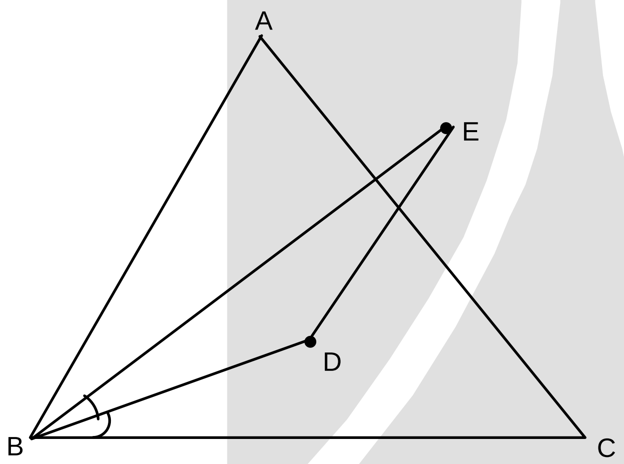
ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA (AMTI)
SUB JUNIOR LEVEL- 2020-2021
Classes VII & VIII
Saturday, 24th April 2021

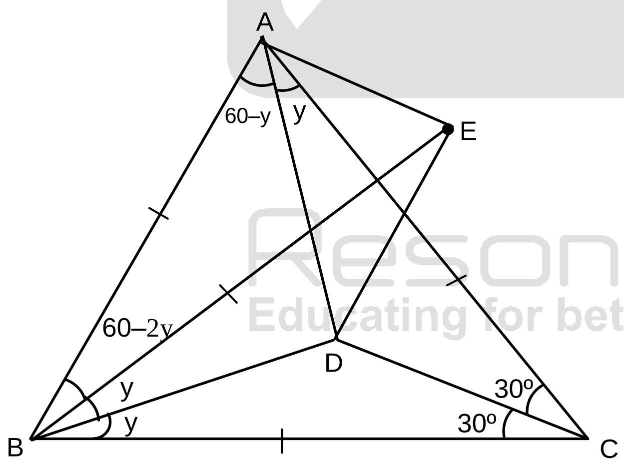
Instructions:
1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

---

1. ΔABC is an equilateral triangle, D is a point inside ΔABC such that AD = BD. choose E such that BE = AB and BD is a bisector of $\angle CBE$. Find $\angle BED$. Diagram given below.



Sol.



In ΔCAD & ΔEBD

CA = EB (Sides of equilateral triangle)

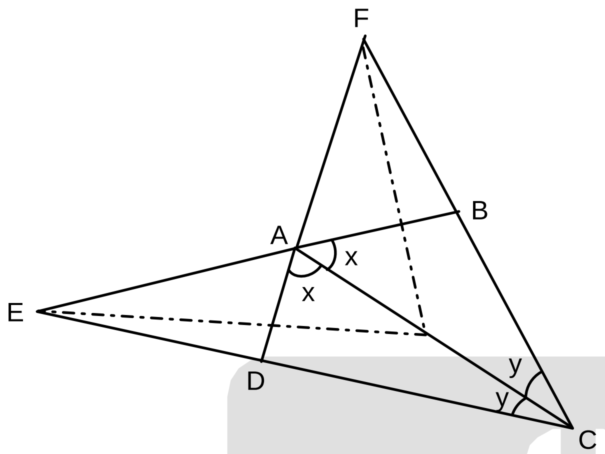
AD = BD

$\angle CAD = \angle EBD = y$

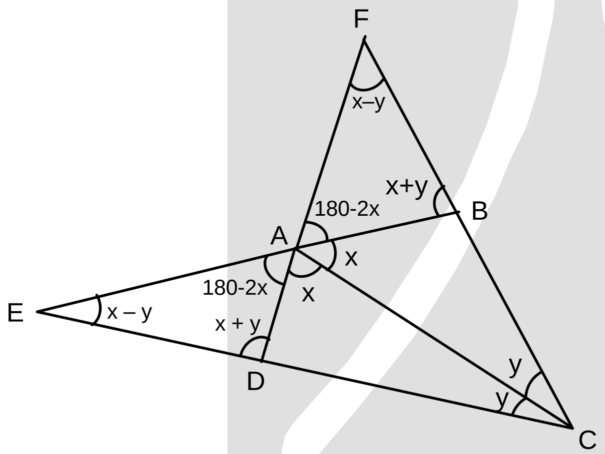
So $\Delta CAD \cong \Delta EBD$

$\angle BED = \angle ACD = 30^o$

2. Given a quadrilateral ABCD, the diagram AC bisects both ∠A and ∠C. If AB and DC extended intersect at E, and AD and BC extended intersect at F, show that for any point P on line AC, PE = PE.



Sol.



∠AED = 180º – 180º + 2x – x – y

= x – y

∠AFC = x – y

in ΔEAC & ΔFAC

AC = AC common

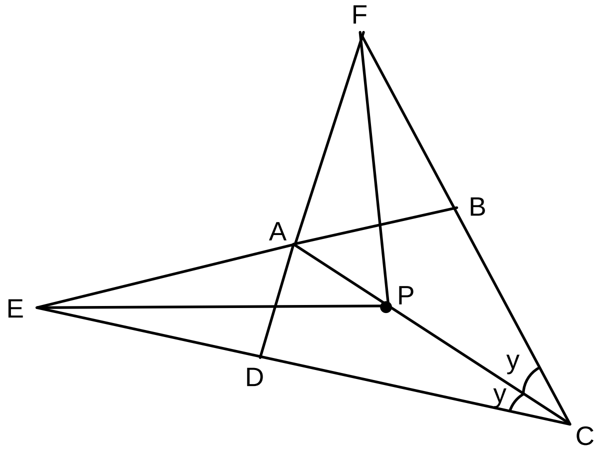
∠EAC = ∠FAC = 180 – x

∠ACE = ∠ACF = x

by ASA rule.

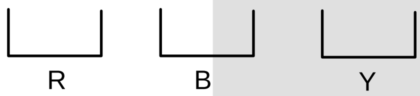
ΔEAC ≅ ΔFAC

EC = FC



in ΔEPC & ΔFPC
PC = PC
EC = FC
$\angle ECP = \angle FCP$
$\Delta EPC \cong \Delta PC$
So EP = FP

3. Twenty seven balls labelled from 1 to 27 are distributed in three bowls red , blue, and yellow. What are the possible values of the numbers of balls in the red bowl, if the average of the labels in red, blue, and yellow bowl are 15, 3 and 18 respectively.



Sol.

Let no. of balls R, B, Y in red, blue and yellow bowl respectively.
So that sum of numbers in red bowl = 15 R.
________________ Blue bowl = 3B
________________ yellow bowl = 18 Y
& 15 R + 3B + 18Y = 1 + 2 + ... +27
$= \frac{27 \times 28}{2} = 378$ ....(1)
& R + B + Y = 27 ....(2)
15R + 3B + 18 (27 – R –B) = 378
– 3R – 15B + 486 = 378
3R + 15B = 108
R + 5B = 36
5B = 36 – R
$B = \frac{36 - R}{5}$

R	**1**	**6**	**11**	**16**	**21**	**26**	**31**
B	**7**	**6**	**5**	**4**	**3**	**2**	**1**
Y	**19**	**15**	**11**	**7**	**4**		

not possible

So possible values of balls in red bowl
= 1, 6, 11, 16, 21

4. Prove that the integer 53 × 83 × 109 + 40 × 66 × 96 is not a prime number.

Sol. 53 × 83 × 109 + 40 × 66 × 96
53 × 83 × 109 + (149 – 109) (149 – 83) (149 –53)
Let 53 = a , 83 = b, 109 = c
So abc +n (149 – a) (149 – b) (149 – c)
abc + $(149)^3 – 149^2(a + b + c)$ + 149 (ab + bc + ca) – abc
= 149 × m.
So number is a multiple of 149, it means it is not a prime number.

5. Let a, b, c & d be real numbers such that $a^3 + b^3 + c^3 + d^3 = a + b + c + d = 0$. Prove that sum of a pair of these numbers is equal to 0.

Sol. $-(a + b) = (c + d)$
& $a^3 + b^3 + c^3 + d^3 = 0$
$(a + b) (a^2 + b^2 – ab) + (c + d) (c^2 + d^2 – cd) = 0$
$(a + b)[(a + b)^2 – 3ab – (a + b) [(c + d)^2 – 3cd] = 0$
$(a + b) [(a + b)^2 – 3ab – (c + d)^2 + 3cd] = 0$

$(a + b) [(a + b)^2 – 3ab – (a + b)^2 + 3cd] = 0$
3 (a + b) (cd – ab) = 0
So a + b = 0 or cd – ab = 0
Similarly b + c = 0 or bc – ad = 0
c + d = 0 or ab – cd = 0

So sum of pairs of number is equal to 0.

6. Find any hundred digit number with non-zero digits, that is divisible by the sum of its digits. Explain the process by which you arrived the answer.

Sol. Let hundred digit number is

1 2 3 4 5 6 7 8 9 9 8 7 6 5 4 3 2 1 - - - - - - | 1 1 2 8 3 8 4 6 7 5

5 times, so 90 digits | 10 digits

Sum of digits = 45 × 10 + 45 = 45 × 11
= 9 × 5 × 11
→ This hundred digit number is divisible by 5 because. last digit is 5.
→ This no. is divisible by 9, because sum of digits is multiple of 9.
→ Also this no. is divisible by 11. Because difference of sum of odd & even place digit is multiple of 11.
Thinking approach :- Sum of 1 to 9 digit is 45 so it is divisible by 9, and is we make unit digit as '5' it also divisible by 5. And if we repeat this pattern in reverse order. it is also divisible by 11. So first 90 digits are divisible by 5,9 & 11.
→ For last 10 digits, we arrange this no. 5 in the form when it is divisible by 11 & sum also 45. according to example.

7. $\frac{a}{b}$ is an improper fraction in its simplest form. If its equivalent mixed fraction is $d\frac{c}{b}$, such that d + c = 9 and a + b = 100, then find all such fractions.

Sol. $\frac{a}{b} = d\frac{c}{b}$ [c < b]
[a > b]
a = db + c ....(1)
& a + b = 100 ....(2)
d + c = 9 ....(3)
From eq. (1) (2) & (3)
100 – b = db + 9 – d.
db + b – d = 91
b (d +1) – (d +1) = 90
(d + 1) (b –1) = 90

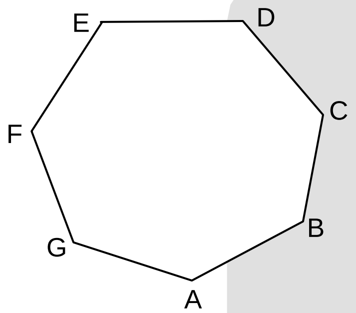
So possible paris of 90 are $90 \times 1, 45 \times 2, 30 \times 3, 18 \times 5, 15 \times 6, 10 \times 9, 9\times 10, 6 \times 15, 5\times 18, 3\times 30, 2 \times 45, 1\times 90$
So possible pairs of (d , b) = (89 ,2), (44,3), (29, 4) (14, 6), (14,7), (9,10), (8, 11), (5, 16), (4,19), (2, 31), (1,46), (0, 91)
But $9 > d > 0$ ($\because$ d + c = 9)
So possible pairs of
(d, b) = (8,11) (5,16) (4,19), (2,31), (1,46)

So $\frac{a}{b} = \frac{89}{11}, \frac{84}{16}, \frac{81}{19}, \frac{69}{31}, \frac{54}{46}$

$\because$ $\frac{84}{16}, \frac{54}{16}$ are not in simplest form so possible pairs of $\frac{a}{b}$, $\frac{89}{11}, \frac{81}{19}, \frac{69}{31}$.

8. ABCDEFG is a regular heptagon. How many quadrilaterals can be obtained from the heptagon satisfying the following properties :
• All the four vertices of the quadrilateral are some vertices of the heptagon.
• Two sides of the quadrilateral are also sides of the heptagon, but not the other two.

Sol.



Case-1 :- When we consider two consecutive sides.
Eg :- By AB & BC
quadrilaterals are ABCE, ABCF (2)
EG :- By BC & CD
quadrilaterals aer BCDF , BCDG (2)
So total $7 \times 2 = 14$ quadrilaterals.

Case-2 When common sides are not consecutive.
Eg :- By AB
quadrilaterals aer BCEF, BCFG.
So total $7 \times 2 = 14$ quad.
So finally total = 14 + 14 = 28 quad.