THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test - Kaprekar Contest

NMTC at SUB JUNIOR LEVEL - VII & VIII Standards

Saturday, 1 September, 2018

Note:

1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.

2. Diagrams are only visual aids; they are NOT drawn to scale.

3. You are free to do rough work on separate sheets.

4. Duration of the test: 2 pm to 4 pm -- 2 hours.

PART—A

Note

- Only one of the choices A. B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering, seek the guidance of the supervisor.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{2}$ mark.**

1. The fraction greater than $8\frac{4}{9}$ is

(A) $8\frac{1}{3}$ (B) $\frac{150}{18}$ (C) $8\frac{2}{3}$ (D) $\frac{216}{27}$

Ans. (C)

Sol. $8\frac{4}{9} = \frac{76}{9} = 8.44$

(A) $8\frac{1}{3} = \frac{25}{3} = 8.33 < 8.44$

(B) $\frac{150}{18} = 8.33 < 8.44$

(C) $8\frac{2}{3} = \frac{26}{3} = 8.66 > 8.44$

(D) $\frac{216}{27} = 8 < 8.44$

2. A car is slowly driven in a road full of fog. The car passes a man who was walking at the rate of 3 km an hour in the same direction. He could see the car for 4 minutes and was visible for up to a distance of 100 meters. The speed of the car is (in km per hours)

(A) $4\frac{1}{2}$ (B) 4 (C) $3\frac{1}{2}$ (D) 3

Ans. (A)

Sol. Let speed of car = x km/hr

Speed of man = 3km/hr

Relative speed = (x – 3) km/hr

D = 100 m = $\frac{1}{10}$ km.

T = 4 min. = $\frac{4}{60}$hr .

Speed = $\frac{D}{T}$

$x - 3 = \frac{\frac{1}{10}}{\frac{4}{60}} \Rightarrow x - 3 = \frac{3}{2}$

$x = 4\frac{1}{2}$.

3. Kiran sells pens at a profit of 20% for Rs. 60. But due to lack of demand he reduced its price to Rs. 55. Then

(A) He gets a profit of 10% (B) He gets a profit of 12%

(C) He incurs a loss of 10% (D) He incurs a loss of 8%

Ans. (A)

Sol. SP = 60

Profit = 20%

$CP = \frac{100 \times 60}{120} = 50.$

If SP = 55

CP = 50

$\therefore$ Profit = 5

Profit % = $\frac{5}{50} \times 100 = 10\%$.

4. If 40% of a number is added to another number then it becomes 125% of itself. The ratio of the second to the first number is

(A) 5 : 8 (B) 7 : 5 (C) 8 : 5 (D) None of these

Ans. (D)

Sol. Let first number is x & second no. is y

ATP

$\frac{40x}{100} + y = \frac{125x}{100}$

$y = \frac{125x}{100} - \frac{40x}{100}$

$y = \frac{85x}{100}$

$\frac{y}{x} = \frac{85}{100} = \frac{17}{20}$.

5. The length of a rectangular sheet of paper is 33 cm. It is rolled along its length into a cylinder so that width becomes height of the cylinder. The volume is 1386 cubic cms. The width of the rectangular sheet (in cm) is

(A) 14 (B) 15 (C) 16 (D) 18

Ans. (C)

Sol. Length of rectangular sheet = 33 cm

Width of rectangle = height of cylinder = x (let)

Volume = 1386 cm^3

Circumference of base of cylinder = Length of rectangular sheet

$2\pi r = 33$

$2 \times \frac{22}{7} \times r = 33$

$r = \frac{21}{4}$

$\pi r^2 h = 1386$

$= \left[\frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times h = 1386\right]$

$h = \frac{1386 \times 7 \times 4 \times 4}{22 \times 21 \times 21} = 16.$

$\therefore$ width of rectangle = 16.

6. If $\frac{1}{1\times 2} + \frac{1}{2\times 3} + ... + \frac{1}{n\times(n+1)} = \frac{19}{20}$ then n =

(A) 18 (B) 19 (C) 20 (D) 25

Ans. (B)

Sol. $\frac{1}{1\times 2} + \frac{1}{2\times 3} + ... + \frac{1}{n\times(n+1)} = \frac{19}{20}$

$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} ..... \frac{1}{n} - \frac{1}{n+1} = \frac{19}{20}$

$1 - \frac{1}{n+1} = \frac{19}{20}$

$\frac{n}{n+1} = \frac{19}{20} \Rightarrow 20n = 19n + 19$

$n = 19.$

7. a, b are natural numbers. If $9a^2 = 12a + 96$ and $b^2 = 2b + 3$, the value of 2018 (a + b) is

(A) 14226 (B) 14128 (C) 14126 (D) 14246

Ans. (C)

Sol. $9a^2 = 12a + 96$

$9a^2 - 12a - 96 = 0$

$3(3a^2 - 4a - 32) = 0$

$3a^2 - 4a - 32 = 0$

$3a^2 - 12a + 8a - 32 = 0$

$3a(a - 4) + 8(a - 4) = 0$

$(a - 4)(3a + 8) = 0$

a = 4 or a = $-\frac{8}{3}$, only possible a = 4

$b^2 = 2b + 3$

$b^2 – 2b – 3 = 0$

$b^2 – 3b + b – 3 = 0$

b (b – 3) + 1 (b – 3) = 0

(b – 3) (b + 1) = 0

b = 3 or b = – 1 , only possible b = 3

∴ 2018 (a + b) = 2018 (4 + 3)

2018 × 7 = 14126.

8. Shanti has three daughters. The average age of them is 15 years. Their ages are in the ratio 3 : 5 : 7. The age of the youngest daughter is (in years)

(A) 8 (B) 9 (C) 10 (D) 12

Ans. (B)

Sol. Let ages be 3x, 5x and 7x

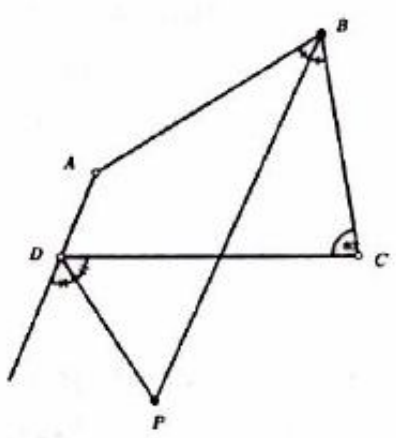
ATP $\frac{3x+5x+7x}{3}$ = 15

$\frac{15x}{3}$ = 15.

x = 3.

∴ Age of youngest daughter = 3 × 3 = 9

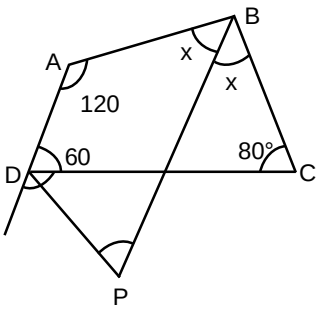
9. In the adjoining figure, ABCD is a quadrilateral. The bisectors of ∠B and the exterior angle at D meet at P. Given ∠C = 80º, ∠ADC = $\frac{1}{2}$∠A and ∠A = ∠C + 40º. Then ∠DPB is



(A) 50º (B) 60º (C) 70º (D) 80º

Ans. (C)

Sol.



$\angle A = \angle C + 40$

$= 80° + 40° = 120°$

$\angle ADC = \frac{1}{2}\angle A$

$= \frac{1}{2} \times 120 = 60°$

$\angle A + \angle B + \angle C + \angle D = 360°$

$\therefore \angle B = 100°$

$x = 50°$

$\angle A + x + \angle ADP + \angle DPB = 360°$

$120 + 50 + 120 + \angle DPB = 360°$

$= 70°$

10. The number of 3-digit number which contain 6 and 7 is

(A) 52 (B) 60 (C) 62 (D) 64

Ans. (A)

Sol. Number of 3-digit number which contain 6 and 7 =

670 , 671 .... 679 $\rightarrow$ 10 numbers

607 , 617 .... 697 $\rightarrow$ 9 numbers

760 , 761 .... 769 $\rightarrow$ 10 numbers

706 , 716 .... 796 $\rightarrow$ 9 numbers

167 , 267 .... 967 $\rightarrow$ 7 numbers

176 , 276 .... 976 $\rightarrow$ 7 numbers

$\therefore$ Total numbers = 52

11. The difference between the biggest and the smallest three digit number each of which has different digits is

(A) 864 (B) 875 (C) 885 (D) 895

Ans. (C)

Sol. Biggest three digit number with distinct digit = 987
Smallest three digit number with distinct digit = 102
Difference = 987 – 102 = 885.

12. If $3x + 1 = 2y - 1 = 5z + 3 = 7w + 1 = 15$, the value of $6x - 3y + 5z - 8w$ is

(A) 1 (B) 2 (C) 3 (D) None of these

Ans. (D)

Sol. $3x + 1 = 2y - 1 = 5z + 3 = 7w + 1 = 15$

$3x + 1 = 15$

$3x = 14$

$x = \frac{14}{3}$

$2y - 1 = 15$

$2y = 16$

$y = 8$

$5z + 3 = 15$

$5z = 12$

$z = \frac{12}{5}$

$7w + 1 = 15$

$7w = 14$

$w = 2.$

$\therefore\ 6x - 3y + 5z - 8w$

$= \frac{6\times14}{3} - 3 \times 8 + \frac{5\times12}{5} - 8 \times 2$

$= 28 - 24 + 12 - 16$

$= 40 - 40 = 0.$

13. Five years ago the average age of Aruna, Roy , David and salman is 45 years. Sita joins them now,. The average age of all the five now is 49 years. The present age of sita is (in years)

(A) 45 (B) 43 (C) 51 (D) 48

Ans. (A)

Sol. Let present ages of Aruna, Roy, David and Salman be x, y, z and w.

Five years ago their ages were $x - 5$, $y - 5$, $z - 5$, $w - 5$

ATP $\frac{x-5+y-5+z-5+w-5}{4} = 45$

$x + y + z + w - 20 = 180$

$x + y + z + w = 200$

Let present age of Sita be 'a'

$\frac{x+y+z+w+a}{5} = 49$

$x + y + z + w + a = 245$

$200 + a = 245$

$a = 45.$

14. The fraction $\frac{B}{3x-1}$ is subtracted from the fraction $\frac{A}{2x+3}$. The resulting fraction is $\frac{-11}{(2x+3)(3x-1)}$. Then A + B =

(A) 11 (B) – 11 (C) 5 (D) – 5

Ans. (C)

Sol. $\frac{A}{2x+3} - \frac{B}{3x-1} = \frac{-11}{(2x+3)\ (3x-1)}$

$\frac{A(3x-1)-B(2x+3)}{(2x+3)\ (3x-1)} = \frac{-11}{(2x+3)\ (3x-1)}$

$3Ax - A - 2Bx - 3B = -11$

$x(3A - 2B) - (A + 3B) = -11$

Comparing we get

$3A - 2B = 0$

$A + 3B = 11$

Solving we get

$B = 3,$

$A = 2$

$A + B = 3 + 2 = 5.$

15. There are some cows and ducks. The total number of legs is equal to 14 more than twice the number of heads. The number of cows is

(A) 5 (B) 6 (C) 7 (D) 8

Ans. (C)

Sol. Number of ducks = x

Number of cows = y

Total number of legs = 2x + 4y

Number of heads = x + y

ATP

2x + 4y = 14 + 2 (x + y)

2x + 4y = 14 + 2x + 2y

2y = 14

y = 7.

16. The sum of 5% of a number and 9% another number is equal to sum of the 8% first number and 7% of the second number. The ratio between the numbers is

(A) 3 : 2 (B) 5 : 7 (C) 7 : 9 (D) 2 : 3

Ans. (D)

Sol. $\frac{5}{100}x+\frac{9}{100}y = \frac{8}{100}x+\frac{7}{100}y$

$\frac{2y}{100} = \frac{3x}{100}$

2y = 3x

x : y = 2 : 3.

17. The length of two sides of an isosceles triangle are 8 cm and 14 cm. The perimeter of the triangle (in cm) is

(A) 30 (B) 36 (C) 19 (D) 30 or 36

Ans. (D)

Sol. Let the sides be 8, 8 and 14

P = 8 + 8 + 14 = 30

Let the sides be 8, 14, 14

P = 8 + 14 + 14 = 36.

18. There are three cell phones A, B, C. A is 50% costlier than C and B is 25% costlier than C.A is a% costlier than B. Then a =

(A) 25 (B) 20 (C) 15 (D) 10

Ans. (B)

Sol. Let the cost price of C be x

$A = x + \frac{50}{100}x = \frac{3x}{2}$

$B = x + \frac{25}{100}x = \frac{5x}{4}$

A is a% costlier than B

$\therefore \frac{3x}{2} = \frac{5x}{4} + \frac{a}{100}\times\frac{5x}{4}$

$\frac{3x}{2} = \frac{5x}{4}\left(1+\frac{a}{100}\right)$

$\frac{6}{5} = \left(1+\frac{a}{100}\right)$

$\frac{a}{100} = \frac{1}{5}$.

a = 20.

19. Sushant wrote a two digit number. He added 5 to the tens digit and subtracted 3 from the unit digit of the number and got a number equal to twice the original number. The original number is

(A) 47 (B) 74 (C) 37 (D) 73

Ans. (A)

Sol. Let unit digit = x

ten digit = y

Number = 10 y + x

Now unit digit = x – 3

ten digit = y + 5

Number = 10 (y + 5) + (x – 3)

10y + 50 + x – 3

10y +x + 47

10y + x + 47 = 2(10y + x)

10y + x + 47 = 20y + 2x

10y + x = 47

Number = 47.

20. The units digit of $5^{2018} - 3^{2018}$ is

(A) 5 (B) 6 (C) 7 (D) 4

Ans. (B)

Sol. $5^{2018} - 3^{2018}$

Unit digit of 5^{2018} = 5

Unit digit of 3^{2018} = 9

Unit digit of $5^{2018} - 3^{2018}$ = 6.

PART - B

Note :

- Write the correct answer in the space provided in the response sheet
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{4}$ marks.

21. The smallest natural number that has to be added to 803642 to get a number which is divisible by 9 is _____.

Ans. 9

Sol. 8 + 0 + 3 + 6 + 4 + 2 = 23

So 4 has to be added to 803642 to get a number divisible by 9.

22. The greatest two digit number that will divided 398, 436, and 542 leaving respectively 7, 11 and 15 as remainders is _____.

Ans. 17

Sol. Required Number = HCF {(398 – 7), (436 – 11), (542 – 15)}

HCF {391, 425, 527} = 17

23. $\frac{2}{3}$ is _____ of $\frac{1}{3}$.

Ans. 2

Sol. $\frac{2}{3} = x \times \frac{1}{3}$

$x = 2$

24. The sum of 5 positive integers is 280. The average of the first 2 number is 40. The average of the third and fourth number is 60. The fifth number is _____

Ans. 80

Sol. $a + b + c + d + e = 280$

$\frac{a+b}{2} = 40 \quad \Rightarrow \quad a + b = 80$

$\frac{c+d}{2} = 60 \quad \Rightarrow \quad c + d = 120$

$\therefore \quad 80 + 120 + e = 280$

$e = 80.$

25. If a : b = 3 : 4 and $\frac{p}{q} = \frac{a^2+b^2+ab}{a^2+b^2-ab}$, where p, q have no common divisors other than 1, p + q is _____.

Ans. 50

Sol. Let a = 3x, b = 4x

$\frac{p}{q} = \frac{(3x)^2+(4x)^2+3x(4x)}{(3x)^2+(4x)^2-3x(4x)} = \frac{9x^2+16x^2+12x^2}{9x^2+16x^2-12x^2} = \frac{37x^2}{13x^2} = \frac{37}{13}$.

p + q = 37 + 13 = 50.

26. a is a natural number such that a has exactly two divisors and (a + 1) has exactly three divisors. The number of divisors of a + 2 is _____.

Ans. 2

Sol. a has exactly 2 divisor

$\therefore$ a must be a prime number

Let a = 3

$\therefore$ a + 1 = 4

Number of divisor of a + 2 = 3 + 2 = 5 = 2.

27. The first term of a series is $\frac{2}{5}$. If x is a term of this series, the next term is $\frac{1-x}{1+x}$. If t_n denotes the n-th term and $t_{2018} - t_{2017} = \frac{p}{q}$, where p, q are integers having no common factors other than 1, p + q is _____.

Ans. 36

Sol. Let $x = \frac{2}{5}$

2^{nd} term $= \frac{1-\frac{2}{5}}{1+\frac{2}{5}} = \frac{3}{7}$.

3^{rd} term $= \frac{1-\frac{3}{7}}{1+\frac{5}{7}} = \frac{4}{10} = \frac{2}{5}$

$\therefore$ Sequence is $\frac{2}{5}, \frac{3}{7}, \frac{2}{5}, \frac{3}{7}$

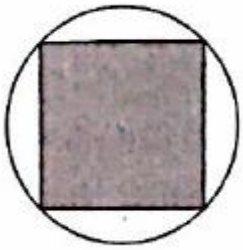
$t_{2018} - t_{2017} = \frac{p}{q}$

$\frac{3}{7} - \frac{2}{5} = \frac{p}{q}$

$\frac{1}{35} = \frac{p}{q}$

$p + q = 1 + 35 = 36.$

28. In the adjoining figure, the side of the square is $\sqrt{\frac{2018}{\pi}}$ cm. The area of the unshaded region is $\left(\frac{\pi-2}{\pi}\right)$A sq. cms. The value of A is _____.



Ans. 1009

Sol. Side of the square = $\sqrt{\frac{2018}{\pi}}$

Diagonal = $\sqrt{2} \times \sqrt{\frac{2018}{\pi}}$

$\therefore$ Diameter = $\sqrt{2} \times \sqrt{\frac{2018}{\pi}}$

Radius = $\frac{\sqrt{2}}{2} \times \sqrt{\frac{2018}{\pi}}$

Area of circle = $\pi \left(\frac{\sqrt{2}}{2} \times \sqrt{\frac{2018}{\pi}}\right)^2$

$\pi \times \frac{2}{4} \times \frac{2018}{\pi} = 1009$

Area of square = $\left(\sqrt{\frac{2018}{\pi}}\right)^2 = \frac{2018}{\pi}$

Area of unshaded region = $1009 - \frac{2018}{\pi}$

$1009 - \frac{2018}{\pi} = \left(\frac{\pi-2}{\pi}\right)$A

$1009\,\pi - 2018 = (\pi - 2)A$

$1009\,(\pi - 2) = (\pi - 2)\,A$

$A = 1009.$

29. n is a natural number. The square root of the sum of the square of n and 19 is equal to the next natural number to n. The value of n is _____.

Ans. 9

Sol. $\sqrt{n^2 + 19} = n + 1$

$n^2 + 19 = (n + 1)^2$

$n^2 + 19 = n^2 + 2n + 1$

$2n = 18 \Rightarrow n = 19$

30. Using only the digits 1,2, 4, 5, two- digit numbers are formed. The digits of the two digit number may be the same or different. The number of such two-digit number is _____

Ans. 16

Sol. Possible two digit numbers =
11, 22, 44, 55
12, 14, 15, 21, 24, 25, 41, 42, 45, 51, 52, 54
Number of possible two digit number = 16.

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