

# **KISHORE VAIGYANIK PROTSAHAN YOJANA - 2013**

### Date : 27-10-2013

**Duration : 3 Hours** 

Max. Marks: 160

# **STREAM - SB/SX**

# **GENERAL INSTRUCTIONS**

- The Test Booklet consists of **120** questions.
- There are Two parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

# **MARKING SCHEME :**

## PART-I:

# MATHEMATICS

Question No. 1 to 20 consist of ONE (1) mark for each correct response.

### PHYSICS

Question No. 21 to 40 consist of ONE (1) mark for each correct response.

### CHEMISTRY

Question No. 41 to 60 consist of ONE (1) mark for each correct response.

### BIOLOGY

Question No. 61 to 80 consist of ONE (1) mark for each correct response.

# PART-II:

#### MATHEMATICS

Question No. 81 to 90 consist of TWO (2) marks for each correct response.

### PHYSICS

Question No. 91 to 100 consist of TWO (2) marks for each correct response.

### CHEMISTRY

Question No. 101 to 110 consist of TWO (2) marks for each correct response.

### BIOLOGY

Question No. 111 to 120 consist of TWO (2) marks for each correct response.

# PART-I One Mark Questions

# MATHEMATICS

1. The sum of non-real roots of the polynomial equation  $x^3 + 3x^2 + 3x + 3 = 0$ . (A) equals 0 (B) lies between 0 and 1 (C) lies between -1 and 0 (D) has absolute value bigger than 1  $f(x) = x^3 + 3x^2 + 3x + 3 = 0$ Sol.  $f'(x) = 3x^2 + 6x + 3 = 3(x + 1)^2 \ge 0$ f(x) is Increasing function Now f(-3) = -6 < 0f(-2) = 1 > 0 $\therefore$  real root  $\gamma$  lies between – 3 and – 2  $\alpha + \beta + \gamma = -3;$   $-3 < \gamma < -2$  $\alpha + \beta - 3 < \alpha + \beta + \gamma < \alpha + \beta - 2$  $\alpha + \beta - 3 < -3 < \alpha + \beta - 2$  $-1 < \alpha + \beta < 0$ Ans.(C) 2. Let n be a positive integer such that Let  $\ell$  be the number of digits in the binary expansion of n. Then the minimum and the maximum possible values of  $\ell$  are (A) 5 and 16 (B) 5 and 17 (C) 4 and 16 (D) 4 and 17 Sol.  $16 < n < 2^{16}$ no. of digits in the binary expansion of 16 is 5 no. of digits in the binary expansion of 2<sup>16</sup> is 17 so no. of digits in the binary expansion of n lies in 5 to 16 Ans.(A) Let  $\omega$  be a cube root of unity not equal to 1. Then the maximum possible value of  $|a + bw + cw^2|$  where a, 3. b,  $c \in \{+1, -1\}$  is (C)  $\sqrt{3}$ (D)  $1 + \sqrt{3}$ (A) 0 (B) 2 Sol.  $|a + bw + cw^2|$ |a - c + (b - c)w|, for maximum value taking a = 1, c = -1, b = 1 $|a + bw + cw^2| = |2 + 2w| = 2|w^2| = 2$ Ans.(B)

4. If a, b are positive real numbers such that the lines ax + 9y = 5 and 4x + by = 3 are parallel, then the least possible value of a + b is (A) 13 (B) 12 (C) 8 (D) 6

Sol.

 $\frac{a}{4} = \frac{9}{b} \Rightarrow ab = 36$ 

using  $AM \ge G.M$ 

$$\frac{a+b}{2} \ge \sqrt{ab} \Rightarrow a+b \ge 12$$
Ans. (B)

5. Two line segments AB and CD are constrained to move along the x and y axes, respectively, in such a way that the points A, B, C, D are concyclic. If AB = a and CD = b, then the locus of the centre of the circle passing through A, B, C, D in polar coordinates is

(A) 
$$r^2 = \frac{a^2 + b^2}{4}$$
  
(B)  $r^2 \cos 2\theta = \frac{a^2 - b^2}{4}$   
(C)  $r^2 = 4(a^2 + b^2)$   
(D)  $r^2 \cos 2\theta = 4(a^2 - b^2)$   
 $2\sqrt{g^2 - c} = a$   
 $2\sqrt{f^2 - c} = b$   
Polar coordinates of centre of circle be (rcos $\theta$ , rsin $\theta$ )

$$g = -r \cos \theta$$
 and  $g^2 - f^2 = \frac{a^2 - b^2}{4}$   
 $f = -r \sin \theta$   $\therefore$   $r^2 \cos 2\theta = \frac{a^2 - b^2}{4}$ 

Ans.(B)

6. Consider a triangle ABC in the xy -plane with vertices A = (0,0), B = (1,1) and C = (9, 1). If the line x = a divides the triangle into two parts of equal area, then a equals (A) 3 (B) 3.5 (C) 4 (D) 4.5

Sol.

Sol.

P(a, 1)  
P(a, 1)  
B(1, 1)  
P(a, 1)  
Q(a, 
$$\frac{a}{9}$$
)  
A(0, 1)  
area of PQC =  $\frac{1}{2}$  area of ABC  
 $x = a$   
 $\frac{1}{2}(9-a)(1-\frac{a}{9}) = \frac{1}{2} \times \frac{1}{2}(8 \times 1)$   
(9 - a)<sup>2</sup> = 36  $\Rightarrow$  a = 3  
Ans. (A)



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7. Let ABC be an acute-angled triangle and let D be the midpoint of BC. If AB = AD, then tan(B)/tan(C) equals



Ans.(D)

The angles  $\alpha$ ,  $\beta$ ,  $\gamma$  of a triangle satisfy the equations  $2\sin\alpha + 3\cos\beta = 3\sqrt{2}$  and  $3\sin\beta + 2\cos\alpha = 1$ . Then 8. angle  $\gamma$  equals (A) 150° (B) 120° (C) 60° (D) 30°  $2\sin\alpha + 3\cos\beta = 3\sqrt{2}$ Sol. ....(1)  $3\sin\beta + 2\cos\alpha = 1$ ....(2) sum of squares of equation (1) and (2)  $4 + 9 + 12 \sin(\alpha + \beta) = 19$  $\sin (\alpha + \beta) = \frac{1}{2} \Rightarrow \alpha + \beta = 150^{\circ} \text{ or } 30^{\circ}$ If  $\alpha + \beta = 30^{\circ} \Rightarrow \beta = 30 - \alpha$ put in equation (1) and (2) we get 7 sin  $\alpha$  + 3 $\sqrt{3}$  cos  $\alpha$  = 6 $\sqrt{2}$  $7 \cos \alpha - 3\sqrt{3} \sin \alpha = 2$  $\cos \alpha = \frac{7 + 9\sqrt{6}}{37} = .8 < \frac{\sqrt{3}}{2}$  $\therefore \alpha > 30^{\circ}$   $\therefore \alpha + \beta \neq 30^{\circ}$  $\cos \alpha < \cos 30^{\circ}$ Ans.(D) Let  $f : R \to R$  be a function such that  $\lim_{x \to \infty} f(x) = M > 0$ . Then which of the following is **false**? 9. (A)  $\lim_{x\to\infty} x\sin(1/x) f(x) = M$ (B)  $\lim_{x \to \infty} \sin(f(x)) = \sin M$ (D)  $\lim_{x \to \infty} \frac{\sin x}{x} f(x) = 0$ (C)  $\lim_{x\to\infty} x\sin(e^{-x}) f(x) = M$ 

 $\lim_{x \to \infty} x \sin(e^{-x}) f(x) = \lim_{x \to \infty} \frac{\sin(e^{-x})}{e^{-x}} \frac{x}{e^x} f(x)$ Sol.  $= 1 \times (0) \times M = 0$ Ans.(C)

 $p_t(x) = (sint)x^2 - (2cost)x + sint$ 

be a family of quadratic polynomials in x with variable coefficients. Let  $A(t) = \int_{a}^{b} p_t(x) dx$ . Which of the

following statements are true? (I) A(t) < 0 for all t. (II) A(t) has infinitely many critical points. (III) A(t) = 0 for infinitely many t. (IV) A'(t) < 0 for all t. (A) (I) and (II) only (B) (II) and (III) only (C) (III) and (IV) only (D) (IV) and (I) only

**Sol.** 
$$A(t) = \int_{0}^{1} \{(\sin t)x^2 - (2\cos t)x + \sin t\} dx$$

$$A(t) = \frac{\sin t}{3} - \cos t + \sin t = \frac{4}{3}\sin t - \cos t$$

$$A'(t) = \frac{4\cos t}{3} + \sin t$$

St. I and IV are False

Ans.(B)

**11.** Let  $f(x) = \sqrt{2 - x - x^2}$  and  $g(x) = \cos x$ . Which of the following statements are true? (I) Domain of  $f((g(x))^2) =$  Domain of f(g(x))(II) Domain of f(g(x)) + g(f(x)) = Domain of g(f(x))(III) Domain of f(g(x)) = Domain of f(g(x))(IV) Domain of  $g((f(x))^3) =$  Domain of f(g(x))(A) Only (I) (B) Only (I) and (II) (C) Only (III) and (IV) (D) Only (I) and (IV)

**Sol.** Domain of f(g(x)) is R

 $\begin{array}{l} \because 2 - \cos x - \cos^2 x \geq 0 \\ (\cos x + 2)(\cos x - 1) \leq 0 \Rightarrow -2 \leq \cos x \leq 1 \\ x \in R \\ \text{Domain of g(f(x)) is [-2, 1]} \\ \because \cos \left(\sqrt{2 - x - x^2}\right) \end{array}$ 

$$\begin{array}{l} 2 - x - x^2 \geq 0\\ \\ \text{Domain of } f(g(x)^2) \text{ is } R\\ \because 2 - \cos^2 x - \cos^4 x \geq 0\\ (\cos^2 x + 2)(\cos^2 x - 1) \leq 0\\ -1 \leq \cos x \leq 1\\ x \in R\\ \\ \\ \text{Domain of } g(f^3(x) \text{ is Domain of } g(f(x))\\ \text{ i.e., } [-2, 1]\end{array}$$

Ans.(B)



- **12.** For real x with  $-10 \le x \le 10$  define  $f(x) = \int_{-10}^{2^{[t]}} dt$ , where for a real number r we denote by [r] the largest integer less than or equal to r. The number of points of discontinuity of f in the interval (-10,10) is (A) 0 (B) 10 (C) 18 (D) 19
- **Sol.** Let r be an integer in (-10, 10)

Now, LHL = 
$$\lim_{x \to r^{-}} \int_{-10}^{x} 2^{[t]} dt$$
  
=  $\lim_{h \to 0^{+}} \left[ \int_{-10}^{-9} 2^{[t]} dt + \int_{-9}^{-8} 2^{[t]} dt + \dots + \int_{r-1}^{r-h} 2^{[t]} dt \right]$   
=  $\lim_{h \to 0} \left[ 2^{-10} + 2^{-9} + \dots + 2^{r-1} (1-h) \right]$   
=  $2^{-10} + 2^{-9} + \dots + 2^{r-1} \dots (1)$   
 $\lim_{x \to r^{+}} \int_{-10}^{x} 2^{[t]} dt$   
=  $\lim_{h \to 0^{+}} \left[ \int_{-10^{2}}^{-9} 2^{[t]} dt + \int_{-9}^{-8} 2^{[t]} dt + \dots + \int_{r}^{r+h} 2^{[t]} dt \right]$   
=  $2^{-10} + 2^{-9} + \dots + 2^{r-1} \dots (2)$   
f(r) =  $\int_{-10}^{r} 2^{[t]} dt$   
=  $2^{-10} + 2^{-9} + \dots + 2^{r-1} \dots (3)$   
From (1), (2) & (3)  
f(x) is continous at all integers.  
**Ans. (A)**

13. For a real number x let [x] denote the largest integer less than or equal to x and  $\{x\} = x - [x]$ . The smallest

possible integer value of n for which  $\int_{1}^{1} [x] [x] dx$  exceeds 2013 is (A) 63 (B) 64 (C) 90 (D) 91 Sol.  $\int_{1}^{n} [r] \{r\} dx = \sum_{r=1}^{n-1} \int_{r}^{r+1} r(x-r) dx$   $\sum_{r=1}^{n-1} r \left[ \frac{x^{2}}{2} - rx \right]_{r}^{r+1}$   $\sum_{r=1}^{n-1} r \left[ \frac{(r+1)^{2} - r^{2}}{2} - r.1 \right]$   $\sum_{r=1}^{n-1} r \left[ \frac{1}{2} \right] = \frac{1}{2} \frac{n(n-1)}{2}$   $\frac{n(n-1)}{4} \ge 2013$   $n(n-1) \ge 4 \times 2013$   $\left( n - \frac{1}{2} \right)^{2} \ge \frac{2013 \times 16 + 1}{4}$   $n \ge \frac{\sqrt{32209}}{2} + \frac{1}{2}$ least n = 91Ans. (D)



14. The area bounded by the curve  $y = \cos x$ , the line joining  $(-\pi/4, \cos(-\pi/4))$  and (0, 2) and the line joining  $(\pi/4, \cos(\pi/4))$  and (0, 2) is



#### Ans (A)

**15.** A box contains coupons labeled 1, 2, 3...,n. A coupon is picked at random and the number x is noted. The coupon is put back into the box and a new coupon is picked at random. The new number is y. Then the probability that one of the numbers x, y divides the other is (in the options below [r] denotes the largest integer less than or equal to r)

(A) 
$$\frac{1}{2}$$
  
(B)  $\frac{1}{n^2} \sum_{k=1}^{n} \left[ \frac{n}{k} \right]$   
(C)  $-\frac{1}{n} + \frac{1}{n^2} \sum_{k=1}^{n} \left[ \frac{n}{k} \right]$   
(D)  $-\frac{1}{n} + \frac{2}{n^2} \sum_{k=1}^{n} \left[ \frac{n}{k} \right]$   
Let  $x = 1$ 

Sol. Let

Sol.

favourable out comes (1, 1), (1, 2) .....(1, n) no. of favourable out comes when x = 1

$$= \left\lfloor \frac{n}{1} \right\rfloor$$

 $\therefore$  no. of favourable out comes when x = 1 or y = 1

$$=2\left[\frac{n}{1}\right]-1$$

no. of favourable out comes when x = 2 or y = 2 but  $x \neq 1$ ,  $y \neq 1$ 

$$2\left[\frac{n}{2}\right]-1$$

Similarly

no. of favourable out comes when x = k or y = k but  $x, y \notin \{1, 2, \dots, k-1\}$ 

$$2\left\lfloor \frac{n}{k} \right\rfloor - 1$$

So probability =  $\frac{\sum_{k=1}^{n} \left[\frac{n}{k}\right] - (1 + 1.....n \text{ times})}{n^2}$ 

$$=\frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]-\frac{1}{n}$$
Ans. (D)

**16.** Let  $n \ge 3$ . A list of numbers  $0 < x_1 < x_2 < ... < x_n$  has mean  $\mu$  and standard deviatiion  $\sigma$ . A new list of numbers is made as follows :  $y_1 = 0$ ,  $y_2 = x_2$ , ...,  $y_{n-1} = x_{n-1}$ ,  $y_n = x_1 + x_n$ . The mean and the standard deviation of the new list are  $\hat{\mu}$  and  $\hat{\sigma}$ . Which of the following is necessarily true?

(A) 
$$\mu = \hat{\mu}, \sigma \le \hat{\sigma}$$
 (B)  $\mu = \hat{\mu}, \sigma \ge \hat{\sigma}$  (C)  $\sigma = \hat{\sigma}$  (D)  $\mu$  may or may not be equal to  $\hat{\mu}$   
Sol.  $\hat{\mu} = \frac{0 + x_2 + x_3 \dots x_{n-1} + (x_1 + x_n)}{n} = \mu$   
 $\hat{\sigma}^2 = \frac{1}{n} \Sigma y_1^2 - \hat{\mu}^2$   
 $= \frac{1}{n} [0^2 + x_2^2 + x_3^2 \dots (x_{n-1})^2 + (x_1 + x_n)^2] - \mu^2$   
 $\hat{\sigma}^2 - \sigma^2 = \frac{2x_1 x_n}{n} > 0$   
So  $\hat{\sigma} > \sigma$ 

Ans.(A)

**17.** Let  $\overrightarrow{v_1, v_2, v_3, v_4}$  be unit vectors in the xy -plane, one each in the interior of the four quadrants. Which of the following statements is necessarily true?

(A) 
$$\overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} + \overrightarrow{v_4} = 0$$

- (B) There exist i, j with  $1 \le i < j \le 4$  such  $\overrightarrow{v_i + v_j}$  is in the first quadrant
- (C) There exist i, j with  $1 \le i < j \le 4$  such that  $\overrightarrow{v_i}, \overrightarrow{v_j} < 0$
- (D) There exist i, j with  $1 \le i < j \le 4$  such that  $\overrightarrow{v_i}, \overrightarrow{v_j} > 0$



In this case B, C, D are not possible. **Ans. (A)** 

- **18.** The number of integers n with  $100 \le n \le 999$  and containing at most two distinct digits is (A) 252 (B) 280 (C) 324 (D) 360
- **Sol.** Total Integers = 999 99 = 900

Total Integers in which all distinct digits

so 900 - 648 = 252

Ans. (A)



- **19.** For an integer n let  $S_n = \{n + 1, n + 2, ...., n + 18\}$ . Which of the following is true for all  $n \ge 10$ ? (A)  $S_n$  has a multiple of 19 (B)  $S_n$  has a prime (C)  $S_n$  has at least four multiples of 5 (D)  $S_n$  has at most six primes
- **Sol.** n + 1, n + 2, ..... n + 18
  - (A) False, if n = 19
  - (C) False if n = 15

16 to 33

20, 25, 30  $\rightarrow$  only three multiples of 5

(D) no. of odd integers in  $S_n = 9$ 

every third odd integer is multiple of 3

so maximum prime no. = 6.

Ans.(D)

**20.** Let P be a closed polygon with 10 sides and 10 vertices (assume that the sides do not intersect except at the vertices). Let k be the number of interior angles of P that are greater than 180°. The maximum possible value of k is



Sum of angles of closed polygon with 10 sides is  $8\pi$ . So maximum number of possible obtuse angle is 7. **Ans. (C)** 

# PHYSICS

21. Consider an initially neutral hollow conducting spherical shell with inner radius r and outer radius 2r. A point charge +Q is now placed inside the shell at a distance r/2 from the centre. The shell is then grounded by connecting the outer surface to the earth. P is an external point at a distance 2r from the point charge +Q on the line passing through the centre and the point charge +Q as shown in the figure.



The magnitude of the force on a test charge +q placed at P will be

A) 
$$\frac{1}{4\pi\epsilon_0} \frac{qQ}{4r^2}$$
 (B)  $\frac{1}{4\pi\epsilon_0} \frac{9qQ}{100r^2}$  (C)  $\frac{1}{4\pi\epsilon_0} \frac{4qQ}{25r^2}$  (D) 0

Sol. Charge on outer most surface is zero Hence force on q is also '0' Ans. (D)



22. Consider the circuit shown in the figure below :



All the resistors are identical. The charge stored in the capacitor, once it is fully charged, is



Sol.

$$V_{AB} = \frac{5\frac{R}{3}}{\frac{5R}{3} + R} \times V = \frac{5}{8}V$$
$$Q = \frac{5}{8}CV$$
Ans (D)

23. A nuclear decay is possible if the mass of the parent nucleus exceeds the total mass of the decay particles. If M(A, Z) denotes the mass of a single neutral atom of an element with mass number A and atomic number Z, then the minimal condition that the  $\beta$  decay

$$X_7^A \rightarrow Y_{7+1}^A + \beta^- + \overline{v}_e$$

will occur is (m denotes the mass of the  $\beta$  particle and the neutrino mass m, can be neglected) : (A) M(A, Z) > M(A, Z + 1) + m (B) M(A, Z) > M(A, Z + 1) (C) M(A, Z) > M(A, Z + 1) + Zm (D) M(A, Z) > M(A, Z + 1) - m (D) M(A, Z) > M(A, Z + 1) - m (D) M(A, Z) = M(A, Z + 1) - m (B) M(A, Z) = M(A, Z + 1) - M (B) M(A, Z) = M(A, Z + 1) -

#### Ans.(A)

24. The equation of state of n moles of a non-ideal gas can be approximated by the equation

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

where a and b are constants characteristic of the gas. Which of the following can represent the equation of a quasistatic adiabat for this gas (Assume that  $C_V$ , the molar heat capacity at constant volume, is independent of temperature)?

(A) 
$$T(V-nb)^{R/C_V} = constant$$
  
(B)  $T(V-nb)^{C_V/R} = constant$   
(C)  $\left(T + \frac{ab}{V^2R}\right) (V-nb)^{R/C_V} = constant$   
(D)  $\left(T + \frac{n^2ab}{V^2R}\right) (V-nb)^{C_V/R} = constant$ 

Ans. (A)



25. A blackbox (BB) which may contain a combination of electrical circuit elements (resistor, capacitor or inductor) is connected with other external circuit elements as shown below in the figure (a). After the switch (S) is closed at time t = 0, the current (I) as a function of time (t) is shown in the figure (b).



From this we can infer that the blackbox contains

- (A) A resistor and a capacitor in series
- (B) A resistor and a capacitor in parallel
- (C) A resistor and an inductor in series
- (D) A resistor and an inductor in parallel
- Sol. I t graph is for L-R series circuit. Ans. (C)
- 26. In a photocell circuit the stopping potential,  $V_0$ , is a measure of the maximum kinetic energy of the photoelectrons. The following graph shows experimentally measured values of stopping potential versus frequency v of incident light.



The values of Plank's constant and the work function as determined from the graph are (taking the magnitude of electronic charge to be  $e = 1.6 \times 10^{-19} \text{ C}$ )

(A)  $6.4 \times 10^{-34}$  Js, 2.0 eV (B)  $6.0 \times 10^{-34}$  Js, 2.0 eV (C)  $6.4 \times 10^{-34}$  Js, 3.2 eV (D)  $6.0 \times 10^{-34}$  Js, 3.2 eV V =  $\frac{hv}{e} - \phi$ 

Hence from graph  $\phi = 2eV$ 

$$\frac{h}{e} = slope = \frac{6}{1.6 \times 10^{15}}$$
$$h = \frac{6 \times 1.6 \times 10^{-19}}{1.6 \times 10^{15}} = 6.0 \times 10^{-34}$$
Ans. (B)



27. An engine moving away from a vertical cliff blows a born at a frequency f. Its speed is 0.5% of the speed of sound in air. The frequency of the reflected sound received at the engine is (A) 0.990 f (B) 0.995 f (C) 1.005 f (D) 1.010 f

Sol. 
$$f_1 = f_0 \left[ \frac{v}{v + v_s} \right]$$
  
 $f_R = f_1 \left[ \frac{v - v_s}{v} \right] = f_0 \left[ \frac{v - v_s}{v + v_s} \right] = \left[ 1 - \frac{2v_s}{v} \right] f$   
= 0.990 f  
Ans. (A)

28. An arangement with a pair of quarter circular coils of radii r and R with a common centre C and carrying a current I is shown.



The permeability of free space is  $\mu_0$ . The magnetic field at C is (A)  $\mu_0 I(1/r - 1/R)/8$  into the page (B)  $\mu_0 I (1/r - 1/R)/8$  out of the page (C)  $\mu_0 I(1/r + 1/R)/8$  out of the page (D)  $\mu_0 I(1/r + 1/R)/8$  into the page

**Sol.** B due to Arc = 
$$\frac{\mu_0 I \theta}{4\pi r}$$

 $\frac{\mu_0 i}{8} \left[ \frac{1}{r} - \frac{1}{R} \right]$  out of the page Ans.(B)

29. The circuit shown has been connected for a long time. The voltage across the capacitor is



**30.** A wheel of radius R with an axle of radius R/2 is shown in the figure and is free to rotate about a frictionless axis through its centre and perpendicular to the page. Three forces (F, F, 2F) are exerted tangentially to the repective rims as shown in the figure.



The magnitude of the net torque acting on the system is nearly (A) 3.5FR (B) 3.2 FR (C) 2.5 FR (D) 1.5 FR

**Sol.** 
$$\tau_{\text{Net}} = F\left(\frac{R}{2}\right) + FR + 2FR = 3.5 FR$$

**31.**Two species of radioactive atoms are mixed in equal number. The disintegration of the first species is  $\lambda$  and<br/>of the second is  $\lambda/3$ . After a long time the mixture will behave as a species with mean life of approximately<br/>(A)  $0.70/\lambda$  (B)  $2.10/\lambda$  (C)  $1.00/\lambda$  (D)  $0.52/\lambda$ 

**Sol.** avg life = 
$$\frac{\int t dN_1 + t dN_2}{2N_0}$$

where  $dN_1 = \lambda N_0 e^{-\lambda t} dt$ 

$$dN_2 = \frac{\lambda}{3} N_0 e^{-\frac{\lambda}{3}t} dt$$

avg. life = 
$$\frac{\int_{0}^{\infty} t(\lambda N_{0}e^{-\lambda t}dt + \frac{\lambda}{3}N_{0}e^{-\frac{\lambda}{3}t}dt}{2N_{0}}$$

Intergrating we got

avg life = 
$$\frac{2}{\lambda} \approx \frac{2.10}{\lambda}$$

Ans.(B)

**32.** The bulk modulus of a gas is defined as B = -VdP/dV. For an adiabatic process the variation of B is proportional to  $P^n$ . For an idea gas, n is

(A) 0 (B) 1 (C) 
$$\frac{5}{2}$$
 (D) 2

**Sol.** For adiabatic process  $PV^{\gamma} = C$ 

$$P\gamma V^{\gamma-1} + \frac{dp}{dv} v^{\gamma} = 0$$
  
$$\gamma P = -V \frac{dp}{dv}$$
  
Hence n = 1  
**Ans. (B)**



**33.** Photons of energy 7 eV are incident on two metals A and B with work functions 6 eV and 3 eV respectively. The minimum de Broglie wavelengths of the emitted photoelectrons with maximum energies are  $\lambda_A$  and  $\lambda_B$ , respectively where  $\lambda_A/\lambda_B$  is nearly (A) 0.5 (B) 1.4 (C) 4.0 (D) 2.0

Ans. (D)

34. An electron enters a chamber in which a uniform magnetic field is present as shown. Ignore gravity.



Magnetic field

During its motion inside the chamber

(A) the force on the electron remains constant

(B) the kinetic energy of the electron remains constant

(C) the momentum of the electron remains constant

(D) the speed of the electron increases at a uniform rate

Sol. ∴ work done = 0 Hence kinetic energy = constant Ans. (B)

**35.** A ray of light incident on a glass sphere (refractive index  $\sqrt{3}$ ) suffers total internal reflection before emerging out exactly parallel to the incident ray. The angle of incidence was (A) 75° (B) 30° (C) 45° (D) 60°



 $\frac{\sin i}{\sin r} = \sqrt{3}$ 

 $2\cos r = \sqrt{3}$ r = 30° & i = 60° Note : But for r = 30° TIR cannot take place at B. **Ans. (D)** 

**36.** Young-Laplace law states that the excess pressure inside a soap bubble of radius R is given by  $\Delta P = 4\sigma/R$  where  $\sigma$  is the coefficient of surface tension of the soap. The Eötvös number  $E_0$  is a dimensionless number that is used to describe the shape of bubbles rising through a surrounding fluid. It is a combination of g, the acceleration due to gravity,  $\rho$ , the density of the surrounding fluid,  $\sigma$  and a characteristic length scale L which could be the radius of the bubble. A possible expression for  $E_0$  is

(A) 
$$\frac{\rho g}{\sigma L^3}$$
 (B)  $\frac{\rho L^2}{\sigma g}$  (C)  $\frac{\rho g L^2}{\sigma}$  (D)  $\frac{g L^2}{\sigma \rho}$ 

Sol. Check dimensionally Ans. (C)



37. A plank is resting on a horizontal ground in the northern hemisphere of the Earth at a 45° latitude. Let the angular speed of the Earth be  $\omega$  and its radius r<sub>e</sub>. The magnitude of the frictional force on the plank will be



 $N = 40 \times 6.023 \times 10^{23} \ \tilde{}\ 24 \times 10^{24}$ Average sep. =  $\left(\frac{1}{N}\right)^{\frac{1}{3}} \approx 1 \text{ nm}$ Ans.(A)

39. A point particle of mass 0.5 kg is moving along the x-axis under a force described by the potential energy V shown below. It is projected towards the right from the origin with a speed v. What is the minimum value of v for which the particle will escape infinitely fasr away from the origin ?



**40.** The figure below shows pressure variation in two different sound waves in air with time at a given position. Both the figures are drawn to the same scale.



Which of the following statements is true?

- (A) Wave 1 has lower frequency and smaller amplitude compared to wave 2
- (B) Wave 1 has higher frequency and greater amplitude compared to wave 2
- (C) Wave 1 has shroter wavelength and greater amplitude compared to wave 2
- (D) Wave 1 has shorter wavelength and smaller amplitude compared to wave 2

Ans. (D)

# CHEMISTRY

41.	Among the following, the set of isoelectronic ions is			
	(A) Na <sup>+</sup> , Mg <sup>2+</sup> , F <sup>-</sup> , Cl <sup>-</sup>	(B) Na⁺, Ca²+, F−, O²−		
	(C) Na⁺, Mg²⁺, F <sup></sup> , O²	(D) Na <sup>+</sup> , K <sup>+</sup> , S <sup>2—</sup> , CI <sup>—</sup>		
Sol.	Na <sup>+</sup> , Mg <sup>2+</sup> , F <sup>-</sup> & O <sup>2-</sup> are isoelectronic(10 electron speceis)			
	Ans. (C)			

- **42.** For a zero-order reaction with rate constant k, the slope of the plot of reactant concentration against time is (A) k/2.303 (B) k (C) -k/2.303 (D) -k
- **Sol.**  $C_t = C_0 kt$ ; slope = -k**Ans. (D)**
- 43.The compound which reacts with excess bromine to produce 2, 4, 6-tribromophenol, is<br/>(A) 1, 3-cyclohexadiene<br/>(C) salicylic acid(B) 1, 3-cyclohexanedione<br/>(D) cyclohexanone



Ans.(C)

-八

44. Ethyl acetate reacts with NH<sub>2</sub>NHCONH<sub>2</sub> to form (B) CH<sub>3</sub>CON(NH<sub>2</sub>)CONH<sub>2</sub> (A) CH<sub>2</sub>CONHCONHNH<sub>2</sub> (D) CH, CH, NHNHCONH, (C) CH<sub>2</sub>CONHNHCONH<sub>2</sub>

more nucleophilic (localised lone pair)

 $CH_{3}-C-OC_{2}H_{5} + H_{2}\dot{N}-\dot{N}H - C - \dot{N}H_{2} \longrightarrow CH_{3} - C - NH - NH - C - NH_{2}$ Sol. Ans.(C)

45. The variation of solubility of four different gases (G1, G2, etc.) in a given solvent with pressure at a constant temperature is shown in the plot.



Ans.(D)

46. For the reaction, A  $\longrightarrow$  n B the concentration of A decreases from 0.06 to 0.03 mol L<sup>-1</sup> and that of B rises from 0 to 0.06 mol L<sup>-1</sup> at equilibrium. The values of n and the equilibrium constant for the reaction, respectively, are (C) 3 and 0.12 (A) 2 and 0.12 (B) 2 and 1.2 (D) 3 and 1.2

Sol.

n B t = 00.06 0 t = t0.03 0.06  $\alpha c = 0.03$  $n\alpha c = 0.06$ n = 2  $K_c = \frac{(0.06)^2}{0.03} = \frac{(0.06)^2}{0.03} = 0.12$ Ans.(A)

The reaction of ethyl methyl ketone with Cl<sub>2</sub>/excess OH<sup>-</sup> gives the following major product 47. (A) CICH, CH, COCH, (B) CH<sub>2</sub>CH<sub>2</sub>COCCI<sub>2</sub> (D) CH<sub>2</sub>CCl<sub>2</sub>COCH<sub>2</sub>Cl (C) CICH, CH, COCH, CI 

Sol.

Ans.(B)

48. The compound that readily tautomerizes is (A) CH<sub>3</sub>COCH<sub>2</sub>CO<sub>2</sub>C<sub>2</sub>H<sub>5</sub> (B) CH<sub>3</sub>COCH<sub>2</sub>CH<sub>2</sub>CH<sub>3</sub>  $(D) (CH_3)_3 CCOC(CH_3)_3$  $CH_3 - C - C - CH_5 = CH_5 = CH_5$ ightarrow CH<sub>3</sub> – C = CH – C – OC<sub>2</sub>H<sub>5</sub> Ĩ Sol. Ĥ. ŚÖ Ö ÓН Ö Aceto acetic ester has active methylene group. Ans.(A) 49. Hydrolysis of BCl, gives X which on treatment with sodium carbonate produces Y, X and Y, respectively, are (B)  $H_3BO_3$  and  $Na_2B_4O_7$ (D)  $B_2O_3$  and  $Na_2B_4O_7$ (A) H<sub>2</sub>BO<sub>2</sub> and NaBO<sub>2</sub> (C)  $B_2O_3$  and  $NaBO_2$  $BCl_3 + 3H_2O \longrightarrow H_3BO_3 + 3HCI$ Sol.  $4H_3BO_3 + Na_2CO_3 \longrightarrow Na_2B_4O_7 + CO_2 + 6H_2O_3$ Ans.(B) The numbers of lone pair(s) on Xe in  $XeF_2$  and  $XeF_4$  are, respectively, 50. (A) 2 and 3 (B) 4 and 1 (C) 3 and 2 (D) 4 and 2  $XeF_2$ ; no. of lp on Xe = 3 XeF<sub>4</sub>; no. of lp on Xe = 2 Ans. (C) Sol. 51. The entropy change in the isothermal reversible expansion of 2 moles of an ideal gas from 10 to 100 L at 300 K is (A) 42.3 JK<sup>-1</sup> (C) 38.3 J K<sup>-1</sup> (B) 35.8 J K<sup>-1</sup> (D) 32.3 J K<sup>-1</sup>  $\Delta S = \frac{nRT}{T} ln \frac{v_2}{v_1}$ Sol. = nR ln  $\frac{V_2}{V_1}$ = 2.303 nR log  $\frac{V_2}{V_1}$ = 2.303 × 2 × 8.314 × 1 = 38.3 J/k Ans.(C) 52. D-Glucose upon treatment with bromine-water gives СНО COOH -OH —он H-H-HO--H HO--H OH н OH (A) (B) H -OH H -OH CH<sub>2</sub>OH ĊHO CHO CHO ⊨o —ОН Н-HO-—Н HO--H H--OH -OH (C) (D) -OH H ·ОН н ĊH,OH ĊOOH COOH CHO Sol. H -OH -OH H HO -H HO--H OH H-·ОН OH H -OH CH<sub>2</sub>OH ĊH,OH **D-Glucose** Ans.(A) **Resonance** Educating for better tomorrow

**53.** In the structure of borax, the numbers of boron atoms and B-O-B units, respectively, are (A) 4 and 5 (B) 4 and 3 (C) 5 and 4 (D) 5 and 3







54. The number of peptide bonds in the compound



**59.** The order of  $S_N^1$  reactivity in aqueous acetic acid solution for the compounds

 $H_3C - CH_2 - CH_2 - CH_2$ H<sub>3</sub>C — -CH<sub>2</sub>--- Cl  $(H_3C)_3C - CI$ 3 is (B) 1 > 3 > 2 (C) 3 > 2 > 1 (D) 3 > 1 > 2 (A) 1 > 2 > 3  $CH_{3} \xrightarrow{C} CH_{3} \xrightarrow{C} CH_{$ Sol.  $CH_3 - CH_2 - CH_2 - CI \longrightarrow CH_3 - CH_2 - CH_2$  $\begin{array}{c} O & O \\ \parallel \\ CH_{3} - C - CH_{2} - CI & \longrightarrow CH_{3} - C - CH_{3} \end{array}$ Carbocation stability order  $CH_3 - C + CH_3 > CH_3 - CH_2 - CH_2 > CH_3 - C - CH_2$ So reactivity order is  $(CH_3)_3 C-CI > CH_3 - CH_2 - CH_2 - CI > CH_3 - C - CH_2 - CI$ Ans.(C) 60. An ionic compound is formed between a metal M and a non-metal Y. If M occupies half the octahedral voids in the cubic close-packed arrangement formed by Y, the chemical formula of the ionic compound is (C) M<sub>2</sub>Y (A) MY (B) MY (D) MY<sub>3</sub> Z (effective no. of atoms/ unit cell) Sol. Υ 4 Μ 2 then formula of ionic compound is MY<sub>2</sub> Ans.(B) BIOLOGY 61. Human fetal haemoglobin differs from the adult haemoglobin in that it has (A) higher affinity for oxygen (B) lower affinity for oxygen (C) two subunits only (D) is glycosylated Sol. HbF have more affinity for oxygen and this property help in extracting O<sub>2</sub> from mother. Ans.(A) 62. Nucleolus is an organelle responsible for the production of (A) carbohydrates (B) messenger RNA (C) lipids (D) ribosomal RNA Sol. In nucleolus RNA-PI is present. Ans.(D)

63.	The sequences of four DNA molecules are given below: i. TATATATATATATATA ii. TTTCCCGGGAAA		
	ATATATATATATATATATATATATATATATATATATAT		
	AACGCAACGGG CGGCCTAGGCCG		
	Which one of these DNA moelcules will have the highest melting temperature (T <sub>m</sub> )? (A) i (B) ii (C) iii (D) iv		
Sol.	In GC Pairing 3 H bonds are present.		
	Ans. (D)		
64.	If DNA codons are ATG GAA, insertion of thymine after the first codon results in, (A) non-sense mutation (B) mis-sense mutation (C) frameshift mutation (D) silent mutation		
Sol.	Entry of T Cause formation of stop codon.		
	Ans.(A)		
65.	Genetic content of a cell reduces to half during		
	(A) meiotic prophase I(B) mitotic prophase(C) meiotic prophase II(D) meiotic telophase		
	Ans.(D)		
66.	Which one of the following techniques is used for the detection of proteins ? (A) Northern blotting (B) Western blotting (C) Southern blotting (D) In-situ hybridization		
	Ans.(B)		
67.	Fission yeasts are (A) Archaebacteria (B) Eubacteria (C) Prokaryotes (D) Eukaryotes		
Sol.	Schizosaccharomyces pombe is called fission yeast.		
	Ans.(D)		
68.	In green leaves, the light and dark reactions occur in (A) stroma and grana respectively (B) grana and stroma respectively (C) cristae and matrix respectively		
Cal	(D) both occur in cytoplasm		
Sol.	Grana bear photosystem and stroma have enzymes of dark reaction. Ans. (B)		
69.	According to Mendel,assort independently. (A) alleles of a gene; alleles of different genes (B) alleles of different genes; alleles of a gene (C) dominanat traits; recessive traits (D) recessive traits; recessive traits		
Sol.	2 alleles of a gene separate in neiosis and Non linked different genes segregate independently. <b>Ans. (A)</b>		
70.	The two enzymatic activities associated with RUBISCO are(A) oxidase and oxygenase(B) oxygenase and carboxylase(C) oxidase and carboxylase(D) oxygenase and carbamylationAns. (B)		
71.	<ul> <li>Chlorofluorocarbons (CFCs) are belived to be associated with cancers because,</li> <li>(A) CFCs react with DNA and cause mutations</li> <li>(B) CFCs react with proteins involved in DNA repair</li> <li>(C) CFCs destroy the ozone layer and permit harmful UV rays to reach the earth</li> <li>(D) CFCs react with DNA polymerase and reduce fidelity of DNA replication</li> </ul>		
Sol.	UV rays damage DNA causing thymine dimerisation.		
	Ans. (C)		



		KVPY QUESTION PAPER - STREAM (SB / SX	7)		
72.	Morphogenetic movements take place predominantly during the following embryonic stage				
	(A) blastula (B) Morula (C) Gastrula (D) Fertilized eggs				
Sol.	In gastrulation cell migration occur forming 3	germ layers.			
	Ans. (C)				
73.	The only organ which is capable of producing Fructose in humans is				
		(B) pancreas			
	(C) seminal vesicles ( Ans. (C)	(D) muscle			
	Alis. (0)				
74.	Stroke could be prevented/treated with				
	(A) balanced diet (B) clotting factors (	(C) insulin (D) blood thinners			
Sol.	Thinners prevent cardiovascular accidents whether the second se	hich can cause stroke.			
	Ans. (D)				
75.	In orange and lemon, the edible part of the fru				
		(B) thalamus			
	(C) hairs of the ovary wall ( Ans. (C)	(D) succulent Mesocap			
	Allo: (0)				
76.	Which one of the following statements about	nitrogenase is correct?			
	(A) It is sensitive to $CO_2$ and therefore present in isolated nodules.				
	-	(B) It requires $O_2$ and therefore functional during the day			
	(C) It is sensitive to $O_2$ and therefore is functional in anaerobic environments				
	<ul> <li>(D) It is sensitive to light and therefore functions only in dark.</li> <li>Ans. (C)</li> </ul>				
	All3. (0)				
77.	Part of epidermis that keeps out unwanted pa	articles is called			
	(A) columnar epithelium (	(B) squamous epithelium			
		(D) cuboidal epithelium			
	Ans. (C)				
78.	Species that are most effective at colonising	now habitate show			
70.		(B) high dispersal ability			
		(D) high competitive ability			
	Ans. (D)				
79.		at a locus are at Hardy Weinberg equilibrium. The frequencies	S		
	are $p = 0.6$ and $q = 0.4$ . The proportion of the (A) 0.24 (B) 1 (				
Sol.	(A) 0.24 (B) 1 ( Heterozygous population = 2pq = .48	(C) 0.48 (D) 0.12			
501.	Ans. (C)				
	All3. (0)				
80.	In vertebrates 'glycogen' is stored chiefly in				
		(B) spleen and stomach			
		(D) liver and muscles			
	Ans. (D)				

# **PART-II Two Mark Questions**

# **MATHEMATICS**

Let f(x) be a non-constant polynomial with real coefficients such that  $f\left(\frac{1}{2}\right) = 100$  and f(x)  $\leq 100$  for all real 81.

x. Which of the following statements is NOT necessarily true ? (A) The coefficient of the highest degree term in f(x) is negative (B) f(x) has at least two real roots (C) If  $x \neq 1/2$  then f(x) < 100(D) At least one of the coefficients of f(x) is bigger than 50.

Sol. Coefficient of highest degree term must be negative becuase if it is positive, then  $x \to \infty$ ,  $y \to \infty$  and it is not possible, since  $f(x) \leq 100$ .

Now, graph will be like



at least two real roots will be there, & if  $x \neq \frac{1}{2}$ , then f(x) < 100, it is not always true, as the graph can be like

this also



Now, let the highest coefficients, it can have is 49

then, 
$$f\left(\frac{1}{2}\right) = 49 + \frac{49}{2} + \frac{49}{2^2} + \dots$$

But the sum cannot be equal to 100.

Ans.(C)

82. Let a, b, c, d be real numbers such that

$$\sum_{k=1}^{n} (ak^{3} + bk^{2} + ck + d) = n^{4}$$
  
for every natural number n. Then | a | + | b | + | C | + | d | is equal to  
(A) 15 (B) 16 (C) 31 (D) 32  
$$\sum_{k=1}^{n} (ak^{3} + bk^{2} + ck + d) = n^{4}$$
$$a\sum_{k=1}^{n} k^{3} + b\sum_{k=1}^{n} k^{2} + c\sum_{k=1}^{n} k + \sum_{k=1}^{n} d = n^{2}$$
$$n^{4}(12 - 3a) - n^{3}(4b + 6a) - n^{2}(6c + 6b + 3a) - n(6c + 2b + 12d) = 0$$
$$12 - 3a = 0, \quad 4b + 6a = 0, \quad 6c + 6b + 3a = 0, \quad 6c + 2b + 12d = 0$$
$$\Rightarrow a = 4, b = -6, c = 4, d = -1$$
$$|a|+|b|+|c|+|d| = 15$$
Ans. (A)

Sol.



The vertices of the base of an isosceles triangle lie on a parabola y<sup>2</sup> = 4x and the base is a part of the line 83. y = 2x - 4. If the third vertex of the triangle lies on the x-axis, its coordinates are

(A)  $\left(\frac{5}{2}, 0\right)$  (B)  $\left(\frac{7}{2}, 0\right)$  (C)  $\left(\frac{9}{2}, 0\right)$  (D)  $\left(\frac{11}{2}, 0\right)$ 

Sol.

Sol.

 $(2x - 4)^2 = 4x$  $(x - 2)^2 = x$  $x^2 - 5x + 4 = 0$ x = 1, 4A(α, 0) C(1, -2) ∵ AB = AC B(4, 4)  $\sqrt{(\alpha - 4)^2 + 16} = \sqrt{(\alpha - 1)^2 + 4}$ y = 2x -On solving, we get  $\alpha = \frac{9}{2}$ In a triangle ABC, let G denote its centroid and let M, N be points in the interiors of the segments AB, AC, (A) r = 1/2(B) r > 1/2(C)  $4/9 \le r < 1/2$ (D) 4/9 < r A(ð)

Ans.(C)

84. respectively, such that M, G, N are collinear. If r denotes the ratio of the area of triangle AMN to the area of ABC then

Let  $\overrightarrow{AB} = \overrightarrow{b} \overrightarrow{AC} = \overrightarrow{c}$  $\overrightarrow{AM} = \lambda \overrightarrow{b}$  $\overrightarrow{AN} = \overrightarrow{m c}$ Let G divides MN in the ratio K : 1 So  $\frac{k\mu c + \lambda b}{k+1} = \frac{\overrightarrow{b} + \overrightarrow{c}}{3}$  $\Rightarrow \frac{k\mu}{k+1} = \frac{1}{3} \qquad \qquad \frac{\lambda}{k+1} = \frac{1}{3}$  $\Rightarrow$  k =  $\frac{\lambda}{\mu}$  $\Rightarrow \frac{1}{\lambda} + \frac{1}{\mu} = 3$  $AM \ge GM$  $\frac{\frac{1}{\lambda} + \frac{1}{\mu}}{2} \ge \frac{1}{\sqrt{\lambda \mu}} \Longrightarrow \left(\frac{2}{3}\right)^2 \le \lambda \mu$ ... (1) Now,  $\frac{\text{area of } \Delta AMN}{\text{area of ABC}} = \frac{\frac{1}{2}\lambda\mu \begin{vmatrix} \overrightarrow{b} \times \overrightarrow{c} \\ \frac{1}{2} \begin{vmatrix} \overrightarrow{b} \times \overrightarrow{c} \end{vmatrix}}{\frac{1}{2} \begin{vmatrix} \overrightarrow{b} \times \overrightarrow{c} \end{vmatrix}$  $= \lambda \mu$ using  $\frac{1}{\lambda} + \frac{1}{\mu} = 3 \Rightarrow$  Ratio =  $\frac{\lambda}{3\lambda - 1}\lambda \in [0, 1]$  maximum value of ratio =  $\frac{\lambda^2}{3\lambda - 1}$  attain when  $\lambda = 1$  using derivative but  $\lambda$  is not 1 becuase M is an interior point. so  $\frac{4}{9} \leq \text{ratio} < \frac{1}{2}$ Ans.(C)

**85.** Let XY be the diameter of a semicircle with centre O. Let A be a variable point on the semicircle and B another point on the semicircle such that AB is parallel to XY. The value of ∠BOY for which the inradius of triangle AOB is maximum, is

triangle AOB is maximum, is  
(A) 
$$\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$$
 (B)  $\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{5}$   
OD = R sin  $\theta$   
AB = 2R cos  $\theta$   
 $r_{OAB} = \frac{ar(AOAB)}{2}$   $= \frac{\frac{1}{2} \times R \sin \theta 2R \cos \theta}{2R + 2R \cos \theta}$   
 $= \frac{\frac{1}{2} \times OD \times AB}{2}$   $= \frac{\frac{1}{2} \times R \sin \theta 2R \cos \theta}{2R + 2R \cos \theta}$   
 $r_{OAB} = \frac{R \sin \theta \cos \theta}{(1 + \cos \theta)}$   
 $\frac{dr_{OAB}}{d\theta} = \frac{(1 + \cos \theta) \cos 2\theta - \sin \theta \cos \theta (-\sin \theta)}{(1 + \cos \theta)^2} = 0$   
at  $\cos \theta = \frac{\sqrt{5}-1}{2}$   
Ans. (A)  
Let  $f(x) = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ . The number of real roots of  $f(x) = 0$  is  
(A) 0 (B) 1 (C) 2 (D) 4  
 $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$   
 $f'(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$   
 $f'(x) = 1 + x + \frac{x^2}{2} > 0$   
 $\Rightarrow f(x)$  is an increasing f<sup>n</sup>  
 $\Rightarrow f(x) = 0$  at  $x = x_0$   
 $\Rightarrow f(x_0) = 0 \Rightarrow 1 + x_0 + \frac{x_0^2}{2} + \frac{x_0^3}{6} = 0$  .... (1)  
 $f(-2)f(-1) < 0$   
 $\Rightarrow x_0 \in (-2, -1)$   
 $f(x_0) = 1 + x_0 + \frac{x_0^2}{4} + \frac{x_0^3}{4} + \frac{x_0^4}{4} > 0$ 

 $I(x_{0}) = 1 + x_{0} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} - \frac{1}{24} = \frac{1}{24} + \frac{1}{24}$ no solution



Sol.

86.

Sol.

**87.** Suppose that the earth is a sphere of radius 6400 kilometers. The height from the earth's surface from where exactly a fourth of the earth's surface is visible, is

(A) 3200 km (B)  $3200\sqrt{2}$  km (C)  $3200\sqrt{3}$  km (D) 6400 km

 $AP = Rsin\theta$ area of ring = ( $2\pi Rsin\theta$ ). Rd $\theta$ 

Total area required, 
$$\pi R^2 = \int_0^{\theta} 2\pi R^2 \sin \theta d\theta$$

$$\frac{1}{2} = 1 - \cos\theta$$
$$\cos\theta = \frac{1}{2}$$

Now,

Sol.



$$\cos\theta = \frac{R}{R+H}$$
$$\frac{1}{2} = \frac{R}{R+H} = R = H = 6400$$
Ans. (D)



88 teger not exceeding x and {x} =

88. Let n be a positive integer. For a real number x, let [x] denote the largest integer n  

$$x - [x]. Then \int_{1}^{n+1} \frac{([x])^{[x]}}{[x]} dx \text{ is equal to}$$
(A)  $\log_{e}(n)$ 
(B)  $\frac{1}{n+1}$ 
(C)  $\frac{n}{n+1}$ 
(D)  $1 + \frac{1}{2} + ... + \frac{1}{n}$ 
Sol.  $\int_{1}^{n+1} \frac{\{x\}^{[x]}}{[x]} dx = \int_{1}^{2} \frac{\{x\}^{[x]}}{[x]} dx + \int_{2}^{3} \frac{\{x\}^{[x]}}{[x]} dx + .... + \int_{1}^{n+1} \frac{\{x\}^{[x]}}{[x]} dx$ 

$$= \sum_{r=1}^{n} \int_{r}^{r+1} \frac{\{x\}^{[x]}}{[x]} dx$$

$$= \sum_{r=1}^{n} \int_{r}^{r+1} \frac{(x-r)^{r}}{r} dx$$

$$= \sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$
Ans. (C)

89. A box contains coupons labelled 1,2,...,100. Five coupons are picked at random one after another without replacement. Let the numbers on the coupons be  $x_1, x_2, \dots, x_5$ . What is the probability that  $x_1 > x_2 > x_3$  and  $x_{2} < x_{4} < x_{5}$ ?

$$(A) 1/120$$
 (B) 1/60
 (C) 1/20
 (D) 1/10

  $\frac{100}{100} C_5 [1 \times 2(3)]}{C_5 \times 5!}$ 
 Suppose 1, 2, 3, 4, 5 are selected coupons.

1

 $=\frac{1}{20}$ 

Sol.

Place of 1 is fixed Total arrangements of 5 is 2



arrangements of 2, 3, 4, are

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**90.** In a tournament with five teams, each team plays against every other team exctly once. Each game is won by one of the playing teams and the winning team scores one point, while the losing team scores zero. Which of the following is NOT necessarily true?

(A) There are at least two teams which have at most two points each.

(B) There are at least two teams which have at least two points each.

(C) There are at most three teams which have at least threee points each

(D) There are at most four teams which have at most two points each

**Sol.** Let teams be  $T_1, T_2, T_3, T_4 \& T_5$ 

Now, we can have 5 teams with the scores of 2 points each matches are

(I) $T_1 T_2$	(II) $T_1 T_3$	(III) $T_1 T_4$	$(IV) T_1 T_5$	$(V) T_2 T_3$	(VI) $T_2 T_4$
(VII) $T_2T_5$	(VIII) $T_3 T_4$	$(IX)T_3T_5$	$(X) T_4 T_5$		

Match No.	<b>T</b> <sub>1</sub>	<b>T</b> <sub>2</sub>	$T_3$	$T_{_4}$	$T_5$
Ι	1	0			
Ш	1		0		
	0			1	
IV	0				1
V		1	0		
VI		1		0	
VII		0			1
VIII			1	0	
IX			1		0
X				1	0
Total Score	2	2	2	2	2

This score board contradicts, option D ∴ D is not always necssarily true. Ans. (D)

# PHYSICS

**91.** A bullet of mass m is fired horizontally into a large sphere of mass M and radius R resting on a smooth horizontal table.



The bullet hits the sphere at a height h from the table and sticks to its surface. If the sphere starts rolling without slippng immediately on impact, then

(A)  $\frac{h}{R} = \frac{4m + 3M}{2(m + M)}$  (B)  $\frac{h}{R} = \frac{m + 3M}{m + 2M}$  (C)  $\frac{h}{R} = \frac{10m + 7M}{5(m + M)}$  (D)  $\frac{h}{R} = \frac{4m + 3M}{m + M}$ Apply conservation of linear momentum  $mv = (m + M) v_0$  $mv \sin\theta R = \left(\frac{2}{5}MR^2 + mR^2\right)\omega_0$ 

$$mv\left(\frac{h-R}{R}\right)R = \frac{(2M+5M)}{5}\omega_0R^2$$
  
(m + M) (h - R) $\omega_0R = \frac{(2M+5M)}{5}\omega_0R^2$   
$$\frac{h}{R} = \frac{(10m+7M)}{5(m+M)}$$
  
Ans. (C)

Sol.

**92.** A small boy is throwing a ball towards a wall 6 in front of him. He releases the ball at a height of 1.4 m from the ground. The ball bounces from the wall at a height of 3 m, rebounds from the ground and reaches the boy's hand exactly at the point of release. Assuming the two bounces (one from the wall and the other from the ground) to be perfectly elastic, how far ahead of the boy did the ball bounce from the ground?



93. In the P-V diagram below the dashed curved line is an adiabat.



For a process that is described by a straight line joining two points X and Y on the adiabat (solid line in the diagram) heat is : (hint : Consider the variations in temperature from X to Y along the straitght line) (A) absorbed throughtout from X to Y (B) released throughout from X to Y (C) absorbed from X up to an intermediate point Z (not shown in the figure) and then released from Z to Y (D) released from X up to an intermediate point Z (not shown in the figure) and then absorbed from Z to Y from graph

Sol. from graph Ans. (C)

Sol.

**94.** A singly ionized helium atom in an excited state (n = 4) emits a photon of energy 2.6 eV. Given that the ground state energy of hydrogen atom is -13.6 eV, the energy (E<sub>t</sub>) and quantum number (n) of the resulting state are respectively,

(A) E <sub>t</sub> = – 13.6eV, n = 1	(B) E <sub>t</sub> = – 6.0 eV, n = 3
(C) $E_t = -6.0 \text{ eV}, n = 2$	(D) E <sub>t</sub> = – 13.6 eV, n = 2
$2.6 = 13.6  \text{z}^2 \left[ \frac{1}{n^2} - \frac{1}{4^2} \right]$	
$\frac{2.6}{13.6\times4} = 2^2 \left[\frac{1}{n^2} - \frac{1}{4^2}\right]$	

$$\frac{2.6}{13.6 \times 4} = \frac{1}{n^2} - \frac{1}{16}$$
  
n = 3 now Energy

$$E = \frac{13.6Z^2}{n^2} eV$$
$$-\frac{13.6 \times 4}{9} eV = -6 eV$$

**95.** The figure below shows a circuit and its input voltage v<sub>i</sub> as function of time t.



Assuming the diodes to be ideal, which of the following graphs depicts the output voltage  $v_0$  as function of time t ?



**96.** A ball is rolling without slipping in a spherical shallow bowl (radius R) as shown in the figure and is executing simple harmonic motion. If the radius of the ball is doubled, the period of oscillation

(A) increases slightly(C) is increased by a factor of 2

mg sin
$$\theta - F_r = ma$$
  
 $F_r = \frac{2}{5}mr^2 \frac{a}{r^2}$   
 $\Rightarrow a = \frac{5}{7}\frac{gsin\theta}{R-r}$   
 $\omega = \sqrt{\frac{5g}{7(R-r)}}$   
 $\Rightarrow T = 2\pi \sqrt{\frac{7(R-r)}{5r}}$ 

Ans.(D)

Sol.

**97.** A solid sphere rolls without slipping, first horizontal and then up to a point X at height h on an inclined plane before rolling down, as shown.



The initial horizontal speed of the sphere is

(A) 
$$\sqrt{10 \text{gh}/7}$$
 (B)  $\sqrt{7 \text{gh}/5}$  (C)  $\sqrt{5 \text{gh}/7}$  (D)  $\sqrt{2 \text{gh}}$   
Sol.  $\text{mgh} = \frac{1}{2} \text{mv}^2 + \frac{1}{2} \frac{2}{5} \text{mR}^2 \frac{\text{v}^2}{\text{R}^2}$   
 $\text{mgh} = \frac{7}{10} \text{mv}^2$   
 $v = \sqrt{\frac{10 \text{gh}}{7}}$ 

**98.** The three processes in a thermodynamic cycle shown in the figure are : Process  $1 \rightarrow 2$  is isothermal; Process  $2 \rightarrow 3$  is isochoric (volume remains constant); Process  $3 \rightarrow 1$  is adiabatic. The total work done by the ideal gas in this cycle is 10 J. The internal energy decreases by 20 J in the isochoric process. The work done by the gas in the adiabatic process is -20 J. The heat added to the system in the isothermal process is

**99.** A block of mass m slides from rest at a height H on a frictionless inclined plane as shown in the figure. It travels a distance d across a rough horizontal surface with coefficient of kinetic friction μ, and compresses a spring of spring k by a distance x before coming to rest momentarily. Then the spring extends and the block travels back attaining a final height of h. Then



**100.** A metallic prong consists of 4 rods made of the same material, cross-section and same lengths as shown. The three forked ends are kept at 100° C and the handle end is at 0°C. The temperature of the junction is





Sol.

# CHEMISTRY

101. The major product obtained in the reaction of aniline with acetic anhydride is



**102.** The maximum number of isomers that can result from monobromination of 2-methyl-2-pentene with N-bromosuccinimide in boiling CCl<sub>4</sub> is





**103**. The compound X ( $C_7H_9N$ ) reacts with benzensulfonyl chloride to give Y ( $C_{13}H_{13}NO_2S$ ) which is insoluble in alkali. The compound X is



Ans. (A)



104. In 108 g of water, 18 g of a non-volatile compound is dissolved. At 100°C the vapor pressure of the solution is 750 mm Hg. Assuming that the compound does not undergo association or dissociation, the molar mass of the compound in g mol<sup>-1</sup> is

(A) 128 (B) 182 (C) 152 (D) 228 Sol.  $\frac{n}{N} = \frac{P_0 - P_S}{P_0}$   $\frac{18}{m} \times \frac{18}{108} = \frac{760 - 750}{760}$   $\frac{18}{6m} = \frac{10}{760}$   $6m = 18 \times 76$  m = 228Ans. (D)

**105.** The standard electrode potential of Zn<sup>2+</sup>/Zn is –0.76 V and that of Ca<sup>2+</sup>/Cu is 0.34 V. The emf (V) and the free energy change (kJ mol<sup>-1</sup>), respectively, for a Daniel cell will be

(A) - 0.42 and 81 (B) 1.1 and -213 (C) -1.1 and 213 (D) 0.42 and -81Sol.  $E^{0}_{Cell} = E^{0}_{Cu^{2+}|Cu} + E^{0}_{Zn|Zn^{2+}}$   $E^{0}_{Cell} = 0.34 + 0.76 = +1.1V$   $\Delta G = -2 \times 96500 \times 1.1 = -213 V$ Ans. (B)

106. Consider the equilibria (1) and (2) with equilibrium constants K<sub>1</sub> and K<sub>2</sub>, respectively

$$SO_{2}(g) + \frac{1}{2}O_{2}(g) \stackrel{\checkmark}{\longrightarrow} SO_{3}(g) \qquad \dots (1)$$
$$2SO_{3}(g) \stackrel{\checkmark}{\longrightarrow} 2SO_{2}(g) + O_{2}(g) \qquad \dots (2)$$

 $K_1$  and  $K_2$  are related as

(A)  $2K_1 = K_2^2$  (B)  $K_1^2 = \frac{1}{K_2}$  (C)  $K_2^2 = \frac{1}{K_1}$  (D)  $K_2 = \frac{2}{K_1^2}$ Sol.  $K_1 = \frac{[SO_3]}{[O_2]_2^1 [SO_2]}$   $K_2 = \frac{[SO_2]^2 [O_2]}{[SO_3]^2}$ Hence:  $K_2 = \frac{1}{K_1^2}$   $K_1^2 = \frac{1}{K_2}$ Ans. (B)

- **107.** Aqueous solution of metallic nitrate X reacts with NH<sub>4</sub>OH to form Y which dissolves in excess NH<sub>4</sub>OH. The resulting complex is reduced by acetaldehyde to deposit the metal. X and Y, respectively, are
  - (A) Cs(NO<sub>3</sub>) and CsOH (B) Zn(NO<sub>3</sub>)<sub>2</sub> and ZnO (C) AgNO<sub>3</sub> and Ag<sub>2</sub>O (D) Mg(NO<sub>3</sub>)<sub>2</sub> and Mg(OH)<sub>2</sub> AgNO<sub>3</sub> + NH<sub>4</sub>OH  $\longrightarrow$  AgOH  $\xrightarrow{NH_4OH}_{(excess)}$  [Ag(NH<sub>3</sub>)<sub>2</sub>]<sup>+</sup>

Tollen's reagent + R – CHO 
$$\longrightarrow$$
 R – COOH Ans. (C)

Sol.

(D) 7.75 g

**108.** The density of eq. wt of a metal are  $10.5 \text{ g cm}^{-3}$  and 100, respectively. The time required for a current of 3 amp to denosit a 0.005 mm thick layer of the same metal on an area of 80 cm<sup>2</sup> is closest to

to deposit a 0.005 mm thick layer of the same metal on an area of 80 cm<sup>2</sup> is closest to (A) 120 s (B) 135 s (C) 67.5 s (D) 270 s Sol.  $w = \frac{E \times i \times t}{96500}$   $w = \frac{100 \times 3 \times t}{96500}$   $d = \frac{m}{V}$  m = dV  $w = 10.5 \times 80 \times 5 \times 10^{-4} = 42 \times 10^{-2}$   $42 \times 10^{-2} = \frac{100 \times 3 \times t}{96500}$  t = 135 sec. Ans. (B)

**109.** The amount of  $Na_2S_2O_3.5H_2O$  required to completely reduce 100 mL of 0.25 N iodine solution, is

(A) 6.20 g (B) 9.30 g (C) 3.10 gSol.  $N_1V_1 = N_2V_2$ mili eq. of hypo =  $0.25 \times 100$ mili eq. of hypo = 25eq. of hypo = 0.025mole of hypo =  $0.025 \times 1$   $\because V_f = 1$ weight of hypo =  $0.025 \times 248 = 6.2 \text{ g}$ Ans. (A)

**110.** In aqueous solution,  $[Co(H_2O)_6]^{2+}$  (X) reacts with molecular oxygen in the presence of excess liquor NH<sub>3</sub> to give a new complex Y. The number of unpaired electrons in X and Y are, respectively

(A) 3, 1 (B) 3, 0 (C) 3, 3 (D) 7, 0  $[CO(H_2O)_6]^{2^+} + 6NH_3 \longrightarrow [CO(NH_3)_6]^{2^+}$ Paramagnetic  $n = 3 ; \mu = 3.9$   $[CO(NH_3)_6]^{3^+}$   $[CO(NH_3)_6]^{3^+}$ Diamagnetic  $\mu = 0 ; n = 0$ 

# BIOLOGY

- **111.** 10<sup>9</sup> bacteria were spread on an agar plate containing penicillin. After incubation overnight at 37°C, 10 bacterial colonies were observed on the plate. That the colonies are likely to be resistant to penicillin can be tested by
  - (A) measuring their growth rate
  - (B) observing the colour of the colonies
  - (C) checking their ability to grow on another plate containing penicillin.
  - (D) checking their ability to cause disease

Ans.(C)

Ans.(B)



Sol.

- 112. Watson and Crick model of DNA is
  - (A) B-form DNA with a spiral length of 34 Å and a diameter of 20 Å
  - (B) A-form DNA with a spiral length of 15 Å and a diameter of 20 Å
  - (C) Z-form DNA with a spiral length of 34 Å and a diameter of 20 Å
  - (D) B-form DNA with a spiral length of 28 Å and a diameter of 14 Å

Ans.(A)

- 113. Eco RI and Rsa I restriction endonucleases require 6 bp and 4 bp sequences respectively for cleavage. In a 10 kb DNA fragment how many probable cleavage sites are present for these enzymes
  - (A) 0 Eco RI and 10 Rsa I
  - (B) 1 Eco RI and 29 Rsa I (C) 4 Eco RI and 69 Rsa I (D) 2 Eco RI and 39 Rsa I
- For EcoR I  $\left(\frac{1}{4}\right)^6 \times 10000 = 2.44$  sites present Sol.

For RsaI 
$$\left(\frac{1}{4}\right)^4 \times 10000 = 39.06$$

Using probability rules Ans.(D)

- 114. From an early amphibian embryo the cells that would give rise to skin in adults were transplanted into the developing brain region of another embryo. The transplanted cells developed into brain tissue in the recipient embryo. What do you infer from this experiment?
  - (A) Cell fate is permanently determined during early embryonic development.
  - (B) Developmental fate of donor cells is influenced by the surrounding cells.
  - (C) Developmental fate of donor cells is not influenced by recipient cells.
  - (D) Any cell which is transplanted into another embryo always develops into a brain.
- Sol. Early embryonic cells have yet not committed and differentiated. Ans.(B)
- 115. Presence of plastids in Plasmodium suggests
  - (A) it is a plant species
  - (B) it is a parasite with a cynobacterium as an endosymbiont
  - (C) it is a parasite with a archebacterium as an endosymbiont
  - (D) it is a plant species with a archebacterium as an endosymbiont
- Sol. (Secondary Endosymbiont) Apicoplast i.e: non photosynthetic plastid is present in plasmodium (protista) Ans.(B)



**116.** The figure below demonstrates the growth curves of two organisms A and B growing in the same area. What kind of relation exists between A and B?



- Sol. Competition is harmful for both species (A is affected more) Ans. (A)
- **117.** A scientist has cloned an 8 Kb fragment of a mouse gene into the Eco RI site of a vector of 6 Kb size. The cloned DNA has no other Eco RI site within. Digestions of the cloned DNA is shown below.



Which one of the following sets of DNA fragments generated by digestion with both Eco RI and Bam HI as shown in (iii) is from the gene?

- (A) 1 Kb and 4 Kb
- (C) 1 Kb and 3 Kb

(B) 1 Kb and 2.5 Kb (D) 1 Kb and 3.5 Kb



Restriction map as per data is shown above. Thus 1 kb, 3 kb, 4 kb are fragments of gene Ans. (A,C)

- 118. Brown fat is a specialised adipose tissue with abundant mitochondria and rich blood supply. Brown fat
  - (A) insulates animals that are acclimatised to cold.
  - (B) is the major source of heat production of birds.
  - (C) provides energy to muscles.
  - (D) produces heat without producing ATP.

Sol. Brown fat is abundant in bibernating mammals and new born babies (mammals only) Ans. (D)

- **119.** In some species, individuals forego reproduction and help bring up another individual's offspring. Such altruistic behaviour CANNOT be explained by which of the following?
  - (A) An individual helps relatives only and gets indirect genetic benefits.
  - (B) The individual benefits because it can later inherit the breeding position.
  - (C) The individual benefits because it gets access to resources, such as food and security from predators, in return.
  - (D) The species benefits from a reduction in competition among offspring.

Ans.(C)

- **120.** Lions in India are currently restricted to Gir, Gujarat. Efforts are being made to move them to other parts of the country. This is because they are MOST susceptible to extinction due to infectious diseases under the following conditions when present as
  - (A) several small, isolated populations
- (B) one large population
- (C) several large, connected populations
- (D) several large, isolated populations

Ans. (B)