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JEE (ADVANCED) 2023

QUESTIONS & TEXT SOLUTION

PAPER-1

DATE & DAY: 4th JUNE 2023, SUNDAY

PAPER-1	PAPER-2
Duration: 3 Hrs.	**Duration:** 3 Hrs.
Time: 09:00 - 12:00 IST	**Time:** 14:30 - 17:30 IST

SUBJECT: MATHEMATICS

REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):
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TARGET: JEE (Adv.) 2024

VIJAY COURSE

For 12th Passed Students

AIR 6
JEE (Adv.) 2022
KARTHIKEYA P.

Course Features*

- Course Duration: **32 Weeks**
- Total No. of Lectures: **533** (P: 178 | C: 177 | M: 178)
- Duration of One Lecture: **1.5 Hrs.** (90 Minutes)
- Classroom Teaching Hours.: **800 Hrs.**
- Testing Duration: **60 Hrs.**
- Total Academic Hours.: **860 Hrs.**

CLASS STARTS
5th & 19th June

SCHOLARSHIP UPTO 100%
Based on JEE (Advanced) 2023 Score, Scholarship Test (ResoNET) & 12th Board

TARGET: JEE (Main) 2024

AJAY COURSE

For 12th Passed Students

AIR 5
JEE (Main) 2023
KAUSHAL V.

Course Features*

- Course Duration: **33 Weeks**
- Total No. of Lectures: **571** (P:1 84 | C: 203 | M: 184)
- Duration of One Lecture: **1.5 Hrs.** (90 Minutes)
- Classroom Teaching Hours.: **857 Hrs.**
- Testing Duration: **33 Hrs.**
- Total Academic Hours.: **890 Hrs.**

CLASS STARTS
5th & 19th June

SCHOLARSHIP UPTO 100%
Based on JEE (Main) 2023 Score, Scholarship Test (ResoNET) & 12th Board

PART : MATHEMATICS

SECTION 1 : 12 Marks

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4 ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : **−2** In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

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1 Let $S = (0,1) \cup (1,2) \cup (3,4)$ and $T = \{0,1,2,3\}$. Then which of the following statements is (are) true?

(A) There are infinitely many functions from S to T

(B) There are infinitely many strictly increasing functions from S to T

(C) The number of continuous functions from S to T is at most 120

(D) Every continuous function from S to T is differentiable

Ans. **(ACD)**

Sol. Here Domain S contains infinite many elements while co-domain T contains only four (finite) elements so there is no strictly increasing function from S to T and there are infinitely many functions from S to T.

Now for continuous function, each interval either (0, 1) or (1, 2) or (1, 3) attend exactly one element from {0, 1, 2, 3} as image so all continuous functions are also differentiable in its domain and number of continuous functions are equal to $4 \times 4 \times 4 = 64$

2 Let T_1 and T_2 be two distinct common tangents to the ellipse $E : \frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola P: $y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2 respectively and the tangent T_2, touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is (are) true?

(A) The area of the quadrilateral $A_1 A_2 A_3 A_4$ is 35 square units

(B) The area of the quadrilateral $A_1 A_2 A_3 A_4$ is 36 square units

(C) The tangents T_1 and T_2 meet the x-axis at the point (–3, 0)

(D) The tangents T_1 and T_2 meet the x-axis at the point (–6, 0)

Ans. (AC)

Sol. $P \equiv y^2 = 12x$ .....(1)

$E \equiv \frac{x^2}{6} + \frac{y^2}{3} = 1$ .....(2)

$\because\ y = mx + \frac{3}{m}$ .....(3) be any tangent of P

For common tangent

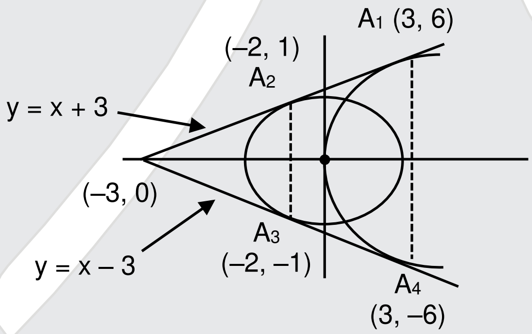
$c^2 = a^2m^2 + b^2$

$\Rightarrow \frac{9}{m^2} = 6 \times m^2 + 3$ $\Rightarrow \frac{3}{m^2} = 2m^2 + 1$

$\Rightarrow 2m^4 + m^2 - 3 = 0$

$2m^4 + 3m^2 - 2m^2 - 3 = 0$

$2m^2 (m^2 - 1) + 3(m^2 - 1) = 0$



$\Rightarrow m = \pm 1$

$\therefore$ Common tangents are : $y = x + 3$ and

$y = -x - 3$

Tangents intersect each other at (–3, 0)

$\because A_1\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv A_1(3, 6)$

$\therefore A_4$ (3, –6)

Finding A_2 :

$\frac{x^2}{6} + \frac{y^2}{3} = 1$

$y = x + 3$ $\Rightarrow$ $x - y + 3 = 0$

$\frac{xx_1}{6} + \frac{yy_1}{3} = 1$

$$\frac{\frac{x_1}{6}}{1} = \frac{\frac{y_1}{3}}{-1} = \frac{1}{-3}$$

$$\frac{x_1}{6} = \frac{y_1}{-3} = \frac{1}{-3}$$

$\therefore$ $x_1 = -2$; $y_1 = 1$

$\therefore$ A_2 (–2, 1)

$\therefore$ A_3 (–2, –1)

$\therefore$ Area of quadrilateral $= \frac{1}{2}(2+12)\times 5$

$= 7 \times 5 = 35$

3. Let f: [0, 1] → [0, 1] be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region S = [0,1] × [0,1].Let G = {(x,y)∈S : y > f(x)} be called the green region and R= {(x, y) ∈ S : y < f(x)} be called the red region. Let L_h ={(x, h) ∈S : x ∈ [0, 1]} be the horizontal line drawn at a height h∈ [0, 1] . Then which of the following statements is(are) true ?

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

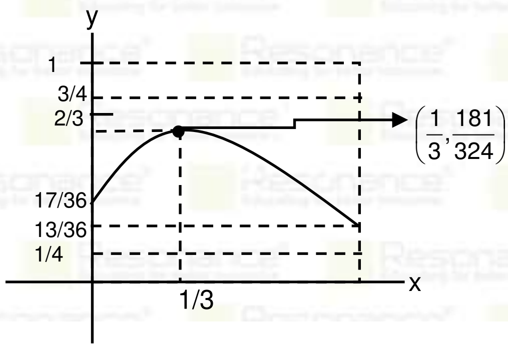
(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

Ans. **(BCD)**

Sol. $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$



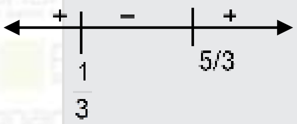
$f'(x) = x^2 - 2x + \frac{5}{9}$

$= (x-1)^2 - 1 + \frac{5}{9}$

$(x-1)^2 - \left(\frac{2}{3}\right)^2$

$= \left(x - 1 - \frac{2}{3}\right)\left(x - 1 + \frac{2}{3}\right)$

$= \left(x - \frac{5}{3}\right)\left(x - \frac{1}{3}\right)$



$f\left(\frac{1}{3}\right) = \frac{1}{81} - \frac{1}{9} + \frac{5}{27} + \frac{17}{36}$

$= \frac{4 - 36 + 60 + 153}{324} = \frac{181}{324}$

$R = \int_0^1 \left(\frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}\right)dx = \frac{1}{12} - \frac{1}{3} + \frac{5}{18} + \frac{17}{36}$

$\frac{3 - 12 + 10 + 17}{36} = \frac{1}{2} = G = R$

Let line is y = h

for option A

$\therefore (1-h) \times 1 = \frac{1}{4}$

$1 - \frac{1}{4} = h = \frac{3}{4}$

$h = \frac{3}{4}$ **option A is INCORRECT**

for option B

$h \times 1 = \frac{1}{4}$

$h = \frac{1}{4}$

option (B) is correct

for option (C) & (D)

let $A(h) = R_h - G_h$, where R_h is area of red region above the line L_h, G_h is area of the green region below the line L_h.

At $h=\frac{13}{36}$, $G_h=0 \Rightarrow A\left(\frac{13}{36}\right)=R_h-0>0$

At $h=\frac{181}{324}$, $R_h=0 \Rightarrow A\left(\frac{181}{324}\right)=0-G_h<0$

so by intermediate value property there exists h = h_1 where $A(h_1)$ = 0 and $h_1 \in \left(\frac{13}{36},\frac{181}{324}\right)$

as $\frac{9}{36}=\frac{1}{4}<\frac{13}{36}$ and $\frac{181}{324}<\frac{216}{324}=\frac{2}{3}$

Option (C) & (D) are correct

SECTION-2 : 12 Marks

- This section contains **FOUR (04)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct option is chosen;
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : **−1** In all other cases.

4. Let f : (0,1) → R be the function defined as $f(x)=\sqrt{n}$ if $x\in\left[\frac{1}{n+1},\frac{1}{n}\right)$ where n∈N. Let g: (0,1)→ R be a function such that $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt<g(x)<2\sqrt{x}$ for all x∈(0, 1). Then $\lim_{x\to 0} f(x)g(x)$

(A) does NOT exist (B) is equal to 1 (C) is equal to 2 (D) is equal to 3

Ans. **(C)**

Sol. $f(x)=\sqrt{n}, x\in\left[\frac{1}{n+1},\frac{1}{n}\right)$

and $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}<g(x)<2\sqrt{x}$

$\Rightarrow$ $\lim_{x\to 0} g(x)=0$

also $\lim_{x\to 0} f(x)=\lim_{n\to\infty} f(x)=\infty$

$\lim_{x\to 0} f(x).\,g(x)$ $\quad 0\times\infty$

since $\frac{1}{n+1}\le x<\frac{1}{n}$ so $n+1\ge\frac{1}{x}>n$

$\sqrt{\frac{1-x}{x}}<f(x)<\frac{1}{\sqrt{x}}$

$\lim_{x\to 0^+}\left(\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt\right)\sqrt{\frac{1-x}{x}}<\lim_{x\to 0^+} f(x)g(x)<\lim_{x\to 0^+} 2\sqrt{x}\frac{1}{\sqrt{x}}$ =2 consider $\lim_{x\to 0^+}\left(\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt\right)\sqrt{\frac{1-x}{x}} = \lim_{x\to 0^+}\frac{\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt}{\sqrt{\frac{x}{1-x}}}$

$$\lim_{x\to 0^+}\frac{1.\sqrt{\frac{1-x}{x}}-2x.\sqrt{\frac{1-x^2}{x^2}}}{\frac{1}{2}\sqrt{\frac{1-x}{x}}.\left(\frac{1.(1-x)-x(-1)}{(1-x)^2}\right)}$$

$$\lim_{x\to 0^+}\frac{2-4x.\sqrt{\frac{1+x}{x}}}{\left(\frac{1}{(1-x)^2}\right)}=2$$

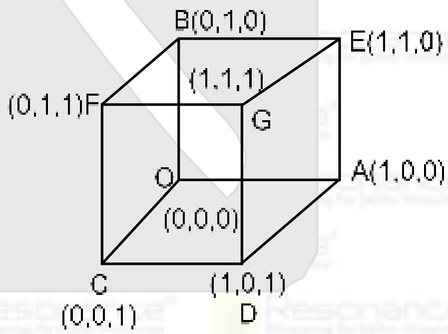
$$\lim_{x\to 0^+} f(x)g(x)=2$$

5 Let Q be the cube with the set of vertices $\{(x_1,x_2,x_3)\in R^3 : x_1,x_2,x_3 \in \{0,1\}\}$ .Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0,0,0) and (1,1,1) is in S. For lines ℓ_1 and ℓ_2 let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

Ans. **(A)**

Sol.



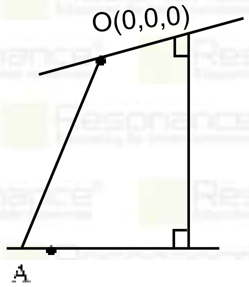
DR'S of OG : 1,1,1
DR'S of AF : –1,1,1
DR'S of CE : 1,1, –1
DR'S of BD : 1, –1,1

equation of line OG : $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

equation of AB : $\frac{x-1}{1}=\frac{y}{-1}=\frac{z}{0}$

Normal to both the line's = $\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$

$= i + j - 2\hat{k}$



$\overrightarrow{OA} = \hat{i}$

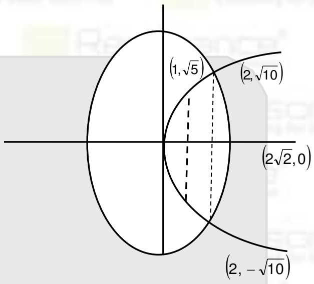
S.D = $\frac{|\,i,(\hat{i}+\hat{j}-2\hat{k})\,|}{|\,i+j-2\hat{k}\,|}=\frac{1}{\sqrt{6}}$

6. Let $X = \left\{(x,y) \in Z \times Z : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

Ans. **(B)**

Sol. $\frac{x^2}{8} + \frac{y^2}{20} < 1$ and $y^2 < 5x$



Solving corresponding equation

$\frac{x^2}{8} + \frac{x}{4} = 1 \Rightarrow x^2 + 2x = 8$

$\Rightarrow x = 2, -4$

x = {(1,1), (1, 0), (1, –1), (1, 2), (1, –2), (2, 3), (2, 2), (2, 1), (2, 0), (2, –1), (2, –2) (2, –3)}

n(s) = $^{12}C_3$

E = Event of selecting three points in which two points are either on x = 1 or x = 2 but distance between them is even.

n(E) = 4 × 7 + 9 × 5 = 28 + 45 = 73

P(E) = $\frac{73}{^{12}C_3} = \frac{73}{220}$

7. Let P be a point on the parabola $y^2 = 4ax$, where a >0. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a,m) is

(A) (2,3) (B) (1,3) (C) (2,4) (D) (3,4)

Ans. **(A)**

Sol. 2yy' = 4a

Slope of normal to the parabola at point P $(at^2, 2at)$ is

$m = -\frac{2at \times 2}{4a}$

m = –t

Now equation of normal at $P(at^2, 2at)$ is

$y - 2at = -t(x - at^2)$

so $Q(2a + at^2, 0)$

Area of Δ PFQ= $\frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ 2a+at^2 & 0 & 1 \\ a & 0 & 1 \end{vmatrix} = 120$

$\left|\frac{2at}{2}\left(a+at^2\right)\right| = 120$

$|a^2t(1 + t^2)| = 120$

$|2^2 \times 3(1 + 3^2)| = 120$

So m = 3, a = 2

SECTION-3 : 24 Marks

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4 ONLY** the correct integer value is entered;

Zero Marks : **0** In all other cases.

8. Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, for $x \in R$. Then the number of real solutions of the equation

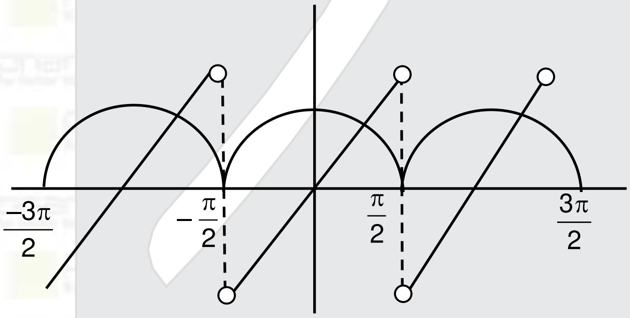
$\sqrt{1+\cos(2x)} = \sqrt{2}\ \tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right)\cup\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ is equal to

Ans. **(3)**

Sol. $\because \sqrt{1+\cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$

$\Rightarrow \sqrt{2}\,|\cos x| = \sqrt{2}\tan^{-}1(\tan x)$

$\Rightarrow |\cos x| = \tan^{-1}(\tan x)$



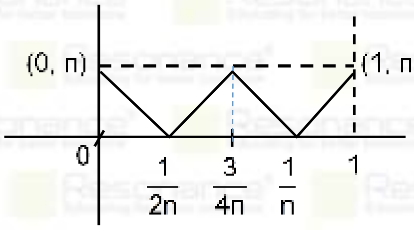
9. Let $n \geq 2$ be a natural number and $f: [0, 1] \to R$ be function defined by

$$f(x)=\begin{cases} n(1-2nx) & \text{if } 0 \le x \le \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \le x \le \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \le x \le \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \le x \le 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function f is

Ans. **(8)**

Sol.



$$4 = \frac{1}{2}.n.\frac{1}{2n} + \frac{1}{2}.\frac{1}{2n}.n + \frac{1}{2}\left(1 - \frac{1}{n}\right).n$$

n = 8

Maximum value of f(x) = n

$\Rightarrow (f(x))_{max} = 8$

10. Let $7\overbrace{5.....5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + ….. + $7\overbrace{5.....5}^{98}7$. If $S = \frac{7\overbrace{5.....5}^{99}7 + m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

Ans. **(1219)**

Sol. LHS = $7(10 + 10^2 + 10^3 + ... + 10^{99}) + 50(1 + 11 + 111 + ... + \underbrace{111.....11}_{98}) + 99 \times 7$

$$= \frac{7 \times 10^{100}}{9} - \frac{70}{9} + \frac{50}{9}\left[(10-1) + (10^2 - 1) + ... + (10^{98} - 1)\right] + 99 \times 7$$

$$= \frac{7 \times 10^{100}}{9} + \frac{50}{9}\left[\frac{10^{99} - 10}{9} - 98\right] + 99 \times 7 - \frac{70}{9}$$

$$= \frac{7 \times 10^{100}}{9} + \frac{50}{9}\left[\frac{10^{99} - 1}{9} - 99\right] + 693 - \frac{70}{9}$$

$$= \frac{7\overbrace{55.....5}^{99}7}{9} + 693 - \frac{5020}{9} - \frac{7}{9}$$

$$= \frac{7\overbrace{55.....5}^{99}7 + 1210}{9}$$

m + n = 1210 + 9 = 1219

11. Let $A = \left\{\dfrac{1967+1686\,i\sin\theta}{7-3i\cos\theta} : \theta \in R\right\}$. If A contains exactly one positive integer n, then the value of n is

Ans. **(281)**

Sol. $\dfrac{1967+1686\sin\theta i}{7-3\cos\theta i} = \dfrac{(1967+1686\sin\theta i)(7+3\cos\theta i)}{49+9\cos^2\theta}$ = integer ....(i)

$1967 \times 3\cos\theta + 1686\sin\theta \times 7 = 0$

$\Rightarrow 2\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = -\dfrac{1}{2}$

$$I = \frac{1967\times7-1686\sin\theta\times3\cos\theta}{49+9\cos^2\theta}$$

$$I = \frac{1967\times7+1686\times6\sin^2\theta}{49+36\sin^2\theta}$$

Put $\sin^2\theta = \dfrac{1}{5}$

$$\Rightarrow I = \frac{1967\times7+1686\times6\times\frac{1}{5}}{49+36\times\frac{1}{5}} = 281$$

12. Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let $S = \{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k} : \alpha^2+\beta^2+\gamma^2 = 1$ and the distance of (α, β, γ) from the plane P is $\dfrac{7}{2}\}$. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\dfrac{80}{\sqrt{3}}V$ is

Ans. **(45)**

Sol. $\sqrt{3}x + 2y + 3z = 16$

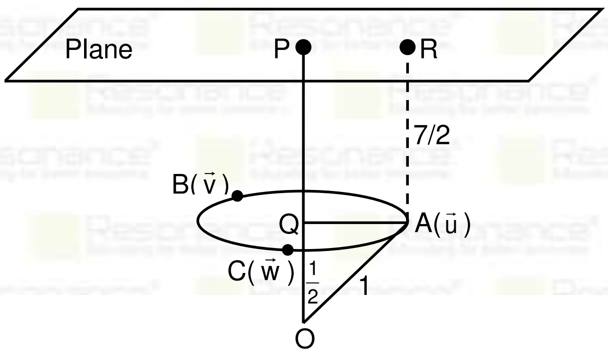
Distance of point from plane = $\dfrac{7}{2}$

$|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$

$\Rightarrow |\vec{u}-\vec{v}|^2 = |\vec{v}-\vec{w}|^2 = |\vec{w}-\vec{u}|^2$

$\because |\vec{v}| = |\vec{u}| = |\vec{w}| = 1$ (given)

$\Rightarrow \vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{u} = \lambda$ (say)



OP = 4

OQ = $4 - \dfrac{7}{2} = \dfrac{1}{2}$

$\Rightarrow AQ = \dfrac{\sqrt{3}}{2}$

$\because$ Point $A(\vec{u})$, $B(\vec{v})$, $C(\vec{c})$ are vertices of an equilateral triangle ΔABC

$\because$ $|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$ and

OA = OB = OC = 1 (given)

In ΔABC, G = O = I = H

$\Rightarrow$ In ΔABC, Q is circumcentre

So $\cos 120° = \dfrac{(QA)^2+(QB)^2-AB^2}{2(QA)(QB)}$

$\Rightarrow AB = \dfrac{3}{2} = |\vec{u}-\vec{v}|$

$\Rightarrow 1 + 1 - 2\vec{u}.\vec{v} = \dfrac{9}{4} \Rightarrow \lambda = -\dfrac{1}{8}$

Volume = $|[\vec{u}\ \vec{v}\ \vec{w}]|$

$\because [\vec{u}\ \vec{v}\ \vec{w}]^2 = \begin{vmatrix} 1 & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{u}\cdot\vec{v} & 1 & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & 1 \end{vmatrix} = (1-\lambda^2) - \lambda(\lambda-\lambda^2) + \lambda(\lambda^2-\lambda)$

$\Rightarrow [\vec{u}\ \vec{v}\ \vec{w}]^2 = (1-\lambda)^2(2\lambda+1)$

$= \dfrac{81}{64}\times\dfrac{3}{4}$

$\Rightarrow$ Volume = $|[\vec{u}\ \vec{v}\ \vec{w}]| = \dfrac{9\sqrt{3}}{16}$

So, $\dfrac{80V}{\sqrt{3}} = \dfrac{80}{\sqrt{3}}\cdot\dfrac{9}{8}\cdot\dfrac{\sqrt{3}}{2} = 45$

13. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2+\dfrac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax-\dfrac{1}{bx^2}\right)^7$, then the value of 2b is

Ans. **(3)**

Sol. $\because \left(ax^2+\dfrac{70}{27bx}\right)^4$

$\because T_{r+1} = {}^4C_r(ax^2)^{4-r}\left(\dfrac{70}{27bx}\right)^r$ ....(1)

for coefficient of x^5 put $8 - 2r - r = 5 \Rightarrow r = 1$

$\because$ coefficient x^5 in $\left(ax^2+\dfrac{70}{27bx}\right)^4$ is $= {}^4C_1.\dfrac{a^3.(70)^1}{27.b}$

$\because$ General term of $\left(ax-\dfrac{1}{bx^2}\right)^7$ is

$T_{r+1} = {}^7C_r\,(ax)^{7-r}\left(-\dfrac{1}{bx^2}\right)^r$

for coefficient of x^{-5} put $7 - r - 2r = -5 \Rightarrow r = 4$

$\therefore$ coefficient x^{-5} in $\left(ax - \frac{1}{bx^2}\right)^7$ is $= {}^7C_4 \frac{a^3(-1)^4}{b^4}$

Now $\because$ ${}^4C_1 \frac{a^3(70)}{27(b)} = {}^7C_4 \frac{a^3}{b^4}$

$\frac{4 \times 70}{27(b)} = \frac{35}{b^4} \Rightarrow b^3 = \frac{27}{8} \Rightarrow b = \frac{3}{2}$

$\therefore$ 2b = 3

SECTION 4 : 12 Marks

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3 ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : **–1** In all other cases.

14. Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II**

List-I	**List-II**
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and γ = 28, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where α = 1 and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where α = 1 and γ = 28, then the system has	(4) x = 11, y = –2 and z = 0 as a solution
	(5) x = –15, y = 4 and z = 0 as a solution

The correct option is :

(A) (P) →(3) (Q) →(2) (R) →(1) (S) →(4)
(B) (P) →(3) (Q) →(2) (R) →(5) (S) →(4)
(C) (P) →(2) (Q) →(1) (R) →(4) (S) →(5)
(D) (P) →(2) (Q) →(1) (R) →(1) (S) →(3)

Ans. (A)

Sol. x + 2y + z = 7

x + αz = 11

2x – 3y + βz = γ

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 1\,(0 + 3\alpha) + 2\,(2\alpha - \beta) + 1(-3-0)$$

= 3α + 4α –2β –3

= 7α –2β –3

$$\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix} = 7(0 + 3\alpha) + 2(\alpha\gamma - 11\beta) + 1(-33 - 0)$$

= 21α + 2αγ –22β –33

$$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix} = 1(11\beta - \alpha\gamma) + 7(2\alpha - \beta) + 1(\gamma - 22)$$

= 11β – αγ + 14α –7β + γ –22

= 4β – αγ + 14α + γ –22

$$\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 1\,(0+33) + 2(22-\gamma) + 7(-3-0)$$

= 33 + 44 – 2γ –21

= 56 –2γ

For unique solution

$\Delta \neq 0 \Rightarrow 7\alpha - 2\beta - 3 \neq 0 \Rightarrow \beta \neq \frac{1}{2}(7\alpha-3)$

For Infinitely many solutions $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$\Delta = 0 \Rightarrow \beta = \frac{1}{2}(7\alpha-3)$

and $\Delta_z = 0 \Rightarrow \gamma = 28$

$\Delta_x = 21\alpha + 56\alpha - 22\beta - 33$

$= 11(7\alpha - 2\beta - 3) = 0$

$\Delta_y = 4\beta - 28\alpha + 14\alpha + 28 - 22$

$= 4\beta - 14\alpha + 6 = 2(2\beta - 7\alpha + 3) = 0$

For no solutions

$\Delta = 0 \Rightarrow \beta = \frac{1}{2}(7\alpha-3)$ and at least one of $\Delta_x, \Delta_y, \Delta_z$ is non zero $\Rightarrow \Delta_z \neq 0 \Rightarrow \gamma \neq 28$.

If $\beta \neq \frac{1}{2}(7\alpha-3) \Rightarrow \Delta \neq 0$ and $\gamma = 28$

$x = \frac{\Delta_x}{\Delta} = 11$, $y = \frac{\Delta_y}{\Delta} = -2$ $z = 0$

If $\beta \neq \frac{1}{2}(7\alpha-3) \Rightarrow \Delta \neq 0, \gamma \neq 28 \Rightarrow \Delta_z \neq 0, z \neq 0. \Rightarrow$ system has a unique solution

15. Consider the given data with frequency distribution

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

The correct option is:

(A) (P) →(3) (Q) →(2) (R) →(4) (S) →(5)

(B) (P) →(3) (Q) →(2) (R) →(1) (S) →(5)

(C) (P) →(2) (Q) →(3) (R) →(4) (S) →(1)

(D) (P) →(3) (Q) →(3) (R) →(5) (S) →(5)

Ans. **(A)**

Sol.

x_i	f_i	$f_i x_i$	C.f	$\lvert x_i - m\rvert$	$f_i \lvert x_i - m\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
3	5	15	5	2	10	15
4	4	16	9	1	4	8
5	4	20	13	0	0	4
8	2	16	15	3	6	4
10	2	20	17	5	10	8
11	3	33	20	6	18	15
	$\sum f_i = 20$	$\sum f_i x_i = 120$			$\sum f_i \lvert x_i - m\rvert = 48$	$\sum f_i \lvert x_i - \bar{x}\rvert = 54$

$$\because \bar{x}(\text{mean}) = \frac{\sum x_i f_i}{\sum f_i} = \frac{120}{20} = 6$$

$$\because \text{median} = \left(\frac{20}{2}\right)^{th} = 10^{th}\,\text{observation}$$

Median = 5

$$\text{Mean deviation about median} = \frac{\sum f_i \,|\, x_i - m\,|}{\sum f_i}$$

$$= \frac{48}{20} = 2.4$$

$$\text{Mean deviation about mean} = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i} = \frac{54}{20} = 2.7$$

Mean = 6

Median = 5

mean deviation about mean = 2.7

mean deviation about median = 2.4

16. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = \lambda(\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct option is:

(A) (P) →(2) (Q) →(4) (R) →(5) (S) →(1)
(B) (P) →(5) (Q) →(4) (R) →(3) (S) →(1)
(C) (P) →(2) (Q) →(1) (R) →(3) (S) →(2)
(D) (P) →(5) (Q) →(1) (R) →(4) (S) →(2)

Ans. **(B)**

Sol. Plane containing ℓ_1 and parallel to ℓ_2 is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow x - z = 0$

(P) $d(H_0)$ = distance of point (0, 1, –1) from x – z = 0 is = $\frac{1}{\sqrt{2}}$

(Q) Distance of point (0,1,2) from H_0 is = $\left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) The distance of origin from H_0 is = $\left|\frac{0-0}{\sqrt{2}}\right| = 0$

(S) Point of intersection of y = z, x = 1 and x –z = 0 is (1,1,1). Its distance from (0, 0,0) = $\sqrt{3}$

17. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non - zero.

Match each entry in List -I to the correct entries in List -II

	List – I		**List –II**
(P)	$\|z\|^2$ is equal to	(i)	12
(Q)	$\|z-\bar{z}\|^2$ is equal to	(ii)	4
(R)	$\|z\|^2 + \|z+\bar{z}\|^2$ is equal to	(iii)	8
(S)	$\|z+1\|^2$ is equal to	(iv)	10
		(v)	7

The correct option is:

(A) (P) →(1) (Q) →(3) (R) →(5) (S) →(4)

(B) (P) →(2) (Q) →(1) (R) →(3) (S) →(5)

(C) (P) →(2) (Q) →(4) (R) →(5) (S) →(1)

(D) (P) →(2) (Q) →(3) (R) →(5) (S) →(4)

Ans. **(B)**

Sol. Let $Z = r(\cos\theta + i\sin\theta)$

$r^3 + 2r^2e^{2i\theta} + 4re^{-i\theta} - 8 = 0$

Comparing real and imaginary part

$r^3 + 2r^2\cos2\theta + 4r\cos\theta = 8$ .....(i)

$4r^2\sin\theta\cos\theta = 4r\sin\theta$ ....(ii)

$\Rightarrow r\cos\theta = 1$ put in (i)

$\Rightarrow \cos2\theta = 2\cos^2\theta - 1 = \frac{2-r^2}{r^2}$

$\Rightarrow r^3 + 2(2 - r^2) + 4 = 8 \Rightarrow r = 2$

(P) $|Z|^2 = 4$

(Q) $|Z - \bar{Z}|^2 = |2ir\sin\theta|^2 = 4r^2(1 - \cos^2\theta) = 16 - 4 = 12$

(R) $|Z|^2 + |Z + \bar{Z}|^2 = 4 + 4 = 8$

(S) $|Z + 1|^2 = (Z + 1)(\bar{Z} + 1) = Z\bar{Z} + Z + \bar{Z} + 1 = 4 + 2 + 1 = 7$