

Note : •

Answer as many questions as possible.

Elegant and novel solution will get extra credits.

Diagrams and explanations should be given wherever necessary.

Fill in FACE SLIP and your rough working should be in the answer book itself

Maximum time allowed is THREE hour.

All questions carry equal marks.

1. A unit fraction is one of the form $\frac{1}{a}$ where 'a' is a natural number ($\neq 1$). Any proper fraction can be written as the sum of two or more unit fractions.

$$\{\text{Ex. } \frac{1}{2} = \frac{1}{3} + \frac{1}{6}, \frac{5}{6} = \frac{1}{2} + \frac{1}{3}, \frac{1}{24} = \frac{1}{54} + \frac{1}{72} + \frac{1}{108}\}$$

In the case of $\frac{1}{2}$, the factors of 2 are 1 and 2 only

$$\frac{1}{2} = \frac{1+2}{2(1+2)} = \frac{1}{2(1+2)} + \frac{2}{2(1+2)} = \frac{1}{6} + \frac{1}{3}$$

Express $\frac{1}{15}$ as the sum of two different unit fraction in 4 different ways.

Ans. $\frac{1}{15} = \frac{1+3}{15(1+3)}$

$$\frac{1}{15 \times 4} + \frac{3}{15 \times 4}$$

$$= \frac{1}{60} + \frac{1}{20} = \frac{1+3}{60} = \frac{4}{60} = \frac{1}{15}$$

$$\frac{1}{15} = \frac{1}{15} \frac{1+15}{(1+15)}$$

$$\frac{1}{15 \times 16} + \frac{15}{15 \times 16}$$

$$\frac{1}{240} + \frac{1}{16} \text{ Ans.}$$

$$\frac{1}{15} = \frac{1}{15} \left(\frac{1+5}{1+5} \right) = \frac{1}{15 \times 6} + \frac{5}{15 \times 6}$$

$$\frac{1}{90} + \frac{1}{18} \text{ Ans.}$$

$$\frac{1}{15} = \frac{1}{15} \frac{(3+5)}{(3+5)} = \frac{3}{15 \times 8} + \frac{5}{15 \times 8}$$

$$= \frac{1}{40} + \frac{1}{24} \text{ Ans.}$$

2. (a) The number 11284 and 7655 when divided by a certain 3 digit number leave the same remainder. Find the number and the remainder.
 (b) What is the least number to be subtracted from 1936 so that when divided by 9, 10 and 15 it will leave the same remainder in each case ?

Ans. (a) The no. 11284 and 7655 are divisible by a 3 digit no. and leaves same remainder so their different is also divisible by that 3 digit no.

$$11284 - 7655 = 3629$$

$$\text{only factors of } 3629 = 19 \times 191$$

so no. is 191 Ans.

Rem is 15 Ans.

(b) The L.C.M. of 9, 10, 15 is 90
 when divided by 1936 by 90
 Rem. = 46
 and Quotient = 21
 $90 \times 21 + 46 = 1936$
 $90 \times 21 = 1890$ is the largest multiple of 90 less than 1936.
 we required least no. which subtracted to give same rem. in each case
 so max. rem. is 8 so no. is
 1898
 so least no. have to subtract is
 $1936 - 1898 = 38$ Ans.

3. A, M, T, I represent different non-zero digits, It is given that

$$A + M + T + I = 11$$

$$A + M + I = 10$$

$$A + M = 1$$

Further in the following addition only one digit is given.

$$\begin{array}{r}
 A M T I A M T I \\
 A M T I A M T \\
 A M T I A M \\
 A M T I A \\
 A M T I \\
 A M T \\
 A M \\
 A \\
 \hline
 * * * * 5 * * *
 \end{array}$$

Sol. Fill up the stars writing proper reasons.

$$\begin{array}{l}
 A + M + T + I = 11 \quad \dots(i) \\
 A + M + I = 10 \quad \dots(ii) \\
 A + M = 1 \quad \dots(iii)
 \end{array}$$

from equation (i) and (ii) $T = 1$

from equation (2) and (3) $I = 5$

$$A + M = 5$$

The first row given addition as = 22

The next row will come by adding $[T + M + A + I + T + M + A + 2] = 19$

The third row = $[I + M + A + I + T + M + A] = 17$

The 4th row = $[I + A + I + T + M + A] = 13 + A = 15$ and $A = 2$ and $M = 3$

\therefore The sum is 35725792

4. ABC is a three digit number in which the digit A is greater than the digit B and C. If the difference between ABC and CBA is 297 and the difference between ABC and BAC is 450, find all possible three digit numbers ABC and find their sum also.

Ans. $A > B$ & $A > C$

$$\begin{array}{r}
 ABC \quad 100A + 10B + C \\
 - CBA \quad 100C + 10B + A \\
 \hline
 297 \quad \quad 297
 \end{array}$$

By solving $A - C = 3 \dots(i)$

$$ABC - BAC = 450$$

By solving $A - B = 5 \dots(ii)$

from i and ii $C - B = 2 \dots(iii)$

Put $A = 9$

so $B = 4$

& $C = 6 \quad ABC = 946$

when $A = 8$

$B = 3$

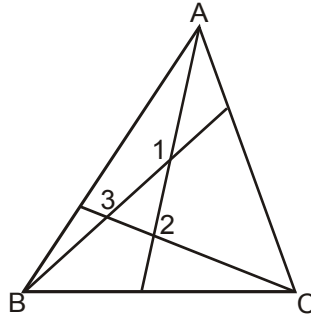
$C = 5 \quad ABC = 835$

When $A = 7$
 $B = 2$
 $C = 4$ $ABC = 724$

When $A = 6$
 $B = 1$
 $C = 3$ $ABC = 613$

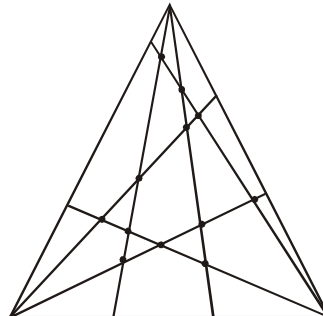
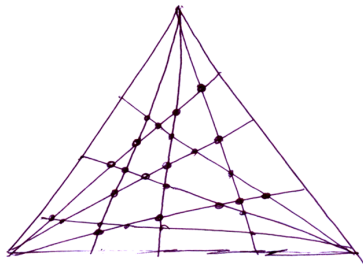
When $A = 5$
 $B = 0$
 $C = 2$ Not possible

5. take a triangle. Three straight lines are drawn through the vertices of the triangle as shown. The maximum number of points of intersection is 3. Draw two lines through each vertex of a triangle to meet the opposite sides. What is the maximum number of points of intersection.



Find again drawing the maximum number of points of intersection when three lines are drawn through each vertex.
 Without drawing can you guess the maximum points of intersection for 4 lines ?

Ans.



→By drawing 2 lines from each vertex we found maximum point of intersection is 12.

→By drawing 3 lines from each vertex we found maximum point of intersection is 27.

so pattern will be

for 1 $3 \times 1^2 = 3$

for 2 $3 \times 2^2 = 12$

for 3 $3 \times 3^2 = 27$

for 4 $3 \times 4^2 = 48$ Ans.