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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Official Based Questions & Solutions

Date: 25 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | facebook.com/ResonanceEdu | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

PART : MATHEMATICS

61. The Mean & Variance of the marks obtained by the student in a test are 10 and 4 respectively. Later, the marks of of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to :

(1) 3.92 (2) 3.96 (3) 4.04 (4) 4.08

Ans. **(2)**

Sol. Let variates $x_1, x_2, x_3 \ldots\ldots\ldots x_{n-1}, 8$

Now, $\frac{x_1 + x_2 + x_3 \ldots\ldots x_{n-1} + 8}{n} = 10 \Rightarrow \sum_{i=1}^{n-1} x_i = 10n - 8$

and $\frac{\sum_{i=1}^{n-1} x_i^2 + (8)^2}{n} - (\bar{x})^2 = 4$

$\frac{\sum x_i^2 + 64}{n} - 100 = 4 \Rightarrow \sum_{i=1}^{n-1} x_i^2 = 104n - 64$

Now, new mean $= \frac{\sum_{i=1}^{n-1} x_i + 12}{n} = 10.2 \Rightarrow 10n - 8 + 12 = 10.2n$

$\Rightarrow n = 20$

New variance $= \frac{\sum_{i=1}^{n-1} x_i^2 + (12)^2}{n} - (10.2)^2$

$= \frac{104n - 64 + 144}{n} - (10.2)^2 = 3.96$

62. If a_r is coefficient of x^{10-r} in the Binomial expansion of $(1 + x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(1) 3025 (2) 1210 (3) 5445 (4) 4895

Ans. **(2)**

Sol. Coeff. of x^{10-r} in $(1 + x)^{10}$

$a_r = {}^{10}C_r$

$\frac{a_r}{a_{r-1}} = \frac{{}^{10}C_r}{{}^{10}C_{r-1}} = \frac{10 - r + 1}{r} = \frac{11 - r}{r}$

$\sum_{r=0}^{10} r^3 \left(\frac{11-r}{r}\right)^2 = \sum_{r=0}^{10} r(11 - r)^2$

$= \sum_{r=0}^{10} \left(r^3 - 22r^2 + 121r\right)$

$= \sum_{r=0}^{10} r^3 - 22\sum_{r=0}^{10} r^2 + 121\sum_{r=0}^{10} r = \left(\frac{10(10+1)}{2}\right)^2 - 22\times \frac{10(10+1)(20+1)}{6} + 121\times \frac{10\cdot 11}{2}$

$= 55^2 - 11 \times 11 \times 70 + 5 \times 11^3 = 11^2[25 - 70 + 55] = 11^2 \times 10 = 1210$

63. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}(1 + xy^2(1 + \log_e x))$, x > 0, y (1) = 3.

Then $\frac{y^2(x)}{9}$ is equal to :

(1) $\frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$

(2) $\frac{x^2}{3x^3(1 + \log_e x^2) - 2}$

(3) $\frac{x^2}{7 - 3x^3(2 + \log_e x^2)}$

(4) $\frac{x^2}{2x^3(2 + \log_e x^3) - 3}$

Ans. **(1)**

Sol. $\frac{dy}{dx} = \frac{y}{x} + y^3(1 + \ln x)$

$y^{-3}\frac{dy}{dx} - \frac{1}{x}y^{-2} = 1 + \ln x$

Put $y^{-2} = t \Rightarrow -2y^{-3}\, dy = dt$

$\Rightarrow -\frac{1}{2}\frac{dt}{dx} - \frac{t}{x} = (1 + \ln x)$

$\Rightarrow \frac{dt}{dx} + \frac{2}{x}t = -2(1 + \ln x)$ ....(i)

I.F. $= e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Hence solution of linear differential equation is given by

$tx^2 = \int -2(1 + \ln x)\cdot x^2 dx + c$

$\frac{x^2}{y^2} = -2\left\{(1 + \ln x)\frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{x^3}{3}dx\right\} + C$

$\frac{x^2}{y^2} = -\frac{2}{3}x^3(1 + \ln x) + \frac{2}{9}x^3 + C$

As, $y(1) = 3 \Rightarrow C = \frac{5}{9} \Rightarrow \frac{x^2}{y^2} = -\frac{2}{3}x^3(1 + \ln x) + \frac{2}{9}x^3 + \frac{5}{9}$

So, $\frac{y^2(x)}{9} = \frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$

64. Let $f(x)=\int\frac{2x}{(x^2+1)(x^2+3)}dx$. If f(3) = $\frac{1}{2}(\log_e 5 - \log_e 6)$, then f(4) is equal to

(1) $\log_e 17 - \log_e 18$ (2) $\log_e 19 - \log_e 20$

(3) $\frac{1}{2}(\log_e 19 - \log_e 17)$ (4) $\frac{1}{2}(\log_e 17 - \log_e 19)$

Ans. **(4)**

Sol. $f(x)=\int\frac{2xdx}{(x^2+1)(x^2+3)}$ Let $x^2=t$, $2xdx=dt$

$= \int\frac{dt}{(t+1)(t+3)}=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$

$f(x)=\frac{1}{2}\ell n\left|\frac{t+1}{t+3}\right|+c$

$f(x)=\frac{1}{2}\ell n\left(\frac{x^2+1}{x^2+3}\right)+c$

$\because f(3)=\frac{1}{2}\ell n\left(\frac{5}{6}\right)$ $\therefore c = 0$

Hence f(x)= $\frac{1}{2}\ell n\left(\frac{x^2+1}{x^2+3}\right)$

$\Rightarrow f(4) = \frac{1}{2}(\log_e 19 - \log_e 17)$

65. The minimum value of the function f(x) = $\int_0^2 e^{|x-t|}dt$ is :

(1) 2 (2) 2 (e – 1) (3) 2 e – 1 (4) e (e – 1)

Ans. **(2)**

Sol. C-1 When x < 0

$F(x) = \int_0^2 e^{t-x}dt = e^{-x}\left(e^t\right)_0^2$

$F(x) = e^{-x}\left(e^2-1\right)$

C-2 when 0 < x < 2

$F(x) = \int_0^x e^{(x-t)}dt+\int_x^2 e^{(t-x)}dt$

$F(x) = e^x\left(-e^{-t}\right)_0^x+e^{-x}\left(e^t\right)_x^2$

$= -e^x\left(e^{-x}-1\right)+e^{-x}\left(e^2-e^x\right)$

$= -1+e^x+e^{-x}e^2-1$

$= e^x+e^{-x}e^2-2$

C-3 $x \geq 2$

$$F(x) = \int_0^2 e^{(x-t)}dt = e^x\left(-e^{-t}\right)_0^2$$

$$= -e^x\left(e^{-2}-1\right)$$

$$= \left(1-e^{-2}\right)e^x$$

Hence

$$F(x) = \begin{cases} e^{-x}(e^2-1) & x<0 \\ e^x + e^{-x}e^2 - 2 & 0 \leq x < 2 \\ \left(1-e^{-2}\right)e^x & x \geq 2 \end{cases}$$

F(x) is decreasing function for $x < 0$ & increasing function for $x \geq 2$

Now for $0 \leq x < 2$, F(x) is minimum at x = 1, by using A.M ≥ G.M

∴ Minimum value of F(x) is F(1) = 2 (e –1)

66. Let $f : (0, 1) \to \mathbb{R}$ be a function defined by $f(x) = \frac{1}{1-e^{-x}}$, and g(x) = f(–x) – f(x)). Consider two statements

(I) g is an increasing function in (0, 1)

(II) g is one-one in (0, 1)

Then,

(1) Only (I) and (II) are true

(2) Both (I) and (II) are true

(3) Neither (I) nor (II) is true

(4) Only (II) is true.

Ans. (2)

Sol. $g(x) = f(-x) - f(x) = \frac{1}{1-e^x} - \frac{1}{1-e^{-x}}$

$$\Rightarrow g(x) = \frac{1+e^x}{1-e^x}$$

$$g'(x) = \frac{\left(1-e^x\right)\left(e^x\right)-\left(1+e^x\right)\left(-e^x\right)}{\left(1-e^x\right)^2}$$

$$g'(x) = \frac{2e^x}{\left(1-e^x\right)^2} > 0$$

g (x) is increasing as well as one-one.

67. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \left\{z \in \mathbb{C} : |z-z_1|^2 - |z-z_2|^2 = |z_1-z_2|^2\right\}$ represents a

(1) straight line with the sum of its intercepts on the coordinate axes equals –18

(2) hyperbola with eccentricity 2

(3) hyperbola with the length of the transverse axis 7

(4) straight line with the sum of its intercepts on the coordinate axes equal to 14

Ans. (4)

Sol. Let moving point be P(z)

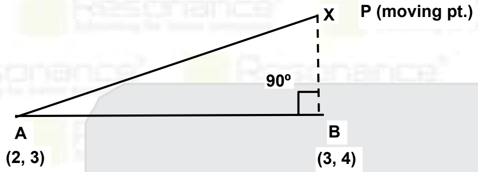
and $z_1 = 2 + 3i = A(z_1)$ and $z_2 = 3+4i = B(z_2)$

Now

$|z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2$

$= PA^2 - PB^2 = AB^2$

$\Rightarrow AB^2 + PB^2 = PA^2$



locus of P is a straight line passing through B and $\perp$ to AB i.e $x + y = 7$

sum of its intercepts on the coordinate axes equals 14

68. The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c$, $m > 0$, of the curve $x = 2y^2$ and $x = 1 + y^2$ is:

(1) $5\sqrt{3}$ (2) $\frac{14}{3}$ (3) $\frac{1}{3}$ (4) 5

Ans. **(4)**

Sol. Equation of Tangent to the curve $y^2 = \frac{1}{2}x$ is of the form $y = mx + \frac{1}{8m}$

It's also tangent of $x = 1 + y^2$

So $x = 1 + \left(\frac{8m^2x + 1}{8m}\right)^2$

$64m^4x^2 - 48m^2x + 1 + 64m^2 = 0$

$D = 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$ $(m > 0)$

Now Common tangent is $x - 2\sqrt{2} + 1 = 0$

Distance from $(6, -2\sqrt{2})$ is

$= \left|\frac{6 + 8 + 1}{3}\right| = 5$ units

69. Let x, y, z > 1 and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. The |adj (adj A^2)| is equal to

(1) 4^8 (2) 2^8 (3) 2^4 (4) 6^4

Ans. **(2)**

Sol. Let $\log x = t_1$, $\log y = t_2$, $\log z = t_3$

$$\Rightarrow |A| = \begin{vmatrix} 1 & \frac{t_2}{t_1} & \frac{t_3}{t_1} \\ \frac{t_1}{t_2} & 2 & \frac{t_3}{t_2} \\ \frac{t_1}{t_3} & \frac{t_2}{t_3} & 3 \end{vmatrix} = 2$$

Now, $|\text{Adj}(\text{Adj}(A^2))| = |\text{Adj } A^2|^2 = (|A^2|^2)^2 = |A|^8$

$= 2^8$

70. Let $y(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$. Then $y' - y''$ at $x = -1$ is equal to :

(1) 976 (2) 944 (3) 496 (4) 464

Ans. (3)

Sol. $y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

$$\Rightarrow y = \frac{(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})}{(1-x)}$$

$$\Rightarrow y = \frac{1-x^{32}}{(1-x)}$$

$\Rightarrow (1-x)y = 1-x^{32}$

$\Rightarrow (1-x)y' - y = -32x^{31}$

$\Rightarrow (1-x)y'' - 2y' = -32.31x^{30}$

Put $x = -1$

$\Rightarrow y' - y'' = 496$

71. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \left\{x \in Z : x(66-x) \geq \frac{5}{9}M\right\}$ and the event A = {$x \in S$: x is a multiple of 3}. The P(A) is equal to

(1) $\frac{15}{44}$ (2) $\frac{1}{3}$ (3) $\frac{7}{22}$ (4) $\frac{1}{5}$

Ans. (2)

Sol. $x + y = 66$, $x, y \in N$

Now Maximum Value of xy is $M = (33)^2$ by using A. M $\geq$ G.M.

$$\frac{x+y}{2} \geq (xy)^{\frac{1}{2}} \Rightarrow xy \leq (33)^2$$

Now $S = \{x \in z, x(66-x) \geq \frac{5}{9}M\}$

$\Rightarrow S = \{x \in z, \quad x(66-x) \geq 55 \times 11\}$

S = { 11, 12, 13, ….., 55}

A = { $x \in s$, x is multiple of 3}

A = {12, 15, 18, …. , 54}

$$P(A) = \frac{n(A)}{n(s)} = \frac{15}{45} = \frac{1}{3}$$

72. Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios 1, –1, –2, intersects L_1 and L_2 at the point P and Q respectively. The the length of the line segment PQ is

(1) $3\sqrt{2}$ (2) 4 (3) $2\sqrt{6}$ (4) $4\sqrt{3}$

Ans. (3)

Sol. Let P on L_1 be P $(2r, +1, r_1 + 3, 2r_1 + 2)$

and Q on L_2 be Q $(r_2 + 2, 2r_2 + 2, 3r_2 + 3)$

$\Rightarrow$ D.R. of PQ = $2r_1 - r_2 - 1, r_1 - 2r_2 + 1, 2r_1 - 3r_2 - 1$

which are equivalent to –1, 1, 2

i.e. $\frac{2r_1 - r_2 - 1}{-1} = \frac{r_1 - 2r_2 + 1}{1} = \frac{2r_1 - 3r_2 - 1}{2}$

$\Rightarrow$ $r_1 = 3 = r_2$

$\Rightarrow$ P(7, 6, 8) and Q (5, 8, 12) $\Rightarrow$ length PQ = $2\sqrt{6}$

73. Let S1 and S2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations

$ax + 2ay - 3az = 1$

$(2a + 1) x + (2a + 3) y + (a + 1) z = 2$

$(3a + 5) x + (a + 5) y + (a + 2) z = 3$

has unique solution and infinitely many solution. The

(1) $S_1 = \phi$ and $S_2 = R - \{0\}$

(2) S_1 is an infinite set and $n(S_2) = 2$

(3) $S_1 = R - \{0\}$ and $S_2 = \phi$

(4) $n(S_1) = 2$ and S_2 is an infinite set

Ans. (3)

Sol. $D = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix} = a(15a^2 + 31a + 37)$

$\Rightarrow D \neq 0$ for all $a \in R - \{0\}$

74. The distance of the point P (4, 6 – 2) from the line passing through the point (–3, 2, 3) and parallel to a line with direction ratios 3, 3, –1 is equal to:

(1) $2\sqrt{3}$ (2) $\sqrt{14}$ (3) 3 (4) $\sqrt{6}$

Ans. (2)

Sol. Equation of line passing through (–3, 2, 3) with direction ratios 3, 3, –1 is

$\frac{x+3}{3} = \frac{y-2}{3} = \frac{z-3}{-1}$ _____ (1)

Consider Q $(3r - 3, 3r + 2, -r + 3)$ on the line (1) such that PQ is $\perp$ to line (1)

D.R. of PQ = $3r - 7, 3r - 4, -r + 5$

PQ $\perp$ line (1)

So $(3r - 7)(3) + (3r - 4)(3) + (-r + 5)(-1) = 0$

$\Rightarrow r = 2$

So Q (3, 8, 1)

$\Rightarrow$ PQ = $\sqrt{1+4+4} = \sqrt{14}$

75. The points of intersection of the line ax + by = 0, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are A $(\alpha, 0)$ and B $(1, \beta)$. The image of the circle with AB as a diameter in the line x + y + 2 = 0 is :

(1) $x^2 + y^2 + 3x + 3y + 4 = 0$

(2) $x^2 + y^2 + 5x + 5y + 12 = 0$

(3) $x^2 + y^2 + 3x + 5y + 8 = 0$

(4) $x^2 + y^2 - 5x - 5y + 12 = 0$

Ans. **(2)**

Sol. If the points of intersection of the line ax + by = 0, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are A $(\alpha, 0)$ and B $(1, \beta)$ $\Rightarrow$ $\alpha = 0$ $\beta = \frac{-a}{b}$ and $\beta^2 = 1$

$\Rightarrow a^2 = b^2 \ (a \neq b) \Rightarrow a + b = 0$

$\Rightarrow \beta = 1$

So centre of circle with diameter AB is $\left(\frac{1}{2}\ \frac{1}{2}\right)$ and radius = $\frac{1}{\sqrt{2}}$, where A(0, 0) B (1, 1)

Now image of $\left(\frac{1}{2}\ \frac{1}{2}\right)$ about the line x + y + 2 = 0

$$\frac{x-\frac{1}{2}}{1} = \frac{y-\frac{1}{2}}{1} = -2\left(\frac{3}{2}\right)$$

$\Rightarrow$ Image $\left(-\frac{5}{2}, -\frac{5}{2}\right)$

equation of Image circle

$$\left(x+\frac{5}{2}\right)^2 + \left(y+\frac{5}{2}\right)^2 = \frac{1}{2}$$

76. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is :

(1) $3\sqrt{2}$ (2) 1 (3) $2\sqrt{3}$ (4) $\sqrt{6}$

Ans. **(1)**

Sol. Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{b} \perp \vec{a} \Rightarrow \vec{b}.\vec{a} = 0 \Rightarrow -x + 2y + z = 0$ ....(1)

$|\vec{a}| = |\vec{b}| \Rightarrow x^2 + y^2 + z^2 = 6$ ....(2)

$\vec{a}, \vec{b}, \hat{j}$ are coplanar $\Rightarrow \begin{vmatrix} x & y & z \\ -1 & 2 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 \Rightarrow x = -z$ ....(3)

Now x = –z, y = –z (from (1))

$z^2 + z^2 + z^2 = 6 = \Rightarrow z = \pm\sqrt{2}$ (from (2))

when $z = \sqrt{2}$, $y = -\sqrt{2}$, $x = -\sqrt{2}$

$z = -\sqrt{2}$, $y = \sqrt{2}$, $x = \sqrt{2}$

but $\vec{b}$ makes an acute angle with positive y axes

$\Rightarrow \vec{b} = \sqrt{2}\,\hat{i} + \sqrt{2}\hat{j} - \sqrt{2}\hat{k}$

$\Rightarrow 3\vec{a} + \sqrt{2}\vec{b} = -\hat{i} + 8\hat{j} + \hat{k}$

Now projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ $= \dfrac{(3\vec{a} + \sqrt{2}\,\vec{b})\vec{c}}{|\vec{c}|} = 3\sqrt{2}$

77. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

(1) equivalent to $p \vee q$

(2) equivalent to $(\sim p) \vee (\sim q)$

(3) a contradiction

(4) a tautology

Ans. **(4)**

Sol. Case I q = T

So $(P \wedge \sim q) \Rightarrow (P \Rightarrow \sim q) \equiv T$

Case II q = F

$\Rightarrow (P \wedge \sim q) \Rightarrow (P \Rightarrow \sim q) = T$

So $(P \wedge \sim q) \Rightarrow (P \Rightarrow \sim q) = T$

78. Let x = 2 be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12$, $x \in (-4, 4)$. If M is local maximum value of the function f in (-4,4), then M =

(1) $18\sqrt{6} - \dfrac{33}{2}$ (2) $12\sqrt{6} - \dfrac{33}{2}$ (3) $12\sqrt{6} - \dfrac{31}{2}$ (4) $18\sqrt{6} - \dfrac{31}{2}$

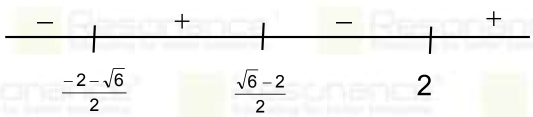
Ans. **(2)**

Sol. $f(x) = 2x^4 - 18x^2 + 8x + 12$

$f'(x) = 8x^3 - 36x + 8$

$= 4(x - 2)(2x^2 + 4x - 1)$

Number line of f'(x) is given by



so maxima point is $x = \dfrac{\sqrt{6} - 2}{2}$

and maximum value of f(x) = $12\sqrt{6} - \dfrac{33}{2}$

79. The value of $\lim\limits_{n\to\infty}\dfrac{1+2-3+4+5-6+.....(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(1) $3(\sqrt{2}+1)$ (2) $\frac{3}{2}(\sqrt{2}+1)$ (3) $\frac{\sqrt{2}+1}{2}$ (4) $\frac{3}{2\sqrt{2}}$

Ans. **(2)**

Sol. $\lim\limits_{n\to\infty}\dfrac{\sum\limits_{r-1}^{n}((3r-1)+(3r-2)-3r)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}=\dfrac{\sum\limits_{r-1}^{n}(3r-3)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$

$\lim\limits_{n\to\infty}\dfrac{3(n)\left(\frac{n+1}{2}\right)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}=\lim\limits_{n\to\infty}\dfrac{\frac{3}{2}\left(1+\frac{1}{n}\right)-\frac{3}{n}}{\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}}=\frac{3}{2}\left(\sqrt{2}+1\right)$

80. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}.\vec{c}=0$ and $\vec{a}\times(\vec{b}\times\vec{c})=\dfrac{\vec{b}-\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b}.\vec{d}$ = $\vec{a}.\vec{b}$, then $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. **(4)**

Sol. $\vec{a}\times(\vec{b}\times\vec{c})=\dfrac{\vec{b}-\vec{c}}{2}\Rightarrow(\vec{a},\vec{c})\vec{b}-(\vec{a},\vec{b})\vec{c}=\dfrac{\vec{b}}{2}-\dfrac{\vec{c}}{2}$

$\Rightarrow \vec{a},\vec{c}=\frac{1}{2},\ \vec{a}.\vec{b}=\frac{1}{2}$ ($\because$ $\vec{b}.\vec{c}=0$)

Given $\vec{b}.\vec{d}$ = $\vec{a}.\vec{b}$

Now $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ = $(\vec{a}.\vec{c})(\vec{b}.\vec{d})-(\vec{a}.\vec{d})(\vec{b}.\vec{c})=\frac{1}{4}$

81. The vertices a hyperbola H are (±6, 0) and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x+y=2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to _______ .

Ans. **(216)**

Sol. Let equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow a = 6$, e = $\frac{\sqrt{5}}{2}$ $\Rightarrow b = 3$

Equation of normal 6 cosθ x + 3 cotθ y = 45 with slope = – $\sqrt{2}$ $\Rightarrow\theta=\frac{\pi}{4}$ and equation of normal

$\sqrt{2}x+y=15$

Now point on y axes is (0 , 15) and point on hyperbola (a sec θ, b tanθ) = $\left(6\sqrt{2},3\right)$

d^2 = 16

82. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. The the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is _____ .

Ans. **(9)**

Sol. since plane passing through the line

$x - 2y - z - 5 = 0 = x + y + 3z - 5$

So equation of plane is

$(x - 2y - z - 5) + \lambda (x + y + 3z - 5) = 0$

$(1 + \lambda) x + (\lambda - 2) y + (3\lambda - 1) z - 5 - 5\lambda = 0$ ___ (1)

And this plane parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ with direction ratio (–5, 3, 1)

Now

$-5 (1 + \lambda) + (\lambda - 2). 3 + (3\lambda - 1). 1 = 0$ [since plane & line are parallel]

$\Rightarrow \lambda = 12$

Hence equation of plane is $13x + 10y + 35z = 65$

Given equation of place is $ax + by + cz = 65$

so, (a, b, c) = (13, 10, 35)

Now we have to find the distance of (13, 10, 35) from $2x + 2y - z + 16 = 0$

$$d = \frac{|26 + 20 - 35 + 16|}{\sqrt{4 + 4 + 1}} = 9$$

83. If the area enclosed by the parabola $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = ax$, $a > 0$, then a^3 is equal to ______ .

Ans. **(600)**

Sol. Intersection points of the curves $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ are $(\pm 2, 10)$

$$\text{area} = \int_{-2}^{2}\left(x^2 + 6 - \frac{5x^2}{2}\right)dx = 16$$

Now area enclosed by P_1 and $y = ax$ is equal to $\int_0^{\frac{2a}{5}}\left(ax - \frac{5x^2}{2}\right)dx = \frac{2a^3}{75} = 16 \Rightarrow a^3 = 600$

84. For some a, b, c ∈ N, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in R$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then (*fog*) (ac) + (*gof*) (b) is equal to _______ .

Ans. **(2039)**

Sol. Now $y = f(g(x)) = ag(x) - 3 = a(x^b + c) - 3$

$$\Rightarrow x = \left(\frac{y + (3 - ca)}{a}\right)^{\frac{1}{b}}$$

$$\Rightarrow [f(g(x))]^{-1} = \left(\frac{x + (3 - ca)}{a}\right)^{\frac{1}{b}} = \left(\frac{x - 7}{2}\right)^{\frac{1}{3}}$$

$\Rightarrow b = 3, a = 2, c = 5$

$\Rightarrow g(x) = x^3 + 5, f(x) = 2x - 3$

$fog(ac) + gof(b) = fog(10) + gof(3) = f(1005) + g(3) = 2039$

85. If the sum of all the solution of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$, $-1 < x < 1$, $x \neq 0$, is $\alpha-\frac{4}{\sqrt{3}}$, then α is equal to ______ .

Ans. **(2)**

Sol. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$ $\quad$ $(x = \tan\theta \Rightarrow \tan^{-1}x = \theta)$

$\left(\frac{-\pi}{4} \le \theta \le \frac{\pi}{4}\right)$

$\tan^{-1}(\tan 2\theta) + \cot^{-1}(\cot 2\theta) = \frac{\pi}{3}$ …..(i) $\quad(-\pi/2 < 2\theta < \pi/2)$

Case-I: $0 < 2\theta < \frac{\pi}{2}$

$2\theta + 2\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{12}$

$\tan^{-1}x = \frac{\pi}{12}$

$x = \tan\frac{\pi}{12}$

Case-II : $-\frac{\pi}{2} < 2\theta < 0$

$2\theta + \pi + 2\theta = \frac{\pi}{3}$

$4\theta = -\frac{2\pi}{3}$

$\theta = -\frac{\pi}{6}$

$x = \tan\left(-\frac{\pi}{6}\right)$

Sum $= \tan\left(\frac{\pi}{12}\right) - \tan\left(\frac{\pi}{6}\right) = \left(2-\sqrt{3}\right) - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}-3-1}{\sqrt{3}}$

$= \left(2-\frac{4}{\sqrt{3}}\right)$

$\therefore \alpha = 2$

86. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that x + y is divisible by 5, is ________ .

Ans. **(120)**

Sol. we divides numbers $1 \le x \le 25$, $1 \le y \le 25$, $x \in N$ in to following five groups

I^{st}	II^{nd}	III^{rd}	IV^{th}	V^{th}
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

x + y is divisible by 5 if $x \in I^{st}$ and $y \in IV^{th} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in II^{nd}$ and $y \in III^{rd} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in III^{rd}$ and $y \in II^{nd} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in IV^{th}$ and $y \in I^{st} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in V^{th}$ and $y \in V^{th} \Rightarrow {}^5C_2 \times 2 = 20$

Total = 120

87. Let A_1, A_2, A_3 be the three A. P. with the common difference d and having their first terms as A, A+1, A+2, respectively. Let a, b, c be the 7^{th}, 9^{th}, 17^{th} terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$. If a = 29, the sum of first 20 terms of an AP whose first term is c–a–b and common difference is $\frac{d}{12}$, is equal to ______ .

Ans. **(495)**

Sol. A + 6 d = a ____ (1)

A + 1 + 8 d = b _____ (2)

A + 2 + 16 d = c _____ (3)

Now

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$$

$\Rightarrow$ c - 2b = 7

Using equation (2) & (3) , we get A = –7

Now a = – 7 + 6d = 29 $\Rightarrow$ d = 6

Now first term = c–a–b = 20 and common difference = $\frac{d}{12}$ = 1/2 and the sum of first 20 terms of an AP = 495

88. Let S = {1, 2, 3, 5, 7, 10, 11}. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is ______ .

Ans. **(43)**

Sol. S = {1, 2, 3, 5, 7, 10, 11}

{3} multiple of 3 = 3 k group

{1 , 7, 10} = 3k+1 group

{2, 5 , 11}= 3k+2 group

Number of sub set

= (one elements) + (two elements)+(there elements)+(four elements)+(five elements) +(6 elements) + (7 elements)

$= {}^1c_1 + ({}^3c_1.{}^3c_1) + ({}^3c_3 + {}^3c_3 + {}^1c_1.{}^3c_1.{}^3c_1) + ({}^3c_3.{}^1c_1 + {}^3c_3.{}^1c_1 + {}^3c_2.{}^3c_2) + ({}^1c_1.{}^3c_2.{}^3c_2) + ({}^3c_3.{}^3c_3) + ({}^1c_1.{}^3c_3.{}^3c_3)$

= 1+9+11+11+9+1 + 1 = 43

89. The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is ______ .

Ans. **(1080)**

Sol. General term = $\frac{5!}{r!_1\, r_2!r_3!}(2x)^{r_1}\left(x^{-7}\right)^{r_2}\left(3x^2\right)^{r_3}$

$\frac{5!}{r!_1\, r_2!r_3!}(2)^{r_1}(3)^{r_3}(x)^{r_1-7r_2+2r_3}$

$\Rightarrow r_1 - 7r_2 + 2r_3 = 0$ and $r_1 + r_2 + r_3 = 5$

$\Rightarrow r_1 = 1, r_2 = 1 \;\; r_3 = 3$

Hence the term independent of 'x' = $5 \times 4 \times 2 \times 3^3 = -13720$

90. Let $S=\left\{\alpha : \log_2(9^{2\alpha-4}+13)-\log_2\left(\frac{5}{2}.3^{2\alpha-4}+1\right)=2\right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{a\in s} a\right)^2 x + \sum_{a\in s}(\alpha+1)^2\beta = 0$ has real roots, is ______ .

Ans. **(25)**

Sol. $S=\left\{\alpha : \log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}.3^{2\alpha-4}+1\right)=2\right\}$

$\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}.3^{2\alpha-4}+1\right)=2$

$\Rightarrow \quad \log_2\left(\frac{9^{2\alpha-4}+13}{\frac{5}{2}9^{\alpha-2}+4}\right)=2$

$\Rightarrow \quad 9^{2\alpha-4}+13=4\left(\frac{5}{2}9^{\alpha-2}+1\right)$

$\Rightarrow \quad 9^{2(\alpha-2)}+13=109^{\alpha-2}+4$

$\Rightarrow \quad 9^2 - 10y + 9 = 0 \qquad$ [where $y = 9^{\alpha-2}$]

$\Rightarrow \quad (y - 9)(y - 1) = 0$

$y = 9$ or $y = 1$

$9^{\alpha-2} = 9$ or $9^{\alpha-2} = 1$

so $\alpha = 2, 3$

$\Rightarrow$ $s = \{2, 3\}$

Now

$$x^2 - 2\left(\sum_{\alpha \in s} \alpha\right)^2 x + \sum_{\alpha \in s} (\alpha + 1)^2 \beta = 0$$

$x^2 - 50x + 25\beta = 0$

for real roots

$D \geq 0$

$50^2 - 100\beta \geq 0$ [$D = b^2 - 4ac$]

$2500 - 100\beta \geq 0$

$25 - \beta \geq 0$

$\beta \leq 25$

$\beta_{max} = 25$

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