THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

NMTC at Sub JUNIOR LEVEL - VII & VIII GRADES
Saturday, the 07 October 2023

ANSWER KEY

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	b	a	b	d	d	d	d	A	d	c	b	c	b	a	a
Ques.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	82	9	48	20	250	3	1	240	3	15	48º	161º	104	4	116

HINTS & SOLUTION

PART-A

1. **b)**

Sol. $\sqrt{2023\sqrt{2022\sqrt{(2021\times 2019)+1}+1}+1}$

$\sqrt{2023\sqrt{2022\sqrt{(2020+1)(2020-1)+1}+1}+1}$

$\sqrt{2023\sqrt{2022\sqrt{2020^2-1^2+1}+1}+1}$

$\sqrt{2023\sqrt{2022\times 2020+1}+1}$

$\sqrt{2023\sqrt{(2021+1)(2021-1)+1}+1}$

$\sqrt{2023\times\sqrt{(2021^2-1^2)+1}+1}$

$\sqrt{(2023\times 2021)+1}$

$\sqrt{(2022+1)(2022-1)+1}$

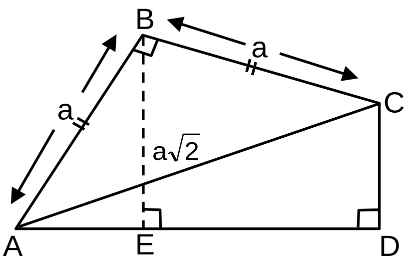
$\sqrt{2022^2-1^2+1}$

$\sqrt{2022^2}$

= 2022

2. **a)**

Sol.



Let AB = BC = x

AC = a $\sqrt{2}$ (because AB = BC)

only and only possible when AD = DC = a

$\therefore$ AB coincide with BE side

So, ABCD is a square.

$\therefore$ area of ABCD = $\text{Side}^2 = BE^2 = 1^2 = 1$

3. **b)**

Sol. $a + b = -c$

$(a + b)^2 = c^2$

$a^2 + b^2 - c^2 = -2ab$

$(a^2 + b^2 - c^2)^2 = 4a^2b^2$

$a^4 + b^4 + c^4 + 2a^2b^2 - 2b^2c^2 - 2a^2c^2 = 4a^2b^2$

$a^4 + b^4 + c^4 = 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - (a^4 + b^4 + c^4)$

$2(a^4 + b^4 + c^4) = (a^2 + b^2 + c^2)^2$

$\Rightarrow \dfrac{(a^2 + b^2 + c^2)^2}{a^4 + b^4 + c^4} = 2$

4. **d)**

Sol. Let the age of Rahim = x

$\therefore$ age of Ram = x + 1

age of Robin = (x + 1) –2 = x – 1

age of Ria = (x + 1) –3 = x – 2

Products of their ages = 5040 given

$x(x+1)(x-1)(x-2) = 5040 = \underline{3 \times 3} \times \underline{2 \times 5} \times \underline{2 \times 2 \times 2} \times \underline{7}$

$= 9 \times 10 \times 8 \times 7$

x = 9

x – 1 = 8

x + 1 = 10

x –2 = 7

$\therefore$ age of Ram = x + 1 = 9 + 1 = 10 years

5. **d)**

Let total number of fruit in his stock = N

Pineapples = 20%

Oranges = 60%

$\therefore$ Apples will be = 20%

given that number of apples are 40.

20% of N = Apples = 40

$N \times \frac{20}{100} = 40$

N = 200

$\therefore$ Pineapples = 40

Orange = 200 – 40 – 40 = 120

If half the oranges are replaced by pineapples the total pineapples = $40 + \frac{120}{2} = 40 + 60 = 100$

6. **d)**

Given that

Carpenter repair 4 tables in 5 hour.

$\therefore$ Carpenter repair 1 tables in $\frac{5}{4}$ hour.

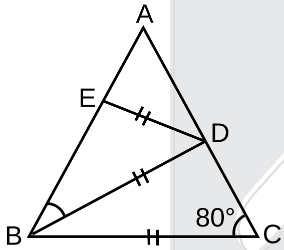
$\therefore$ time for him to repair 7 tables = $7 \times \frac{5}{4}$ hour

$= \frac{35}{4}$ hour

$= 8\frac{3}{4}$ hour

7. **d)**

Sol.



Given AB = AC

$\angle C = \angle B = 80^\circ$

BC = BD also given then $\angle BDC = \angle BCD = 80^\circ$

Angle BDC is an exterior angle for triangle ABD

$\angle BDC = \angle DAB + \angle ABD$

$80^\circ = 20^\circ + \angle ABD$

$\angle ABD = 60^\circ$

Given BD = DE

$\angle EBD = \angle DEB = 60^\circ$

triangle BDE is an equilateral triangle $\angle BDE = 60^\circ$

$\therefore\ \angle ADE = 180° - \angle BDE - \angle BDC$

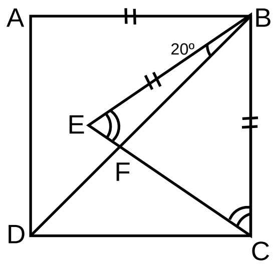
$= 180° - 60° - 80°$

$= 180° - 140°$

$\angle ADE = 40°$

8. **a)**

Sol.



Given AB = BE = BC

$\therefore$ $\angle BEC = \angle ECB = y$

$y + y + 65^o = 180^o$

$y = \frac{115^o}{2} = 57.5^o$

$\therefore$ In ΔEFB

$57.5^o + x + 20^o = 180^o$

$x = 180^o$

$57.5 + x + 20^o = 180^o$

$x = 180^o - 77.5^o$

$x = 102.5^o$

$2x = 205^o$

9. **d)**

Sol. Given a + b = 4(a–b)

$a + b = 4a - 4b$

$3a = 5b$

$a = \frac{5b}{3}$

Value of $\frac{2ab}{3(a^2 - b^2)} = \frac{2 \times \frac{5b}{3} \times b}{3\left[\left(\frac{5b}{3}\right)^2 - b^2\right]} = \frac{\frac{10}{3}b^2}{3\left[\frac{25b^2 - 9b^2}{9}\right]}$

$= \frac{10b^2 \times 9}{3 \times 3 \times 16b^2} = \frac{10}{16} = \frac{5}{8}$

10. **c)**

Given average of three number is x

$\frac{N_1 + N_2 + N_3}{3} = x$

$N_1 + N_2 + N_3 = 3x$

Given $N_1 = y$, $N_2 = z$

$y + z + N_3 = 3x$

$\therefore$ third is $N_3 = 3x - y - z$

11. **b)**

Sol. Let the natural number is = N

$$\frac{\frac{N-3}{4}+4}{5}=2$$

$$\frac{\frac{N-3+16}{4}}{5}=2$$

N + 13 = 40

N = 27 a perfect cube

12. **c)**

Sol. Number = $\dfrac{9n^2-64}{n-1-\dfrac{1}{1-\dfrac{n}{n+4}}}=\dfrac{(3n-8)(3n+8)}{(n-1)-\dfrac{(n+4)}{(n+4)-n}}$

$$=\frac{(3n-8)(3n+8)\times 4}{(4n-4)-n-4}$$

$$=\frac{(3n-8)(3n+8)\times 4}{3n-8}$$

Number = (3n + 8) × 4

For total permissible natural number n

(3n + 8) × 4 number always divisible by 4.

13. **b)**

Sol. $\sqrt{x+5}+\sqrt{3x+4}=\sqrt{12x+1}$ Squaring on both

(x + 5) + (3x + 4) + 2 $\sqrt{3x^2+4x+15x+20}$ = 12x + 1

4x + 9 + 2 $\sqrt{3x^2+19x+20}$ = 12x + 1

2 $\sqrt{3x^2+19x+20}$ = 8x – 8

$\sqrt{3x^2+19x+20}$ = 4x – 4

Squaring on both sides

$3x^2 + 19x + 20 = 16x^2 + 16 - 32x$

$13x^2 - 51x - 4 = 0$

$13x^2 - 52x + x - 4 = 0$

13x(x – 4) +1(x – 4) = 0

(x – 4) (13x +1) = 0

x = 4 and x = $-\dfrac{1}{13}$

x = $-\dfrac{1}{13}$ (not possible)

x = 4 is only one solution.

14. **a)**

Sol. $\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+......\left(\frac{1}{n-3}-\frac{1}{n-1}\right)+\left(\frac{1}{n-2}-\frac{1}{n}\right)+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$

$= \frac{3553}{4830}$

$\frac{1}{2}\left[1+\frac{1}{2}+\left(-\frac{1}{n+1}\right)+\left(-\frac{1}{n+2}\right)\right]=\frac{3553}{4830}$

$\frac{1}{2}\left[\frac{3}{2}-\frac{(n+2)+(n+1)}{(n+1)(n+2)}\right]=\frac{3553}{4830}$

$\frac{3}{2}-\frac{3553}{2415}=\frac{2n+3}{(n+1)(n+2)}$

$\frac{7245-7106}{4830}=\frac{2n+3}{(n+1)(n+2)}$

$\frac{139}{4830}=\frac{2n+3}{(n+1)(n+2)}$

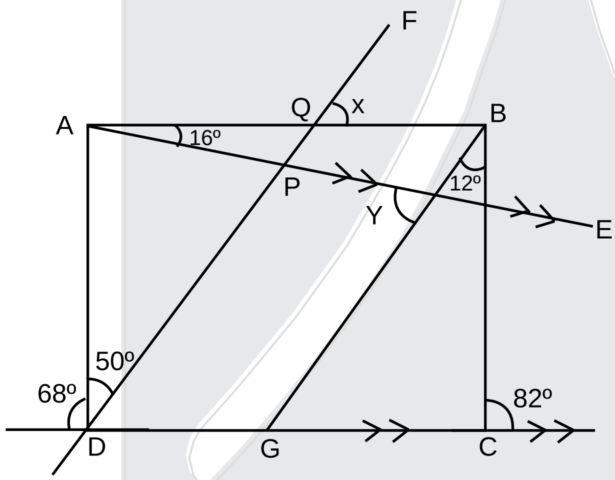
$\frac{2\times 68+3}{69\times 70}=\frac{2n+3}{(n+1)(n+2)}$

n + 1 = 69

$\therefore$ n = 68

15. **a)**

Sol.



In ΔBGC

$\angle$B + $\angle$G = 82º

12 + $\angle$G = 82º

$\angle$G = 82º – 12º = 70º

CD | | AE

$\angle$G = $\angle$y = 70º

CD is st. line

$\therefore$ 68 + 50 + $\angle$FDG = 180º

$\angle$FDG = 180º – 118º = 62º

CD | | AE

$\angle APD = 62^o = \angle PDG$ (Alternate interior angle)

$\angle APF = 180 - 62 = 118^o$

In ΔAPQ $16 + 118^o + \angle AQP = 180^o$

$134 + \angle AQP = 180$

$\angle AQP = 180 - 134$

$\angle AQP = 46^o$

$\angle AQP = 46 = x \therefore$ opp. angle

Now x + y

$46 + 70 = 116^o$

PART-B

16. **(82)**

Sol. Apple $\rightarrow x$ Banana $\rightarrow y$ Pineapple $\rightarrow z$

in 1st row $2x + 2y = 44$

$x + y = 22$ ....(1)

3rd row $z + x + y + y = 54$

$z + 22 + y = 54$

$y + z = 32$ ....(2)

1st column $x + x + z + y = 72$

$x + x + y + z = 72$

$x + 22 + z = 72$

$x + z = 72 - 22$

$x + z = 50$ ....(3)

equation (3) – equation (2)

$x - y = 18$

$\underline{x + y = 22}$

$2x = 40$ $x \rightarrow 20$ Apple

$x + y = 22$

$20 + y = 22$ $y \rightarrow 20$ banana

$y + z = 32$

$2 + z = 32$

$z = 30$ $z \rightarrow$ Pineapple $= 30$

2nd row $x + 2z + y \Rightarrow 20 + 60 + 2 = 82$

17. **(9)**

Sol. $2^{2^{x^2-1}} = 16$

$2^{2^{x^2-1}} = 2^4$

$2^{x^2-1} = 4$

$2^{x^2-1} = 2^2$

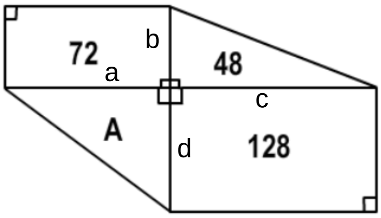
$x^2 - 1 = 2$

$x^2 = 3$

$x = +\sqrt{3}$

$x^4 = 9$ Ans.

18. **(48)**



$A = \frac{1}{2} \times a \times d$

$a \times b = 72$ ......(1) also $\frac{1}{2} \times b \times c = 48$ ... (2)

$c \times d = 128$ ... (3)

From (3) $\Rightarrow c = \frac{128}{d}$ ... (4)

from $\frac{1}{2} \times b \times c = 48$

$\frac{1}{2} \times b \times \frac{128}{d} = 48$

$\Rightarrow$ $b = \frac{48}{64} d = \frac{3}{4} d$ ... (5)

Now from (1)

$a \times b = 72$

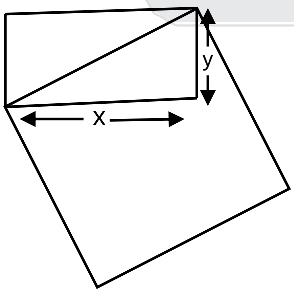
$a \times \frac{3}{4} d = 72$

$a = \frac{72 \times 4}{3} = \frac{96}{d}$

So from (1) $A = \frac{1}{2} \times \frac{96}{d} \times d = 48$

19. **(20)**

Sol. Let length and breadth of a rectangle are x and y respectively.



$\therefore$ Length of digonal of rectangle = $\sqrt{x^2 + y^2}$ = side of square area of square = $\left(\sqrt{x^2+y^2}\right)^2 = x^2 + y^2$

area of rectangle = xy.

given sum of length and breadth of rectangle = 6

$x + y = 6$

$y = 6 - x$

Given $\frac{\text{area of square}}{\text{area of rectangle}} = \frac{5}{2}$

$\frac{x^2+y^2}{xy} = \frac{5}{2}$

$2x^2 + 2y^2 = 5xy$

$2x^2 + 2(6 - x)^2 = 5x(6 - x)$

$2x^2 + 2(36 + x^2 - 12x) = 30x - 5x^2$

$9x^2 - 54x + 72 = 0$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

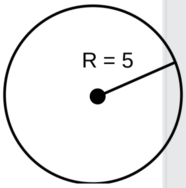
$x = 2, 4$ $\therefore$ Length $= x = 4$

$y = 4, 2$ Breadth $= y = 2$

area of square = $x^2 + y^2 = 4^2 + 2^2 = 16 + 4 = 20\text{cm}^2$

20. **(250)**

Sol.



area of circle = $\pi r^2 = 25\pi$

circumference of circle = $2\pi R = 10\pi$

given area of circle is x% of circumference

25π = x% of circumference = $10\pi \times \frac{x}{100}$

$25\pi = \frac{\pi x}{10}$

$25 \times 10 = x$

$x = 250$

21. **(3)**

Sol. Let there no are N_1, N_2 and N_3

given that $N_2 = N_1 + (N_3 - N_2)$

$2N_2 = N_1 + N_3$

$N_2 = \frac{N_1+N_3}{2}$ ....(1)

and product of two smaller no = 85

$N_1 N_2 = 85$

$N_1 = \frac{85}{N_2}$ ....(2)

Product of two bigger numbers = 115

$N_2 N_3 = 115$

$N_3 = \frac{115}{N_2}$ .... (3)

Put the value of N_1 and N_3 in equation (1)

$N_2 = \frac{\frac{85}{N_2} + \frac{115}{N_2}}{2}$

$2N_2 = \frac{200}{N_2}$

$N_2^2 = 100$

$N_2 = 10$

$\therefore$ $N_1 = \frac{85}{10} = 8.5$ $N_3 = \frac{115}{10} = 11.5$

$\therefore$ 3 Positive real no are = 8.5, 10, 11.5

$\therefore$ Difference of smallest and the greatest no = 11.5 – 8.5 = 3

22. **(1)**

Sol. Given $T_n = \frac{n(n+1)}{2}$ ....(1)

n is replace by 3n+1 in equation number (1)

$\therefore$ $T_{3n+1} = \frac{(3n+1)(3n+1+1)}{2}$ ....(2)

$\therefore$ $T_{3n+1} - 9T_n = \frac{(3n+1)\,(3n+2)}{2} - 9\frac{(n)(n+1)}{2}$

$= \frac{9n^2 + 6n + 3n + 2 - 9n^2 - 9n}{2}$

$= \frac{9n^2 - 9n^2 + 9n - 9n + 2}{2}$

$= \frac{2}{2}$

$T_{3n+1} - 9T_n = 1$

23. **(240)**

Sol. Let total no. of pages be x.

one day read = $\frac{3}{8} \times x$ ....(1)

remainder = $x - \frac{3x}{8} = \frac{5x}{8}$ ....(2)

2nd day page read = $\frac{5x}{8} \times \frac{4}{5} = \frac{x}{2}$

total page read in two days = $\frac{3x}{8} + \frac{x}{2} = \frac{7x}{8}$

Reming page = $x - \frac{7x}{8} = \frac{x}{8}$

$\frac{x}{8} = 30 \Rightarrow x = 8 \times 30$

x = 240 Pages

24. **(3)**

Sol.

A : B	A : C	B : C
	1000 : 625	
t : t + 60	Distance 8 : 5	(t + 60) : (t + 90)
	Time 5 : 8	

$\frac{t}{t+90} = \frac{5}{8}$

3t = 450

t = 150 Sec.

Time of B = 150 + 60 = 210 Sec.

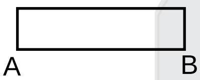
Time of B = 3 Min. 30 Sec.

Given time of B = x min. 30 Sec.

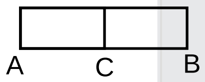
$\therefore$ x = 3

25. **(15)**

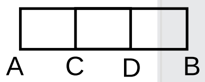
Sol. A ruler with no. marks length AB



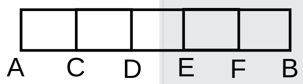
A ruler with one marks length AC, AB, CB (3)



A ruler with two marks AC, AD, AB, CD, CB, DB (6)

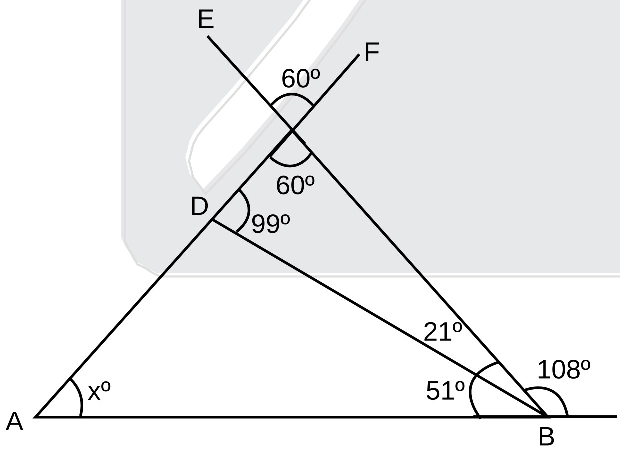


A ruler with marks AC, AD, AE, AF, AB CD, CE, CF, CB, DE, DF, DB, EF, EB, FB total (15)



26. **(48º)**

Sol.



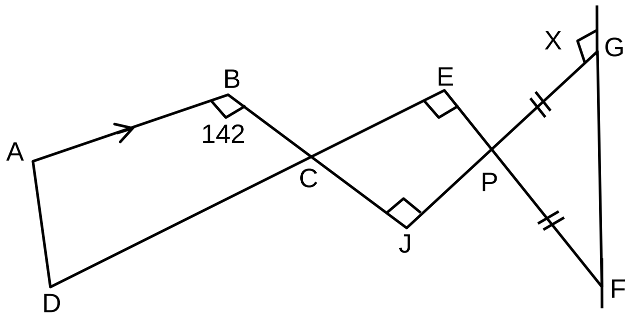
x + 51º = 99º

x = 99º – 51º

x = 48º

27. **(161º)**

Sol.



$\angle BCD + \angle ABC = 180^\circ$ (Co-interior angle)

$\therefore$ $\angle BCD = 180^\circ - 142^\circ = 38^\circ$

$\angle ECJ = \angle BCD = 38^\circ$ (Vertically opposite angle)

$\angle CJPE$ is a quadrilateral

$\therefore$ $\angle ECJ + \angle CJP + \angle JPE + \angle PEC = 360^\circ$

$\angle GPF = \angle EPJ = 360^\circ - 90^\circ - 90^\circ - 38^\circ$

$\angle GPF = 142^\circ$

given PG = PF

$\therefore$ $\angle PGF = \angle PFG = x$(let

$\therefore$ $x + y + 142^\circ = 180^\circ$ sum of all integer angle of ΔPFG

$x = 19^\circ$

$\therefore$ $x = 180^\circ - 19^\circ = 161^\circ$

$x = 161^\circ$

28. **(104)**

Sol. Let four consecutive odd integers are $2x + 1$, $x + 3$, $2x + 5$ and $2x + 7$

given, sum of first and last odd integer is 52.

$(2x + 1) + (2x + 7) = 52^\circ$

$4x + 8 = 52^\circ$

$4x = 44^\circ$

$x = \frac{44^\circ}{4} = 11^\circ$

$x = 11$

$\therefore$ $N_1 = 2x + 1 = 2 \times 11 + 1 = 23$

$N_2 = 25$

$N_3 = 27$

$N_4 = 9$

sum of all odd integer $= N_1 + N_2 + N_3 + N_4$

$= 23 + 25 + 27 + 29 = 104$

29. **(4)**

Sol. By given condistion -

$\sqrt{N} + 2 = N$

$N = (N - 2)^2$

$N = N^2 + 4 - 4N$

$N^2 - \sqrt{N} + 4 = 0$

$N^2 - 4N - N + 4 = 0$

$(N - 4)(N - 1)$

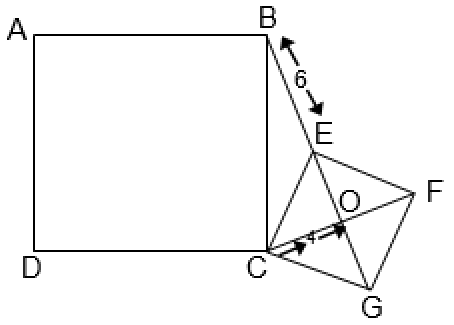
$N = 1, 4$

$N = 4$

$\sqrt{N} + 2 = 4$ only 4 satisfied

$2 + 2 = 4$

30. **(116)**



Sol.

diagnoal of small square (CF) = side × $\sqrt{2}$

$= 4\sqrt{2} \times \sqrt{2}$

$= 8$ cm

ΔBCO is a right angle triangle

BC2 = $CO^2 + BO^2$

$= 4^2 + 10^2$

$= 16 + 100$

$BC^2 = 116$

area of square ABCD = $side^2 = BC^2 = 116$