



Resonance[®]
Educating for better tomorrow

JEE (MAIN) 2026

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 06th April 2026 & Monday

PAPER-1

Duration: 3 Hrs.

Time: 03:00 PM – 06:00 PM

SUBJECT: MATHEMATICS

Selections in JEE (Advanced)/
IIT-JEE Since 2002

52979

Classroom: 35901 | Distance: 17078

Selections in JEE (Main)/
AIEEE Since 2009

262693

Classroom: 194471 | Distance: 68222

Selections in NEET (UG)/
AIPMT/AIIMS Since 2012

22733

Classroom: 15409 | Distance: 7324

Admission Open for 2026-27

Target: JEE (Advanced) | JEE (Main) | NEET (UG) | PCCP (Class V to X)

100% Scholarship on the basis of Class 10th, 12th
& JEE (Main) 2026 %ile / AIR

☎ 0744-2777777 | 📞 73400 10345 | Follow Us: @ResonanceEdu | @Resonance_Edu

REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):

CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Rajasthan) - 324005

☎ 0744-2777777 | 📞 73400 10345 | 📧 contact@resonance.ac.in | 🌐 www.resonance.ac.in | Follow Us: @ResonanceEdu | @Resonance_Edu

This Solution was download from Resonance JEE (Main) 2026 Solution Portal

PART : MATHEMATICS

1. Let R be a relation such that $R = \{\ell n(x + y) \leq 2, (x, y) \in \mathbb{N} \times \mathbb{N}\}$.
Minimum no. of elements to be added in R so that it becomes transitive is
(1) 10 (2) 15 (3) 14 (4) 16

Ans. (2)

Sol. $x + y \leq e^2 (e^2 \approx 7.29)$
 $R = \{(1,1), (1,2), (1,4), (1,5), (1,6)\}$
 $(2,1), (2,2), (2,3), (2,4), (2,5)$
 $(3,1), (3,2), (3,3), (3,4)$
 $(4,1), (4,2), (4,3)$
 $(5,1), (5,2)$
 $(6,1)$

For R to be transitive we have to add

$(6,2), (6,3), (6,4), (6,5), (6,6)$
 $(5,3), (5,4), (5,5), (5,6)$
 $(4,4), (4,5), (4,6)$
 $(3,5), (3,6)$
 $(2,3)$

= 15 elements

2. $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + |x| + 1} dx$ is equal to

(1) $\ln 3 - \frac{\pi}{3\sqrt{3}}$ (2) $\frac{\pi}{3\sqrt{3}}$ (3) $\ln 3 + \frac{\pi}{3\sqrt{3}}$ (4) $-\frac{\pi}{3\sqrt{3}}$

Ans. (1)

Sol. $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + |x| + 1} dx$
 $= \int_{-1}^1 \frac{x^3}{x^2 + |x| + 1} dx + \int_{-1}^1 \frac{|x| + 1}{x^2 + |x| + 1} dx$
 $= 0 + 2 \int_0^1 \frac{x + 1}{x^2 + x + 1} dx$
 $= \int_0^1 \frac{(2x + 1) + 1}{x^2 + x + 1} dx$
 $= \int_0^1 \frac{2x + 1}{x^2 + x + 1} dx + \int_0^1 \frac{1}{x^2 + x + 1} dx$
 $= [\ell n|x^2 + x + 1|]_0^1 + \left[\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) \right]_0^1$
 $= \ell n 3 - \frac{2}{\sqrt{3}} \left[\tan^{-1} \sqrt{3} - \tan^{-1} \frac{1}{\sqrt{3}} \right]$
 $= \ell n 3 - \frac{2}{\sqrt{3}} \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$
 $= \ell n 3 - \frac{\pi}{3\sqrt{3}}$

3. A bag contains 6 Red and 6 black balls. 6 pair of balls are selected one by one without replacement then the probability that each of the 6 pairs contains 1 red and 1 black ball.

(1) $\frac{15}{231}$ (2) $\frac{14}{231}$ (3) $\frac{13}{231}$ (4) $\frac{16}{231}$

Ans. (4)

Sol. probability = $\frac{6 \times 6}{\binom{12}{2} \times 6} = \frac{16}{231}$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  7340010333  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

4. Let $\lim_{x \rightarrow 2} \frac{\tan(x-2)[rx^2+(p-2)x-2p]}{(x-2)^2} = 5$ for some $r, p \in \mathbb{R}$. If the set of all possible values of q , such that the roots of the equation $rx^2 - px + q = 0$ lie in $(0, 2)$ be the interval $(\alpha, \beta]$, then $4(\alpha + \beta)$ is equal to
 (1) 11 (2) 21 (3) 17 (4) 13

Ans. (3)

Sol. $\lim_{x \rightarrow 2} \frac{\tan(x-2)}{x-2} \cdot \frac{[rx^2+(p-2)x-2p]}{x-2} = 5$
 $\Rightarrow \lim_{x \rightarrow 2} \frac{rx^2 - 2x + p(x-2)}{x-2} = 5$
 $\because D^r \rightarrow 0 \therefore N^r \rightarrow 0 \Rightarrow r = 1$
 $\Rightarrow \lim_{x \rightarrow 2} \frac{x(x-2) + p(x-2)}{x-2} = 5$
 $\Rightarrow 2 + p = 5$
 $\Rightarrow p = 3$

Now, quadratic equation is $x^2 - 3x + q = 0$ its both roots lie in $(0, 2)$



$$D \geq 0, 0 < \frac{-b}{2a} < 2, f(0) > 0, f(2) > 0$$

$$\Rightarrow 9 - 4q \geq 0 \text{ and } q > 2$$

$$\Rightarrow q \in \left(2, \frac{9}{4}\right]$$

$$\alpha = 2, \beta = \frac{9}{4} \Rightarrow 4\left(2 + \frac{9}{4}\right) = 17$$

5. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}, \vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. If $2\vec{a} + 3\vec{b}$ and $\vec{a} - \vec{b}$ are two adjacent sides of a triangle then square of area of the triangle is
 (1) 1800 (2) 1600 (3) 2000 (4) 2200

Ans. (1)

Sol. $2\vec{a} + 3\vec{b} = 2(2\hat{i} + 3\hat{j} + 3\hat{k}) + 3(6\hat{i} + 3\hat{j} + 3\hat{k})$
 $\Rightarrow 22\hat{i} + 15\hat{j} + 15\hat{k}$
 $\vec{a} - \vec{b} = -4\hat{i}$
 $\text{Area} = \frac{1}{2} |(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})|$
 $= \frac{1}{2} |-60\hat{j} + 60\hat{k}| = \frac{1}{2} \sqrt{(60)^2 + (60)^2}$
 $A = \frac{60}{\sqrt{2}}$

Square of area is $A^2 = 1800$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

6. If matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{bmatrix}$ is singular. Given a function $f(x) = \int_0^x (t^2 + 2t + 3)dt \forall x \in [1, \alpha]$ & m and n are maximum and minimum value of function f(x), then the value of $3(m - n)$ is
 (1) 550 (2) 510 (3) 490 (4) 540

Ans. (4)

Sol. $|m| = 7 - \alpha = 0 \Rightarrow \alpha = 7$ given $f(x) = \int_0^x (t^2 + 2t + 3)dt$

$$\Rightarrow f(x) = \frac{x^3}{3} + x^2 + 3x \text{ and } f'(x) = x^2 + 2x + 3 > 0 \forall x$$

$\therefore y = f(x)$ is strictly increasing function

$$\Rightarrow m = f(7) = \frac{553}{3}$$

$$\& n = f(1) = \frac{13}{3}$$

$$\therefore 3(m - n) = 540$$

7. The sum of series $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \dots$ upto 10 terms is
 (1) $\frac{313}{2}$ (2) $\frac{315}{2}$ (3) $\frac{325}{2}$ (4) $\frac{335}{2}$

Ans. (2)

Sol. $T_n = \frac{1}{n}(1^2 + 2^2 + \dots + n^2)$

$$T_n = \frac{n(n+1)(2n+1)}{6n}$$

$$T_n = \frac{1}{6}[2n^2 + 3n + 1]$$

$$S_n = \Sigma T_n = \frac{1}{6}[2\Sigma n^2 + 3\Sigma n + \Sigma 1]$$

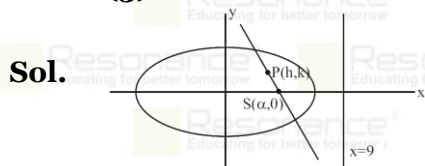
$$\Sigma T_n = \frac{1}{6}\left[2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n\right]$$

$$S_{10} = \frac{1}{6}\left[\frac{2 \times 10 \times 11 \times 21}{6} + \frac{3}{2} \times 10 \times 11 + 10\right]$$

$$= \frac{10}{6}\left[77 + \frac{33}{2} + 1\right] = \frac{315}{2}$$

8. Let $x = 9$ be a directrix of an ellipse centred at $(0,0)$ and having eccentricity $\frac{1}{3}$. If focus at $(\alpha, 0)$ ($\alpha > 0$), then locus of the mid-point of the chord passing through the focus $(\alpha, 0)$ is
 (1) $8y^2 = 9x(1+x)$ (2) $9y^2 = 8x(1+x)$
 (3) $9y^2 = 8x(1-x)$ (4) $8y^2 = 9x(1-x)$

Ans. (3)



$$\frac{a}{e} = 9 \Rightarrow a = 3$$

$$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 8$$

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 7340010333 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

$$S(\alpha, 0) \equiv S(1, 0)$$

$$T = S_1 \Rightarrow \frac{hx}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}$$

$$(1, 0) \Rightarrow \frac{h}{9} + 0 = \frac{h^2}{9} + \frac{k^2}{8}$$

$$h = h^2 + \frac{9}{8}k^2$$

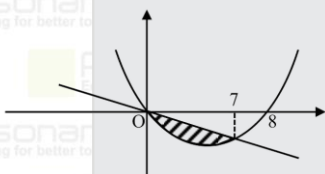
$$9y^2 = 8x(1-x)$$

9. The area (in square units) of the region $\{(x, y): x^2 - 8x \leq y \leq -x\}$, is

- (1) $\frac{343}{6}$ (2) $\frac{343}{2}$ (3) $\frac{1715}{6}$ (4) $\frac{340}{3}$

Ans. (1)

Sol. Required area = $\int_0^7 (7x - x^2) dx = \left[\frac{7x^2}{2} - \frac{x^3}{3} \right]_0^7 = \frac{343}{6}$



10. The value of x which satisfies the equation $\sin^{-1}\left(\frac{2}{3}\sqrt{1-x^2}\right) = \cot^{-1}(2\sqrt{x})$, is

- (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{9}$

Ans. (1)

Sol. Taking sine on both the sides,

$$\sin\left(\sin^{-1}\frac{2}{3}\sqrt{1-x^2}\right) = \sin(\cot^{-1} 2\sqrt{x})$$

$$\Rightarrow \frac{2}{3}\sqrt{1-x^2} = \frac{1}{\sqrt{1+4x}}$$

$$\Rightarrow (1-x^2)(1+4x) = \frac{9}{4}$$

$$\Rightarrow x = \frac{1}{2}$$

11. Let $(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0$ be a quadratic equation. If α is the minimum value of product of roots and β is the maximum value of sum of roots, then the sum of first six terms of geometric progression whose first term is α and common ratio is $\left(\frac{\alpha}{\beta}\right)$, is

- (1) $\frac{364}{243}$ (2) $\frac{343}{243}$ (3) $\frac{256}{81}$ (4) $\frac{364}{81}$

Ans. (1)

Sol. $\alpha = n^2 - 2n + 2$

$$\alpha = (n-1)^2 + 1$$

\therefore minimum value of α is 1

Similarly $\beta = \frac{3}{(n-1)^2 + 1}$

\therefore maximum value of β is 3 required G.P. is $1, \frac{1}{3}, \frac{1}{3^2}, \dots$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

$$S_6 = \frac{1 \left(1 - \left(\frac{1}{3} \right)^6 \right)}{1 - \frac{1}{3}} = \frac{3 \left[1 - \left(\frac{1}{3} \right)^6 \right]}{2} = \frac{364}{243}$$

12. Let $f(x)$ be a function satisfying the functional rule $f(xy) = f(x)f(y)$, $f(0) \neq 0$.

If $x^2 g(x) = \int_1^x (t^2 f(t) + t g(t)) dt$, then $g(2)$ is equal to

- (1) $\frac{15}{32}$ (2) $\frac{3}{4}$ (3) $\frac{4}{3}$ (4) $\frac{32}{15}$

Ans. (2)

Sol. $\because f(xy) = f(x)f(y)$
 $\therefore f(x) = x^n$ or $f(x) = k$
 But $f(0) \neq 0 \Rightarrow f(x) \neq x^n$

For $f(x) = k \Rightarrow k = k \cdot k \Rightarrow k = 1 (\because f(0) \neq 0)$

$f(x) = 1$

Now, $x^2 \cdot g(x) = \int_1^x (t^2 \cdot 1 + t \cdot g(t)) dt$

Differentiating both the sides

$$x^2 \cdot g'(x) + g(x) \cdot 2x = x^2 + xg(x)$$

$$x^2 g'(x) + xg(x) = x^2$$

$$xg'(x) + g(x) = x$$

$$x \cdot g(x) = \frac{x^2}{2} + c$$

$$\because g(1) = 0 \Rightarrow c = -\frac{1}{2}$$

$$\Rightarrow x \cdot g(x) = \frac{x^2 - 1}{2}$$

$$\Rightarrow g(2) = \frac{3}{4}$$

13. The shortest distance between the lines $\frac{x-3}{-1} = \frac{y-2}{4} = \frac{z-1}{2}$ & $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{5}$

- (1) $\sqrt{6}$ (2) $6\sqrt{6}$ (3) $\sqrt{5}$ (4) $5\sqrt{5}$

Ans. (1)

Sol. $\vec{a} = 3i + 2j + k$ & $\vec{b} = i + j + 2k$

$$\vec{r}_1 = -i + 4j + 2k$$

$$\vec{r}_2 = 2i + j + 5k$$

$$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} i & j & k \\ -1 & 4 & 2 \\ 2 & 1 & 5 \end{vmatrix} = 18i + 9j - 9k$$

$$\Rightarrow |\vec{r}_1 \times \vec{r}_2| = 9\sqrt{6}$$

Now

$$|\vec{a} - \vec{b} \cdot \vec{r}_1 \cdot \vec{r}_2| = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 4 & 2 \\ 2 & 1 & 5 \end{vmatrix} = 54$$

$$\text{shortest distance} = \frac{|\vec{a} - \vec{b} \cdot \vec{r}_1 \cdot \vec{r}_2|}{|\vec{r}_1 \times \vec{r}_2|} = \frac{54}{9\sqrt{6}} = \sqrt{6}$$

14. A lift of a 10 floor building contains 9 persons and group of 4 and 5 leave the lift on different floor and there is no stoppage of lift at 1st and 2nd floor, then find number of ways this can be

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

done.

- (1) 7056 (2) 7656 (3) 7066 (4) 7057

Ans. (1)

Sol. Form group of 4 and 5 person $\frac{9!}{4!5!} = 126$

choose two floor out of 8 floor ${}^8C_2 = 28$

Total ways in which 4 & 5 persons can leave at different floor $126 \times 28 \times 2 = 7056$

15. If system of equations

$$x \cos 3\theta - 8y - 12z = 0$$

$$x \cos 2\theta + y + 3z = 0$$

$x + y + 3z = 0$ has non-trivial solution, then find sum of values of θ (where $\theta \in [0, 2\pi]$)

- (1) π (2) 2π (3) 3π (4) 4π

Ans. (3)

Sol. For Non-trivial solution

$$\begin{vmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 1 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \cos 3\theta & -8 & 12 \\ \cos 2\theta & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$$\cos 2\theta = 1$$

$$\theta = 0, \pi, 2\pi$$

\therefore sum of values of θ is 3π

16. Consider the observations : $2, 4, \alpha, \beta, 6, 12, 14$. If their mean is 8 and variance = 16, then the quadratic equation whose roots are $3\alpha + 2$ & $2\beta + 1$, is

(1) $x^2 - 49x + 544 = 0$

(2) $x^2 - 49x - 544 = 0$

(3) $x^2 - 23x - 512 = 0$

(4) $x^2 + 23x - 512 = 0$

Ans. (1)

Sol. $S = 2 + 4 + \alpha + \beta + 6 + 12 + 14$

$$= \alpha + 38 + \beta$$

$$\text{Mean} = \frac{\alpha + \beta + 38}{7} = 8$$

$$\alpha + \beta = 18$$

$$\text{Variance} = \sum \frac{x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$\Rightarrow \frac{1}{7}(4 + 16 + \alpha^2 + \beta^2 + 36 + 144 + 196) - 64 = 16$$

$$\Rightarrow \frac{1}{7}(\alpha^2 + \beta^2 + 396) = 80$$

$$\alpha^2 + \beta^2 + 396 = 560$$

$$\alpha^2 + \beta^2 = 164$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 164$$

$$324 - 2\alpha\beta = 164 \Rightarrow \alpha\beta = 80$$

$$\alpha = 10, \beta = 8$$

$$\text{Sum} = (3\alpha + 2) + (2\beta + 1) = 32 + 17 = 49$$

$$\text{Product} = (3\alpha + 2)(2\beta + 1) = 32 \times 17 = 544$$

Required quadratic equation is $x^2 - 49x + 544 = 0$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

17. Let a hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & ellipse be $\frac{x^2}{9} + \frac{y^2}{8} = 1$. If length of latus rectum of hyperbola is equal to minor axis of ellipse & eccentricity of hyperbola is equal to semi-major axis of ellipse, then $2ae$ is equal to (where 'e' is the eccentricity of hyperbola)

- (1) $3\sqrt{2}$ (2) $\frac{3\sqrt{2}}{2}$ (3) $2\sqrt{2}$ (4) $\frac{\sqrt{2}}{3}$

Ans. (2)

Sol. $\frac{2b^2}{a} = 4\sqrt{2} = b^2 = 2\sqrt{2}a$

$$e_H = \sqrt{1 + \frac{b^2}{a^2}} = 3 \Rightarrow 1 + \frac{b^2}{a^2} = 9 \Rightarrow b^2 = 8a^2$$

$$a = \frac{\sqrt{2}}{4}, b^2 = 1 \Rightarrow b = 1$$

$$\therefore 2ae = 2 \times \frac{\sqrt{2}}{4} \times 3 = \frac{3\sqrt{2}}{2}$$

18. Let $f: R \rightarrow R, f(x) = \frac{2x^2 - 3x + 2}{3x^2 + x + 3}$, then $f(x)$ is

- (1) one-one and onto (2) one-one and into
(3) many-one and into (4) many-one and onto

Ans. (3)

Sol. $\frac{2x^2 - 3x + 2}{3x^2 + x + 3} = y$

$$(3y - 2)x^2 + (y + 3)x + (3y - 2) = 0$$

$$D \geq 0$$

$$(7y - 1)(5y - 7) \leq 0$$

$$y \in \left[\frac{1}{7}, \frac{7}{5} \right]$$

Hence $f(x)$ is into obviously $f(x)$ is many-one because $f(x)$ is non-monotonic.

19. If $(1 - x^3)^{10} = \sum_{r=0}^{10} a_r x^r \cdot (1 - x)^{30 - 2r}$, then find $\frac{9a_9}{a_{10}}$.

Ans. (30)

Sol. $(1 - x)^3 = 1 - x^3 - 3x(1 - x)$

$$(1 - x)^3 = 1 - x^3 - 3x + 3x^2$$

$$(1 - x)^3 + 3x - 3x^2 = 1 - x^3$$

$$(1 - x^3)^{10} = [(1 - x)^3 + 3x(1 - x)]^{10}$$

$$\Rightarrow {}^{10}C_r (3x(1 - x))^r \cdot ((1 - x)^3)^{10 - r}$$

$$(1 - x^3)^{10} = {}^{10}C_r \cdot 3^r \cdot x^r \cdot (1 - x)^{30 - 2r}$$

$$\therefore a_r = {}^{10}C_r \cdot 3^r$$

$$a_9 = {}^{10}C_9 \cdot 3^9$$

$$\frac{a_9}{a_{10}} = \frac{{}^{10}C_9 \cdot 3^9}{{}^{10}C_{10} \cdot 3^{10}}$$

$$\frac{a_9}{a_{10}} = \frac{10}{3}$$

$$\frac{9a_9}{a_{10}} = 30$$

20. The line $x - y = 4$ is a chord of the circle $(x - 4)^2 + (y + 3)^2 = 9$, which cuts the circle at points Q & R . If $P(\alpha, \beta)$ lies on the circle such that $PQ = PR$, then find $(6\alpha + 8\beta)^2$.

Ans. (18)

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

Sol. Solving $x - y = 4$ & $(x - 4)^2 + (y + 3)^2 = 9$

We get Q(4,0) & R(1, -3)

$$PQ^2 = PR^2$$

$$\Rightarrow (\alpha - 4)^2 + \beta^2 = (\alpha - 1)^2 + (\beta + 3)^2$$

$$\Rightarrow \alpha + \beta = 1 \quad (1)$$

Also $(\alpha - 4)^2 + (\beta + 3)^2 = 9$

Solving (1) & (2), we get

$$\alpha = 4 + \frac{3}{\sqrt{2}}, \beta = -3 - \frac{3}{\sqrt{2}}$$

$$\text{and } \alpha = 4 - \frac{3}{\sqrt{2}}, \beta = -3 + \frac{3}{\sqrt{2}}$$

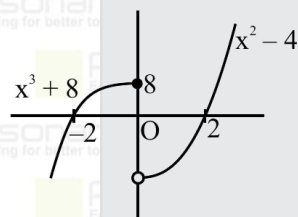
For both pairs we get $(6\alpha + 8\beta)^2 = 18$

21. Let $f(x) = \begin{cases} x^3 + 8 & x < 0 \\ x^2 - 4 & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} (x - 8)^{1/3} & x < 0 \\ (x + 4)^{1/2} & x \geq 0 \end{cases}$

then find number of points of discontinuity of $g(f(x))$.

Ans. (3)

Sol. $g(f(x)) = \begin{cases} (f(x) - 8)^{1/3} & f(x) < 0 \\ (f(x) + 4)^{1/2} & f(x) \geq 0 \end{cases}$



$$g(f(x)) = \begin{cases} (x^3)^{1/3}, & x < -2 \\ (x^2 - 12)^{1/3}, & 0 < x < 2 \\ (x^3 + 12)^{1/2}, & -2 \leq x \leq 0 \\ (x^2)^{1/2}, & x \geq 2 \end{cases}$$

$$g(f(x)) = \begin{cases} x, & x < -2 \\ (x^3 + 12)^{1/2}, & -2 \leq x \leq 0 \\ (x^2 - 12)^{1/3}, & 0 < x < 2 \\ x, & x \geq 2 \end{cases}$$

Number of points of discontinuity is equal to 3.

22. If α & β are the roots of the equation $z^2 - \sqrt{6}iz - 3 = 0$, then find $\alpha^8 + \beta^8$.

Ans. (162)

Sol. $z = \frac{\sqrt{6}i \pm \sqrt{-6+12}}{2}$

$$= \frac{\sqrt{6}i \pm \sqrt{6}}{2}$$

$$= \frac{\sqrt{6}}{2} (\pm 1 + i)$$

$$\alpha = \frac{\sqrt{6}}{2} (1 + i), \beta = \frac{-\sqrt{6}}{2} (1 - i)$$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

$$\alpha = \frac{\sqrt{6}}{2} \times \sqrt{2} e^{\frac{i\pi}{4}}, \beta = -\frac{\sqrt{6}}{2} \times \sqrt{2} e^{-\frac{i\pi}{4}}$$

$$\alpha = \sqrt{3} e^{\frac{i\pi}{4}}, \beta = -\sqrt{3} e^{-\frac{i\pi}{4}}$$

$$\alpha^8 + \beta^8 = 81 \cdot e^{i2\pi} + 81 \cdot e^{i(-2\pi)}$$

$$= 81(1 + 1) = 162$$

23. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$. If $B = [b_{ij}]_{3 \times 3}$ and $B = A^{99} - I$, then find $\frac{b_{31} - b_{21}}{b_{32}}$.

Ans. (149)

Sol. $A = P + I$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = P + I \text{ where } P^3 = 0$$

$$A^{99} = I + 99P + {}^{99}C_2 P^2$$

$$A^{99} - I = 99 \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + {}^{99}C_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 99 \times 3 & 0 & 0 \\ 44550 & 99 \times 3 & 0 \end{bmatrix}$$

$$b_{31} = 44550, b_{21} = 99 \times 3, b_{32} = 99 \times 3$$

$$\frac{44550 - 297}{297} = 149$$

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | FAX No. : +91-022-39167222

To Know more : sms RESO at 56677 | Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | [facebook.com/ResonanceEdu](https://www.facebook.com/ResonanceEdu) | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in