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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 29 July, 2022 (SHIFT-2) | TIME : (3.00 a.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$

A $\sqrt{2}$

B 1

C $\sqrt{2} - 1$

D $\sqrt{2} + 1$

NTA Ans. **(D)**

Reso Ans. **(D)**

Sol. $\big||z_1| - |z_2|\big| \leq |z_1 - z_2|$

$\left||z| - \frac{1}{|z|}\right| \leq 2$ Let $|z| = t$

$\left|t - \frac{1}{t}\right| \leq 2$

Squaring both sides

$t^4 - 6t^2 + 1 \leq 0$

$(t^2 - 3)^2 - \left(2\sqrt{2}\right)^2 \leq 0$

$t \in \left[\sqrt{3 - 2\sqrt{2}}, \sqrt{3 + 2\sqrt{2}}\right]$

$t \in \left[\sqrt{2} - 1, \sqrt{2} + 1\right]$

Hence $|z|_{max} = \sqrt{2} + 1$

2 Which of the following matrices can **NOT** be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation ?

A $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

B $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

D $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

NTA Ans. **(C)**

Reso Ans. **(C)**

3 If the system of equations

$$x + y + z = 6$$
$$2x + 5y + \alpha z = \beta$$
$$x + 2y + 3z = 14$$

has infinitely many solutions, then $\alpha + \beta$ is equal to

(A) 8 (B) 36 (C) 44 (D) 48

NTA Ans. (C)

Reso Ans. (C)

Sol. For infinitely many solution's $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $1(15 - 2\alpha) - 1(6 - \alpha) + 1(4 - 5) = 0$

$\Rightarrow$ $15 - 2\alpha - 6 + \alpha - 1 = 0$

$\Rightarrow$ $\alpha = 8$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & 8 \\ 14 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $6(15 - 16) - 1(3\beta - 112) + 1(2\beta - 70) = 0$

$\Rightarrow$ $-6 - 3\beta + 112 + 2\beta - 70 = 0$

$\Rightarrow$ $\beta = 36$

So $\alpha + \beta = 44$

4 Let the function $f(x) = \begin{cases} \dfrac{\log_e(1+5x) - \log_e(1+\alpha x)}{x} & ; if\ x \neq 0 \\ 10 & ; if\ x = 0 \end{cases}$ be continuous at $x = 0$.

Then α is equal to

(A) 10 (B) – 10 (C) 5 (D) –5

NTA Ans. (D)

Reso Ans. (D)

Sol. f(x) is continuous at x = 0 so $\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x} = 10$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{\ln(1+5x)}{x} - \frac{\ln(1+\alpha x)}{x}\right) = 10$$

$\Rightarrow 5 - \alpha = 10$

$\Rightarrow \alpha = -5$

5 If $[t]$ denotes the greatest integer $\le t$, then the value of $\int_0^1 \left[2x - \left|3x^2 - 5x + 2\right| + 1\right] dx$ is :

A $\frac{\sqrt{37}+\sqrt{13}-4}{6}$

B $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

C $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$

D $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

NTA Ans. (A)

Reso Ans. (A)

6 Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \ge 0$.

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

(A) 483 (B) 528 (C) 575 (D) 624

NTA Ans. (B)

Reso Ans. (B)

7 $\sum_{r=1}^{20} (r^2+1)(r!)$ is equal to

A $22! - 21!$

B $22! - 2(21!)$

C $21! - 2(20!)$

D $21! - 20!$

NTA Ans. (B)

Reso Ans. (B)

Sol. $\sum_{r=1}^{20} (r^2+1)\, r! = \sum_{r=1}^{20} \left((r+1)^2 r! - 2r.r!\right)$

$= \sum_{r=1}^{20} \left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20} r.r!$

$= (21.\ (21!) - 1) - \sum_{r=1}^{20} \left((r+1)! - r!\right)$

$= (21.\ (21!) - 1) - [(21!) - 1]$

$= (21)\ (21!) - (21!)$

$= (22 - 1)\ (21!) - (21!)$

$= (22)(21!) - (21!) - (21!)$

$= (22)\ (21!) - 2(21!) = 22! - 2(21!)$

8 For $I(x)=\int \frac{\sec^2 x - 2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then

A $3^{1010}\, I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

B $3^{1010}\, I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

C $3^{1011}\, I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

D $3^{1011}\, I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

NTA Ans. (A)

Reso Ans. (A)

Sol. $I(x) = \int (\sin x)^{-2022} \sec^2 x\, dx - 2022\int (\sin x)^{-2022} dx$

$\Rightarrow I(x) = (\sin x)^{-2022} \tan x - \int (-2022)(\sin x)^{-2023}.\cos x.\ \tan x\ dx - 2022\int (\sin x)^{-2022}\ dx$

$\Rightarrow I(x) = (\sin x)^{-2022} \tan x + 2022 \int (\sin x)^{-2022}\ dx - 2022\int (\sin x)^{-2022}\ dx$

$\Rightarrow\ I(x) = (\sin x)^{-2022} \tan x + C$

put $x = \frac{\pi}{4}$

$2^{1011} = \left(\frac{1}{\sqrt{2}}\right)^{-2022} (1) + C$

$\Rightarrow C = 0$

So , $I(x) = (\sin x)^{-2022} \tan x$

$I\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}$

$\Rightarrow\ I\left(\frac{\pi}{3}\right) = \frac{2^{2022}}{\sqrt{3}^{2021}} = \frac{2^{2022}}{3^{1010}.\sqrt{3}}$

$I\left(\frac{\pi}{6}\right) = \frac{\frac{1}{\sqrt{3}}}{\left(\frac{1}{2}\right)^{2022}} = \frac{2^{2022}}{\sqrt{3}}$

So, $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

9. If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then

A $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

B $\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

C $2\tan^{-1}\left(\frac{1}{k+1}\right) = \log_e\left(k^2+2k+2\right)$

D $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(\frac{k^2+1}{k^2}\right)$

NTA Ans. **(A)**

Reso Ans. **(A)**

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ ______(1)

put x = X + h, y = Y + k

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$ ______(2) where h + k – 2 = 0 and h – k = 0

$\Rightarrow$ h = k = 1

equation (2) is homogenous differential equation so

put Y = vX in equation (2)

$v + X\frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow \frac{v-1}{v^2+1}dv = \frac{-1}{X}dX$ ______(3)

Integrate

$\ln(v^2+1) - 2\tan^{-1}v = -2\ln X + c$

$\ln(v^2X^2 + X^2) = 2\tan^{-1}v + c$

$\ln(Y^2 + X^2) = 2\tan^{-1}\left(\frac{Y}{X}\right) + c$

$\ln\left[(x-1)^2 + (y-1)^2\right] = 2\tan^{-1}\left(\frac{y-1}{x-1}\right) + c$ ______(4)

It is passing through(2, 1) so c = 0

$\Rightarrow \ln((x-1)^2 + (y-1)^2) = 2\tan^{-1}\left(\frac{y-1}{x-1}\right)$ ______(5)

It is also passing through point (k + 1, 2) so

$\ln(1 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right)$

10 Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{(x+3)}{x+1}$, $x > -1$, which passes through the point (0, 1). Then $y(1)$ is equal to :

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

NTA Ans. (B)

Reso Ans. (B)

Sol. $\frac{dy}{dx} + \frac{(2x^2+11x+13)}{(x+1)(x+2)(x+3)} y = \frac{x+3}{x+1}$

I.F. $= e^{\int \frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}dx} = e^{\int\left(\frac{2}{x+1}+\frac{1}{x+2}-\frac{1}{x+3}\right)dx} = e^{2\ln(x+1)+\ln(x+2)-\ln(x+3)} = e^{\ln\frac{(x+1)^2(x+2)}{(x+3)}}$

I.F $= \frac{(x+1)^2(x+2)}{(x+3)}$

So, curve $y.\frac{(x+1)^2\,(x+2)}{(x+3)} = \int \frac{(x+1)^2\,(x+2)}{(x+3)} . \frac{(x+3)}{(x+1)} dx + c$

$\Rightarrow \quad y\frac{(x+1)^2(x+2)}{(x+3)} = \int (x^2+3x+2)dx + c$

$\Rightarrow \quad y\frac{(x+1)^2(x+2)}{(x+3)} = \frac{x^3}{3}+\frac{3x^2}{2}+2x+c$

$\because$ Passes through (0, 1) $\Rightarrow c = \frac{2}{3}$

So solution curve is

$y\frac{(x+1)^2\,(x+2)}{(x+3)} = \frac{x^3}{3} + \frac{3x^3}{2} + 2x + \frac{2}{3}$

for $x = 1 \Rightarrow y = \frac{3}{2}$

11 Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\,(m_1^2+m_2^2)=220$. If one vertex of the square is $(10(\cos\alpha-\sin\alpha), 10\,(\sin\alpha+\cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha-\sin\alpha)\,x + (\sin\alpha+\cos\alpha)\,y = 10$, then $72\,(\sin^4\alpha+\cos^4\alpha)+a^2-3a+13$ is equal to :

(A) 119 (B) 128 (C) 145 (D) 155

NTA Ans. (B)

Reso Ans. (B)

12 The number of elements in the set $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x+4^{-x}\right\}$ is :

(A) 1 (B) 3 (C) 0 (D) infinite

NTA Ans. (A)

Reso Ans. (A)

Sol. $\cos\left(\frac{x^2+x}{6}\right) = \frac{4^x+4^{-x}}{2}$ ________(1)

Here $\cos\left(\frac{x^2+x}{6}\right) \in [-1,1]$

$\frac{4^x + 4^{-x}}{2} \in [1, \infty)$

It is possible only x = 0 so, S = {0}

So number of elements in set S is one only

13 Let A (α, –2), B (α, 6) and C $\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is **NOT** correct about ΔABC

A area is 24

B perimeter is 25

C circumradius is 5

D inradius is 2

NTA Ans. (B)

Reso Ans. (B)

Sol.

14 Let *Q* be the foot of perpendicular drawn from the point *P* (1, 2, 3) to the plane $x + 2y + z = 14$. If *R* is a point on the plane such that $\angle PRQ = 60°$, then the area of ΔPQR is equal to :

A $\frac{\sqrt{3}}{2}$

B $\sqrt{3}$

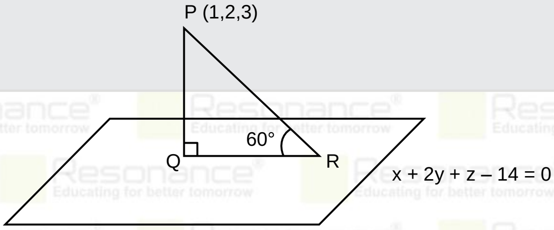
C $2\sqrt{3}$

D 3

NTA Ans. (B)

Reso Ans. (B)

Sol.



$$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$$

$$\text{In } \Delta\text{ PQR, } \tan 60° = \frac{PQ}{QR} = \frac{\sqrt{6}}{QR} \Rightarrow QR = \sqrt{2}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}(PQ)(QR) = \frac{1}{2}\left(\sqrt{6}\right)\left(\sqrt{2}\right) = \sqrt{3} \text{ square units}$$

15 If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is :

A $\frac{21}{2}$

B $\frac{59}{8}$

C $\frac{57}{8}$

D $\frac{95}{8}$

NTA Ans. (D)

Reso Ans. (D)

16 Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is :

(A) $\frac{4}{9}$ (B) $\frac{5}{18}$ (C) $\frac{1}{6}$ (D) $\frac{3}{10}$

NTA Ans. (B)

Reso Ans. (B)

Sol.

17 Let $S = \{z = x+iy : |z-1+i| \ge |z|, |z| < 2, |z+i| = |z-1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

A $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$

B $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

C $\left(-\sqrt{2}, \frac{1}{2}\right]$

D $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

NTA Ans. (B)

Reso Ans. (B)

18 Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b}) = 168$, then $|\vec{a}|+|\vec{b}|+|\vec{c}|$ is equal

(A) 10 (B) 14 (C) 16 (D) 18

NTA Ans. (C)

Reso Ans. (C)

Sol. Since $\vec{a}$, $\vec{b}$, $\vec{c}$ are concurrent and coplanar and equally inclined.

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{a} \wedge \vec{c} = 2\pi/3$

$\vec{a} \times \vec{b} \,||\, \vec{b} \times \vec{c} \,||\, \vec{c} \times \vec{a}$

$(\vec{a} \times \vec{b}) . (\vec{a} \times \vec{c}) = |\vec{a} \times \vec{b}|\,|\vec{b} \times \vec{c}| \cos(0^o)$

$= \left(|\vec{a}|\,|\vec{b}| \sin\frac{2\pi}{3}\right)\left(|\vec{b}|\,|\vec{c}| \sin\frac{2\pi}{3}\right)$

$= \frac{3}{4}|\vec{a}|\,|\vec{b}|^2\,|\vec{c}|$

Similarly $(\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) = \frac{3}{4}|\vec{a}|\,|\vec{b}|\,|\vec{c}|^2$

$(\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = \frac{3}{4}|\vec{a}|^2\,|\vec{b}|\,|\vec{c}|$

Now

$(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$

$\frac{3}{4}|\vec{a}|\,|\vec{b}|\,|\vec{c}|\,(|\vec{a}| + |\vec{b}| + |\vec{c}|) = 168$

$|\vec{a}| + |\vec{b}| + |\vec{c}| = 168 \times \frac{4}{3} \times \frac{1}{14} = 16$

19 The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is :

A $[1, \infty)$

B $[-1, 2]$

C $[-1, \infty)$

D $(-\infty, 2]$

NTA Ans. (C)

Reso Ans. (C)

Sol. $-1 \le \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \le 1$

$-x^2 - 2x - 7 \le x^2 - 3x + 2 \le x^2 + 2x + 7$

$0 \le 2x^2 - x + 9$ and $0 \le 5x + 5$

Domain = $x \in [-1, \infty)$

20 The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is **NOT** equivalent to

A $(p \wedge (\sim r)) \Rightarrow q$

B $(\sim q) \Rightarrow ((\sim r) \vee p)$

C $p \Rightarrow (q \vee r)$

D $(p \wedge (\sim q)) \Rightarrow r$

NTA Ans. **(B)**

Reso Ans. **(B)**

21 The sum and product of the mean and variance of a binomial distribution are 82·5 and 1350 respectively. Then the number of trials in the binomial distribution is

NTA Ans. **(96)**

Reso Ans. **(96)**

Sol. Mean = np, Variance = npq

$np + npq = 82.5 \Rightarrow np(1 + q) = 82.5$ ....(1)

$np \cdot npq = 1350 \Rightarrow n^2p^2 \cdot q = 1350$ ....(2)

from (1) & (2)

$$\frac{q}{(1+q)^2} = \frac{24}{121}$$

$24q^2 - 73q + 24 = 0$

$(3q - 8)(8q - 3) = 0$

$q = \frac{8}{3}$ or $\frac{3}{8} \Rightarrow q = \frac{3}{8}$ $(\because q \le 1)$

$\Rightarrow p = 1 - q = \frac{5}{8}$

$n = 96$

22 Let $\alpha, \beta\ (\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\dfrac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to ______.

NTA Ans. **(16)**

Reso Ans. **(16)**

Sol. $\alpha^2 - \alpha + 4 = 0 \Rightarrow \alpha^n - \alpha^{n-1} + 4\alpha^{n-2} = 0$ ....(1)

Similarly $\beta^n - \beta^{n-1} + 4\beta^{n-2} = 0$ .... (2)

Subtract (2) from (1)

$\Rightarrow P_n - P_{n-1} + 4P_{n-2} = 0$

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}} = \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \left(\frac{P_{15} - P_{14}}{P_{13}}\right)\left(\frac{P_{16} - P_{15}}{P_{14}}\right)$$

$= (-4)(-4) = 16$

23 Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if $X'A^kX = 33$, then k is equal to

NTA Ans. **(10)**

Reso Ans. **(10)**

24. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is ________.

NTA Ans. **(6)**

Reso Ans. **(6)**

Sol. **Case I :** when number is four digit number

a	b	c	5

for divisibility by 11

a + c = b + 5

When b = 2 a = 3, c = 4

a = 4, c = 3

b = 3 a = 2, c = 6

a = 6, c = 2

b = 4 a = 3, c = 6

a = 6, c = 3

So possible number of natural numbers is 6.

Case II :

No five digit number is possible since least 5 digit number is 23456 which is greater than 23421

25 If $\sum_{k=1}^{10} K^2\left(10_{C_K}\right)^2 = 22000L$, then L is equal to ________.

NTA Ans. **(221)**

Reso Ans. **(221)**

Sol. $\sum_{k=1}^{10} \left(k\ {}^{10}C_k\right)^2 = \sum_{k=1}^{10} (10)^2\left({}^9C_{k-1}\right)^2$

$= 100\left[\left({}^9C_0\right)^2 + \left({}^9C_1\right)^2 \ldots\ldots + \left({}^9C_9\right)^2\right]$

$= 100\ ({}^{18}C_9) = 22000\ L$

$\Rightarrow \frac{18!}{9!\ 9!} = 220\ L \Rightarrow L = 17 \times 13 = 221$

26 If $[t]$ denotes the greatest integer $\le t$, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x + \frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval $(-20, 20)$, is ________.

NTA Ans. **(79)**

Reso Ans. **(79)**

27 If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to ________

NTA Ans. (195)

Reso Ans. (195)

28 Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$.

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ________.

NTA Ans. (72)

Reso Ans. (72)

29 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a}\cdot\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to ________.

NTA Ans. (14)

Reso Ans. (14)

Sol. $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2 \Rightarrow |\vec{b}|^2 = 2\vec{a}.\vec{b} \Rightarrow |\vec{b}|^2 = 6$ ______(1)

$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2 \Rightarrow 6|\vec{a}|^2 - 9 = 75$

$\Rightarrow 6|\vec{a}|^2 = 84 \Rightarrow |\vec{a}|^2 = 14$

30 Let $S = \{(x, y) \in \mathbb{N}\times\mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \le 144\}$ and $T = \{(x, y) \in \mathbb{R}\times\mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\}$. Then $n(S\cap T)$ is equal to ________

NTA Ans. (27)

Reso Ans. (27)