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50th
NMTC-2018

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA (Regd.)

Screening Test – Ramanujan Contest
NMTC at INTER LEVEL – XI & XII Standards

TEST PAPER WITH SOLUTION & ANSWER KEY

Date: 1st September, 2018 | Duration: 2 Hours

National Mathematics Talent Competitions (NMTC) 2018

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»Instructions to Candidates «

1. Do not open he Question booklet until you are told to do so.
2. a) This is a Multiple choice and fill in the blanks test. In multiple choices each Question is followed by four answer (A), (B), (C) and (D) Only one of them it correct. When you have decided on your choice, shade the corresponding letter in the box against the question number. For example, if you decide that for a question, the correct response is (A), shade of the box marked A (Eg.[] on A in the box shading that box.
   b) In the case of fill in the blanks questions write the correct answer in the space provided.
3. Each question carries 1 marks.
4. a) If you are unable to solve a problem, it is better not to answer the question. Avoid guessing. Since you are penalized for each wrong answer.
   b) ½ mark will be deducted for wrong answers in part A
   c) ½ mark will be deducted for wrong answers in part B
5. Be certain that you understand thoroughly the coding system for your answer sheet. If you are not sure, ask your supervisor to clarify it.
6. You are permitted to use rough paper. No other aid, like the instrument box, calculator, etc, are permitted.
7. Diagrams are not drawn to scale. They are intended as aids only.
8. Before commencing to write your answers. Fill in the details of your bio-data (Name, School/address etc.,) in the appropriate places in the response sheet. If you are writing in a centre other than your school write the agency through whom you write the exam. Eg,., AMTI for open quota.
9. After completion, return only the response sheet. The question-booklet and the rough-worksheets may be retained by you or as directed by your supervisor.
10. When your supervisor instructs you to begin, you have 120 minutes if working time.

NOTE:

1. **Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.**
2. **Diagrams are only visual aids, they are NOT drawn to scale.**
3. **You are free to do rough work on separate sheets.**
4. **Duration of the test: 2 pm to 4 pm (2 hours).**

1. In the addition shown, each of the letters T,H,I,S represents a non zero digit. What is T + H + I + S ?

T	H	I	S
		I	S
2	0	1	8

(A) 34 (B) 32 (C) 24 (D) 22

Ans. (C)

Sol. T H I S

I S

---

2 0 1 8

Clearly S = 4 or S = 9

If S = 4 $\Rightarrow$ I can not take any value

If S = 9 $\Rightarrow$ I can take value 5 only

Hence

T H 5 9

5 9

T(H+1) 1 8 = 2018

$\Rightarrow$ T = 1 and H = 9

So the number T H I S = 1959

Hence T + H + I + S = 1 + 9 + 5 + 9 = 24

2. We have four sets S_1, S_2, S_3, S_4 each containing a number of parallel lines. The set S_i contains i + 1 parallel lines i = 1,2,3,4. A line in S_i is not parallel to lines in S_j when $i \neq j$. In how many points do these lines intersect ?

(A) 54 (B) 63 (C) 71 (D) 95

Ans. (C)

Sol. Number of point of intersection = ${}^2C_1\,({}^3C_1 + {}^4C_1 + {}^5C_1) + {}^3C_1\,({}^4C_1 + {}^5C_1) + {}^4C_1 \times {}^5C_1$

= 2(12) + 3(9) + 20

= 24 + 27 + 20

= 71

3. An old tanker is 100 km due north of a cruise liner. The tanker sails Southeast at a speed of 20 kilometers per hour and the liner sail Northwest at a speed of 10 kilometres per hour. What is the shortest distance between the two boats during the subsequent motion ?

(A) $50\sqrt{2}$ km (B) 60 km (C) 80 km (D) 100 km

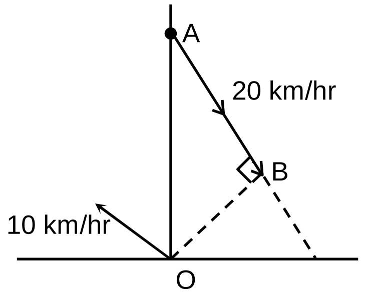
Ans. (A)

Sol. From relative motion

Let initially oil tanker at A and d cruise liner at 0

Both move parallely opposite directions

Shortest distance = OB = $50\sqrt{2}$



4. Volume A equals one fourth of the sum of the volumes B and C, while volume B equals one sixth of the sum of the volumes A and C. The ratio of volume C to the sum of volumes of A and B is

(A) 2 : 3 (B) 9 : 10 (C) 7 : 12 (D) 12 : 23

Ans. (D)

Sol. $V_A = \frac{1}{4}(A_B + V_C)$ .... (1)

$V_B = \frac{1}{6}(V_A + V_C)$ .... (2)

To find $\frac{V_A + V_B}{V_C}$

from (1) and (2)

$4V_A - 6V_B = V_B - B_A$

$5V_A = 7V_B$

Put in (1) $4V_A = \frac{5}{7}V_A + V_C$

$\Rightarrow 23V_A = 7V_C$

Hence $\frac{V_C}{V_A + V_B} = \frac{\frac{23}{7}V_A}{V_A + \frac{5}{7}V_A} = \frac{23}{7} \times \frac{7}{12} = \frac{23}{12}$

5. In the ninety-nine shop every item costs some whole number of rupees plus 99 paise. Rhea spent sixty five rupees and seventy six paise in buying some items from the shop. How many items did she buy ?

(A) 23 (B) 24 (C) 65 (D) 66

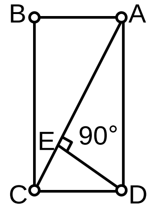
Ans. (B)

Sol. Let p items cost Rs x_1.99, Rs x_2.99 and so on

$\Rightarrow x_1 + x_2 + ... + x_p + 0.99p = 65.76$

$p = 24 \Rightarrow 0.99 \times 24 = 23.76$

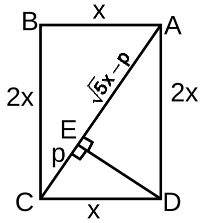
6. The diagram shows a rectangle ABCD where AB : AD = 1 : 2. Point E on AC is such that DE is perpendicular to AC. What is the ratio of the area of the triangle DCE to the rectangle ABCD?



(A) $1 : 4\sqrt{2}$ (B) 1 : 6 (C) 1 : 8 (D) 1 : 10

Ans. (D)

Sol.



$AC = \sqrt{5}x$

$ED^2 = x^2 - p^2 = 4x^2 - (\sqrt{5}x - p)^2$

$\Rightarrow 2x^2 = 2\sqrt{5}\,px \Rightarrow x = \sqrt{5}\,p$

$$\frac{\text{Area of } \Delta DCE}{\text{Area of rectangle } ABCD} = \frac{\frac{p}{2}\sqrt{x^2 - p^2}}{2x^2} = \frac{\frac{x}{2\sqrt{5}}\sqrt{x^2 - \frac{x^2}{5}}}{2x^2} = \frac{1}{10}$$

7. The numbers 2, 3, 12, 14, 15, 20, 21 may be divided into two sets so that the product of the numbers in each set is the same. What is this product?

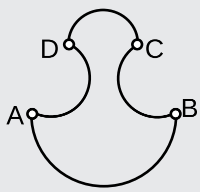
(A) 420 (B) 1260 (C) 2520 (D) 6720

Ans. (C)

Sol. $P^2 = 2 \times 3 \times 12 \times 14 \times 15 \times 20 \times 21$

$\Rightarrow P = 2520$

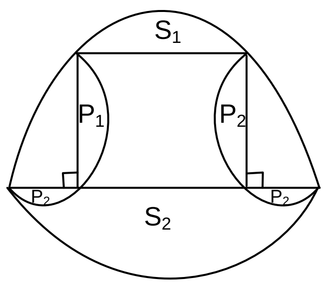
8. ABCD is a trapezium with AD = DC = CD = 10 units and AB = 22 units. Semi circles are drawn as shown in the figure. The area of the region bounded by these semi circles in square units is



(A) $128 + 48\pi$ (B) $128 + 24\pi$ (C) $116 + 48\pi$ (D) $116 + 24\pi$

Ans. (A)

Sol.



Required Area = area of trapezium – $(2\Delta + 2P_1) + S_1 + S_2 - 2P_2$

= area of trapezium + $S_1 + S_2 - 2(\Delta + P_1 + P_2)$

= area of trapezium + $S_2 + S_1$

$= \frac{1}{2}(10 + 22) \times 8 + \frac{\pi}{2}(11^2 - 5^2) = 128 + 48\pi$

9. Consider the number of ways in which five girls and five boys sit in ten seats that are equally spaced around a circle. The proportion of the seating arrangements in which no two girls sit at the ends of a diameter is

(A) $\frac{1}{2}$ (B) $\frac{8}{63}$ (C) $\frac{55}{63}$ (D) None of the above

Ans. (B)

Sol. Proportion = $\dfrac{\dfrac{2^5 \times 5! \times !}{10}}{(10-1)!} = \dfrac{8}{63}$

10. Let A = $1^{-4} + 2^{-4} + 3^{-4} + \ldots\ldots\ldots\ldots$, the sum of reciprocals of fourth powers of integers and B = $1^{-4} + 3^{-4} + 5^{-4} + \ldots\ldots\ldots\ldots$, the sum of reciprocals of fourth powers of odd positive integers. The value of A/B as a fraction is

(A) $\frac{16}{15}$ (B) $\frac{32}{31}$ (C) $\frac{64}{63}$ (D) $\frac{128}{127}$

Ans. (A)

Sol. B = $\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\ldots\ldots\right)-\left(\frac{1}{2^4}+\frac{1}{4^4}+\ldots\ldots\right)$

$= A - \frac{1}{2^4}(A) = \frac{15}{16}(A)$

11. The number $5^{(6^7)}$ is written on the board (in base 10). Gia takes two of the digits at a time, erases them but appends the sum of those digits at the end. She repeats this till she ends up with one digit on the board. What is the digit that remains on the board?

(A) 1 (B) 5 (C) 6 (D) 7

Ans. (A)

Sol. obviously

12. Seven points are marked on the circumference of a circle and all pairs of points are joined by straight lines. No three of these lines have a common point and any two intersect at a point inside the circle. Into how many regions is the interior of the circle divided by these lines?

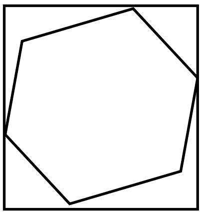
(A) 64 (B) 63 (C) 57 (D) 56

Ans. (C)

Sol.

Points	Section
2	2
3	$2+2=4$
4	$2+2+4=8$
5	$2+2+4+8=16$
6	$2+2+4+8+15=31$
7	$2+2+4+8+15+26=57$

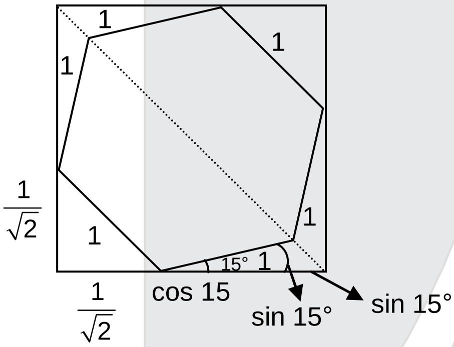
13. The diagram below shows a regular hexagon with side length 1, insceibed in a square. Two of the vertices lie on the diagonal of the square and the remaining vertices lie on its sides. What is the area of the square ?



(A) $\frac{7}{2}$ (B) 4 (C) $2 + \sqrt{3}$ (D) $3 + \sqrt{2}$

Ans. (C)

Sol.



Length = $\frac{1}{\sqrt{2}}$ + cos 15° + sin 15°

$= \frac{1}{\sqrt{2}} + \sqrt{1 + \sin 30^\circ}$

$= \frac{1}{\sqrt{2}} + \sqrt{1 + \frac{1}{2}}$

$= \frac{1}{\sqrt{2}} + \sqrt{\frac{3}{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}$

$= \frac{3+1+2\sqrt{3}}{2} = 2 + \sqrt{3}$ Ans.

14. AB is a diameter of a semicircle of centre O. C is the midpoint of the arc AB. AC and the tangent at B to the semicircle meet at P. D is the midpoint of BP. If ACDO is a parallelogram and $\angle PAD = \theta$, then $\sin\theta$ is

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{10}}$ (C) $\frac{2}{\sqrt{10}}$ (D) $\frac{3}{\sqrt{10}}$

Ans. (B)

Sol. AO = OB = BO = PD = CD = r

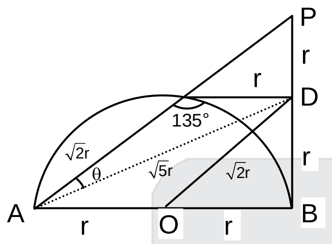
OD = AC = $\sqrt{2}$ r

AD = $\sqrt{5}$ r

$\angle ACD = 135°$

$\Rightarrow \frac{\sin\theta}{r} = \frac{\sin 135°}{\sqrt{5}\ r}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{10}}$



15. The real valued function f(x) satisfies the equation $2f(1 - x) + 1 = xf(x)$ for all x. Then $(x^2 - x + 4)$ f(x) equals

(A) $x - 1$ (B) x (C) $x + 1$ (D) $x - 3$

Ans. (D)

Sol. $2f(1 - x) + 1 = xf(x)$

Replace x by $1 - x$

$2f(x) + 1 = (1 - x)\, f(1 - x)$

$2f(x) + 1 = (1 - x)\left[\frac{xf(x) - 1}{2}\right]$

$4f(x) + 2 = (1 - x)x\, f(x) - (1 - x)$

$(x^2 - x + 4)\, f(x) = x - 3$

PART – B

Note :

- Write the correct answer in the space provided in the response sheet.
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{4}$ Mark.

16. The number of ways in which 26 identical chocolates be distributed between Amy, Bob, Cathy and Daniel so that each receives at least one chocolate and Amy receives more chocolates than Bob is _______.

Ans. (1078)

Sol. Total number of natural solution of equation $x_1 + x_2 + x_3 + x_4 = 26$ is ${}^{25}C_3 = 2300$. total number of natural solution of equation $2x_1 + x_3 + x_4 = 26 = 1 + 3 + \ldots\ldots\ldots + 23 = 144$

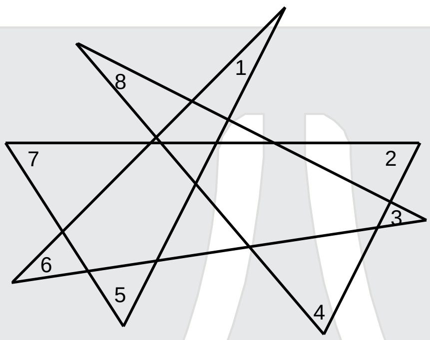
Total number of natural solution of equation $x_1 + x_2 + x_3 + x_4 = 26$ when $x_1 > x_2$ is $\frac{2300 - 144}{2} = 1078$

17. A set S contains 11 numbers. The average of the numbers in S is 302. The average of the six smallest numbers of S is 100 and the average of the six largest of the numbers is 300. What is the median of the numbers in S__________.

Ans. (360)

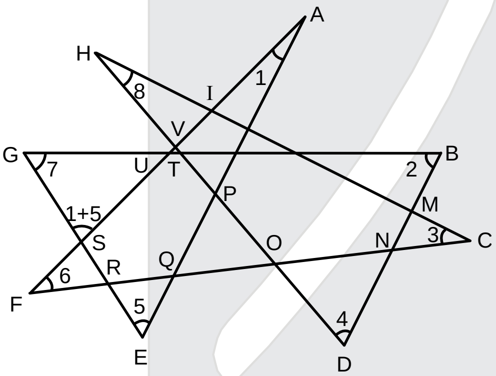
Sol. Bonus (wrong question)

18. The sum of the angles 1,2,3,4,5,6,7,8 in degrees shows in the following figure is ______.



Ans. (360)

Sol.



$\angle GST = 1 + 5$

$\angle HIV = 3 + 6$

$\angle HUV = 2 + 4$

$\angle TVJ = \angle UVI = 1 + 5 + 7$

In quadrilateral UVIH

sum of angle = 1 + 5 + 7 + 2 + 4 + 3 + 6 + 8 = 360

19. The number of positive integers less than 2018 that are divisible by 6 but are not divisible by at least one of the numbers 4 or 9 is _______.

Ans. (280)

Sol. Total number divisible by 6 but less than 2018 are 336

Total number divisible by 36 but less than 2018 are 56

Required number equals to 336 – 56 = 280

20. If $x(x+1)(x+2)........(x+23) = \sum_{n=1}^{24} a_n x^n$ the number of coefficients a_n that are multiples of 3 is ________.

Ans. (15)

Sol. In elements of set {0,1,2,........23}

8 elements are of the form 3λ

8 elements are of the form $3\lambda + 1$

8 elements are of the form $3\lambda - 1$

Coefficient of x^{24} is 1 (not divisible by 3)

Coefficient of x^{23} is $(1 + 2 + ...+.23) = \frac{24 \times 23}{2}$ = multiple of 3

Coefficient of x^{22} is not multiple of 3

Coefficient of x^{21} is multiple of 3

Coefficient of x^{8} is not multiple of 3

Coefficient of x^{7} is multiple of 3

Now all coefficient of power of x less than 7 are multiple of 3

Coefficient of x^{24} , Coefficient of x^{22} ,......... Coefficient of x^{8} are not multiple of 3

Number of coefficient which are multiple of 3 are 24 – 9 = 15

21. A square is cut into 37 squares of which 36 have area 1 square cms. The length of the side of the original square is _______.

Ans. (10)

Sol. Let a square is of side length a is taken which cuts in 36 square of side length 1 and one of side length b.

$\Rightarrow a^2 = 36(1) + b^2$

$\Rightarrow a^2 - b^2 = 36$

$\Rightarrow (a-b)(a+b) = 36$ clearly a + b and a – b both even

$\left.\begin{array}{l} a+b=18 \\ a-b=2 \end{array}\right\} \Rightarrow a = 10 \text{ and } b = 8$

Hence length of original square 10.

22. There are 4 coins in a row and all are showing heads to start with. The coins can be flipped with the following rules :

(a) The fourth coin (from the left) can be flipped any time

(b) An intermediate coin can be changed to tail only if its immediate neighbor on the right is heads and all other coins (if any) to its right are tails.

(c) Only one coin can be flipped in one step.

The minimum number of steps required to bring all coins to show tails is _______.

Ans. (8)

Sol. HHHH, HHTH, HHTT, THTT, THHT,

TTHT, TTHH, TTTH, TTTT

23. A poet met a tortoise sitting under a tree. When the tortoise was the poet's age, the poet was only a quarter of his current age. When the tree was the tortoise's age, the tortoise was only a seventh of its current age. If all the ages are in whole number of years, and the sum of their ages is now 264, the age of the tree in years is ______,

Ans. (143)

Sol. p = poet, t = tortoise, T = tree

p, t, T

$p - (t - p) = \frac{p}{4}$ and $t - (T - t) = \frac{t}{7}$

$\Rightarrow p = \frac{4t}{7}$ and $t = \frac{7T}{13}$

$T + t + p = 264$

$\Rightarrow T + \frac{7T}{13} + \frac{4T}{7} = 264 \Rightarrow T = 143$

24. The sum of all real value of x satisfying $\left(x+\frac{1}{x}-17\right)^2 = x + \frac{1}{x} + 17$ is ______.

Ans. (35)

Sol. $\left(x+\frac{1}{x}-17\right)^2 = \left(x+\frac{1}{x}+17\right)$

Let $x + \frac{1}{x} - 17 = t$

$t^2 = t + 34$

$\Rightarrow t^2 - t - 34 = 0$

$\Rightarrow t = \frac{1 \pm \sqrt{1+136}}{2}$

$\Rightarrow t = \frac{1+\sqrt{137}}{2}$ or $\frac{1-\sqrt{137}}{2}$

$\Rightarrow x + \frac{1}{x} = 17 + \frac{1+\sqrt{137}}{2}$ or $17 + \frac{1-\sqrt{137}}{2}$

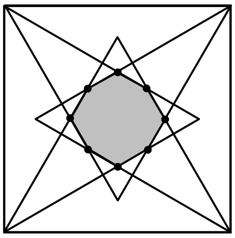
$\Rightarrow x + \frac{1}{x} = \frac{35+\sqrt{137}}{2}$ or $\frac{35-\sqrt{137}}{2}$

$\Rightarrow 2x^2 - (35 + \sqrt{137})x + 1 = 0 \begin{cases} x_1 \\ x_2 \end{cases} \Rightarrow x_1 + x_2 = \frac{35+\sqrt{137}}{2}$

and $2x^2 - (35 - \sqrt{137})x + 1 = 0 \begin{cases} x_3 \\ x_4 \end{cases} \Rightarrow x_3 + x_4 = \frac{35-\sqrt{137}}{2}$

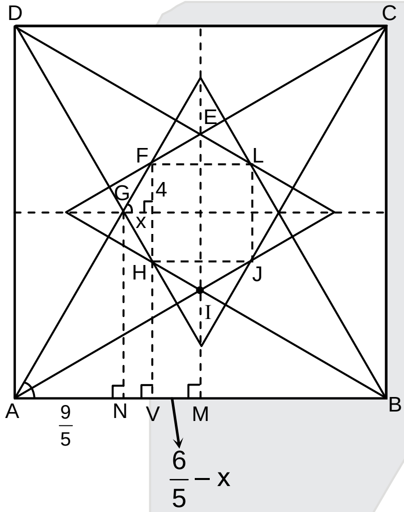
$\Rightarrow x_1 + x_2 + x_3 + x_4 = 35$ Ans. 35

25. On the inside of a square with side length 6, construct four congruent isosceles triangles each with base 6 and height 5, and each having one side coinciding with a different side of the square. The area of the octagonal region common to the interiors of all four triangles is _____.



Ans. (3.6)

Sol.



$\tan\theta = \frac{PM}{MA} = \frac{5}{3} \Rightarrow \frac{GN}{AN} = \frac{5}{3} \Rightarrow \frac{3}{AN} = \frac{5}{3} \Rightarrow AN = \frac{9}{5}$

$\tan\theta = \frac{y+3}{\frac{9}{5}+x} = \frac{y}{x} \Rightarrow xy + 3x = \frac{9}{5}y + xy \Rightarrow 5x = 3y$

Now $FL = y \Rightarrow \frac{6}{5} - x = y \Rightarrow 6 - 5x = 5\left(\frac{5x}{3}\right) \Rightarrow x = \frac{9}{20}$

Area = area of square + 4(area of ΔEFL)

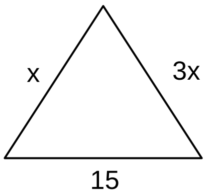
$= 4y^2 + 2(2y)(x)$

$= \frac{160x^2}{9} = \frac{160 \times 81}{9 \times 400} = \frac{18}{5}$

26. In a triangle with integer side lengths, one side is thrice the other. The third side is 15 cm. The greatest possible perimeter of the triangle is (in cm) _______.

Ans. (43)

Sol.



$3x - x < 15$

and 3x + x > 15

$\Rightarrow$ $x < \frac{15}{2}$ and $x > \frac{15}{4}$

$\Rightarrow$ greatest value of x can be 7

$\Rightarrow$ perimeter = 7 + 21 + 15 = 43

27. A cube has edge length x (an integer). three faces meeting at a corner are painted blue. The cube is then cut into smaller cubes of unit length. If exactly 343 of these cubes have no faces painted blue, then the value of x is ________ .

Ans. (8)

Sol. $(x - 1)^3 = 343$

$\Rightarrow x - 1 = 7$

$\Rightarrow x = 8$

28. If $f(x) = ax^4 - bx^2 + x + 5$ and f(3) = 8, the value of f(–3) is ______.

Ans. (2)

Sol. f(3) – f(–3) = 6

$\Rightarrow$ f(– 3) = 2

29. Archana has to choose a three-digit code for her bike lock. The digits can be chosen from 1 to 9. To help her remember them, she decides to choose three different digits in increasing order, for example 278. The number of such codes she can choose is ______.

Ans. (84)

Sol. Number of ways = $^9C_3 = \frac{9 \times 8 \times 7}{3!} = 84$

30. Let S be a set of five different positive integers, the largest of which is n. It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of S. The smallest possible value of n is ________.

Ans. (11)

Sol. S = {1,2,3,6,11}

In quadrilateral with side a,b,c,d

we have a + b + c > d etc which is not possible in set S.