 2017

Test Dates: 27.11.2016, 11.12.2016

Study Center Cities (29): Rajasthan: Kota, Jaipur, Jodhpur, Udaipur, Ajmer, Sikar; **Bihar:** Patna; **Chattisgarh:** Raipur; **Delhi; Gujarat:** Ahmedabad, Surat, Rajkot, Vadodara; **Jharkhand:** Ranchi; **Madhya Pradesh:** Bhopal, Gwalior, Indore, Jabalpur; **Maharashtra:** Aurangabad, Mumbai, Nagpur, Nanded, Nashik, Chandrapur; **Odisha:** Bhubaneswar; **Uttar Pradesh:** Agra, Allahabad, Lucknow; **West Bengal:** Kolkata;

Test Dates: 20.11.2016, 25.12.2016, 15.01.2017

Study Center Cities (27): Rajasthan: Kota, Jaipur, Jodhpur, Udaipur; **Bihar:** Patna; **Chattisgarh:** Raipur; **Delhi; Gujarat:** Ahmedabad, Surat, Rajkot, Vadodara; **Jharkhand:** Ranchi; **Madhya Pradesh:** Bhopal, Gwalior, Indore, Jabalpur; **Maharashtra:** Aurangabad, Mumbai, Nagpur, Nanded, Nashik, Chandrapur; **Odisha:** Bhubaneswar; **Uttar Pradesh:** Agra, Allahabad, Lucknow; **West Bengal:** Kolkata;
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5111
CCP: 3554 | DLP/ e-LP: 1557

JEE (Main) 2016
28090
CCP: 20429 | DLP/ e-LP: 7661

AIIMS 2016
213
CCP: 32 | DLP/ e-LP: 181

NEET 2016
1787
CCP: 1155 | DLP/ e-LP: 632

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