SJ

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
KAPREKAR CONTEST - FINAL - SUB JUNIOR
Classes VII & VIII
Saturday, 28th October_2017.

Instructions:
1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

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1. **(a)** Find all three digit numbers in which any two adjacent digits differ by 3.

(b) There are 5 cards. Five positive integers (may be different or equal) are written on these cards, one on each card. Abhiram finds the sum of the numbers on every pair of cards. He obtains only three different totals 57, 70, 83. Find the largest integer written on a card.

Sol. **(a)** Let the no. be xyz

$x \rightarrow 1$ to 9

$y \rightarrow 0$ to 9

z = 0 to 9

x & y are

x = (1 to 2) or (7 to 9) y = (0 to 2) or (7 to 9) (0 cases)

x = (3 to 6) , y = (0 to 2) or (7 to 9) (6 cases)

(30, 41, 47, 52, 58, 69)

x = (1 to 2) or (7 to 9) y = (3 to 6) (5 cases)

(14,25,74,85,96)

x = (3 to 6) y = (3 to 6) (2 cases)

(36, 63)

No. are

141,147,252,258,303,363,369,414,474,525,585,630,636,696,741,747,852,
858, 963,969 **20 Number of there**

(b) As the three sum are obtained 57,70,83

We have combination of three numbers same and two numbers different

a, a, a, b, c

a + b = 57

b + c = 70

c + a = 83

2a = 70

a = 35

b = 22

c = 48

Largest integer is 48

2. **(a)** ABC is a triangle in which AB = 24, BC = 10 and CA = 26 . P is a point inside the triangle. Perpendiculars are drawn to BC, AB and AC. Length of these perpendiculars respectively are x, y and z. Find the numerical value of 5x + 12y + 13z.

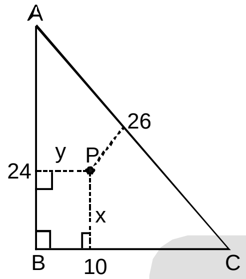
(b) If $x^2(y + z) = a^2$, $y^2(z + x) = b^2$, $z^2(x + y) = c^2$, xyz = a b c prove that $a^2 + b^2 + c^2 + 2abc = 1$

Sol. **(a)** $\Delta = \frac{1}{2}$ 24 × 10 = 20

Δ = ar ΔPAB + ΔPBC + or ΔPAC

$120 = \frac{1}{2}x \times 10 + \frac{1}{2}z \times 26 + \frac{1}{2}y + 24$

120 = 5x + 13z + 12 y



(b) If $x^2(y + z) = a^2$, $y^2(z + x) = b^2$, $z^2(x + y) = c^2$, xyz = a b c

$x^2 (y + z)\, y^2 (z + x)\, z^2 (x + y) = a^2b^2c^2$

$x^2y^2z^2 (x + y) (y + z) (z + x) = a^2b^2c^2$

xyz = abc

(x + y) (y + z) (z + x) = 1

$a^2 + b^2 + c^2 + 2abc \Rightarrow$

$x^2(y + z) + y^2 (z + x) + z^2(x + y) + 2xyz$

Put y = – z

$0 + z^2(z + x) + z^2(x - z) + 2x (-z)z$

$z^3 + z^2x + z^2x - z^3 - 2z^2x = 0$

$\therefore$ (y + z) is factors

Similarly (x + y) & (x + z) are factors

$x^2(y + z) + y^2(z + x) + z^2(x + y) + 2xyz$

= k (x + y) (y + z) (x + z)

For k put x = 0 y = 1 z = 1

0 + 1 + 1 + 0 = k (1) (2) (1)

2 = 2 k

k = 1

$x^2(y + z) + y^2(z + x) + z^2(x + y) + 2xyz$

= (x + y) (y + z) (z + x)

and (x + y) (y + z) (z + x) = 1

$\therefore$ $x^2(y + z) + y^2(z + x) + z^2 (x + y) + 2xyz = 1$

3. If

$$X = \frac{a^2 - (2b - 3c)^2}{(3c + a)^2 - 4b^2} + \frac{4b^2 - (3c - a)^2}{(a + 2b)^2 - 9c^2} + \frac{9c^2 - (a - 2b)^2}{(2b + 3c)^2 - a^2}$$

$$Y = \frac{9y^2 - (4z - 2x)^2}{(2x + 3y)^2 - 16z^2} + \frac{16z^2 - (2x - 3y)^2}{(3y + 4z)^2 - 4x^2} + \frac{4x^2 - (3y - 4z)^2}{(4z + 2x)^2 - 9y^2}$$

Find 2017 (X + Y)

Sol.

$$x = \frac{a^2 - (2b - 3c)^2}{(3c + a)^2 - 4b^2} + \frac{4b^2 - (3c - a)^2}{(a + 2b)^2 - 9c^2} + \frac{9c^2 - (a - 2b)^2}{(2b + 3c)^2 - a^2}$$

$$x = \frac{(a + 2b - 3c)(a - 2b + 3c)}{(3c + a + 2b)(3c + a - 2b)} + \frac{(2b + 3c - a)(2b - 3c + a)}{(a + 2b + 3c)(a + 2b - 3c)} + \frac{(3c + a - 2b)(3c - a + 2b)}{(2b + 3c + a)(2b + 3c - a)}$$

$$x = \frac{a + 2b - 3c}{a + 2b + 3c} + \frac{2b + 3c - a}{a + 2b + 3c} + \frac{3c + a - 2b}{a + 2b + 3c}$$

$$x = \frac{a + 2b - 3c + 2b + 3c - a + 3c + a - 2b}{a + 2b + 3c}$$

$$x = \frac{a + 2b + 3c}{a + 2b + 3c} = 1$$

$$Y = \frac{9y^2 - (4z - 2x)^2}{(2x + 3y)^2 - 16z^2} + \frac{16z^2 - (2x - 3y)^2}{(3y + 4z)^2 - 4x^2} + \frac{4x^2 - (3y - 4z)^2}{(4z + 2x)^2 - 9y^2}$$

$$Y = \frac{(3y + 4z - 2x)(3y - 4z + 2x)}{(2x + 3y + 4z)(2x + 3y - 4z)} + \frac{(4z + 2x - 3y)(4z - 2x + 3y)}{(3y + 4z + 2x)(3y + 4z - 2x)} + \frac{(2x + 3y - 4z)(2x - 3y + 4z)}{(4z + 2x + 3y)(4z + 2x - 3y)}$$

$$Y = \frac{3y + 4z - 2x}{2x + 3y + 4z} + \frac{4z + 2x - 3y}{2x + 3y + 4z} + \frac{2x + 3y - 4z}{2x + 3y + 4z}$$

$$Y = \frac{3y + 4z - 2x + 4z + 2x - 3y + 2x + 3y - 4z}{2x + 3y + 4z}$$

$$Y = \frac{2x + 3y + 4z}{2x + 3y + 4z} = 1$$

So, 2017 (x + y)

2017 (1 + 1)

2017 × 2 = 4034 **Ans.**

4. The sum of the ages of a man and his wife is six times the sum of the ages of their children. Two years ago the sum of their ages was ten times the sum of the ages of their children. Six years hence the sum of their ages will be three times the sum of the ages of their children. How many children do they have?

Sol. Let present age of man = M

Let present age of wife = W

Let the no. of children = x

Let the sum of the ages of children = C

ATQ

M + W = 6 C … (1)

M– 2 + W – 2 = 10 (C – 2x) … (2)

M + 6 + W + 6 = 3 (C + 6x) … (3)

From (2) M + W – 4 = 10 C – 20 x

By using (1) 6 C – 4 = 10 C – 20 x

– 4C + 20x = 4

–C + 5x = 1 … (5)

From (3) M + W + 12 = 3C + 18x

By using (1) 6C + 12 = 3C + 18x

3C – 18x = – 12

C – 6x = – 4 … (6)

From (5) & (6)

–C + 5x = 1

C – 6x = – 4

∴ – x = – 3

x = 3 **Ans.**

5. **(a)** a, b, c are three natural numbers such that a x b x c = 27846. If $\frac{a}{6}$ = b + 4 = c - 4, find a + b + c.

(b) ABCDEFGH is a regular octagon with side length equal to a. Find the area of the trapezium ABDG.

Sol. **(a)** a ×b × c = 27846 … (1)

$$\frac{a}{b} = b + 4 = c - 4$$

a = 6b + 24

b = b

c = b + 8

Put value of a, b, c in (1)
(6b + 24) (b) (b + 8) = 27846
6(b + 4) (b) (b + 8) = 27846
(b + 4) (b) (b + 8) = 4641 = 13 × 17 × 3 × 7 = 13 × 17 × 21

$\therefore$ b = 13

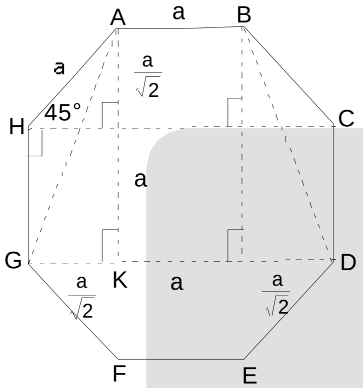
a = 6 × 13 + 24
= 78 + 24
= 102

c = b + 8 = 13 + 8 = 21

$\therefore$ a + b + c = 102 + 13 + 21 = 136

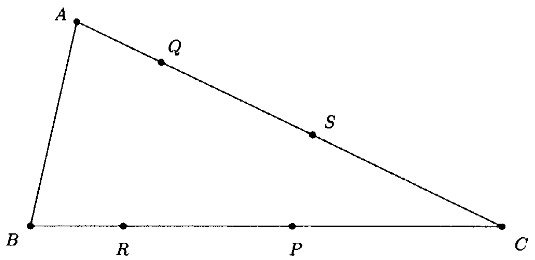
(b) Area of trapezium = $\frac{1}{2}$(AB + GD) AK

$$= \frac{1}{2}\left(a + a\left(1+\sqrt{2}\right)\right)\left(a + \frac{a}{\sqrt{2}}\right) = \frac{a^2}{2\sqrt{2}}\left(4 + 3\sqrt{2}\right)$$



6. **(a)** If a, b, c are positive real number such that no two of them are equal, show that a(a – b) (a – c) + b(b – c) (b – a) + c(c– a) (c – b) is always positive

(b) In the figure below, P, Q, R, S are point on the sides of the triangle ABC such that CP = PQ = QB = BA = AR = RS = SC



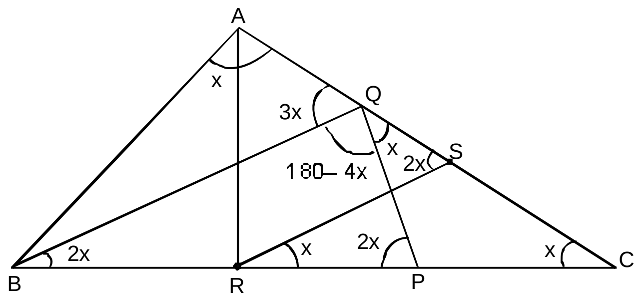
Sol. **(a)** Let a > b > c

a (a – b) (a – c) – b (b – c) (a – b) + c(c – a) (c – b)
(a – b) [a (a – c) – b (b – c)] + c(c – a) (c – b)
(a – b) [a^2 – ac – b^2 + bc] + c(c – a) (c – b)
(a – b) [a^2 – b^2 – (ac – bc)] + c(c – a) (c – b)
(a – b) [(a + b) (a – b) – c(a – b)] + c(c – a) (c – b)

$$\underbrace{(a-b)^2[a+b-c]}_{T_1} + \underbrace{c(c-a)(c-b)}_{T_2}$$

$T_1 > 0$, $T_2 > 0$
$\therefore$ $T_1 + T_2 > 0$.

(b)



∠ PCS = x

PC = PQ

∴ ∠PQC = x

∠QPR= 2x (Exterior angle)

∠QPR = ∠QBP = 2x

In ΔSRC

SC = SR

∠ SRC = ∠SCR = x

In ΔSRC

At ∠QSR = 2x

In ΔBQP

∠BQP = 180 – 4x

AQS is a straight line

So, ∠AQB = 3x

QB = AB

∠BAQ = 3x

AR = RS

∠ RAS = 2x

∠BAR = x

In ΔARS

∠ ARS = 180 – 4x

∠ ARB = 3x

AR = AB

∴ ∠ ABQ = x

∴ ∠ BAC = 3x

∠ ABC = 3x

∠ACB = x

In ΔABC

3x + 3x + x = 180

7x = 180

$x = \frac{180}{7}$.

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for **VIII** **IX** **X**

Test Dates: **on 29th October 2017 or 12th November 2017**

What are the Vardaan Schemes?

- **VARDAAN-1:** Above 90% marks in interview + test
- **VARDAAN-2:** Above 80% & less than 90% marks in interview + test
- **VARDAAN-3:** Above 75% and less than 80% marks in interview + test

Note: Valid for few study centers