JEE (Main) 2020

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 09 January, 2020 (SHIFT-1) | TIME: (9.30 am to 12.30 pm)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS
Success at JEE Main 2019

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AIR-100 Ananjan Nandi
Classroom student since class 9th
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AIR-77 Anubhav Kalyani
Classroom student since class 9th
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AIR-96 Suhas Jain
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AIR-45 Atreyo Goswami
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AIR-30 Rajdeep Paul
Classroom student since class 9th
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AIR-12 Shubhankar Gambhir
Classroom student since class 9th
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Atin Bainada, Raja, Kuldeep Meena
AIR-11 (SC)
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PART : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)
Straight Objective Type (सीधे वक्तृत्व प्रकार)

This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

इस खंड में 20 बहु-विकल्पी प्रश्न हैं। प्रत्येक प्रश्न के 4 विकल्प (1), (2), (3) तथा (4) हैं, जिनमें से सिर्फ एक सही है।

1. The product $\frac{1}{2^4} \cdot \frac{1}{4^4} \cdot \frac{1}{8^4} \cdot \frac{1}{16^4} \cdots$ to $\infty$ is equal to

Ans. (2)

Sol. $\frac{1}{2^4} \cdot \frac{1}{4^4} \cdot \frac{1}{8^4} \cdot \frac{1}{16^4} \cdots = \frac{1}{4^\infty} = \sqrt{2}$

2. If $f(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to:

Ans. (4)

Sol. $f(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 - \cos(\frac{\pi}{2} + x)}{\sin(\frac{\pi}{2} + x)}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\pi + x}{4}}{2\sin\frac{\pi + x}{4} \cos\frac{\pi + x}{4}}\right)

f(x)dx = \frac{\pi + x}{4} dx

f(x) = \frac{\pi}{4} x + \frac{x^2}{4} + c

f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4} x + \frac{x^2}{4}

So $f(1) = \frac{\pi + 1}{4}$
3. The value of \( \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} \, dx \) is equal to:

\[
\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} \, dx = 2\pi \quad \text{(1)} \quad \pi^2 \quad \text{(2)} \quad 4\pi \quad \text{(3)} \quad 2\pi^2 \quad \text{(4)}
\]

\[\text{Ans. (2)}\]

\[\text{Sol.} \quad \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} \, dx = \int_0^{\pi/2} 2\pi \, dx = \pi^2 \quad \text{(1)} \quad \frac{\pi^2}{2} \quad \text{(2)} \quad 2\pi \quad \text{(3)} \quad \pi^2 \quad \text{(4)} \]

4. The value of \( \cos^3 \left( \frac{\pi}{8} \right) \cdot \cos \left( \frac{3\pi}{8} \right) + \sin^3 \left( \frac{\pi}{8} \right) \cdot \sin \left( \frac{3\pi}{8} \right) \) is:

\[
\cos^3 \left( \frac{\pi}{8} \right) \cdot \cos \left( \frac{3\pi}{8} \right) + \sin^3 \left( \frac{\pi}{8} \right) \cdot \sin \left( \frac{3\pi}{8} \right) \quad \text{का मान है}--
\]

\[\text{Ans. (1)}\]

\[\text{Sol.} \quad \cos^3 \frac{\pi}{8} \left[ 4 \cos^3 \frac{\pi}{8} \right] + \sin^3 \frac{\pi}{8} \left[ 3 \sin^3 \frac{\pi}{8} \right] = 4 \cos \frac{\pi}{8} - 4 \sin \frac{\pi}{8} - 3 \cos^4 \frac{\pi}{8} + 3 \sin^4 \frac{\pi}{8}
\]

\[= 4 \left[ \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[ \sin^4 \frac{\pi}{8} + \cos \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right] - 3 \left[ \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right]
\]

\[= \cos \frac{\pi}{4} \left[ 4 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} - 3 \right] = \frac{1}{\sqrt{2}} \left[ 1 - \frac{1}{2} \right] = \frac{1}{2\sqrt{2}}
\]

5. If the matrices \( A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \), \( B = \text{adj} A \) and \( C = 3A \), then \( \frac{\text{adj} B}{\text{C}} \) is equal to:

\[\text{यदि} \quad A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}, \quad B = \text{adj} A \] \( \text{तथा} \quad C = 3A \), \( \text{तब} \quad \frac{\text{adj} B}{\text{C}} \) \( \text{बराबर है}--
\]

\[\text{Ans. (2)}\]

\[\text{(1) 72} \quad \text{(2) 8} \quad \text{(3) 16} \quad \text{(4) 2}\]
6. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle?

एक वृत्त y-अक्ष को बिन्दु (0, 4) पर स्पर्श करता है तथा बिन्दु (2, 0) से होकर जाता है। निम्न में से कौन सी रेखा इस वृत्त की स्पर्श रेखा नहीं है?

(1) 4x − 3y + 17 = 0  
(2) 3x − 4y − 24 = 0  
(3) 3x + 4y − 6 = 0  
(4) 4x + 3y − 8 = 0

Ans. (4)

Sol.

![Circle Touching Y-axis](image)

equation of family of circle वृत्त निकाय का समीकरण

\[(x - 0)^2 + (y - 4)^2 + \lambda x = 0 \Rightarrow \text{passes गुजरता है (2, 0)}\]

\[4 + 16 + 2\lambda = 0 \Rightarrow \lambda = -10\]

\[x^2 + y^2 - 10x - 8y + 16 = 0\]

centre केंद्र (5, 4). \(R = \sqrt{25 + 16 - 16} = 5\)

Check the options. विकल्पों की जीव करें।

Option (4)

\[4 \times 5 + 3 \times 4 - 8 = \frac{24}{5} \neq 5\]

7. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm³/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which the thickness of ice decreases, is:

एक 10 cm व्रत्त गोलाकार लोहे की गेंद को बर्फ की एक समान मोटाई गोलाकार पोल तथा तैयार किया गया है, जो कि 50 cm³/min. की दर से घटती है। जब बर्फ की परत की मोटाई 5 cm है, उस समय बर्फ की मोटाई के घटने की दर (in cm/min. में) है?

(1) \(\frac{5}{6\pi}\)  
(2) \(\frac{1}{36\pi}\)  
(3) \(\frac{1}{18\pi}\)  
(4) \(\frac{1}{54\pi}\)

Ans. (3)
Sol. Let thickness माना मोटाई = x cm

Total volume कुल आयतन \( v = \frac{4}{3}\pi(10 + x)^3 \)

\[
\frac{dv}{dt} = 4\pi (10 + x)^2 \frac{dx}{dt} \quad \text{.........(i)}
\]

Given दिया है \( \frac{dv}{dt} = 50 \text{cm}^3/\text{min} \)

At \( x = 5 \text{cm} \)

\[
50 = 4\pi (10 + 5)^2 \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = \frac{1}{18\pi} \text{cm/min}
\]

8. Negation of the statement:
\( \sqrt{5} \) is an integer or 5 is irrational is:

(1) \( \sqrt{5} \) is not an integer or 5 is not irrational

(2) \( \sqrt{5} \) is an integer and 5 is irrational

(3) \( \sqrt{5} \) is irrational or 5 is an integer.

(4) \( \sqrt{5} \) is not an integer and 5 is not irrational

कथन:
\( \sqrt{5} \) एक पूर्णांक है या 5 अपरिमेय है का निवेदन है:

(1) \( \sqrt{5} \) एक पूर्णांक नहीं है और 5 अपरिमेय नहीं

(2) \( \sqrt{5} \) एक पूर्णांक है और 5 अपरिमेय है

(3) \( \sqrt{5} \) अपरिमेय है या 5 एक पूर्णांक है।

(4) \( \sqrt{5} \) एक पूर्णांक नहीं है या 5 अपरिमेय नहीं है।

Ans. (4)

Sol. \( \sqrt{5} \) is not an integer and 5 is not an irrational Number \(~(p \lor q) = \neg p \land \neg q\)

\( \sqrt{5} \) एक पूर्णांक नहीं है तथा 5 एक अपरिमेय संख्या नहीं है \(~(p \lor q) = \neg p \land \neg q\)

9. The integral \( \int \frac{dx}{(x + 4)^{8/7}(x - 3)^{6/7}} \) is equal to:

(where \( C \) is a constant of integration)

समाकल \( \int \frac{dx}{(x + 4)^{8/7}(x - 3)^{6/7}} \) वरोकर है:

(जहाँ \( C \) एक समाकल अचर है)

(1) \( \left( \frac{x - 3}{x + 4} \right)^{1/7} + C \)

(2) \( \frac{1}{13}\left( \frac{x - 3}{x + 4} \right)^{13/7} + C \)

(3) \( \frac{1}{2}\left( \frac{x - 3}{x + 4} \right)^{3/7} + C \)

(4) \( \left( \frac{x - 3}{x + 4} \right)^{-1/7} + C \)

Ans. (1)
Sol. \[ \int \left( \frac{x-3}{x+4} \right)^{-6} \frac{1}{(x+4)^2} \, dx \]

Let \( \frac{x-3}{x+4} = y^7 \),

\[ \frac{7}{(x+4)^2} \, dx = 7t^6 dt \]

\[ \int t^{-6} t^6 dt = t + c \]

10. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336 k, then k is equal to:

(1) 7 (2) 4 (3) 6 (4) 8

Ans. (4)
Sol.

Number of numbers संख्याओं की संख्या = 8 \times 8 \times 7 \times 6 = 2688 = 336 \times k \Rightarrow k = 8

11. In an an box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is:

(1) \( \frac{13}{16} \) (2) \( \frac{15}{16} \) (3) \( \frac{9}{16} \) (4) \( \frac{11}{16} \)

Ans. (4)
Sol. \[ AA + ABA + BAA + ABBA + BBAA + BABBA = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16} \]

12. Let \( z \) be a complex number such that \( \frac{z-1}{z+2i} = 1 \) and \( |z| = \frac{5}{2} \). Then the value of \( |z+3i| \) is:

माना \( z \) एक ऐसी सम्भव संख्या है, कि \( \frac{z-1}{z+2i} = 1 \) है तथा \( |z| = \frac{5}{2} \) है, तो \( |z+3i| \) का मान है—

(1) \( \sqrt{10} \) (2) \( 2\sqrt{3} \) (3) \( \frac{7}{2} \) (4) \( \frac{15}{4} \)

Ans. (3)
Sol. \( x^2 + (y-1)^2 = x^2 + (y+2)^2 \)

\[-2y + 1 = 4y + 4 \]

\[6y = -3 \Rightarrow y = -\frac{1}{2} \]
13. If for some \( \alpha \) and \( \beta \) in \( \mathbb{R} \), the intersection of the following three planes
\[
\begin{align*}
&x + 4y - 2z = 1 \\
x + 7y - 5z = \beta \\
x + 5y + az = 5
\end{align*}
\]
is a line in \( \mathbb{R}^3 \), then \( \alpha + \beta \) is equal to:

\( \text{Ans. (3)} \)

**Sol.**
\[
\Delta = 0 \Rightarrow \begin{vmatrix}
1 & 4 & -2 \\
1 & 7 & -5 \\
1 & 5 & \alpha
\end{vmatrix} = 0
\]

\[
(7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0
\]

\[
3\alpha + 9 = 0 \Rightarrow \alpha = -3
\]

Also \( D_z = 0 \Rightarrow \begin{vmatrix}
1 & 4 & 1 \\
1 & 7 & \beta \\
1 & 5 & 5
\end{vmatrix} = 0
\]

\[
1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0
\]

\[
\beta = 13
\]

14. The number of real roots of the equation, \( e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0 \) is:

\( \text{समीकरण } e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0 \text{ के वास्तविक मूलों की संख्या है} \)

\( \text{Ans. (2)} \)

**Sol.**
Let \( e^x = t \in (0, \infty) \)

Given equation दी गई समीकरण
\[
t^4 + t^3 - 4t^2 + t + 1 = 0
\]

\[
t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0
\]

\[
\left( t^2 + \frac{1}{t^2} \right) + \left( t + \frac{1}{t} \right) - 4 = 0
\]
15. Let \( f \) be any function continuous on \([a, b]\) and twice differentiable on \((a, b)\). If for all \( x \in (a, b)\), \( f'(x) > 0 \) and \( f''(x) < 0 \), then for any \( c \in (a, b)\), \( \frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c} \) is greater than

\[ f'(x) = \frac{f(c) - f(a)}{c - a} = f'(\alpha), \quad \alpha \in (a, c) \]

also use LMVT for \( x \in [c, b] \)

\[ \frac{f(b) - f(c)}{b - c} = f'(\beta), \quad \beta \in (c, b) \]

\[ f'(x) < 0 \Rightarrow f'(x) \text{ is decreasing} \]

\[ f''(\alpha) > f''(\beta) \]

\[ \frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c} \]

\[ f'(x) = \frac{c - a}{b - c} \quad (\because f(x) \text{ is increasing}) \]

Hindi.

LMVT का उपयोग करने पर \( x \in [a, c] \) के लिये

\[ \frac{f(c) - f(a)}{c - a} = f'(\alpha), \quad \alpha \in (a, c) \]

LMVT का उपयोग करने पर \( x \in [c, b] \) के लिये

\[ \frac{f(b) - f(c)}{b - c} = f'(\beta), \quad \beta \in (c, b) \]

\[ f''(\alpha) > f''(\beta) \]

\[ \frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c} \]

\[ f'(x) = \frac{c - a}{b - c} \quad (\because f(x) \text{ वर्धमान है}) \]
16. If for all real triplets (a, b, c), \( f(x) = a + bx + cx^2 \); then \( \int_{0}^{1} f(x) \, dx \) is equal to:

\[ \text{यदि सभी वास्तविक त्रिभुज (a, b, c) के लिए, } f(x) = a + bx + cx^2 \text{ है, तो } \int_{0}^{1} f(x) \, dx \text{ बराबर है} - \]

\[
\begin{align*}
(1) & \quad \frac{1}{3} \left[ f(0) + f\left(\frac{1}{2}\right) \right] \\
(2) & \quad \frac{1}{3} \left[ f(1) + 3f\left(\frac{1}{2}\right) \right] \\
(3) & \quad \frac{1}{6} \left[ f(0) + f(1) + 4f\left(\frac{1}{2}\right) \right] \\
(4) & \quad 2f(1) + 2f\left(\frac{1}{2}\right)
\end{align*}
\]

Ans. (3)

Sol. \[
\int_{0}^{1} (a + bx + cx^2) \, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} \bigg|_{0}^{1} = a + \frac{b}{2} + \frac{c}{3}
\]

\[ f(1) = a + b + c \]

\[ f(0) = a \]

\[ f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4} \]

Now \[
\frac{1}{6} \left( f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right) = \frac{1}{6} \left( a + b + c + a + \frac{b}{2} + \frac{c}{4} \right) = \frac{1}{6} \left( 6a + 3b + 2c \right) = a + \frac{b}{2} + \frac{c}{3}
\]

17. Let the observations \( x_i (1 \leq i \leq 10) \) satisfy the equations, \( \sum_{i=1}^{10} (x_i - 5) = 10 \) and \( \sum_{i=1}^{10} (x_i - 5)^2 = 40 \). If \( \mu \) and \( \lambda \) are the mean and the variance of the observations, \( x_1 - 3, x_2 - 3, \ldots, x_{10} - 3 \), then the ordered pair \((\mu, \lambda)\) is equal to:

\[ \text{माना प्रेक्षण } x_i(1 \leq i \leq 10) \text{ समीकरण } \sum_{i=1}^{10} (x_i - 5) = 10 \text{ तथा } \sum_{i=1}^{10} (x_i - 5)^2 = 40 \text{ का संतुष्टि करते है } \]

\[ \text{यदि } \mu \text{ तथा } \lambda, \text{ प्रेक्षण } x_1 - 3, x_2 - 3, \ldots, x_{10} - 3 \text{ के क्रमशः माध्य तथा प्रभावण है, तो क्रमित गुण (} \mu, \lambda ) \text{ बराबर है} - \]

\[
\begin{align*}
(1) & \quad (3, 3) \\
(2) & \quad (3, 6) \\
(3) & \quad (6, 6) \\
(4) & \quad (6, 3)
\end{align*}
\]

Ans. (1)

Sol. Mean माध्य \( (x_i - 5) = \frac{\sum (x_i - 5)}{10} = 1 \) \[ \therefore \lambda = \{\text{Mean माध्य } (x_i - 5)\} + 2 = 3 \]

\[ \mu = \text{var चरिता } (x_i - 5) = \frac{\sum (x_i - 5)^2}{10} - \frac{\sum (x_i - 5)^2}{10} = 3 \]
18. If \( e_1 \) and \( e_2 \) are the eccentricities of the ellipse, \( \frac{x^2}{18} + \frac{y^2}{4} = 1 \) and the hyperbola, \( \frac{x^2}{9} - \frac{y^2}{4} = 1 \) respectively and \( (e_1, e_2) \) is a point on the ellipse, \( 15x^2 + 3y^2 = k \), then \( k \) is equal to

\[
\begin{align*}
\text{Ans.} & \quad (1) \ 14 \\
\text{Sol.} & \quad e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3} \\
e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3} \\
15e_1^2 + 3e_2^2 = k \Rightarrow k = 15(\frac{7}{9}) + 3(\frac{13}{9}) \quad \therefore k = 16
\end{align*}
\]

19. Let \( C \) be the centroid of the triangle with vertices \( (3, -1), \ (1, 3) \) and \( (2, 4) \). Let \( P \) be the point of intersection of the lines \( x + 3y - 1 = 0 \) and \( 3x - y + 1 = 0 \). Then the line passing through the points \( C \) and \( P \) also passes through the point

\[
\begin{align*}
\text{Ans.} & \quad (1) \ (7, 6) \\
\text{Sol.} & \quad D (2, 2) \\
\text{Point of intersection} P \left( -\frac{1}{5}, \frac{2}{5} \right) \\
equation \text{ of line DP} 8x - 11y + 6 = 0
\end{align*}
\]

\[
\begin{align*}
\text{Sol.} & \quad D (2, 2) \\
\text{pratibhandh vidhu} P \left( -\frac{1}{5}, \frac{2}{5} \right) \\
\text{reka} \ DP \ \text{ka} \ \text{samikaran} \ 8x - 11y + 6 = 0
\end{align*}
\]
20. If \( f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}; & x < 0 \\ b; & x = 0 \\ \frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}}; & x > 0 \end{cases} \)

is continuous at \( x = 0 \), then \( a + 2b \) is equal to:

\[ \text{Ans. (2)} \]

\[ \text{Sol. LHL = a + 3} \]
\[ f(0) = b \]
\[ \text{RHL = } \lim_{h \to 0} \left( \frac{1+3h^{1/3} - 1}{h} \right) = 1 \]
\[ \therefore \quad a = -2 \]
\[ b = 1 \]
\[ \therefore \quad a + 2b = 0 \]

SECTION – 2 : (Maximum Marks : 20)

- This section contains FIVE (05) questions. The answer to each question is NUMERICAL VALUE with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.
  - Full Marks : +4 If ONLY the correct option is chosen.
  - Zero Marks : 0 In all other cases

खंड 2 (अध्यक्षतम अंक: 20)

- इस खंड में पांच (05) प्रश्न है। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान (NUMERICAL VALUE) है, जो ही-अंकीय पूर्णांक तथा दशमलव एकल-अंक में है।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के दो स्थानों तक ट्रांसल्यूजेंट/रांड ऑफ (truncate/round-off) करें।
- अंकन की योजना:
  - पूर्ण अंक : +4 यदि सिद्ध सही फिक्तता ही चुना गया है।
  - शून्य अंक : 0 अन्य सभी परिप्रेक्ष्यों में।

21. The number of distinct solutions of the equation, \( \log_{1/2} |\sin x| = 2 – \log_{1/2} |\cos x| \) in the interval \([0, 2\pi]\), is

\[ \text{Ans. (8)} \]

\[ \text{Sol. } \log_{1/2} |\sin x| = 2 – \log_{1/2} |\cos x| \]
\[ \log_{1/2} |\sin x \cos x| = 2 \]
|sin x cos x| = \frac{1}{4}

\sin 2x = \pm \frac{1}{2}

Number of solution हलों की संख्या = 8.

22. The projection of the line segment joining the points (1, -1, 3) and (2, -4, 11) on the line joining the points (-1, 2, 3) and (3, -2, 10) is:

किन्नरों (1, -1, 3) तथा (2, -4, 11) को मिलाने वाले रेखाखण्ड का किन्नरों (-1, 2, 3) तथा (3, -2, 10) को मिलाने वाली रेखा पर प्रक्षेप है

Ans. (8)

Sol. Let A(1, -1, 3), B(2, -4, 11)

\overrightarrow{AB} = \hat{i} - 3\hat{j} + 8\hat{k}

C(-1, 2, 3), D(3, -2, 10)

\overrightarrow{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}

\text{projection of } \overrightarrow{AB} \text{ on } \overrightarrow{CD} = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{CD}|} = \left( \frac{4 + 12 + 56}{\sqrt{16 + 16 + 49}} \right) = \frac{72}{9} = 8

23. The coefficient of \(x^4\) in the expansion of \((1 + x + x^2)^{10}\) is...

\((1 + x + x^2)^{10}\) के प्रसार में \(x^4\) का गुणांक है

Ans. 615

Sol. General term महत्म पद \(\frac{10!}{\alpha!\beta!\gamma!} x^{\alpha+2\gamma}\) for coefficient of \(x^4\) का गुणांक \(\Rightarrow \beta + 2\gamma = 4\)

\(\gamma = 0, \beta = 4, \alpha = 6 \Rightarrow \frac{10!}{6!4!0!} = 210\)

\(\gamma = 1, \beta = 2, \alpha = 7 \Rightarrow \frac{10!}{7!2!1!} = 360\)

\(\gamma = 2, \beta = 0, \alpha = 8 \Rightarrow \frac{10!}{8!0!2!} = 45\)

Total कुल = 615
24. If for \( x \geq 0 \), \( y = y(x) \) is the solution of the differential equation,

\[
(x + 1)dy = ((x + 1)^2 + y - 3)dx \quad y(2) = 0
\]

then \( y(3) \) is equal to:

\[\text{Ans. (3)}\]

\[
\text{Sol. } \frac{dy}{dx} = \frac{y - 3}{1 + x}
\]

\[
\frac{dy}{dx} - \frac{1}{1 + x}y = \frac{3}{1 + x}
\]

I.F. = \( e^\int \frac{1}{1 + x} dx = e^{\ln(1 + x)} = 1 + x \)

\[
\therefore \quad \frac{d}{dx} \left( \frac{y}{1 + x} \right) = 1 - \frac{3}{(1 + x)^2}
\]

\[
y = (1 + x) \left( x + \frac{3}{1 + x} + c \right)
\]

\[\text{at } x = 2, \quad y = 0 \Rightarrow 0 = 3(2 + 1 + c) \Rightarrow c = -3 \Rightarrow \text{at } x = 3, \quad y = 3 \]

25. If the vectors, \( \vec{p} = (a + 1) \hat{i} + a \hat{j} + a \hat{k}, \quad \vec{q} = a \hat{i} + (a + 1) \hat{j} + a \hat{k} \) and \( \vec{r} = a \hat{i} + a \hat{j} + (a + 1) \hat{k}, (a \in \mathbb{R}) \) are coplanar and \( 3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \cdot \vec{q}|^2 = 0 \), then the value of \( \lambda \) is:

\[\text{Ans. 1}\]

\[
\begin{vmatrix}
 a+1 & a & a \\
 a & a+1 & a \\
 a & a & a+1
\end{vmatrix} = 0 \Rightarrow a + 1 + a + a = 0 \quad \Rightarrow a = -\frac{1}{3}
\]

\[\vec{P} = \frac{2}{3} \hat{i} - \frac{2}{3} \hat{j} - \frac{1}{3} \hat{k}
\]

\[\vec{Q} = \frac{1}{3}(-\hat{i} + 2 \hat{j} - \hat{k})
\]

\[\vec{R} = \frac{1}{3}(-\hat{i} - \hat{j} + 2 \hat{k})
\]

\[\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}
\]

\[\vec{R} \times \vec{Q} = \frac{1}{9} \begin{vmatrix}
 i & j & k \\
 -1 & 2 & -1 \\
 2 & -1 & 1
\end{vmatrix} = \frac{1}{9} ((4 - 1) - j(-2 - 1) + k(1 + 2)) = \frac{1}{9} (3i + 3j + 3k) = \frac{i + j + k}{3}
\]

\[|\vec{R} \times \vec{Q}| = \frac{1}{3} \sqrt{3} \Rightarrow |\vec{R} \times \vec{Q}|^2 = \frac{1}{3}
\]

\[3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0
\]

\[3 \left( \frac{1}{9} \right) - \lambda \left( \frac{1}{3} \right) = 0 \Rightarrow \lambda = 1
\]
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