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JEE (MAIN) 2026

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 04th April 2026 & Saturday

PAPER-1

Duration: 3 Hrs.

Time: 03:00 PM – 06:00 PM

SUBJECT: MATHEMATICS

Selections in JEE (Advanced)/
IIT-JEE Since 2002

52979

Classroom: 35901 | Distance: 17078

Selections in JEE (Main)/
AIEEE Since 2009

262693

Classroom: 194471 | Distance: 68222

Selections in NEET (UG)/
AIPMT/AIIMS Since 2012

22733

Classroom: 15409 | Distance: 7324

Admission Open for 2026-27

Target: JEE (Advanced) | JEE (Main) | NEET (UG) | PCCP (Class V to X)

100% Scholarship on the basis of Class 10th, 12th
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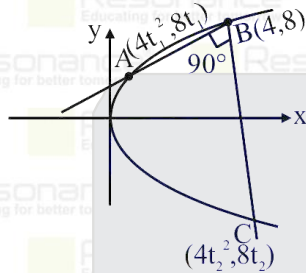
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PART : MATHEMATICS

1. From point B(4,8) on the parabola $y^2 = 16x$, two perpendicular chords BA and BC are drawn. Given that the locus of centroid of triangle BAC is another parabola with length of the latus rectum equal to ℓ , then 3ℓ is equal to
 (1) 14 (2) 15 (3) 12 (4) 16

Ans. (4)



Sol.

Variable point A & C are shown in the figure

$$\therefore m_{AB} \cdot m_{BC} = -1$$

$$\Rightarrow t_1 + t_2 + t_1 t_2 = -5 \quad (1)$$

Suppose locus of centroid of $\triangle ABC$ is (h, k)

$$\therefore 3h = 4 + 4t_1^2 + 4t_2^2 \text{ \& } 3k = 8 + 8t_1 + 8t_2$$

by eliminating t_1 & t_2 using equation (1) also

$$\text{we get } h = \frac{9}{48}k^2 + \frac{40}{3}$$

\therefore locus of centroid of parabola is

$$x = \frac{9}{48}y^2 + \frac{40}{3}$$

$$\therefore \ell(\text{L.R.}) = \frac{48}{9}$$

$$\therefore 3\ell(\text{L.R.}) = \frac{48}{3}$$

2. Area bounded between the curves $x = -2y^2$ and $x = 1 - 4y^2$ is

- (1) $\frac{\sqrt{2}}{3}$ (2) $\frac{2\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3\sqrt{3}}{3}$

Ans. (2)

Sol. Solving both the curves

$$-2y^2 = 1 - 4y^2$$

$$y = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$$\text{Required area} = \left| \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (x_2 - x_1) dx \right|$$

$$= \left| \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 4y^2 + 2y^2) dy \right|$$

$$= \left| 2 \int_0^{1/\sqrt{2}} (1 - 2y^2) dy \right|$$

$$= \left| 2 \left[\left(y - \frac{2y^3}{3} \right) \right]_0^{1/\sqrt{2}} \right|$$

$$= \left| 2 \left(\frac{1}{\sqrt{2}} - \frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right) \right| = \frac{2\sqrt{2}}{3}$$

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3. If $\sum_{i=1}^{10} (x_i + 2)^2 = 180$ and $\sum_{i=1}^{10} (x_i - 1)^2 = 90$, then the Standard Deviation is equal to
 (1) 3 (2) 2 (3) 4 (4) 5

Ans. (1)

Sol. $\sum_{i=1}^{10} (x_i + 2)^2 = 180$
 $\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 4 = 180$

$$\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i = 180 - 40 \quad (1)$$

Also $\sum_{i=1}^{10} (x_i - 1)^2 = 90$
 $\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 1 = 90$

$$\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i = 90 - 10 \quad (2)$$

$$\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i = 140$$

$$\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i = 80$$

$$\sum_{i=1}^{10} x_i^2 = 100 \text{ and } \sum_{i=1}^{10} x_i = 10$$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{N} - \left(\frac{\sum_{i=1}^{10} x_i}{N} \right)^2$$

$$= \frac{100}{10} - \left(\frac{10}{10} \right)^2$$

$$\sigma^2 = 10 - 1 = 9$$

$$\Rightarrow \sigma = 3$$

4. If the system of equations

$$x + y + z = 5$$

$$x + 2y + 3z = 9$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions, then value of $\lambda + \mu$ is

- (1) 13 (2) 20 (3) 18 (4) 26

Ans. (3)

Sol. $D = \lambda - 5$

$$D_1 = \lambda + \mu - 18$$

$$D_2 = 4\lambda - 2\mu + 6$$

$$D_3 = \mu - 13$$

$$\text{Here } x = \frac{D_1}{D}; y = \frac{D_2}{D}; z = \frac{D_3}{D}$$

For infinitely many solutions

$$D = D_1 = D_2 = D_3 = 0$$

$$\Rightarrow \lambda = 5; \mu = 13$$

$$\therefore \lambda + \mu = 18$$

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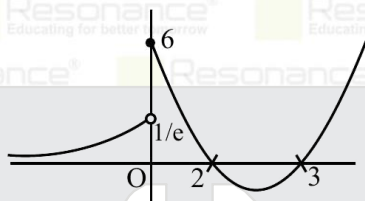
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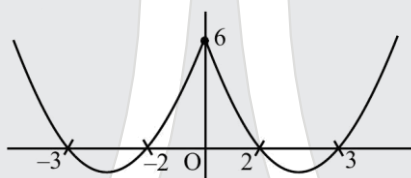
5. Let $f(x) = \begin{cases} e^{x-1} & x < 0 \\ x^2 - 5x + 6 & x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$.
If $\alpha =$ number of points of discontinuity of $g(x)$ and $\beta =$ number of points of non-differentiability of $g(x)$, then $\alpha + \beta =$
(1) 2 (2) 4 (3) 3 (4) 5

Ans. (2)

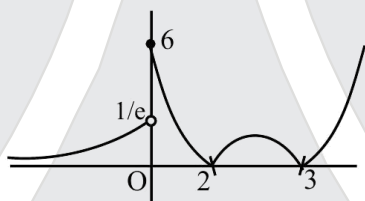
Sol. Graph of $f(x)$



Now graph of $f(|x|)$



$f(|x|)$ is continuous function and it is non diff. at $x = 0$ Graph of $|f(x)|$



$|f(x)|$ is discontinuous at $x = 0$

$|f(x)|$ is non-diff. at $x = 0, 2, 3$

$g(x) = f(|x|) + |f(x)|$

$g(x)$ will be discontinuous at $x = 0$

$g(x)$ will be non diff at $x = 0, 2, 3$

$\alpha = 1, \beta = 3$

$\alpha + \beta = 1 + 3 = 4$

6. If z_1, z_2 and z_3 are roots of $x^3 + ax^2 + bx + c = 0$. Let $z_1 = 1, z_2 = 1 + i\sqrt{2}$ and $a, b, c \in R$ then the value of $\int_{-1}^1 (x^3 + ax^2 + bx + c) dx$

(1) -8

(2) 8

(3) 6

(4) -4

Ans. (1)

Sol. $\int_{-1}^1 (x^3 + ax^2 + bx + c) dx$

$$\Rightarrow \int_{-1}^1 (ax^2 + c) dx = 2 \int_0^1 (ax^2 + c) dx \quad (1)$$

Roots are $1, 1 + i\sqrt{2}$ and $1 - i\sqrt{2}$ because $a, b, c \in R$

$$1 + 1 + i\sqrt{2} + 1 - i\sqrt{2} = -a$$

$$a = -3$$

$$1(1 + i\sqrt{2})(1 - i\sqrt{2}) = -c$$

$$c = -4$$

$$2 \int_0^1 (-3x^2 - 3) dx \Rightarrow -2[x^3 + 3x]_0^1$$

$$= -2[4] = -8$$

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7. The maximum value of $E = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$ where $x \in [0, \pi]$, is

- (1) 3 (2) $3\sqrt{3}$ (3) $6\sqrt{3}$ (4) 6

Ans. (2)

Sol. $E = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$

$E = 4\sin x[1 + \cos x]$

$\frac{dE}{dx} = 4[\cos x + \cos 2x] = 8\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow x = \left\{\frac{\pi}{3}, \pi\right\}$ are critical points of the function

$E(0) = 0$

$E(\pi) = 0$

$E\left(\frac{\pi}{3}\right) = 3\sqrt{3}$

\therefore maximum value of $E = 3\sqrt{3}$

8. If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$ (where $\lambda \neq -2$) has two positive roots then the numbers of possible integral values of λ is

- (1) 2 (2) 4 (3) 1 (4) 3

Ans. (1)

Sol. $f(x) = (\lambda + 2)x^2 - 3\lambda x + 4\lambda$

C-1 $af(0) > 0$

$(\lambda + 2)4\lambda > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

C-2 $-\frac{b}{2a} > 0$

$\frac{3\lambda}{2(\lambda + 2)} > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

C-3 $D \geq 0$

$(-3\lambda)^2 - 4(\lambda + 2) \times 4\lambda \geq 0$

$\lambda(7\lambda + 32) \leq 0$

$\Rightarrow \lambda \in \left[-\frac{32}{7}, 0\right]$

C-1 and C-2 and C-3

$\Rightarrow \lambda \in \left[-\frac{32}{7}, -2\right)$

$\lambda \in [-4.57, -2)$

$\Rightarrow \lambda = -4, -3$

\therefore Number of values of $\lambda = 2$

9. 3 numbers are selected randomly from numbers 1,2,3 ...,31. The probability that they are in A.P. is

- (1) $\frac{15}{31}$ (2) $\frac{7}{31}$ (3) $\frac{8}{17}$ (4) $\frac{45}{899}$

Ans. (4)

Sol. Total numbers of ways of selecting 3 numbers = ${}^{31}C_3 = \frac{31 \times 30 \times 29}{3 \times 2 \times 1} = 4495$ a, b, c are in A.P.

\Rightarrow either a & c both odd numbers or both even numbers

$= {}^{16}C_2 + {}^{15}C_2 = 120 + 105 = 225$

Probability = $\frac{225}{4495} = \frac{45}{899}$

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10. If function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} + \left[\frac{6x^2 + e^{-2x}(3x^2 + 2x^3 + 4)}{(x^3 + 2)(2 + e^{-2x})} \right] y = e^{-2x} + 2 \text{ such that } y(0) = \frac{3}{2} \text{ \& } y(1) = \alpha(e^{-2} + 2), \text{ then } \alpha \text{ is equal to}$$

- (1) $\frac{13}{12}$ (2) $\frac{12}{13}$ (3) $\frac{4}{13}$ (4) $\frac{17}{12}$

Ans. (1)

Sol. I.F. = $e^{\int \frac{6x^2 + e^{-2x}(3x^2 + 2x^3 + 4)}{(x^3 + 2)(2 + e^{-2x})} dx} = e^{\int \frac{6x^2 + e^{-2x}(3x^2 - 2x^3 - 4) + (4x^3 + 8)e^{-2x}}{(x^3 + 2)(2 + e^{-2x})} dx}$

Let $(x^3 + 2)(2 + e^{-2x}) = t$
 $(6x^2 + e^{-2x}(3x^2 - 2x^3 - 4))dx = dt$

I.F. = $e^{\ln(x^3 + 2)(2 + e^{-2x}) - \int \frac{4e^{-2x} dx}{2 + e^{-2x}}} = e^{\ln|(x^3 + 2)(2 + e^{-2x})| - 2\ln|2 + e^{-2x}|} = e^{\ln \left| \frac{x^3 + 2}{2 + e^{-2x}} \right|} = \frac{x^3 + 2}{2 + e^{-2x}}$

$\Rightarrow y \cdot \left(\frac{x^3 + 2}{2 + e^{-2x}} \right) = \int \left(\frac{x^3 + 2}{2 + e^{-2x}} \right) (e^{-2x} + 2) dx + C$

$y \cdot \left(\frac{x^3 + 2}{2 + e^{-2x}} \right) = \frac{x^4}{4} + 2x + C$ at $y(0) = \frac{3}{2} \Rightarrow C = 1$

So $y \cdot \left(\frac{x^3 + 2}{2 + e^{-2x}} \right) = \frac{x^4}{4} + 2x + 1$

at $x = 1$

$y \cdot \left(\frac{3}{2 + e^{-2}} \right) = \frac{1}{4} + 2 + 1$

$\Rightarrow y = \frac{13}{12} (e^{-2} + 2)$

So $\alpha = \frac{13}{12}$

11. $\int_0^1 \cot^{-1}(1 + x + x^2) dx$ is equal to

- (1) $2\tan^{-1} 2 - \frac{1}{2} \log_e \left(\frac{5}{4} \right) - \frac{\pi}{2}$ (2) $2\tan^{-1} 2 - \frac{1}{2} \log_e \left(\frac{5}{4} \right) + \frac{\pi}{2}$
 (3) $2\tan^{-1} 2 + \frac{1}{2} \log_e \left(\frac{5}{4} \right) - \frac{\pi}{2}$ (4) $2\tan^{-1} 2 + \frac{1}{2} \log_e \left(\frac{5}{4} \right) + \frac{\pi}{2}$

Ans. (1)

Sol. = $\int_0^1 \tan^{-1} \left(\frac{1}{1 + x + x^2} \right) dx$

= $\int_0^1 \tan^{-1} \left(\frac{(x+1) - x}{1 + x(x+1)} \right) dx$

= $\int_0^1 (\tan^{-1}(x+1) - \tan^{-1} x) dx$

= $\left(x \tan^{-1}(x+1) - \frac{1}{2} \ln|1 + (1+x)^2| + \tan^{-1}(x+1) \right)_0^1$

$- \left(x \tan^{-1} x - \frac{1}{2} \ln|1 + x^2| \right)_0^1$

= $\left(2\tan^{-1} 2 - \frac{\pi}{4} - \frac{1}{2} \ln \frac{5}{2} \right) - \left(\frac{\pi}{4} - \frac{1}{2} \ln 2 \right)$

= $2\tan^{-1} 2 - \frac{1}{2} \ln \left(\frac{5}{4} \right) - \frac{\pi}{2}$

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12. Number of non-negative integral solutions of the equation $a + b + 2c = 22$ is

- (1) 144 (2) 121 (3) 168 (4) 99

Ans. (1)

Sol. $c = 0 \Rightarrow a + b = 22 \rightarrow {}^{22+2-1}C_{2-1} = {}^{23}C_1$

$c = 1 \Rightarrow a + b = 20 \rightarrow {}^{20+2-1}C_{2-1} = {}^{21}C_1$

\vdots

$c = 10 \Rightarrow a + b = 2 \rightarrow {}^{2+2-1}C_{2-1} = {}^3C_1$

$c = 11 \Rightarrow a + b = 0 \rightarrow {}^{0+2-1}C_{2-1} = {}^1C_1$

Number of non-negative integral solutions

${}^1C_1 + {}^3C_1 + {}^5C_1 + \dots + {}^{23}C_1$

$1 + 3 + 5 \dots \dots + 23$

$= 144$

13. If $f(x)$ satisfies the functional equation $f(x + y) = f(x) + 2y^2 + y + \alpha xy$ (where x, y are whole numbers), such that $f(0) = -1$ & $f(1) = 2$, then the value of $\sum_{i=1}^5 (f(i) + \alpha)$ is

- (1) 130 (2) 145 (3) 120 (4) 140

Ans. (4)

Sol. Given $f(x + y) = f(x) + 2y^2 + y + \alpha xy$ (1)

put $x = 1, y = 1 \Rightarrow f(2) = 5 + \alpha$

put $x = 2, y = 1 \Rightarrow f(3) = 8 + 3\alpha$ (2)

put $x = 1, y = 2 \Rightarrow f(3) = 12 + 2\alpha$ (3)

from (2) & (3) $\Rightarrow \alpha = 4$

put $x = 3, y = 1 \Rightarrow f(4) = 35$

put $x = 4, y = 1 \Rightarrow f(5) = 54$

$\therefore \Rightarrow$ value of $\sum_{i=1}^5 (f(i) + \alpha) = \sum_{i=1}^5 f(i) + 5\alpha$

$= f(1) + f(2) + f(3) + f(4) + f(5) + 5\alpha$

$= 140$

14. If $f(x) = \int_0^x \tan(t - x) dt + \int_0^x f(t) \tan t dt$, then the value of $f''\left(\frac{\pi}{6}\right) - 12f'\left(-\frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$ is equal to

- (1) $-7 - \frac{16}{3\sqrt{3}}$ (2) $7 + \frac{5}{3\sqrt{3}}$ (3) $7 - \frac{16}{3\sqrt{3}}$ (4) $\frac{1}{3\sqrt{3}}$

Ans. (1)

Sol. $f(x) = \int_0^x \tan(x - t - x) dt + \int_0^x f(t) \tan t dt$

$f(x) = -\int_0^x \tan t dt + \int_0^x f(t) \tan t dt$

$f'(x) = -\tan x + f(x) \tan x$

$\frac{dy}{y-1} = \tan x dx$

$y - 1 = \sec x \cdot c$

$y = 1 + c \sec x$

$f(0) = 0$

$0 = 1 + c \sec 0$

$c = -1$

$y = 1 - \sec x$

$f(x) = 1 - \sec x$

$f\left(\frac{\pi}{6}\right) = 1 - \frac{2}{\sqrt{3}}$

$f'(x) = -\sec x \tan x$

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$$f' \left(\frac{-\pi}{6} \right) = + \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$$

$$f''(x) = -(\sec x \tan^2 x + \sec^3 x)$$

$$f'' \left(\frac{\pi}{6} \right) = - \left(\frac{2}{\sqrt{3}} \times \frac{1}{3} + \frac{8}{3\sqrt{3}} \right) = \frac{-10}{3\sqrt{3}}$$

$$f'' \left(\frac{\pi}{6} \right) - 12f' \left(\frac{-\pi}{6} \right) + f \left(\frac{\pi}{6} \right)$$

$$= \frac{-10}{3\sqrt{3}} - 12 \times \left(\frac{2}{3} \right) + 1 - \frac{2}{\sqrt{3}}$$

$$= -7 - \frac{16}{3\sqrt{3}}$$

15. Let $S = \{z: z^2 + 4z + 16 = 0, z \in C\}$, then value of $\sum_{z \in S} |z + \sqrt{3}i|^2$ is equal to
 (1) 34 (2) 35 (3) 38 (4) 41

Ans. (3)

Sol. $z^2 + 4z + 16 = 0$
 $\Rightarrow (z + 2)^2 = -12$
 $\Rightarrow z = -2 \pm 2\sqrt{3}i$
 $\Rightarrow S = \{-2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i\}$

$$\therefore \sum_{z \in S} |z + \sqrt{3}i|^2$$

$$= |-2 + 2\sqrt{3}i + \sqrt{3}i|^2 + |-2 - 2\sqrt{3}i + \sqrt{3}i|^2$$

$$= |-2 + 3\sqrt{3}i|^2 + |-2 - \sqrt{3}i|^2$$

$$= 38$$

16. If $\alpha = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ up to infinity
 $\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ up to infinity
 Then value of $(0.2)^{\log_{\sqrt{5}} \alpha} + (0.04)^{\log_5 \beta}$ is
 (1) $\frac{1}{2}$ (2) 8 (3) 3 (4) $\frac{3}{4}$

Ans. (2)

Sol. $\alpha = \frac{1/4}{1 - \frac{1}{2}} = \frac{1}{2}$
 $\beta = \frac{1/3}{1 - \frac{1}{3}} = \frac{1}{2}$

$$(0.2)^{\log_{\sqrt{5}} \alpha} = \alpha^{\log_{\sqrt{5}} \frac{1}{2}} = \left(\frac{1}{2} \right)^{-2} = 4$$

$$(0.04)^{\log_5 \beta} = 5^{-2 \log_5 \beta} = \left(\frac{1}{2} \right)^{-2} = 4$$

$$\Rightarrow 4 + 4 = 8$$

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17. The shortest distance between the lines is:

$$\vec{r} = \frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k} + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k}) \text{ and } \vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu(\hat{j} - \hat{k}), \lambda, \mu \in \mathbb{R}$$

(1) $2\sqrt{3}$ (2) 3 (3) $\sqrt{15}$ (4) $\sqrt{5}$

Ans. (2)

Sol. $\vec{a}_1 = \frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}$

$$\vec{a}_2 = \frac{-2}{3}\hat{i} - \frac{1}{3}\hat{k}$$

$$\vec{b}_1 = 2\hat{i} - 5\hat{j} + 6\hat{k}, \vec{b}_2 = \hat{j} - \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{a}_1 - \vec{a}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$S.D = \frac{|(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$S.D = \frac{|(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} + 2\hat{k})|}{3}$$

$$= \frac{9}{3} = 3$$

18. If $f(x) = (x - 1)^4 + 1 \forall x \in [1, \infty)$.

Statement-1 : $f(x) = f^{-1}(x)$ has only two solutions.

Statement-2 : $f^{-1}(x + 1) = f(x)$ has no solution.

- (1) Statement 1 and Statement 2 both are true
 (2) Statement 1 is false and Statement 2 is true
 (3) Statement 1 is true and Statement 2 is false
 (4) Statement 1 and Statement 2 both are false

Ans. (3)

Sol. $f(x) = (x - 1)^4 + 1$

$$f'(x) = 4(x - 1)^3; f'(x) \geq 0$$

$\therefore f(x)$ is increasing

$$\therefore (x - 1)^4 + 1 = x$$

$$(x - 1)[(x - 1)^3 - 1] = 0$$

$x = 1, x = 2$ are two solution



Now

$$f^{-1}(x) = (x - 1)^{1/4} + 1$$

$$f^{-1}(x + 1) = x^{1/4} + 1$$

$$f^{-1}(x + 1) = f(x)$$

$$x^{1/4} + 1 = (x - 1)^4 + 1$$

$$x = (x - 1)^{16}$$

Above equation has one solution using graph. Hence option (3) is correct.

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19. If \hat{u}, \hat{v} are unit vectors and $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$ and $\hat{A} = \lambda\hat{u} + \hat{v} + \hat{u} \times \hat{v}$ then find λ [angle between \hat{u} & \hat{v} is acute]

$$(1) \lambda = \frac{1}{3} \hat{A} \cdot \hat{u} - \frac{1}{3} \hat{A} \cdot \hat{v}$$

$$(2) \lambda = \frac{4}{3} \hat{A} \cdot \hat{u} - \frac{2}{3} \hat{A} \cdot \hat{v}$$

$$(3) \lambda = \frac{8}{3} \hat{A} \cdot \hat{u} - \frac{2}{3} \hat{A} \cdot \hat{v}$$

$$(4) \lambda = \frac{2}{3} \hat{A} \cdot \hat{u} - \frac{4}{3} \hat{A} \cdot \hat{v}$$

Ans. (2)

Sol. $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$

$$|\hat{u}||\hat{v}|\sin\theta = \frac{\sqrt{3}}{2} \therefore \theta = \frac{\pi}{3}$$

$$\text{and } \hat{u} \cdot \hat{v} = |\hat{u}||\hat{v}|\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\hat{A} = \lambda\hat{u} + \hat{v} + \hat{u} \times \hat{v} \quad (A)$$

Dot with \hat{u}

$$\hat{A} \cdot \hat{u} = \lambda(1) + \hat{u} \cdot \hat{v} + \hat{u} \cdot (\hat{u} \times \hat{v})$$

$$\hat{A} \cdot \hat{u} = \lambda + \frac{1}{2}$$

$$\text{Or } 2\hat{A} \cdot \hat{u} = 2\lambda + 1 \quad (1)$$

Dot equation (A) with \hat{v}

$$\hat{A} \cdot \hat{v} = \lambda(\hat{u} \cdot \hat{v}) + \hat{v} \cdot \hat{v} + \hat{v} \cdot (\hat{u} \times \hat{v})$$

$$\hat{A} \cdot \hat{v} = \frac{\lambda}{2} + 1$$

$$\hat{A} \cdot \hat{v} - \frac{\lambda}{2} = 1 \quad (2)$$

From (1) and (2)

$$2\hat{A} \cdot \hat{u} - 2\lambda = \hat{A} \cdot \hat{v} - \frac{\lambda}{2}$$

$$\therefore \lambda = \frac{4}{3} \hat{A} \cdot \hat{u} - \frac{2}{3} \hat{A} \cdot \hat{v}$$

20. In the expansion of $(9x - \frac{1}{3\sqrt{x}})^{18}$, if coefficient of term independent of x is 221 k, then value of k is

Ans. (84)

Sol. General term in above expression is

$$T_{r+1} = {}^{18}C_r (9x)^{18-r} \left(-\frac{1}{3\sqrt{x}}\right)^r$$

$$= \left(-\frac{1}{3}\right)^r {}^{18}C_r 9^{18-r} \cdot x^{18-\frac{3r}{2}}$$

For term independent of x, $18 - \frac{3r}{2} = 0 \Rightarrow r = 12$

\therefore Coefficient of term independent of x

$$= \left(-\frac{1}{3}\right)^{12} {}^{18}C_{12} 9^{18-12}$$

$$= 18564$$

$$= 221k \text{ (given)}$$

$$\Rightarrow k = 84$$

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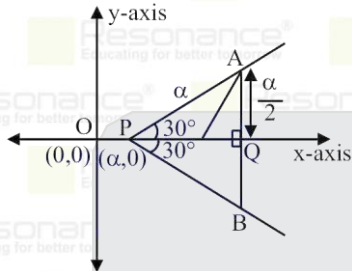
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21. P is point of intersection of these two set of half lines $x - \sqrt{3}y = \alpha, \alpha > 0$. A and B are two points on these lines at distance ' α ' from 'P'. If length of perpendicular from P on AB is $\frac{9}{2}$ and radius of circumcircle of $\triangle PAB$ is R, then $\frac{\alpha^2}{R}$ is

Ans. (9.00)

Sol. $\triangle PAB$ is equilateral



$$PQ = \frac{\sqrt{3}\alpha}{2} = \frac{9}{2} \therefore \alpha = \frac{9}{\sqrt{3}}$$

$$\alpha = 3\sqrt{3}$$

$$\frac{\alpha}{2r} = \cos 30^\circ$$

$$\alpha = \frac{\sqrt{3}}{2} \cdot 2R$$

$$\alpha = \sqrt{3}R$$

$$R = 3$$

$$\frac{\alpha^2}{R} = \frac{(3\sqrt{3})^2}{3} = 9$$

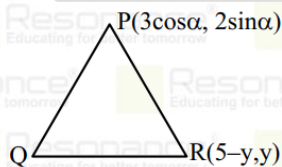
22. P is a point on $\frac{x^2}{9} + \frac{y^2}{4} = 1$ as $(3\cos\alpha, 2\sin\alpha)$ Q is a point on $x^2 + y^2 - 14x + 14y + 82 = 0$ R is a point on line $x + y = 5$ their centroid is on $(\cos\alpha + 2, \frac{2\sin\alpha}{3} + 3)$. Find the sum of possible ordinates of R.

Ans. (22)

Sol. Centroid :

$$\left(\frac{3\cos\alpha + 7 + 4\cos\theta + 5 - y}{3}, \frac{2\sin\alpha - 7 + 4\sin\theta + y}{3} \right)$$

$$= \left(\cos\alpha + 2, \frac{2}{3}\sin\alpha + 3 \right)$$



$$(7+4\cos\theta, -7+4\sin\theta)$$

On comparison

$$\cos\theta = \frac{y-6}{4} \text{ \& \ } \sin\theta = \frac{16-y}{4}$$

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow (y-6)^2 + (y-16)^2 = 16$$

$$y^2 - 22y + 138 = 0$$

Sum of possible values of $y = 22$

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23. If $(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha-1}{2})$ is image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the value of α is

Ans. (3)

Sol. \therefore P is mid point of AB and lies on the given line

$$\Rightarrow P\left(\frac{3\alpha+1}{2}, \frac{\alpha^2-\alpha}{2}, \frac{\alpha+1}{4}\right)$$

A($\alpha, 2\alpha, 1$)

B($2\alpha+1, \alpha^2-3\alpha, \frac{\alpha-1}{2}$)

Now put coordinates of P in the line

$$\Rightarrow \frac{\frac{3\alpha+1}{2} - 2}{3} = \frac{\frac{\alpha^2-\alpha}{2} - 1}{2} = \frac{\alpha+1}{4}$$

$$\Rightarrow \frac{\alpha-1}{2} = \frac{\alpha+1}{4} = \frac{\alpha^2-\alpha-2}{4}$$

$$\Rightarrow \alpha = 3$$

Now for $\alpha = 3$, clearly AB is perpendicular to $3\hat{i} + 2\hat{j} + \hat{k}$ for $\alpha = 3$ point B is image of point A.

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