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JEE (ADVANCED) 2023

QUESTIONS & TEXT SOLUTION

PAPER-2

DATE & DAY: 4th JUNE 2023, SUNDAY

PAPER-1	PAPER-2
Duration: 3 Hrs.	**Duration:** 3 Hrs.
Time: 09:00 - 12:00 IST	**Time:** 14:30 - 17:30 IST

SUBJECT: MATHEMATICS

REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):
CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Rajasthan) - 324005
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TARGET: JEE (Adv.) 2024

VIJAY COURSE

For 12th Passed Students

Course Features*

- Course Duration: **32 Weeks**
- Total No. of Lectures: **533** (P: 178 | C: 177 | M: 178)
- Duration of One Lecture: **1.5 Hrs.** (90 Minutes)
- Classroom Teaching Hours.: **800 Hrs.**
- Testing Duration: **60 Hrs.**
- Total Academic Hours.: **860 Hrs.**

CLASS STARTS
5th & 19th June

AIR 6
JEE (Adv.) 2022
KARTHIKEYA P.

SCHOLARSHIP UPTO 100%
Based on JEE (Advanced) 2023 Score, Scholarship Test (ResoNET) & 12th Board

TARGET: JEE (Main) 2024

AIR 5
JEE (Main) 2023
KAUSHAL V.

SCHOLARSHIP UPTO 100%
Based on JEE (Main) 2023 Score, Scholarship Test (ResoNET) & 12th Board

AJAY COURSE

For 12th Passed Students

Course Features*

- Course Duration: **33 Weeks**
- Total No. of Lectures: **571** (P:1 84 | C: 203 | M: 184)
- Duration of One Lecture: **1.5 Hrs.** (90 Minutes)
- Classroom Teaching Hours.: **857 Hrs.**
- Testing Duration: **33 Hrs.**
- Total Academic Hours.: **890 Hrs.**

CLASS STARTS
5th & 19th June

PART-I : MATHEMATICS

SECTION 1 : 12 Marks

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If **ONLY** the correct option is chosen;
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : **−1** In all other cases.

1. Let $f:[1, \infty) \to R$ be a differentiable function such that $f(1) = \frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x) - \frac{x^3}{3}, x \in [1,\infty)$. Let e denote the base of the natural logarithm. Then the value of f(e) is

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$ (C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Ans. **(C)**

Sol. $3\int_1^x f(t)dt = xf(x) - \frac{x^3}{3}$

Differentiate w.r.t. x

$3(f(x).1-0) = 1.f(x) + xf'(x) - \frac{3x^2}{3}$

$\Rightarrow 2f(x) = x f'(x) - x^2$

$x\frac{dy}{dx} - 2y = x^2 \Rightarrow \frac{dy}{dx} - \frac{2}{x}y = x$

I. F . = $e^{\int -\frac{2}{x}dx} = e^{-2\ell nx} = \frac{1}{x^2}$

Solution of D.E ;

$y.\left(\frac{1}{x^2}\right) = \int (x)\frac{1}{x^2}dx$

$\frac{y}{x^2} = \ell nx + c$

$y = x^2 \ell nx + cx^2$

$\because f(1) = \frac{1}{3}$

$\Rightarrow \frac{1}{3} = 0 + c.\ 1^2 \Rightarrow c = \frac{1}{3}$

$f(x) = x^2 \ell nx + x^2/3$

$f(e) = e^2 + \frac{e^2}{3} = \frac{4e^2}{3}$

2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$ (C) $\frac{4}{21}$ (D) $\frac{2}{7}$

Ans. **(B)**

Sol. Probability = (HH+HTHH+HTHTHH+...... ∞) + (THH+THTHH+THTHTHH+...... ∞)

$$= \frac{HH}{1-HT} + \frac{THH}{1-TH} = \frac{HH+THH}{1-TH}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{3}+\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}{1-\frac{2}{3}\times\frac{1}{3}} = \frac{\frac{5}{27}}{\frac{7}{9}} = \frac{5}{21}$$

3. For any y ∈ R, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3}-3$ (B) $3-2\sqrt{3}$ (C) $4\sqrt{3}-6$ (D) $6-4\sqrt{3}$

Ans. **(C)**

Sol. **Case-1**

When $-3 < y < 0$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \pi + \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow 2\tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{-\pi}{3}$$

$$\Rightarrow \frac{6y}{9-y^2} = \frac{-1}{\sqrt{3}}$$

$$\Rightarrow y^2 - 6\sqrt{3}y - 9 = 0$$

$$y^2 - 6\sqrt{3}y - 9 = 0$$

$\because -3 < y < 0$

$$\Rightarrow y = \frac{6\sqrt{3}-12}{2}$$

$$\Rightarrow y = 3\sqrt{3}-6$$

case-2 when 0 < y < 3

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$\Rightarrow 2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$\Rightarrow \frac{6y}{9-y^2}=\sqrt{3}$$

$$\sqrt{3}y^2+6y-9\sqrt{3}=0$$

$$\sqrt{3}y^2+9y-3y-9\sqrt{3}=0$$

$$\sqrt{3}y\left(y+3\sqrt{3}\right)-3\left(y+3\sqrt{3}\right)=0$$

So $y=\sqrt{3}$ is selected

Hence sum of all solutions = $4\sqrt{3}-6$

4. Let the position vector of the points P,Q,R and S be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true ?

(A) The points P,Q,R and S are **NOT** coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5:4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5:4

(D) The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

Ans. **(B)**

Sol. $\overrightarrow{PQ}=2\hat{i}+4\hat{j}+8\hat{k}$

$$\overrightarrow{PR}=\frac{12}{5}\hat{i}+\frac{6}{5}\hat{j}+12\hat{k}$$

$$\overrightarrow{PS}=\hat{i}-\hat{j}+6\hat{k}$$

now $$\left[\overrightarrow{PQ}\ \overrightarrow{PR}\ \overrightarrow{PS}\right]=\frac{1}{5}\begin{vmatrix}2 & 4 & 8\\ 12 & 6 & 60\\ 1 & -1 & 6\end{vmatrix}=\frac{12}{5}\begin{vmatrix}1 & 2 & 4\\ 2 & 1 & 10\\ 1 & -1 & 6\end{vmatrix}$$

$$=\frac{12}{5}(16-4-12)=0$$

So option (A) is **incorrect**

$$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}$$

$$\frac{5\vec{c}+4\vec{a}}{9}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}=\frac{\vec{b}+2\vec{d}}{3}$$

So option (B) is **correct and option (C) is incorrect**

Now $\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 6 & 3\\ 2 & 1 & 1\end{vmatrix}=3\hat{i}+3\hat{j}-9\hat{k}$

$\Rightarrow|\vec{b}\times\vec{d}|^2=9+9+81=99$

So option (D) is **incorrect**

SECTION 2 : 12 Marks

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4** If only (all) the correct option(s) is(are) chosen;

  Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : **0** If unanswered;

  Negative Marks : **−2** In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

5. Let $M = [a_{ij}]$, $i,j \in \{1,2,3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j+1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is(are) true ?

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in R^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$ where $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(D) The matrix $(M-2I)$ is invertible, where I is the 3×3 identity matrix

Ans. (BC)

Sol. $M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$|M| = 1(0-1) - (0-1) = -1 + 1 = 0$

$\Rightarrow$ M is singular

option (A) is wrong

Now, $M\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$\Rightarrow a_1 + a_2 + a_3 = -a_1$

$a_1 + a_3 = -a_2$

$a_2 = -a_3$

$\Rightarrow a_1 = 0$, $a_2 + a_3 = 0$ infinite solutions exists

option (B) is correct

Now $MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} x+y+z=0 \\ x+z=0 \\ y=0 \end{matrix}$

Infinite solution

Option (C) is correct

Now $M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

$\therefore |M-2I| = -3 + 3 = 0$

option (D) is wrong

6. Let f: (0,1) → R be the function defined as $f(x) = [4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$, where [x] denotes the greatest integer less than or equal to x. Then which of the following statements is(are) true?

(A) The function f is discontinuous exactly at one point in (0,1)

(B) There is exactly one point in (0,1) at which the function f is continuous but **NOT** differentiable.

(C) The function f is **NOT** differentiable at more than three points in (0,1).

(D) The minimum value of the function f is $\frac{-1}{512}$

Ans. **(AB)**

Sol. $f:(0,1) \to R,\ f(x) = [4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$ where [.] is G.I.F

$$\begin{cases} 0, & 0 < x < \frac{1}{4} \\ \left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{1}{4} \le x < \frac{1}{2} \\ 2\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{3}{4} \le x < 1 \end{cases}$$

Clearly f(x) continuous at $x = \frac{1}{4}, \frac{1}{2}$ but not continuous at $x = \frac{3}{4}$

f(x) is continuous at $x = \frac{1}{2}$ but not differentiable at $x = \frac{1}{2}$

clearly min $f(x) = -\frac{1}{432}$ at $x = \frac{5}{12}$

7. Let S be the set of all twice differentiable functions f from R to R such that $\frac{d^2f}{dx^2}(x) > 0$ for all $x \in (-1,1)$.

For $f \in S$, let X_f be the number of points $x \in (-1,1)$ for which f(x) = x. Then which of the following statements is(are) true ?

(A) There exists a function $f \in S$ such that $X_f = 0$

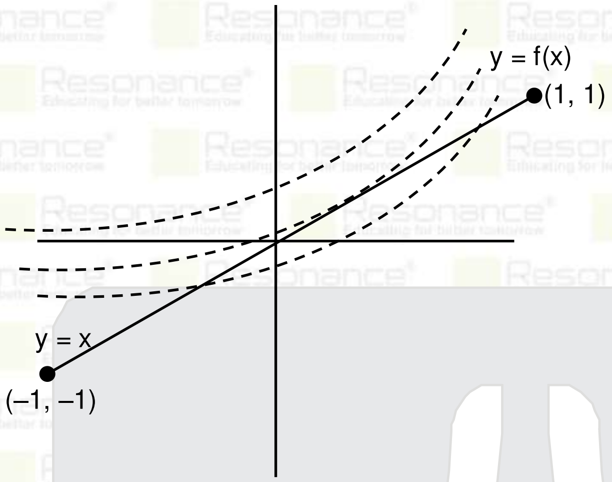
(B) For every function $f \in S$, we have $X_f \le 2$

(C) There exists a function $f \in S$ such that $X_f = 2$

(D) There does **NOT** exist any function f in S such that $X_f = 1$

Ans. **(ABC)**

Sol.



f(x) is concave function.
So intersection point of f(x) and y = x at most two and minimum zero.

SECTION-3 : 24 Marks

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4 ONLY** if the correct numerical value is entered;
Zero Marks : **0** In all other cases.

8. For $x \in R$, let $\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : R \to R$ defined by

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}}\,dt \text{ is}$$

Ans. **(0)**

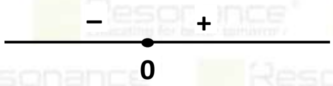
Sol. $$f'(x) = \frac{e^{x\tan^{-1}x-\cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}}\left[\frac{x}{1+x^2}+\tan^{-1}x\right]$$

$f'(x) = 0$

$\frac{x}{1+x^2}+\tan^{-1}x = 0$

$x + (1+x^2)\tan^{-1}x = 0$

$x = 0$



So x = 0 will be point of minima.

So minimum value of f(x)

f(0) = 0

9. For $x \in R$, let y(x) be a solution of differential equation $(x^2-5)\frac{dy}{dx}-2xy=-2x(x^2-5)^2$ such that y(2) = 7. Then the maximum value of the function y(x) is

Ans. **(16)**

Sol. $(x^2-5)\frac{dy}{dx}-2xy=-2x(x^2-5)^2$

$\Rightarrow \frac{dy}{dx}-\frac{2x}{x^2-5}y=-2x(x^2-5)$

$\Rightarrow$ I.F. = $e^{-\int\frac{2x}{x^2-5}dx}=\frac{1}{|x^2-5|}$

So solution

$$y.\frac{1}{|x^2-5|}=\int\frac{1}{|x^2-5|}\{-2x(x^2-5)\}dx+C$$

If $x^2>5$ then $\frac{y}{x^2-5}=-x^2+C$

If $x^2<5$ then $\frac{y}{5-x^2}=x^2+C$

using y(2) = 7 $\Rightarrow$ C = 3

$\Rightarrow$ $y = (3 + x^2)(5 - x^2) = -x^4 + 2x^2 + 15$

$\Rightarrow \frac{dy}{dx}=-4x^3+4x=0 \Rightarrow x = 0, 1, -1$

Maximum at x = 1 or –1 and $y_{maximum}$ = 16 Ans.

10. Let X be the set of all five digit numbers formed using digits 1, 2, 2, 2, 4, 4, 0. For example 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to

Ans. **(31)**

Sol. A = Number of elements in 'X' which is multiple of 5

1, 2, 2, 2

				0

$\rightarrow \frac{4!}{3!}=4$

1, 4, 2, 2

				0

$\rightarrow \frac{4!}{2!}=12$

1, 2, 4, 4

				0

$\rightarrow \frac{4!}{2!} = 12$

4, 2, 2, 2

				0

$\rightarrow \frac{4!}{3!} = 4$

2, 2, 4, 4

				0

$\rightarrow \frac{4!}{2!2!} = 6$

n(A) = 4 +12 +12 +4 +6= 38

B' = Number of elements in X which is not multiple of 20

2, 2, 2

			1	0

$\rightarrow 1$

4, 2, 2

			1	0

$\rightarrow \frac{3!}{2!} = 3$

2, 4, 4

			1	0

$\rightarrow \frac{3!}{2!} = 3$

So number of elements in X which is multiple of 20 = n(B) = (4 –1) + (12 -3) +(12–3) + 4+6 = 31

$P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)}$ $\quad [\therefore n(A \cap B) = n(B)]$

$\Rightarrow p = \frac{31}{38} = \Rightarrow 38p = 31$

11. Let $A_1, A_2, A_3 \ldots\ldots A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for i = 1, 2 …… 8. If P varies over the circle, then the maximum value of the product $PA_1.PA_2 \ldots.PA_8$ is

Ans. **(512)**

Sol. Let $z^8 - 2^8 = 0 \Rightarrow Z = 2, 2\alpha, 2\alpha^2, \ldots\ldots\ldots\ldots 2\alpha^7$, where $\alpha = e^{\frac{2\pi i}{8}}$

$Z^8 - 2^8 = (Z-2)(Z-2\alpha)(Z-2\alpha^2) \ldots\ldots\ldots\ldots (Z-2\alpha^7)$

$|Z^8 - 2^8| = |Z-2|\,|Z-2\alpha| \ldots\ldots\ldots\ldots |Z-2\alpha^7| \le |Z^8| + |2^8|$

$\le |Z|^8 + 2^8$

$\le 2^8 + 2^8$

$\le 2^9$

Max $PA_1 . PA_2 \ldots\ldots\ldots\ldots PA_8 = 2^9$, Equality will hold when $\arg(z^8) = \pi$

12. Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a,b,c,d \in \{0,3,5,7,11,13,17,19\} \right\}$. Then the number of invertible matrices in R is

Ans. **(3780)**

Sol. $\Delta = \begin{vmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{vmatrix} = -5\,(ad - bc)$

Number of invertible matrices = total matrices – non invertible matrices (Δ=0)

Total matrices = $8 \times 8 \times 8 \times 8 = 4096$

Now $\Delta = 0 \Rightarrow ad = bc$

Case(1)

(i)When all four a, b, c, d are zero =1

(ii) When three zero , one non-zero = $7\,\dfrac{4!}{3!} = 28$

(iii) When two zero two non zero $= {}^2C_1 . {}^2C_1 . 7 . 7 = 196$

Case(2) When all 4 non zero

(i) All 4 same =7

(ii) 2same + 2other same = ${}^7C_2 . {}^2C_1 . {}^2C_1 = 21 \times 4 = 84$

Total non invertible matrices = 1 + 28 + 196 + 7 + 84 = 316

Number of invertible matrices = 4096 – 316 = 3780

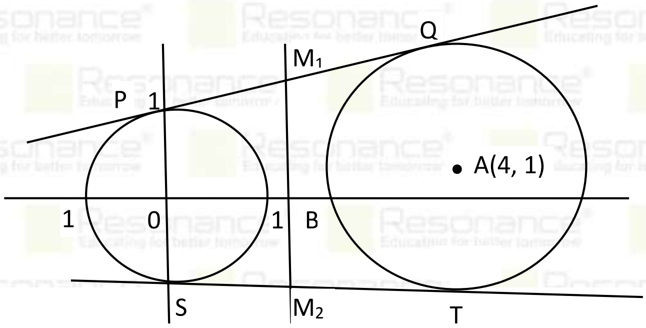
13. Let C_1 be the circle of radius 1 with centre at origin. Let C_2 be the circle of radius r with centre at the point A = (4, 1), where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If AB = $\sqrt{5}$, then the value of r^2 is

Ans. **(2)**

Sol. C_1 ; $x^2 + y^2 = 1$ C_2 ; $(x - 4)^2 + (y - 1)^2 = r^2$ $(1 < r < 3)$

A (4, 1)

Clearly M_1M_2 is radical axis



∴ Equation of M_1M_2 is $S_1 - S_2 = 0$

$\Rightarrow 8x + 2y - 18 + r^2 = 0$

$B\left(\frac{18-r^2}{8}, 0\right)$, A (4, 1)

$AB = \sqrt{5}$

$\therefore \left(\frac{18-r^2}{8} - 4\right)^2 + (0-1)^2 = 5$

$\Rightarrow r^2 = 2$

SECTION-4 : 12 Marks

- This section contains **TWO (02)** question paragraphs.
- Based on each paragraph, there are **TWO (02**) questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If **ONLY** the correct numerical value is entered at the designated place;
  Zero Marks : **0** In all other cases

PARAGRAPH 'I'

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

There are two questions based on PARAGRAPH 'I', the question given below is one of them

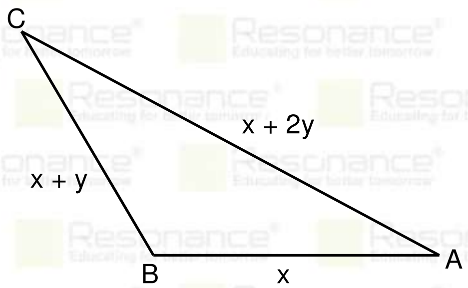
14. Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

Ans. **(1008)**

15. Then the inradius of the triangle ABC is

Ans. **(00.25)**

Sol **for (14 & 15)**



Let B be the greatest angle and C be the smallest angle with AB = x, BC = x + y and CA = x + 2y

Now $B - C = \frac{\pi}{2}$ (given)

$\Rightarrow B = \frac{\pi}{2} + C$

Since, $A + B + C = \pi$

$\Rightarrow A + \frac{\pi}{2} + C + C = \pi$

$\Rightarrow A = \frac{\pi}{2} - 2C$

Since, AB, BC, CA are in AP

$\Rightarrow 2BC = AC + AB$

$\Rightarrow 4R \sin A = 2R \sin B + 2R \sin C$

$\Rightarrow 2 \sin A = \sin B + \sin C$

$\Rightarrow 2 \sin\left(\frac{\pi}{2} - 2C\right) = \sin\left(\frac{\pi}{2} + C\right) + \sin C$

$\Rightarrow 2 \cos 2C = \cos C + \sin C$

$\Rightarrow 2 (\cos^2 C - \sin^2 C) = \cos C + \sin C$

$\Rightarrow \cos C - \sin C = \frac{1}{2}$

$\Rightarrow 1 - \sin 2C = \frac{1}{4} \Rightarrow \sin 2C = \frac{3}{4}$

(14) Area of $\Delta ABC = \frac{AB.BC.CA}{4R}$

$= \frac{8 \sin A \sin B \sin C}{4}$

$= 2 \sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) \sin C$

$= 2 \cos 2C \cos C \sin C$

$= \cos 2C \cdot \sin 2C$

Area a $= \sqrt{1 - \frac{9}{16}} \cdot \frac{3}{4} = \frac{3\sqrt{7}}{16}$

$\Rightarrow 64a = 12\sqrt{7} \Rightarrow (64a)^2 = 1008$

(15) Inradius $r = \frac{\Delta}{s} = \frac{a}{2R \sin A + 2R \sin B + 2R \sin C}$

$= \frac{a}{(\sin A + \sin B + \sin C)} = \frac{a}{\sin\left(\frac{\pi}{2} - 2C\right) + \sin\left(\frac{\pi}{2} + C\right) + \sin C}$

$= \frac{a}{\cos 2C + \cos C + \sin C} = \frac{a}{\cos 2C + \sqrt{1 + \sin 2C}}$

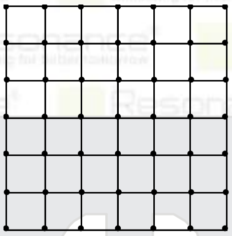
$= \frac{\frac{3\sqrt{7}}{16}}{\sqrt{1 - \frac{9}{16}} + \sqrt{1 + \frac{3}{4}}} = \frac{\frac{3\sqrt{7}}{16}}{\frac{\sqrt{7}}{4} + \frac{\sqrt{7}}{2}} = \frac{3\sqrt{7}}{16} \times \frac{4}{3\sqrt{7}} = \frac{1}{4}$

$r = 0.25$

PARAGRAPH 'II'

Consider the 6 × 6 square in the figure. Let $A_1, A_2.....A_{49}$ be the points in intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.

There are two questions based on PARAGRAPH 'II', the question given below is one of them



16. Let p_i be the probability that a randomly chosen point has i many friends, i = 0,1,2,3,4. Let X be a random variable such that for i = 0,1,2,3,4, the probability $P(X = i) = p_i$. Then the value of 7E(X) is

Ans. **(24)**

Sol. $P(x = 0) = 0$ $\qquad$ $P(x = 1) = 0$

$P(x = 2) = \frac{4}{49}$ $\qquad$ $P(x = 3) = \frac{20}{49}$

$P(x = 4) = 1 - \frac{24}{49} = \frac{25}{49}$

$$E(X) = \sum_{i=0}^{4} i.\,P(x = i) = 2.\frac{4}{49} + 3.\frac{20}{49} + 4.\frac{25}{49}$$

$$= \frac{8+60+100}{49} = \frac{168}{49} = \frac{24}{7}$$

$7E(X) = 24$

17. Two distinct points are chosen randomly out of the points $A_1, A_2.....A_{49}$. Let p be the probability that they are friends. Then the value of 7p is

Ans. **(00.50)**

Sol. $$p = \frac{6\times7+6\times7}{^{49}C_2} = \frac{2\times6\times7\times2}{49\times48} = \frac{1}{14}$$

$7p = \frac{1}{2} = 0.5 = 00.50$