ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
Screening Test – Bhaskara Contest

NMTC AT JUNIOR LEVEL - IX & X GRADES

Saturday, the 07 October 2023

ANSWER KEY

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	d	b	d	b	*	c	b	Bonus	c	b	c	d	c	b	a
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	Bonus	∞	0	19	4	8	114°	2	32	12	84	36	1	5	7

HINTS & SOLUTION

SECTION-A

1. $x^3 + 6x^2 + ax + b = (x + c)^3$

$x^3 + 6x^2 + ax + b = x^3 + 3cx^2 + 3cx + c^3$

on compairing

$3c = 6$ $\quad$ $3c^2 = a$ $\quad$ $b = c^3$

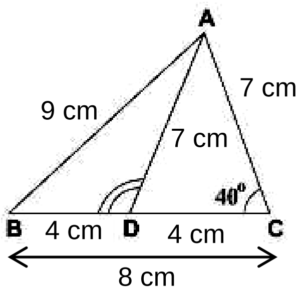
$c = 2$ $\quad$ $a = 12$ $\quad$ $b = 8$

(c)

$a + b + c= 8 + 12 + 2 = 22$

a + b + c is divisible by 11.

2.



AD $\rightarrow$ a median

$BD = DC = \frac{8}{2} = 4$ cm

Using apollonius theorem:

$AB^2 + AC^2 = 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]$

$81 + 49 = 2[AD^2 + 16]$

$130 = 2(AD^2 + 16)$

$AD^2 + 16 = \frac{130}{2} = 65$

$AD^2 = 65 – 16$

$AD^2 = 49 \Rightarrow AD = 7$ cm

So, In ΔADC

AC = AD

So, $\angle ADC = 40°$

then $\angle ADB = 180 – 40 = 140°$

3. Given: $x^2 + 6x + 1 = 0$

$\frac{x^4 + kx^2 + 1}{3x^3 + kx^2 + 3x} = 2$

$x^4 + kx- + 1 = 6x^3 + 2kx^2 + 6x$

$x^4 - 6x^3 + (k - 2k)x^2 - 6x + 1 = 0$

$x^4 - 6x^3 - kx^2 - 6x + 1 = 0$

$-6x^3 - 6x^3 - x^2 - kx^2 - 6x + 1 = 0$

$-12x^3 - (k + 1)x^2 - 6x - 1 = 0$

$12x^3 + (k + 1)x^2 + 6x - 1 = 0$

$-72x^2 - 72x + (k + 1)x^2 + 6x - 1 = 0$

$x^2(k + 1 - 72) - 6x - 1 = 0$

$(k - 71)x^2 - 6x - 1 = 0$

$(k - 71)x^2 = 6x + 1$

$(k - 71)x^2 = - x^2$

$k - 71 = -1$

$k = 70$

using $x^2 + 6x + 1 = 0$

$x^4 + 6x^3 + x^2 = 0$

$x^4 = - 6x^3 - x^2$

$x2 + 6x + 1 = 0$

$12x^3 = - 72x^2 - 72x$

4. $x = \sqrt[3]{49} + \sqrt[3]{42} + \sqrt[3]{36}$

$x - \sqrt[3]{42} = \sqrt[3]{49} + \sqrt[3]{36}$

$(x - \sqrt[3]{42})^3 = \left(\sqrt[3]{49} + \sqrt[3]{36}\right)^3$

$x^3 - 42 - 3x\sqrt[3]{42}\left(x - \sqrt[3]{42}\right) = 49 + 36 + 3\sqrt[3]{49 \times 36}\left(\sqrt[3]{49} + \sqrt[3]{36}\right)$

$x^3 - 3x^2\sqrt[3]{42} + 3x\left(\sqrt[3]{42 \times 42}\right) = 127 + 3\left(\sqrt[3]{49 \times 36}\right)\left(x - \sqrt{42}\right)$

$= 127 + 3x\sqrt[3]{49 \times 36} - 3\sqrt[3]{49 \times 36 \times 42}$

$x^3 - 3x^2\sqrt[3]{42} + 3x\sqrt[3]{42^2} = 127 + 3x\sqrt[3]{49 \times 36} - 126$

$x^3 - 3x^2\sqrt[3]{42} + 3x\sqrt[3]{1764} = 127 + 3x\sqrt[3]{1764} - 126$

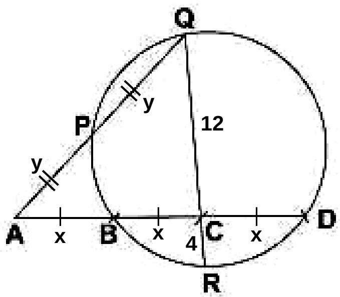
$x^3 - 3x^2\sqrt[3]{42} = 1$

$x^3 - 1 = 3x^2\sqrt[3]{42}$

$\frac{x^3 - 1}{x^2} = 3\sqrt[3]{42}$

$x - \frac{1}{x^2} = 3\sqrt[3]{42}$

5.



Using property

$BC \times CD = CR \times QC$

$x \times x = 4 \times 12$

$x^2 = 48$

$x = \sqrt{48}$

Again:

$AB \times AD = AP \times AQ$

$x \times 3x = y \times 2y$

$3x^2 = 2y^2$

$3 \times 48 = 2y^2$ $\quad$ $PQ = y = \sqrt{72} = 6\sqrt{2}$

$3 \times 24 = y2$

$y^2 = 72 \Rightarrow y = \sqrt{72}$

6. A is the midpoint of BAC

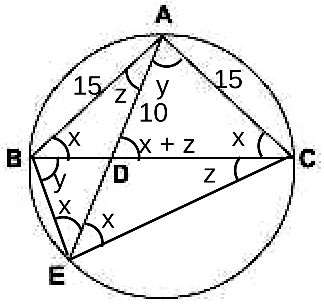
arc AB = arc AC

So AB = AC

So $\angle BEA = \angle BCA = x$

By chord CE

$\angle EBC = \angle EAC$



In ΔABC

AB = AC = 15 cm

$\angle ABC = \angle ACB = x$

In ΔADC and ΔAEC

$\angle ACD = \angle ACE$

$\angle ADC = \angle ACE$

By AA similarity

$\Delta CDA \sim \Delta ECA$

$$\frac{AC}{AD} = \frac{AE}{AC}$$

$$\frac{15}{10} = \frac{AE}{15} \Rightarrow \frac{225}{10} = AE$$

AE = 22.5 m

7.
$$\frac{8^x\left(1+\left(\frac{27}{8}\right)^x\right)}{12^x\left(1+\left(\frac{10}{12}\right)^x\right)} = \frac{7}{6}$$

$$\left(\frac{2}{3}\right)^x \left(\frac{1+\left(\frac{3}{2}\right)^{3x}}{1+\left(\frac{3}{2}\right)^x}\right) = \frac{7}{6}$$

Let $\left(\frac{8}{2}\right)^x = t$

$$\frac{1}{t}\left(\frac{1+t^3}{1+t}\right) = \frac{7}{6}$$

$$\frac{1}{t}\ \frac{(1+t)(1+t^2-t)}{(1+t)} = \frac{7}{6}$$

$$\frac{1+t^2-t}{t} = \frac{7}{6}$$

$6t^2 - 13t + 6 = 0$

$t = \frac{3}{2}$, $t = \frac{2}{3}$

$\left(\frac{3}{2}\right)^x = \frac{3}{2}$	$\left(\frac{3}{2}\right)^x = \frac{2}{3}$
x = 1	x = –1

9. $x - y^2 = 4$ | $x^2 + y^4 = 26$

$(x - y^2)^2 = 16$

$x^2 + y^4 - 2xy^2 = 16$

$26 - 2xy^2 = 16$

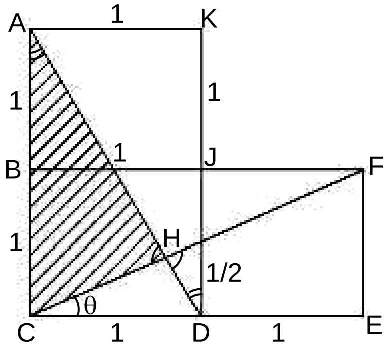
$xy^2 = 5$

$x = \frac{5}{y^2}$

y = 1   y = –1

x = 5   x = 5

10.



ar(ACDK) = 2 cm^2

ar(BCEF) = 2 cm^2

In ΔCEF

D is the mid point of CE

By converse mid point then

$DG = \frac{1}{2}EF$

$DG = \frac{1}{2}$, $DG = \frac{1}{2}$

$\therefore BI = \frac{1}{2}$, $IJ = \frac{1}{2}$

In ΔAKD

by Pythagoras theorem

(1)^2 + (2)2 = AD2

$\sqrt{5}$ = AD

$\Rightarrow AH = \frac{4\sqrt{5}}{5}$, $DH = \frac{\sqrt{5}}{5}$

Similarly, ΔCDG

$\Rightarrow (CG)^2 = (1)^2 + \left(\frac{1}{2}\right)^2$

$= 1 + \frac{1}{4}$

Also, ΔACH & ΔDGH

$\Rightarrow \frac{AC}{DG} = \frac{AH}{DH} = \frac{CH}{GH}$

$\Rightarrow \frac{2}{\left(\frac{1}{2}\right)} = \frac{AH}{DH} = \frac{CH}{GH}$

$\frac{4}{1} = \frac{AH}{DH}$

$CG = \frac{5}{4}$

$CG = \frac{\sqrt{5}}{2}$

$CH = \frac{4\sqrt{5}}{2\times5}$, $GH = \frac{\sqrt{5}}{2\times5}$

In ΔACH

$AC = 2$, $CH = \frac{2}{\sqrt{5}}$, $AH = \frac{4}{\sqrt{5}}$

Heron's, formula.

$S = \frac{\left(2+\frac{2}{\sqrt{5}}+\frac{4}{\sqrt{5}}\right)}{2}$

$S = \frac{2\sqrt{5}+6}{2\sqrt{5}}$

$\text{Area} = \sqrt{\left(\frac{2\sqrt{5}+6}{2\sqrt{5}}\right)\left(\frac{2\sqrt{5}+6}{2\sqrt{5}}-2\right)\left(\frac{2\sqrt{5}+6}{2\sqrt{5}}-\frac{2}{\sqrt{5}}\right)\left(\frac{2\sqrt{5}+6}{2\sqrt{5}}-\frac{4}{\sqrt{5}}\right)}$

$= \sqrt{\left(\frac{\sqrt{5}+3}{\sqrt{5}}\right)\left(\frac{\sqrt{5}+3-2\sqrt{5}}{\sqrt{5}}\right)\left(\frac{\sqrt{5}+3-2}{\sqrt{5}}\right)\left(\frac{\sqrt{5}+3-4}{\sqrt{5}}\right)}$

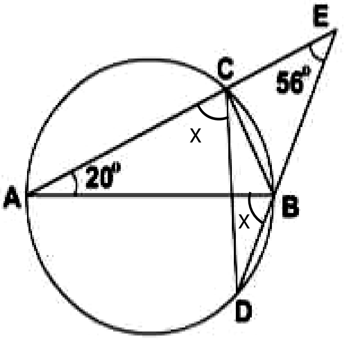
$= \sqrt{\left(\frac{\sqrt{5}+3}{\sqrt{5}}\right)\left(\frac{3-\sqrt{5}}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{\sqrt{5}}\right)\left(\frac{\sqrt{5}-1}{\sqrt{5}}\right)}$

$= \frac{1}{5}\sqrt{\left((3)^2-(\sqrt{5})^2\right)\left((\sqrt{5})^2-(1)^2\right)}$

$= \frac{1}{5} \times 4$

$= \frac{4}{5}$ cm^2

11. AB + Diameter



$\angle ACB = 90^o$

By chord AD

Let $\angle ACD = \angle ABD = x$

So $\angle DCB = 90 - x$

In ΔABE

$x = 20 + 56$

$x = 76$

So, $\angle DCB = 90 - x = 90 - 76 = 14$

12. (d) 1250

13. $x^2 + 4y = 3x + 16$

$x^2 = 3x,$	$4y = 16$	$x^2 = 16$	$4y = 3x$
$x^2 - 3x = 0$	$y = 4$	$x = 4$	$y = 3$
$x(x - 3) = 0$			
$x = 0$			
$x = 3$			

Ordered pair three (0, 4), (3, 4), (4, 3)

14. $(a + b + ab + 2)^2 + (a - ab + 2 - b)^2 - 2b^2(1 + a^2)$

$[(a + 2) + b(a + 1)]^2 + [(a + 2) - b(a + 1)]^2 - 2b^2(1 + a^2)$

$(a + 2)^2 + b^2(a + 1)^2 + 2(a + 2)(a + 1)b + (a + 2)^2 + b^2(a + 1)^2 - 2b(a + 1)(a + 2) - 2b^2(a^2 + 1)$

$2(a + 2)^2 + 2b^2(a + 1)^2 - 2a^2b^2 - 2b^2$

$2(a + b)^2 + 2b^2(a + 1)^2 - 2a^2b^2 - 2b^2$

$2(a + b)^2 + 2a^2b^2 + 2b^2 + 4ab^2 - 2a^2b^2 - 2b^2$

$2((a + 2)^2 + 2ab^2]$

$2(a + 2)^2 + 4ab^2$

15. $(1 \times 4) + (2 \times 7) + (3 \times 10) + (9 \times 13) + \ldots\ldots$ 49 term

$(1 \times (1 \times 3 + 1)] (2 \times (2 \times 3 + 1)] + \ldots\ldots. 49$

$$\sum_{n=1}^{49} n(3n+1)$$

$$\sum_{n=1}^{49} 3n^2 + n$$

$$3\sum_{n=1}^{49} n^2 + \sum_{n=1}^{49} n$$

$$3\left(\frac{n(n+1)(2n+1)}{6}\right) + \left(\frac{n(n+1)}{2}\right)$$

$$3\left(\frac{49 \times 50 \times 99}{6}\right) + \frac{49 \times 50}{2}$$

$49 \times 25 \times 99 + 49 \times 25$

$121275 + 1225$

122500

Section B

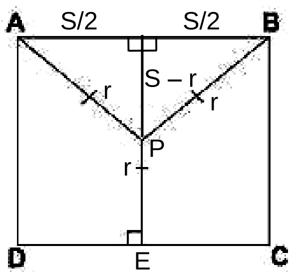
17. Infinite

$$\frac{a^2 + b^2 + c^2 + d^2}{4} \geq \sqrt[4]{a^2b^2c^2d^2}$$

$a^2 + b^2 + c^2 + d^2 = 4$

18. Product of two consecutive number cannot be a perfect sequare.

19.



Let PA = PB = PE = x

AB = BC = CD = DA = S

$\Delta ASP \cong \Delta BSP$ (By RHS)

So, $AS = BS = \frac{AB}{2} = \frac{S}{2}$

SP = SE – PE

= S – x

In ΔASP

$AP^2 = SP^2 + AS^2$

$x^2 = (S-x)^2 + \left(\frac{S}{2}\right)^2$

$x^2 = S^2 + x^2 - 2Sx + \frac{S^2}{4}$

$2Sx = \frac{5}{4}S^2$

$x = \frac{5}{4}S \cdot \frac{1}{2}$

$x = \frac{5}{8}S$

$\frac{ar(\Delta PAB)}{ar(ABCD)} = \frac{\frac{1}{2}\times S\times(S-x)}{S^2}$

$= \frac{(S-x)}{2S} = \frac{1-\frac{x}{S}}{2}$

$= \frac{\left(1-\frac{5}{8}\right)}{2} = \frac{3}{16}$

$\frac{ar(\Delta PAB)}{ar(ABCD)} = \frac{m}{n} = \frac{3}{16}$

m + n = 3 + 16 = 19

20. $4^{x/y} = 32 \cdot 8^{y/x}$

$2^{\frac{2x}{y}} = 2^{5+\frac{3x}{x}}$

$\frac{2x}{y} = 5 + \frac{3y}{x}$

$\frac{2x}{y} = \frac{5x+3y}{x}$

$2x^2 = 5xy + 3y^2$

$3^{x/y} = 3 \cdot 9^{\frac{1-y}{y}}$

$3^{x/y} = 3^{1+2\left(\frac{1-y}{y}\right)}$

$\frac{x}{y} = 1 + \frac{2-2y}{y}$

x = y + 2 – 2y

$\Rightarrow x = 2 - y \Rightarrow x + y = 2$

$2x^2 = 5x(2-x) + 3(2-x)^2$

$2x^2 = 10x - 5x^2 + 3(4 + x^2 - 4x)$

$2x^2 = 10x - 5x^2 + 12 + 3x^2 - 12x$

$0 = -4x^2 - 2x + 12$

$4x^2 + 2x - 12 = 0$

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x + 2) - 3(x + 2) = 0$

$(x + 2)(2x - 3) = 0$

$x = -2$	$x = 3/2$
$y = 2 - x$	$y = 2 - x = 2 - 3/2$
$y = 4$	$y = 1/2$

Sum of roots of the simultaneous = $-2 + 4 + \frac{3}{2} + \frac{1}{2}$

$= 2 + 2 = 4$

21. $2\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}$

$2\sqrt{3+\sqrt{5-\sqrt{12+1+4\times\sqrt{3}}}}$

$2\sqrt{3+\sqrt{5-\sqrt{\left(2\sqrt{3}\right)^2+(1)^2+2\times 2\sqrt{3}}}}$

$2\sqrt{3+\sqrt{5-\sqrt{\left(2\sqrt{3}+1\right)^2}}}$

$2\sqrt{3+\sqrt{5-2\sqrt{3}-1}}$

$2\sqrt{3+\sqrt{4-2\sqrt{3}}}$

$2\sqrt{3+\sqrt{\left(\sqrt{3}\right)^2+1-\sqrt{3}}}$

$2\sqrt{3+\sqrt{\left(\sqrt{3}-1\right)^2}}$

$2\sqrt{3+\sqrt{\sqrt{3}-1}}$

$2\sqrt{3+\sqrt{3}-1}$

$2\sqrt{2+\sqrt{3}}$

$\sqrt{8+4\sqrt{3}}$

$\sqrt{\frac{16+8\sqrt{3}}{2}}$

$\sqrt{\frac{(2)^2+\left(2\sqrt{3}\right)^2+2\times 2\times 2\sqrt{3}}{2}}$

$\sqrt{\frac{\left(2+2\sqrt{3}\right)^2}{2}}$

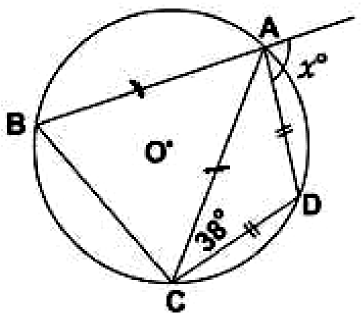
$\frac{2+2\sqrt{3}}{\sqrt{2}}$

$\sqrt{2}+2\sqrt{\frac{3}{2}} = \sqrt{2}+\sqrt{\frac{3}{2}+4} = \sqrt{2}+\sqrt{6} = \sqrt{a}+\sqrt{b}$

So, a = 2, b = 6

a + b = 2 + 6 = 8

22.



In ΔACD
AD = AC
So, $\angle$CAD = 38°
In ΔACD
$\angle$ADC = 180 – (38 + 38)
= 180 – 76
= 104
So, $\angle$ABC + $\angle$ADC = 180
$\angle$ABC + 104 = 180
$\angle$ABC = 180 – 104
$\angle$ABC = 76°
In ΔABC
AB = AC
$\angle$ACB = $\angle$ABC = 76°
So, In cyclic quadriteral ABCD
x = 76 + 38
x = 114°

23. $\frac{a}{b+c} + \frac{c}{a+b} = \frac{2b}{c+a}$

$\frac{a}{b+c} + 1 + \frac{c}{a+b} + 1 = \frac{2b}{c+a} + 2$

$\frac{a+b+c}{b+c} + \frac{a+b+c}{a+b} = \frac{2(a+b+c)}{c+a}$

$\frac{1}{b+c} + \frac{1}{a+b} = \frac{2}{a+c}$

So, $\frac{1}{b+c}, \frac{1}{a+c}, \frac{1}{a+b}$ in A.P.

Let $\frac{1}{b+c} = 1$, $\frac{1}{a+c} = \frac{3}{2}$ $\frac{1}{a+b} = 2$

b + c = 1 a + c = $\frac{2}{3}$ a + b = $\frac{1}{2}$

$2(a + b + c) = 1 + \frac{2}{3} + \frac{1}{2} = \frac{6+4+3}{6} = \frac{13}{6}$

$a + b + c = \frac{13}{12}$	$c = \frac{2}{3} - a$	$b = \frac{1}{2} - a$
$a = \frac{1}{12}$	$c = \frac{2}{3} - \frac{1}{12}$	$b = \frac{1}{2} - \frac{1}{12}$
	$c = \frac{8-1}{12}$	$b = \frac{6-1}{12}$
	$c = \frac{7}{12}$	$b = \frac{5}{12}$

$$\frac{a^2 + c^2}{b^2} = \frac{\frac{1}{144} + \frac{49}{144}}{\frac{25}{144}} = \frac{50}{25} = 2$$

24. Given:

$\sqrt{ab} = 8$

ab = 64 $\quad\quad \frac{a+b}{2} = 17 \Rightarrow a + b = 34$

$(a - b)^2 = (a + b)^2 - 4ab$
$= (34)^2 - 4(64)$
$= 1156 - 256$
$(a - b)^2 = 900$

a – b = 30,	a – b = –30
a + b = 34	a + b = 34
2a = 64	2a = 4
a = 32	a = 2
b = 2	b = 32 (Answer number → 32)

25. Let two digit number ab
a → 1, 3, 5, 7
b → 1, 3, 5, 7

if a = 1 then b = 3, 5, 7	total no. = 3
a = 3 then b = 1, 5, 7	total no. = 3
a = 5 then b = 1, 3, 7	total no. = 3
a = 7 then b = 1, 3, 5	total no. = 3

total number = 3 + 3 + 3 + 3 = 12

26. $\left(5\sqrt[3]{4} - 3\sqrt[3]{\tfrac{1}{2}}\right)\left(12\sqrt[3]{2} + \sqrt[3]{16} - 2\sqrt[3]{2}\right)$

Let $\sqrt[3]{2} = x \Rightarrow x^3 = 2$

$\left(5x^2 - \frac{3}{x}\right)(10x + x^4)$

$50x^3 + 5x^6 - 30 - 3x^3$
$50(2) + 5(4) - 30 - 3 \times 2$
$100 + 20 - 30 - 6$
$120 - 36 = 84$

27. $\frac{xy}{x+y} = 1, \ \frac{yz}{y+z} = 2, \ \frac{zx}{z+x} = 3$

$\frac{x+y}{xy} = 1, \ \frac{y+z}{yz} = \frac{1}{2}, \ \frac{z+x}{zx} = \frac{1}{3}$

$\frac{1}{x} + \frac{1}{y} = 1, \ \frac{1}{y} + \frac{1}{z} = \frac{1}{2}, \ \frac{1}{x} + \frac{1}{z} = \frac{1}{3}$

$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 1 + \frac{1}{2} + \frac{1}{3}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{12}$ ....(i)

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{12}$	$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{12}$
$1 + \frac{1}{z} = \frac{11}{12}$	$\frac{1}{x} + \frac{1}{z} = \frac{11}{12}$
$\frac{1}{z} = \frac{11}{12} - 1$	$\frac{1}{x} = \frac{11}{12} - \frac{1}{2}$
$\frac{1}{z} = \frac{-1}{12}$	$\frac{1}{x} = \frac{11-6}{12}$
z = –12	$\frac{1}{x} = \frac{5}{12}$ (similarly $y = \frac{12}{7}$)
	$x = \frac{15}{5}$

$15x - 7y - z$

$15 \times \frac{12}{5} - 7 \times \frac{12}{7} - (-12)$

$36 - 12 + 12$

36

28. $x + 3 < 4 + 2x$ $\quad$ $5x - 3 < 4x - 1$

$3 - 4 < 2x - x$ $\quad$ $x < 2$

$-1 < x$

$-1 < x < 2$

Only one natural number between –1 and 2 is 1.

So sum = 1

29. Given:

$ar^3 - a = 52$

$a(r^3 - 1) = 52$

$a(r - 1)(r^2 + r + 1) = 52$ ......(i)

$T_1 + T_2 + T_3 = 26$

$a + ar + ar^2 = 26$

$a(1 + r + r^2) = 26$ ....(ii)

equation (i) ÷ (ii)

$\frac{a(r-1)(1+r+r^2)}{a(1+r+r^2)} = \frac{52}{26} = 2$

$r - 1 = 2$

$r = 3$

So, $a(13) = 26$

$a = 2$

Value of $\frac{t_{2024}}{t_{2023}} + \frac{a^{2024}}{a^{2023}}$

$\frac{ar^{2023}}{ar^{2022}} + a$

$r + a$

$3 + 2 = 5$

30. Let base of $\Delta = x - 4$

Altitude = x

Area = 96

$\frac{1}{2} x \times (x - 4) = 96$

$x_2 - 4x = 192$

$x^2 - 4x - 192 = 0$

$(x - 16)(x + 12) = 0$

$x = 16$	$x = -12$ (rejected)
$x - 4 = 12$	

$\frac{\text{Base}}{\text{Height}} = \frac{12}{16} = \frac{3}{4} = \frac{p}{q}$

$p + q = 3 + 4 = 7$