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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 11 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If equation of the plane that contains the point (–2, 3, 5) and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$

(1) 15 (2) 16 (3) 17 (4) 18

NTA. (1)

RESO (1)

Sol. Let plane be $a(x+2) + b(y-3) + c(z-5) = 0$

So $2a + 4b + 5c = 0$

$3a - 2b + 3c = 0$

$\Rightarrow \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$

So dr's 22, 9, –16

$\Rightarrow$ $22x + 9y - 16z + 97 = 0$

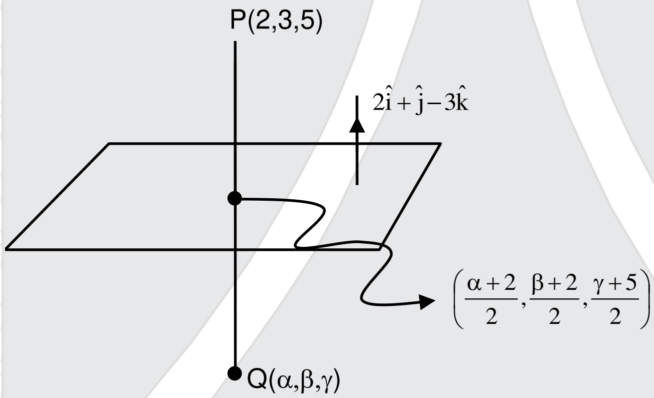
So $\alpha+\beta+\gamma = 22 + 9 - 16 = 15$

2. Let (α, β, γ) be the image of the point P(2, 3, 5) in the plane $2x + y - 3z = 6$. Then $\alpha+\beta+\gamma$ is equal to

(1) 10 (2) 12 (3) 9 (4) 5

NTA. (1)

RESO (1)



Sol.

$\alpha + 2 + \frac{\beta+3}{2} - \frac{3}{2}(\gamma+5) = 6$

$2\alpha + \beta - 3\gamma + 2 + \frac{3}{2} - \frac{15}{2} = 6$

$2\alpha + \beta - 3\gamma = 10$ .... (1)

$\frac{\alpha-2}{2} = \frac{\beta-3}{1} = \frac{\gamma-5}{-3} = \lambda$ .... (2)

from equation (1) & (2)

$2(2\lambda + 2) + (\lambda + 3) - 3(-3\lambda + 5) = 10$

$15\lambda = 18 \Rightarrow \lambda = \frac{6}{5}$

$\alpha + \beta + \gamma = 2\lambda + 2 + \lambda + 3 - 3\lambda + 5 = 10$

3. Let y = y(x) be a solution curve of the differential equation.
$(1 - x^2y^2)dx = ydx + xdy$.
If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at y = α, then a value of α is

(1) $\frac{1-3e^2}{2(3e^2+1)}$ (2) $\frac{3e^2}{2(3e^2-1)}$ (3) $\frac{3e^2}{2(3e^2+1)}$ (4) $\frac{1+3e^2}{2(3e^2-1)}$

NTA. (4)
RESO (4)

Sol. $dx = \frac{d(xy)}{1-(xy)^2} \Rightarrow 2dx = \frac{d(xy)}{1-xy} + \frac{d(xy)}{1+xy}$

$2x+c = \ell n\left|\frac{1+xy}{1-xy}\right| \Rightarrow \left|\frac{xy+1}{xy-1}\right| = e^c.e^{2x}$

$\because$ y(1) = 2

$\Rightarrow 3 = e^c . e^2$

$\Rightarrow e^c = \frac{3}{e^2}e^{2x}$

So at x = 2

$\Rightarrow \left|\frac{2\alpha+1}{2\alpha-1}\right| = 3e^2$

Case : 1 for +ve sign

$\Rightarrow \frac{2\alpha+1}{2\alpha-1} = \frac{3e^2}{1}$

$\Rightarrow \frac{4\alpha}{2} = \frac{3e^2+1}{3e^2-1}$

$\Rightarrow \alpha = \frac{1}{2}\left(\frac{3e^2+1}{3e^2-1}\right)$

Case: 2 for (–ve)sign not required

4. Let A be a 2 × 2 matrix with real entries such that A' = αA + I where a ∈ R – {–1, 1}. If det(A² – A) = 4, then the sum of all possible values of α is

(1) $\frac{3}{2}$ (2) 0 (3) $\frac{5}{2}$ (4) 2

NTA. (3)
RESO (3)

Sol. $(A^T)^T = (\alpha A + I)^T = \alpha A^T + I^T$

$A = \alpha(\alpha A + I) + I$

$\Rightarrow (1-\alpha^2) A = (\alpha+1) I$

$\because \alpha \neq 1, -1$

$\Rightarrow A = \left(\frac{1}{1-\alpha}\right)I$

$\Rightarrow |A| = \frac{1}{1-\alpha}$

$\therefore |A^2 - A| = 4$

$\Rightarrow |A|\,|A - I| = 4$

$\Rightarrow |A|\,|A^T - I| = 4$

$\Rightarrow |A|\,|\alpha A| = 4$

$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$

Case1: when $\frac{\alpha}{(1-\alpha)^2} = 2$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$

$\alpha_1 + \alpha_2 = 5/2$ ($\because \alpha \in R$)

Case2: when $\frac{\alpha}{(1-\alpha)^2} = -2$

$\Rightarrow 2\alpha^2 - 3\alpha + 2 = 0$

D< 0

$\Rightarrow \alpha$ is imaginary

So required sum = 5/2

5. Let f : [2, 4] $\Rightarrow$ R be a differentiable function such that $(x \log_e x) f'(x) + (\log x) f(x) + f(x) \geq 1$, $x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

Consider the following two statements:

(A) : $f(x) \leq 1$, for all $x \in [2, 4]$

(B) : $f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$

Then,

(1) Neither statement (A) nor statement (B) is true

(2) Both the statements (A) and (B) are true

(3) Only statement (B) is true

(4) Only statement (A) is true

NTA. (2)

RESO (2)

Sol. $\frac{d}{dx}(x\ell nx.f(x) - x) \geq 0, \ \forall x \in [2, 4]$

let $g(x) = x\ell nx\, f(x) - x, \ \forall x \in [2, 4]$

$\Rightarrow g'(x) \geq 0, \ \forall x \in [2, 4]$

$\Rightarrow g'(x)$ is increasing function in [2,4]

$\Rightarrow g(2) \leq g(x) \leq g(4)$

$\Rightarrow 2.\ell n - \frac{1}{2} - 2 \leq x\ell nx f(x) - x \leq 4.\ell n4.\frac{1}{4} - 4$

$\Rightarrow \ell n2 - 2 + x \leq x.\ \ell nx.\ f(x) \leq 2(\ell n2 - 2) + x$

$\Rightarrow \frac{\ell n2 - 2}{x\ell na} + \frac{1}{\ell nx} \leq f(x) \leq \frac{2(\ell n2 - 2)}{x\ell nx} + \frac{1}{\ell nx}$

Since numerator is constant

$\Rightarrow \frac{\ell n2 - 2}{4\ell n4} + \frac{1}{\ell n4} \leq f(x) \leq \frac{2(\ell n2 - 2)}{2\ell n2} + \frac{1}{\ell n2}$

$\Rightarrow \frac{1}{8} + \frac{1}{4\ell n2} \leq f(x) \leq 1 - \frac{1}{\ell n2}$

$2 \Rightarrow \frac{1}{8} < f(x) < 1, \ \forall x \in [2,4]$

6. for any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1$, i = 1, 2, 3, consider the following statements:

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \le |\vec{a}|$

(B) : $|\vec{a}| \le 3\max\{|a_1|, |a_2|, |a_3|\}$

(1) Neither (A) nor (B) is true
(2) Only (A) is true
(3) Both (A) and (B) are true
(4) Only (B) is true

NTA. (3)
RESO (3)

Sol. $\vec{a} = a_1 i + a_2\hat{j} + a_3\hat{k},\ |a_i| < \frac{1}{10}, i = 1,2,3$

$\because |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \Rightarrow |\vec{a}| = \sqrt{|a_1|^2 + |a_2|^2 + |a_3|^2} \ge \max(|a_1|,|a_2|,|a_3|)$

Similarly $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} < \sqrt{\frac{1}{100} + \frac{1}{100} + \frac{1}{100}}$

$|\vec{a}| < \frac{\sqrt{3}}{10} < \frac{3}{10}$

$\Rightarrow |\vec{a}| < 3\max(|a_1|,|a_2|,|a_3|) \Rightarrow$ A and b both are correct

7. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx$ is equal to

(1) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$
(2) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$
(3) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$
(4) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

NTA. (3)

RESO (3)

Sol. $I = \int_{-\ell n 2}^{\ell n 2} e^x\left(\ell n\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx \Rightarrow I = \left[e^x \ell\left(e^x + \sqrt{1+e^{2x}}\right)\right]_{-\ell n 2}^{\ell n 2} - \int_{-\ell n 2}^{\ell n 2} e^x \frac{e^{2x}}{\sqrt{1+e^{2x}}}dx$

$= \left(2\ell n(2+\sqrt{5}) - \frac{1}{2}\ell n\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)\right) - \left[\sqrt{1+e^{2x}}\right]_{-\ell n 2}^{\ell n 2} = \ell n\left(\frac{(2+\sqrt{5})^2}{\sqrt{\sqrt{5}+1}}.\sqrt{2}\right) - \left(\sqrt{5} - \frac{\sqrt{5}}{2}\right)$

$\ell n\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{\sqrt{5}+1}}\right) - \frac{\sqrt{5}}{2}$

8. Let S = {M = [a_{ij}], $a_{ij} \in \{0, 1, 2\}, 1 \le i, j \le 2$} be a sample space and A = {M ∈ S : M is invertible} be an event. Then P(A) is equal to

(1) $\frac{47}{81}$ (2) $\frac{50}{81}$ (3) $\frac{16}{27}$ (4) $\frac{49}{81}$

NTA. (2)
RESO (2)

Sol. n(S) = 3×3×3×3 = 81

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad |M| = a_{11}a_{22} - a_{12}a_{21}$$

If |M| = 0 ⇒ $a_{11}a_{22} = a_{12}\,a_{21}$

Case – I 1 = 1 One way
Case – II 4 = 4 One way
Case – III 2 = 2 4 way
Case – IV 0 = 0 25 way

Number of ways for |M| = 0 all 31 ways

Required probability = $1 - \frac{31}{81} = \frac{50}{81}$

9. Consider ellipse $E_k : kx^2 + k^2y^2 = 1$, k = 1, 2, ….., 20. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

(1) 3320 (2) 3210 (3) 3080 (4) 2870

NTA. (3)
RESO (3)

Sol. $E_k : \frac{x^2}{\frac{1}{k}} + \frac{y^2}{\frac{1}{k^2}} = 1$

$a = \frac{1}{\sqrt{k}}, b = \frac{1}{k}$

$\sqrt{k}x + ky = 1$

clearly circle be $x^2 + y^2 = r_k^2$

$\Rightarrow \left|\frac{0+0-1}{\sqrt{k+k^2}}\right| = r_k \Rightarrow r_k = \frac{1}{\sqrt{k^2+k}}$

So $\sum_{k=1}^{20}(k^2+k) = \frac{20.21.41}{6} + \frac{20.21}{2} = 2870+210 = 3080$

10. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \ge 0$ is

(1) 7 (2) 8 (3) 5 (4) 6

NTA. (4)
RESO (4)

Sol. Case 1: When $x + \frac{7}{2} > 1$

$\Rightarrow x > -\frac{5}{2}$

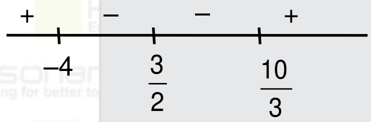
$\Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$\Rightarrow \frac{(x-7)^2-(2x-3)^2}{(2x-3)^2} \geq 0$

$\Rightarrow \frac{(3x-10)(x+4)}{(2x-3)^2} \leq 0$

$\Rightarrow x \in \left[-4, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

So $x \in \left(-\frac{5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$



Case – 2 When $0 < x + \frac{7}{2} < 1$

$\Rightarrow -\frac{7}{2} < x < -\frac{5}{2}$

$\Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \leq 1$ and $x \neq 7, \frac{3}{2}$

$\Rightarrow \frac{(3x-10)(x+4)}{(2x-3)^2} \geq 0$ and $x \neq 7, \frac{3}{2}$

$\Rightarrow x \leq -4 \cup x \geq \frac{10}{3}$ and $x \neq 7, \frac{3}{2}$

$\Rightarrow x \in \phi$

So $x \in \left(-\frac{5}{2}, \frac{10}{3}\right) - \left\{\frac{3}{2}\right\}$

hence number of integral values of x is 6

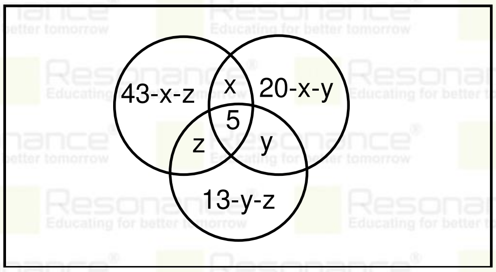
11. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

(1) 15 (2) 10 (3) 9 (4) 21

NTA. (4)

RESO (4)

Sol.



(20–x–y) + (43 – x –z) + (13 – y – z) + x + y + z + 5 = 60

21= x + y + z

12. The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying x + y + z = 15, is

(1) 114 (2) 80 (3) 136 (4) 92

NTA. (1)

RESO (1)

Sol. Le x < y < z

$x = t_1$

$y = x + 1 + t_2 = t_1 + t_1 + 1$

$z = y + 1 + t_3 = 2 + t_1 + t_2 + t_3$

$\Rightarrow t_1 + (1+t_1 + t_2) + (2 + t_1 + t_2 + t_3) = 15$

$\Rightarrow 3t_1 + 2t_2 + t_3 = 12$

When $t_1 = 0 \Rightarrow 2t_2 + t_3 = 12$ $\Rightarrow$ No. of Solution = 7

$t_1 = 1 \Rightarrow 2t_2 + t_3 = 9$ $\Rightarrow$ No. of Solution = 5

$t_1 = 2 \Rightarrow 2t_2 + t_3 = 6$ $\Rightarrow$ No. of Solution = 4

$t_1 = 3 \Rightarrow 2t_2 + t_3 = 3$ $\Rightarrow$ No. of Solution = 2

$t_1 = 4 \Rightarrow 2t_2 + t_3 = 0$ $\Rightarrow$ No. of Solution = 1

Total = 19

But we know that $x \neq y \neq z$

hence total number of such solutions = 19.(3!)

= 114

13. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of $y_1, y_2, \ldots, y_{100}$ is

(1) 10049.50 (2) 10101.50 (3) 10051.50 (4) 10100

NTA. (1)

RESO (1)

Sol. 2 + (2 +d) + ………… + (2+ 99d) = 20000

$\Rightarrow 200 + \frac{99}{2} \times 100d = 20000$

$\Rightarrow$ 99d = 396

d = 4

$y_1 + y_2 + \ldots\ldots\ldots y_{100} = \sum_{i=1}^{100} i(2+(i-1)4-i) = \sum_{i=1}^{100}(3i^2 - 2i) = \frac{3.100.101.201}{6} - 100.101$

$= 10100\left(\frac{201}{2} - 1\right) = (10100).\left(\frac{199}{2}\right)$

So mean $= \frac{(10100)(99.5)}{100} = 10049.5$

14. Area of the region $\{(x, y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is

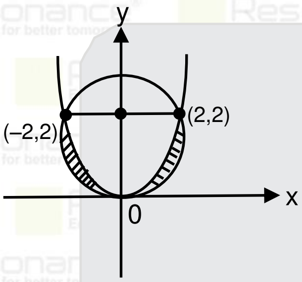
(1) $\pi - \frac{8}{3}$ (2) $2\pi - \frac{16}{3}$ (3) $2\pi + \frac{16}{3}$ (4) $\pi + \frac{8}{3}$

NTA. (2)
RESO (2)

Sol. on solving $x^2 + (y-2)^2 = 4$ and $x^2 = 2y$

We get $y = 0, 2$

So points are (0, 0), (2, 2), (–2, 2)



So area shaded region $= 2\left[\frac{1}{4}\pi(4) - \int_0^2 \sqrt{2}\sqrt{y}dy\right]$

$= 2\left(\pi - \sqrt{2}\left(\frac{y^{3/2}}{3/2}\right)_0^2\right)$

$= 2\left(\pi - \frac{2\sqrt{2}}{3}..2\sqrt{2}\right)$

$= 2\left(\pi - \frac{8}{3}\right)$

15. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to

(1) $-\pi + \tan^{-1}\frac{33}{5}$ (2) $-\pi + \tan^{-1}\frac{8}{9}$ (3) $\pi - \tan^{-1}\frac{8}{9}$ (4) $\pi - \tan^{-1}\frac{33}{5}$

NTA. ()
RESO ()

Sol. $w_1 = z_1 e^{i\frac{\pi}{2}}$, $w_2 = z_2 e^{-i\frac{\pi}{2}}$

$= -4 + 5i$, $= 5 - 3i$

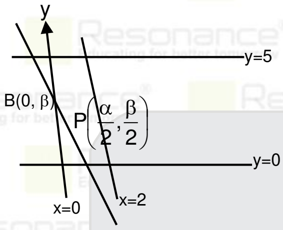
$w_1 - w_2 = -9 + 8i$

$\Rightarrow$ principal arg $(w_1 - w_2) = \pi - \tan^{-1}\frac{8}{9}$

16. Let R be a rectangle given by the lines x = 0, x = 2, y = 0 and y = 5. Let A(α, 0) and B(0,β), $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the mid-point of AB lies on a

(1) circle (2) parabola (3) straight line (4) hyperbola

NTA. (4)
RESO (4)
Sol



$\therefore \frac{1}{2}(\alpha)(\beta) = 2$

$\alpha\beta = 4$

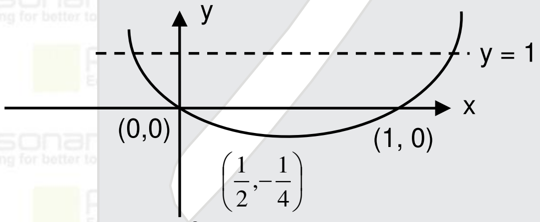
$\Rightarrow \frac{\alpha}{2}.\frac{\beta}{2} = 1 \Rightarrow xy = 1$

So P lies on a rectangular hyperbola

17. Let f(x) = $[x^2 - x] + |-x + [x]|$, where $x \in R$ and [t] denotes the greatest integer less than or equal to t, Then f is

(1) continuous at x = 0, but not continuous at x = 1
(2) continuous at x = 0 and x = 1
(3) continuous at x = 1, but not continuous at x = 0
(4) not continuous at x = 0 and x = 1

NTA. (3)
RESO (3)
Sol.



$f(x) = [x^2 - x] + |x - [x]|$
$= [x^2 - x] + |\{x\}|$

So $f(x) = [x^2 - x] + \{x\}$

for x = 0
$f(0) = 0$
$f(0^+) = -1 + 0 = -1$
$\Rightarrow$ f(x) is discontinuous at x = 0
For x = 1
$f(1) = 0$
$f(1^-) = -1 + 1 = 0$
$f(1^+) = 0 + 0 = 0$

$\Rightarrow$ f(x) is continuous at x= 1

18. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is

(1) 10 (2) 12 (3) 9 (4) 8

NTA. **(3)**

RESO **(3)**

Sol. $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 5\cos^2\theta + 2(1-(\sin^2\theta)^3) = 0$

$\Rightarrow 3\cos^4\theta - 5\cos^2\theta + 2((1-\sin^2\theta)^3 + 3\sin^2\theta\cos^2\theta) = 0$

$\Rightarrow 2\cos^6\theta + 3\cos^4\theta - 5\cos^2\theta + 6(1-\cos^2\theta)\cos^2\theta = 0$

$\Rightarrow 2\cos^6\theta - 3\cos^4\theta + \cos^2\theta = 0$

$\Rightarrow \cos^2\theta\ (2\cos^4\theta + 3\cos^2\theta + 1) = 0$

$\Rightarrow \cos^2\theta = 0$ or $2\cos^4\theta - \cos^2\theta - 2\cos^2\theta + 1 = 0$

$\cos^2\theta(2\cos^2\theta - 1) - 1(2\cos^2\theta - 1) = 0$

$\Rightarrow \cos\theta = 0$ or $\cos\theta = \pm\frac{1}{\sqrt{2}}$ or $\cos\theta = \pm 1$

$\because \theta \in [0, 2\pi]$

so total number of elements = 9

19. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ______.

(1) 36 (2) 32 (3) 40 (4) 38

NTA. **(4)**

RESO **(4)**

Sol. A

mean $(x_1, x_2, \ldots\ldots x_5) = 5$

$\Rightarrow (x_1-3, x_2-3, \ldots\ldots x_5-3) = 2$

var.$(x_1, x_2, \ldots\ldots x_5) = 12$

var.$(x_1-3, x_2-3, \ldots\ldots x_5-3) = 12$

$$\frac{\sum(x_i-3)^2}{5} - 4 = 12$$

B

mean $(y_1, y_2, \ldots.. y_5) = 8$

$\Rightarrow$ mean $(y_1+2, y_2+2, \ldots.. y_5+2) = 10$

var.$(y_1, y_2, \ldots.. y_5) = 20$

var. $(y_1+2, y_2+2, \ldots.. y_5+2) = 20$

$$\frac{\sum(y_i+2)^2}{5} - 100 = 20$$

Combined mean = $\dfrac{\sum_{i=1}^{5}(x_i-3) + \sum_{i=1}^{5}(y_i+2)}{10} = \dfrac{10+50}{10} = 6$

Combined variance = $\dfrac{\sum(x_i-3)^2 + \sum(y_i+2)^2}{10} - 6^2$

$= \dfrac{80+120\times 5}{10} - 36 = 68 - 36 = 32$

So combined mean + Combined variance = 6 +32 = 38

20. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}$, $\hat{i} + \hat{k}$ and $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then the ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to

(1) $\left(\frac{\pi}{4}, 6\right)$ (2) $\left(\frac{\pi}{3}, 6\right)$ (3) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$ (4) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$

NTA. (1)
RESO (1)

Sol. $\vec{a} = \lambda((\vec{n}_1 \times \vec{n}_2) \times (\vec{n}_3 \times \vec{n}_4))$

$= \lambda([\vec{n}_1\vec{n}_2\vec{n}_4]\vec{n}_3 - [\vec{n}_1\vec{n}_2\vec{n}_3]\vec{n}_4)$

$= \left(\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}(\hat{i} - \hat{j}) - \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix}(\hat{j} - \hat{k})\right)\lambda$

$(0 - 2(\hat{j} - \hat{k}))\lambda$

So $\vec{a} = 2\lambda(-\hat{j} + \hat{k})$

$\because \vec{a}.\vec{b} = 6$

$\Rightarrow 2\lambda\ (2 + 1) = 6$

$\lambda = 1$

$\Rightarrow \vec{a} = -2\hat{j} + 2\hat{k}$

$\Rightarrow \cos\theta = \left|\frac{6}{(3)(2\sqrt{2})}\right| = \theta = \frac{\pi}{4}$

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2$

$= (9)\ (8) - 36$

$= 36$

$|\vec{a} \times \vec{b}| = 6$

21. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is

NTA. (44)

RESO (44)

Sol. Clearly Derangement of 5 persons = $5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

= 60–20+5–1= 40+4 = 44

22. Let a line l pass through the origin and be perpendicular to the lines

$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in R$ and

$l_2 : \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in R.$

If P is the point of intersection of l and l_1, and Q (α, β, γ) is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to ______.

NTA. (5)

RESO (5)

Sol. Since $\ell \perp \ell_1$ and $\ell \perp \ell_2$

Let dr's of ℓ be a, b, c

$\Rightarrow \quad a + 2b + 3c = 0$

$2a + 2b + c = 0$

$\Rightarrow \frac{a}{-4} = \frac{b}{5} = \frac{c}{-2}$

hence dr's of ℓ can be 4, –5, 2

So any point P on ℓ can be taken as $(4\alpha, -5\alpha, 2\alpha)$

$\Rightarrow 4a = \lambda + 1$ and $2\lambda - 11 = -5\alpha$, $3\lambda - 7 = 2\alpha$

$\Rightarrow 2(3\lambda - 7) = \lambda + 1$

$\Rightarrow 5\lambda = 15 \Rightarrow \lambda = 3$

so P (4, – 5, 2)

dr's of PQ : $2\mu - 5, 2\mu + 5, \mu - 1$

$\because PQ \perp \ell_2$

$= 2(2\mu - 5) + 2(2\mu + 5) + \mu - 1 = 0$

$\Rightarrow 9\mu = 1 \Rightarrow \mu = \frac{1}{9}$

$\Rightarrow Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

hence $9(\alpha + \beta + \gamma) = 5$

23. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ________.

NTA. (51)

RESO (51)

Sol. $a^2 - 7a - 1 = 0$

$\Rightarrow \quad a^{21} = 7a^{20} + a^{19}$

$b^{21} = 7b^{20} + b^{19}$

So $\quad a^{21} + b^{21} = 7(a^{20} + b^{20}) + a^{19} + b^{19}$ ________ (1)

also $\quad a^{20} + b^{20} = 7(a^{19} + b^{19}) + a^{18} + b^{18}$ ________ (2)

$a^{19} + b^{19} = 7(a^{18} + b^{18}) + a^{17} + b^{17}$ ________ (3)

(1) + 7 (2)

$a^{21} + b^{21} = (a^{19} + b^{19}) + 49(a^{19} + b^{19}) + 7(a^{18} + b^{18})$

$= 50(a^{19} + b^{19}) + (a^{19} + b^{19}) - (a^{17} + b^{17})$

$\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}} = 51$

24. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where a, c $\in$ R. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in N$, then n is equal to ______.

NTA. **(2)**
RESO **(2)**

Sol. characteristic equation of matrix A is

$|A - xI| = 0$, where x is scalar

$\Rightarrow \begin{vmatrix} -x & 1 & 2 \\ a & -x & 3 \\ 1 & c & -x \end{vmatrix} = 0$

$\Rightarrow x^3 - (3c + a + 2)x + 2ac + 3 = 0$

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$A^3 - (3c + a + 2)A + (2ac + 3)I = 0$

$\because A^3 = A$

$\Rightarrow (3c + a + 1)A = (2ac + 3)I$

This is possible only when

$3c + a + 1 = 0$ and $2ac + 3 = 0$

$\Rightarrow 3\left(\frac{-3}{2a}\right) + a + 1 = 0$

$\Rightarrow 2a^2 + 2a - 9 = 0$

$\Rightarrow a = \frac{\sqrt{119} - 1}{2} (\because a > 0)$

So $a \in (1, 2]$

$\Rightarrow n = 2$

25. For m, n > 0, let $\alpha(m, n) = \int_0^2 t^m (1 + 3t)^n dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$, then p is equal to ______.

NTA. **(32)**
RESO **(32)**

Sol. $\alpha(m,n) = \left[\frac{(1+3t)^n \times t^{m+1}}{m+1}\right]_0^2 - \frac{3n}{m+1}\int_0^2 t^{m+1}.(1+3t)^{n-1} dt$

$\Rightarrow \alpha(m, n) + \frac{3n}{m+1}.\alpha(m+1, n-1) = \frac{2^{m+1}.7^n}{m+1}$

put m = 10, n = 6

$\Rightarrow 11.\alpha(10, 6) + 18\alpha(11, 5) = 2^{11}.7^6$

$= 32(14)^6$

So P = 32

26. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots\ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to ______.

NTA. **(2175)**

RESO **(2175)**

Sol. $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots.. + \frac{1}{5^{108}}$

$$\frac{1}{5}S = \frac{109}{5} + \frac{108}{5^2} + \frac{107}{5^3} + \ldots\ldots + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4}{5}S = 109 - \frac{1}{5} - \frac{1}{5^2} - \ldots\ldots - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$\frac{4}{5}S = 109 - \left[\frac{\frac{1}{5}\left(1-\left(\frac{1}{5}\right)^{109}\right)}{\left(1-\frac{1}{5}\right)}\right]$$

$$S = \frac{5}{4}\left[109 - \frac{1}{4}\left(1-\left(\frac{1}{5}\right)^{109}\right)\right]$$

$16S = 20 \times 109 - 5 + 5^{-108}$

$16S - (25)^{-54} = 2180 - 5$

$= 2175$

27. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____.

NTA. **(7)**

RESO **(7)**

p	q	r	$q \wedge r$	$S_1 : p \vee (q \wedge r)$	$S_2 : (q \vee r)$	$S_1 \to S_2$
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	F	F	T	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Alt. $\equiv p \vee (q \wedge r) \Rightarrow (q \vee r)$

$P \to Q$ is false only when P is T and Q is F

So in $p \vee (q \wedge r) \Rightarrow (q \vee r)$

Clearly truth value of q and r must be "F" and P must be "T"

so remaining (2^3-1) give truth value T

28. The mean of the coefficients of x, x^2, …., x^7 in the binomial expansion of $(2 + x)^9$ is ________.
NTA. **(2736)**
RESO **(2736)**
Sol. $T_{r+1} = {}^9C_r.2^{9-r}.\, x^r$

Sum of coefficients = ${}^9C_1.2^8 + {}^9C_2\, 2^7 + \ldots\ldots\ldots\ldots + {}^9C_7\, 2^2$

$= (2+1)^9 - \{{}^9C_0.2^9 + {}^9C_8.2^1 + {}^9C_9.2^0\}$

$= 3^9 - (2^9 + 18 + 1)$

$= 3^9 - 2^9 - 19$

So mean = $\frac{3^9 - 2^9 - 19}{7} = 2736$

29. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to
NTA. **(171)**
RESO **(171)**
Sol. $T_{r+1} = {}^{680}C_r 3^{\frac{680-r}{2}}.5^{\frac{r}{4}}$

for rational terms $\frac{680-r}{2}$ and $\frac{r}{4}$ must be integer where $r \in W$, $0 \le r \le 680$

$\Rightarrow r = 0, 4, 8, \ldots\ldots., 680$

so number of rational terms = 171

30. Let $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$, $n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If ℓ is the length of the latus rectum of H_k, then 21ℓ is equal to _____.
NTA. **(306)**
RESO **(306)**
Sol. $e = \sqrt{1 + \frac{3+n}{n+1}}$

$\Rightarrow e = \sqrt{\frac{2n+4}{n+1}}$

$\sqrt{2 + \frac{2}{n+1}}$

$\Rightarrow$ n smallest = 48, for which $e \in Q$

Hence $21\ell = \frac{21 \times 2 \times 51}{7} = 306$