

Questions & Solutions

PAPER – 2

SUBJECT: MATHEMATICS

MAX. MARKS: 186

TIME: 3 HRS.

PAPER-2 : INSTRUCTIONS TO CANDIDATES

- Question paper-2 has three (03) parts : Physics, Chemistry and Mathematics.
- Each part has a total of eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).
- Total number of questions in Question Paper-2 are : Fifty Four (54) and Maximum Marks are One Hundred Eighty Six (186).

Type of Questions and Marking Schemes

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen.
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen and both of which are correct.
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : -1 In all other cases.

SECTION 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : +3 If **ONLY** the correct numerical value is entered.
Zero Marks : 0 In all other cases.

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SECTION 3 (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (I),(II), (III) and (IV) **List-II** has **Six** entries (P),(Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : **+3** If **ONLY** the option corresponding to the correct combination is chosen.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **-1** In all other cases.

Answering Questions :

- To select the option(s), use the mouse to click on the corresponding button(s) of the option(s).
- To deselect the chosen option(s) for the questions of **SECTION-1** click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.
- To deselect the chosen option for the questions of **SECTION-3**, click on the button of the chosen option again or click on the **Clear Response** button to clear the chosen option.
- To change the option(s) of a previously answered question of **SECTION-1** and **SECTION-3** first deselect as given above and then select the new option(s)
- To answer questions of **SECTION-2** use the mouse to click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value in the space provided for answer.
- To change the answer of a question of **SECTION-2** first click on the **Clear Response** button to clear the correct answer and then enter the new numerical value.
- To mark a question **ONLY** for review (i.e. without answering it). Click on the **Mark for Review & next** button.
- To mark is question for review (after answering it), click on **Mark for Review & Next** button - the answered question which is also marked for review will be evaluated.
- To save the answer click on the **Save & Next** button, the answered question will be evaluated.

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Classroom student since class 11th
Reso Roll No.: 17174485

AIR-77
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AIR-96
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Reso Roll No.: 14400705

AIR-82
Saptarshi Dasgupta
Classroom student since class 11th
Reso Roll No.: 17107807

AIR-45
Atreya Goswami
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GAMBHIR**
Classroom student since class 10th
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*Based on the information collected from
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AIR-11 (SC)

Anshul Navphule

Students qualified for
JEE Advanced 2019

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students from Repeaters' Batches

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students from Freshers' Batches

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PART-III : MATHEMATICS

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
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Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option.

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Negative Marks : -1 In all other cases.

खंड 1 (अधिकतम अंक: 32)

- इस खंड में **आठ (08)** प्रश्न हैं।
- प्रत्येक प्रश्न के लिए चार विकल्प दिए गए हैं। इन चार विकल्पों में से **एक या एक से अधिक** विकल्प सही हैं(हैं)।
- प्रत्येक प्रश्न के लिए, दिए हुए विकल्पों में से सही उत्तर (उत्तरों) से संबंधित विकल्प (विकल्पों) को चुनिए।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्न योजना के अनुसार होगा :
 - पूर्ण अंक : +4 यदि केवल (सारे) सही विकल्प (विकल्पों) को चुना गया है।
 - आंशिक अंक : +3 यदि चारों विकल्प सही हैं परन्तु केवल तीन विकल्पों को चुना गया है।
 - आंशिक अंक : +2 यदि तीन या तीन से अधिक विकल्प सही हैं परन्तु केवल दो विकल्पों को चुना गया है और दोनों चुने हुए विकल्प सही विकल्प हैं।
 - आंशिक अंक : +1 यदि दो या दो से अधिक विकल्प सही हैं परन्तु केवल एक विकल्प को चुना गया है और चुना हुआ विकल्प सही विकल्प है।
 - शून्य अंक : 0 यदि किसी भी विकल्प को नहीं चुना गया है (अर्थात् प्रश्न अनुत्तरित है)।
 - ऋण अंक : -1 अन्य सभी परिस्थितियों में।

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (x - 1)(x - 2)(x - 5)$. Define $F(x) = \int_0^x f(t) dt, x > 0$. Then which of the

following options is/are correct ?

- (1) $F(x)$ has a local maximum at $x = 2$
- (2) $F(x)$ has a local minimum at $x = 1$
- (3) $F(x)$ has two local maxima and one local minimum in $(0, \infty)$
- (4) $F(x) \neq 0$, for all $x \in (0, 5)$

माना कि $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = (x - 1)(x - 2)(x - 5)$ द्वारा दिया गया है। परिभाषित करें $F(x) = \int_0^x f(t) dt, x > 0$.

तब निम्न में से कौन सा (से) विकल्प सही है(हैं) ?

- (1) F का एक स्थानीय उच्चतम (local maximum) $x = 2$ पर है।
- (2) F का एक स्थानीय निम्नतम (local minimum) $x = 1$ पर है।
- (3) F के दो स्थानीय उच्चतम और एक स्थानीय निम्नतम, $(0, \infty)$ में हैं।
- (4) सभी $x \in (0, 5)$ के लिए $F(x) \neq 0$ है।





Ans. (1,2,4)

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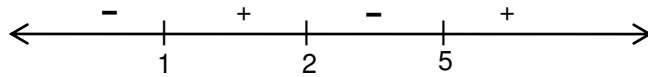
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Sol. $f(x) = (x-1)(x-2)(x-5)$

$F(x) = \int_0^x f(t) dt$ as $F(1)$ is negative and $F(2)$ is also negative so $F(x)$ cannot be zero for $x \in (0, 5)$. So

option (A) is correct.

$\therefore F'(x) = f(x) = (x-1)(x-2)(x-5)$



$\Rightarrow F(x)$ has two local minima points at $x = 1$ and $x = 5$

$F(x)$ has one local maxima point at $x = 2$

2. Three lines

$L_1: \vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$

$L_2: \vec{r} = \hat{k} + \mu \hat{j}, \mu \in \mathbb{R}$ and

$L_3: \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \nu \in \mathbb{R}.$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear.

तीन रेखाएँ

$L_1: \vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$

$L_2: \vec{r} = \hat{k} + \mu \hat{j}, \mu \in \mathbb{R}$ तथा

$L_3: \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \nu \in \mathbb{R}.$

दी गयी है। L_2 के किस बिन्दु (किन बिन्दुओं) Q के लिए हम L_1 पर एक बिन्दु P, और L_3 पर एक बिन्दु R प्राप्त कर सकते हैं। ताकि P, Q और R सरेख (collinear) हो जाएँ ?

(1) $\hat{k} + \frac{1}{2} \hat{j}$

(2) $\hat{k} + \hat{j}$

(3) \hat{k}

(4) $\hat{k} - \frac{1}{2} \hat{j}$

Ans. (1,4)

Sol. $P(\lambda, 0, 0)$

$Q(0, \mu, 1)$

$R(1, 1, \gamma)$

$\vec{PQ} = k\vec{PR} \Rightarrow \frac{\lambda}{\lambda-1} = \frac{-\mu}{-1} = \frac{-1}{-\gamma}$

$\Rightarrow 1 + \frac{1}{\lambda-1} = \mu = \frac{1}{\gamma} \Rightarrow \mu$ cannot take value of 1 and 0

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3. Let $x \in \mathbb{R}$ and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}.$$

Then which of the following is/are correct

(1) there exists a real number x such that $PQ = QP$

(2) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$ for all $x \in \mathbb{R}$

(3) For $x = 1$ there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which are $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(4) For $x = 0$ if $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ then $a + b = 5$

माना कि $x \in \mathbb{R}$ और माना कि

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ तथा } R = PQP^{-1} \text{ तो निम्न में से कौन सा (से) विकल्प सही है (हैं)–}$$

(1) एक ऐसी वास्तविक संख्या x सम्भव है जिसके लिए $PQ = QP$

(2) सभी $x \in \mathbb{R}$ के लिए $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$

(3) $x = 1$ के लिए, एक इकाई सदिश $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ सम्भव है, जिसके लिए $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(4) $x = 0$ के लिए यदि $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, तो $a + b = 5$

Ans. (2, 4)

Sol.

$$\det R = \det P \times \det Q \times \det (P^{-1})$$

$$\Rightarrow \det R = \det Q = 4(12 - x^2) = 48 - 4x^2$$

$$\text{Now } \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} = 4(10 - x^2) = 40 - 4x^2$$

$$\Rightarrow \det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8 \quad \forall x \in \mathbb{R}$$

$$\text{At } x = 1, \det Q = 48 - 4 = 44 = \det R$$





Because $\det R \neq 0$ so

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$$R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \alpha = \beta = \gamma = 0$$

So $\alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$ is not a unit vector

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q(x=0) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$R = P Q P^{-1}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 0 & 8 & 12 \\ 0 & 0 & 18 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 6 & -3 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 & 6 & 4 \\ 0 & 24 & 8 \\ 0 & 0 & 36 \end{bmatrix}$$

$$R = \begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$(R - 6I) \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = \begin{bmatrix} -4 & 1 & 2/3 \\ 0 & -2 & 4/3 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$$

$$-4 + a + \frac{2b}{3} = 0$$

$$-2a + \frac{4}{3}b = 0$$

$$-8 + \frac{8}{3} = 0 \Rightarrow b = 3$$

$$a = 2$$

$$PQ = QP \Rightarrow a_{12} \text{ for both are same}$$

$$\Rightarrow x + 4 + x = 2 + 2x + 0 \Rightarrow x \in \phi \Rightarrow \text{No value exist}$$

4. For non-negative integer n, let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$ which of the following options is/are correct?

(1) $f(4) = \frac{\sqrt{3}}{2}$

(2) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

(3) $\sin(7 \cos^{-1} f(5)) = 0$





(4) $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$

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अऋणात्मक पूर्णांकों (non-negative integers) n के लिए माना कि

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

माना कि $\cos^{-1}x$ का मान $[0, \pi]$ में है, तब निम्न में से कौन सा (से) कथन सत्य है (हैं)–

(1) $f(4) = \frac{\sqrt{3}}{2}$

(2) यदि $\alpha = \tan(\cos^{-1} f(6))$, तब $\alpha^2 + 2\alpha - 1 = 0$

(3) $\sin(7 \cos^{-1} f(5)) = 0$

(4) $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$

Ans. (1,2,3)

Sol. $f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)} = \frac{\sum_{k=0}^n \left(\cos\frac{\pi}{n+2} - \cos\left(\frac{2k+3}{n+2}\pi\right)\right)}{\sum_{k=0}^n 2\sin^2\left(\frac{k+1}{n+2}\pi\right)}$

$$= \frac{(n+1)\cos\frac{\pi}{n+2} - \frac{\cos\left(\frac{n+3}{n+2}\pi\right)\sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\frac{\pi}{n+2}}}{(n+1) - \frac{\cos\pi \cdot \sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\frac{\pi}{n+2}}}$$

$$= \frac{(n+1)\cos\frac{\pi}{n+2} + \cos\left(\frac{n+3}{n+2}\pi\right)}{(n+1)+1} = \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{n+2} = \cos\left(\frac{\pi}{n+2}\right)$$

(1) $f(4) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ correct

(2) $\alpha = \tan(\cos^{-1} f(6)) = \tan\left|\cos^{-1}\left(\cos\frac{\pi}{8}\right)\right| = \tan\frac{\pi}{8}$

$$\tan\frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} \Rightarrow 1 = \frac{2\alpha}{1-\alpha^2} \Rightarrow \alpha^2 + 2\alpha - 1 = 0 \quad (\text{A) correct}$$

(3) $\sin(7\cos^{-1} f(5)) = \sin(7\cos^{-1}(\cos\frac{\pi}{7})) = \sin\pi = 0$ correct

(A), (C), (D) correct





(4) $\lim_{n \rightarrow \infty} f(n) = \cos\left(\frac{\pi}{n+2}\right) = 1$ (B) Incorrect

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5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function we say that f has

PROPERTY 1 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exist and is finite and

PROPERTY 2 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$ exist and is finite.

Then which of the following options is/are correct?

- (1) $f(x) = x^{2/3}$ has property 1
- (2) $f(x) = \sin x$ has property 2
- (3) $f(x) = |x|$ has property 1
- (4) $f(x) = x|x|$ has property 2

माना कि $f : \mathbb{R} \rightarrow \mathbb{R}$ एक फलन है। हम कहते हैं कि f में

गुण 1 है यदि $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ का अस्तित्व है और वह परिमित है,

गुण 2 है यदि $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$ का अस्तित्व है और वह परिमित है।

तब निम्न में से कौन सा (से) विकल्प सही है (हैं)–

- (1) $f(x) = x^{2/3}$, गुण 1 को प्रदर्शित करता है
- (2) $f(x) = \sin x$, गुण 2 को प्रदर्शित करता है
- (3) $f(x) = |x|$, गुण 1 को प्रदर्शित करता है
- (4) $f(x) = x|x|$, गुण 2 को प्रदर्शित करता है

Ans. (1,3)

Sol. (1) $\lim_{h \rightarrow 0} \frac{h^{2/3} - 0}{\sqrt{|h|}} = 0$

(2) $\lim_{h \rightarrow 0} \frac{\sin h - 0}{h^2}$ does not exist

(3) $\lim_{h \rightarrow 0} \frac{|h| - 0}{\sqrt{|h|}} = 0$

(4) $f(x) = x|x|$
 $\Rightarrow \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2} = \lim_{h \rightarrow 0} \frac{h|h| - 0}{h^2}$ does not exist

6. For $a \in \mathbb{R}$, $|a| > 1$, let

$$\lim_{n \rightarrow \infty} \frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left(\frac{1}{(na+1)^2} + \frac{1}{(na+2)^2} + \dots + \frac{1}{(na+n)^2} \right)} = 54. \text{ Then possible value(s) } a \text{ is/are -}$$

माना कि $a \in \mathbb{R}$, $|a| > 1$ के लिए

$$\lim_{n \rightarrow \infty} \frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left(\frac{1}{(na+1)^2} + \frac{1}{(na+2)^2} + \dots + \frac{1}{(na+n)^2} \right)} = 54, \text{ तब } a \text{ का (के) संभावित मान है (हैं)–}$$





- Ans. (1) 8 (2) -9 (3) 7 (4) -6

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Sol.
$$\lim_{n \rightarrow \infty} \frac{\frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n}\right)^{1/3}}{\frac{1}{n} \left(\frac{n^2}{(na+1)^2} + \frac{n^2}{(na+2)^2} + \dots + \frac{1}{(na+1)^2} \right)} =$$

$$\frac{\int_0^1 x^{1/3} dx}{\int_0^1 \frac{dx}{(a+x)^2}} = \frac{\left[\frac{3}{4} X^{4/3} \right]_0^1}{\left[-\frac{1}{a+x} \right]_0^1} = \frac{\frac{3}{4}}{\frac{1}{a} - \frac{1}{a+1}} = 54$$

$$\Rightarrow \frac{1}{a} - \frac{1}{a+1} = \frac{3}{4 \times 54} \Rightarrow \frac{1}{a(a+1)} = \frac{1}{72}$$

$a = 8$ or $a = -9$

7. Let

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

and $X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$.

Where P_k^T denotes the transpose of matrix P_k . Then which of the following options is/are correct ?

- (1) X is a symmetric matrix
 (2) The sum of diagonal entries of X is 18.

(3) If $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

- (4) $X - 30I$ is an invertible matrix

माना कि

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$





और
$$X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

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जहाँ आव्यूह (matrix) P_k के परिवर्त (transpose) को P_k^T से दर्शाया गया है। तब निम्न में से कौनसा (से) विकल्प सही है (हैं)–

- (1) X एक सममित आव्यूह है।
- (2) X के विकर्णों के अवयवों का योग 18 होगा।
- (3) यदि $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, तो $\alpha = 30$
- (4) $X - 30I$ एक व्युत्क्रमणीय आव्यूह होगा।

Ans. (1,2,3)

Sol.

Clearly $P_1 = P_1^T = P_1^{-1}$

$P_2 = P_2^T = P_2^{-1}$

⋮

⋮

$P_6 = P_6^T = P_6^{-1}$

and $A^T = A$, where $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

Using formula $(A + B)^T = A^T + B^T$

$X^T = (P_1 A P_1^T + \dots + P_6 A P_6^T)^T = P_1 A^T P_1^T + \dots + P_6 A^T P_6^T = X \Rightarrow X$ is symmetric.

Let $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$XB = P_1 A P_1^T B + P_2 A P_2^T B + \dots + P_6 A P_6^T B = P_1 A B + P_2 A B + \dots + P_6 A B$

$XB = (P_1 + P_2 + \dots + P_6) \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 6 \times 2 + 3 \times 2 + 6 \times 2 \\ 6 \times 2 + 3 \times 2 + 6 \times 2 \\ 6 \times 2 + 3 \times 2 + 6 \times 2 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 30B \Rightarrow \alpha = 30$

since $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow (X - 30I)B = 0$ has a non trivial solution $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow |X - 30I| = 0$

$X = P_1 A P_1^T + \dots + P_6 A P_6^T$

trace (x) = $\text{tr}(P_1 A P_1^T) + \dots + \text{tr}(P_6 A P_6^T) = (2 + 0 + 1) + \dots + (2 + 0 + 1) = 3 \times 6 = 18$

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8. Let

$$f(x) = \frac{\sin \pi x}{x^2}, x > 0.$$

Let $x_1 < x_2 < x_3 \dots < x_n < \dots$ be all points of local maximum of f and $y_1 < y_2 < y_3 < \dots < y_n < \dots$ be all the points of local minimum of f . Then which of the following options is/are correct ?

- (1) $|x_n - y_n| > 1$ for every n (2) $x_1 < y_1$
 (3) $x_{n+1} - x_n > 2$ for every n (4) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

माना

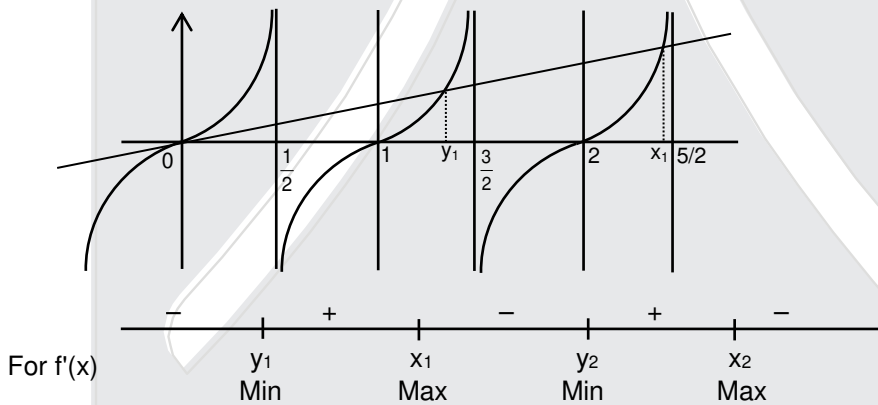
$$f(x) = \frac{\sin \pi x}{x^2}, x > 0.$$

माना कि f के सभी स्थानीय उच्चिष्ठ बिन्दु $x_1 < x_2 < x_3 \dots < x_n < \dots$ है और f के सभी स्थानीय न्यूनतम बिन्दु $y_1 < y_2 < y_3 < \dots < y_n < \dots$ हैं। तब निम्न में से कौन सा(से) विकल्प सही है (हैं)–

- (1) $|x_n - y_n| > 1$, प्रत्येक n के लिए (2) $x_1 < y_1$
 (3) $x_{n+1} - x_n > 2$, प्रत्येक n के लिए (4) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$, प्रत्येक n के लिए

Ans. (1,3,4)

Sol. $f'(x) = \frac{2x \cos \pi x \left(\frac{\pi x}{2} - \tan \pi x\right)}{x^4}$



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SECTION 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : **+3** If **ONLY** the correct numerical value is entered.
Zero Marks : **0** In all other cases.

खंड 2 (अधिकतम अंक: 18)

- इस खंड में **छः (06)** प्रश्न हैं। प्रत्येक प्रश्न का उत्तर एक संख्यात्मक मान (**NUMERICAL VALUE**) है।
- प्रत्येक प्रश्न के उत्तर के सही संख्यात्मक मान को माउज़ (mouse) और ऑन-स्क्रीन (on-screen) वर्चुअल न्यूमेरिक कीपेड (virtual numeric keypad) के प्रयोग से उत्तर के लिए चिन्हित स्थान पर दर्ज करें। यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान हैं, तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्न योजना के अनुसार होगा :-
पूर्ण अंक : **+3** यदि दर्ज किया गया संख्यात्मक मान (**Numerical value**) ही सही उत्तर है।
शून्य अंक : **0** अन्य सभी परिस्थितियों में।

1. Suppose

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n . Then $\sum_{k=0}^n \frac{{}^n C_k}{k+1}$ equals.

माना कि

किसी धनात्मक पूर्णांक n के लिए

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0, \text{ तो } \sum_{k=0}^n \frac{{}^n C_k}{k+1} \text{ का मान है—}$$

Ans. (6.20)

$$\text{Sol. } \begin{vmatrix} \frac{n(n+1)}{2} & n2^{n-1} + n(n-1)2^{n-2} \\ n2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2}{4} - \frac{n^2(n-1)}{8} = 0$$





$$n = 0 \quad \text{or} \quad 4(n+1) - 2n - n(n-1) = 0$$

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$$4n + 4 - 2n - n^2 + n = 0$$

$$3n - n^2 + 4 = 0 \Rightarrow n^2 - 3n - 4 = 0$$

$$(n-4)(n+1) = 0$$

$$n = 4$$

$$\sum_{r=0}^4 \frac{{}^4C_r}{r+1} = \sum_{r=0}^4 \frac{{}^5C_{r+1}}{5} = \frac{2^5 - 1}{5} = \frac{31}{5} = 6.20$$

2. The value of

$$\sec^{-1} \left(\frac{1}{4} \sum_{k=0}^{10} \sec \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \sec \left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2} \right) \right) \text{ in the interval } \left[-\frac{\pi}{4}, \frac{3\pi}{4} \right] \text{ equals}$$

अन्तराल $\left[-\frac{\pi}{4}, \frac{3\pi}{4} \right]$ में $\sec^{-1} \left(\frac{1}{4} \sum_{k=0}^{10} \sec \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \sec \left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2} \right) \right)$ का मान है-

Ans. (0.00)

Sol. Evaluate $\sec^{-1} \left[\frac{1}{4} \sum_{k=0}^{10} \sec \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \sec \left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2} \right) \right]$

$$\text{Given exp} = \sec^{-1} \left[-\frac{1}{4} \sum_{k=0}^{10} \sec \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \operatorname{cosec} \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \right]$$

$$= \sec^{-1} \left[-\frac{1}{4} \sum_{k=0}^{10} \frac{2}{\sin \left(\frac{7\pi}{6} + k\pi \right)} \right] = \sec^{-1} \left[-\frac{1}{2} \sum_{k=0}^{10} \frac{1}{(-1)^{k+1} \sin \frac{\pi}{6}} \right] = \sec^{-1} \left(-\sum_{k=0}^{10} \frac{1}{(-1)^{k+1}} \right) = \sec^{-1} (1) = 0$$

3. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in \mathbb{R}$. If the projection of \vec{c} on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equal to
यदि $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ तथा $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ दो सदिश दिये हुए है माना एक सदिश \vec{c} इस प्रकार है कि $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in \mathbb{R}$. यदि सदिश \vec{c} का सदिश $(\vec{a} + \vec{b})$ पर प्रक्षेप $3\sqrt{2}$ है तो $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ का न्यूनतम मान होगा-

Ans. (18.00)

Sol. $\vec{c} = \alpha(2\hat{i} + \hat{j} - \hat{k}) + \beta(\hat{i} + 2\hat{j} + \hat{k})$

$$\Rightarrow \vec{c} = (2\alpha + \beta)\hat{i} + (\alpha + 2\beta)\hat{j} + (\beta - \alpha)\hat{k}$$

$$\frac{\vec{c} \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = 3\sqrt{2} \Rightarrow 9(\alpha + \beta) = 18 \Rightarrow \alpha + \beta = 2$$

$$(\vec{c} - \vec{a} \times \vec{b}) \cdot \vec{c} = (\alpha\vec{a} + \beta\vec{b} - \vec{a} \times \vec{b}) \cdot (\alpha\vec{a} + \beta\vec{b}) = 6\alpha^2 + 6\alpha\beta + 6\beta^2 = 6[\alpha^2 + \alpha(2 - \alpha) + (2 - \alpha)^2]$$

$$= 6(\alpha^2 - 2\alpha + 4) = \text{min value } 18$$

4. Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the numbers of ways of distributing the hats such that the person seated in adjacent seats get different coloured hats is

पाँच व्यक्तियों A,B,C,D तथा E को वृत्तीय क्रम में बैठाया जाता है। यदि प्रत्येक व्यक्ति को तीन रंगों (लाल, नीला तथा हरा) में से एक रंग की टोपी दी जाती है, तो टोपियों को कितने तरीकों से बाँटा जा सकता है जबकि पास-पास बैठे व्यक्तियों के पास भिन्न-भिन्न रंग की टोपियाँ हो-





Ans. (30.00)

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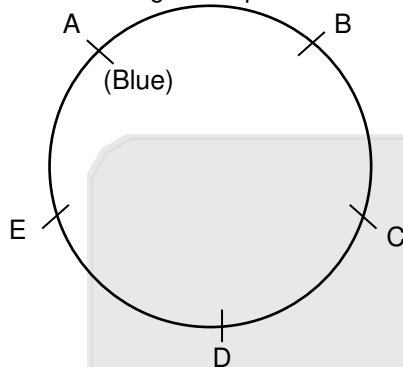
Sol. Maximum number of hats used of same colour are 2. They can not be 3 otherwise atleast 2 hats of same colour are consecutive.

Now, Let hats used are R, R, G, G, B

(Which can be selected in 3 ways. It can be RGGGB or RRGGB also)

Now, numbers of ways of distributing blue hat (single one) in 5 person equal to 5

Let blue hat goes to person A.



Now either position B & D are filled by green hats and C & E are filled by Reds hats

Or B & D are filled by Red hats and C & E are filled by Green hats

⇒ 2 ways are possible

Hence total number of ways = $3 \times 5 \times 2 = 30$ ways

5. The value of the integral

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} d\theta$$

equals

समाकलन $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})^5} d\theta$ का मान है—

Ans. (0.50)

Sol.
$$I = 3 \int_0^{\pi/2} \frac{\sqrt{\cos\theta} d\theta}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})^5} = 3 \int_0^{\pi/2} \frac{3\sqrt{\sin\theta} d\theta}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5}$$

$$\Rightarrow \int_0^{\pi/2} \frac{3\sqrt{\cos\theta} d\theta}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})^5} = 3 \int_0^{\pi/2} \frac{\sqrt{\sin\theta} d\theta}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} \Rightarrow 2I = 3 \int_0^{\pi/2} \frac{d\theta}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})^4}$$

$$\frac{2I}{3} = \int_0^{\pi/2} \frac{\sec^2\theta d\theta}{(\sqrt{\tan\theta} + 1)^4}$$

let $\tan\theta = t^2 \Rightarrow \sec^2\theta d\theta = 2t dt$

$$\Rightarrow \frac{2I}{3} = \int_0^\infty \frac{2t dt}{(t+1)^4} \Rightarrow \frac{I}{3} = \int_0^\infty \left[\frac{1}{(t+1)^3} - \frac{1}{(t+1)^4} \right] dt$$

$$\Rightarrow I = \left[\frac{-3}{2(t+1)^2} + \frac{1}{(t+1)^3} \right]_0^\infty = \frac{3}{2} - 1 = \frac{1}{2}$$

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6. Let $|X|$ denote the number of elements in a set X . Let $S = \{1,2,3,4,5,6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S , then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$ equals
 एक समुच्चय S इस प्रकार है कि $S = \{1,2,3,4,5,6\}$. $|X|$ किसी समुच्चय X में अवयवों की संख्या को दर्शाता है। समुच्चय S के उपसमुच्चय A तथा B इस प्रकार चुने जाते हैं कि S का प्रत्येक अवयव समान प्रायिकता से चुना जा सकता है तथा यदि A और B , S के साथ स्वतंत्र घटनाएँ हैं, तब $1 \leq |B| < |A|$ तो क्रमित युग्म (A, B) की संख्या होगी—

Ans. (422)

Sol. **A and B are independent events**

$$P(A)P(B) = P(A \cap B) \Rightarrow \frac{a}{6} \times \frac{b}{6} = \frac{c}{6} \Rightarrow ab = 6c$$

$$|A| = a, |B| = b, |A \cap B| = c$$

$(a, b, c) = (3, 2, 1)$	so	${}^6C_1 {}^5C_2 {}^3C_1 = 180$
$= (4, 3, 2)$	so	${}^6C_2 {}^4C_2 {}^2C_1 = 180$
$= (6, 1, 1)$	so	${}^6C_1 = 6$
$= (6, 2, 2)$	so	${}^6C_2 = 15$
$= (6, 3, 3)$	so	${}^6C_3 = 20$
$= (6, 4, 4)$	so	${}^6C_4 = 15$
$= (6, 5, 5)$	so	${}^6C_5 = 6$

$$\text{Total} = 360 + 62 = 422$$

SECTION 3 (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**.
- List-I** has **Four** entries (I),(II), (III) and (IV) **List-II** has **Six** entries (P),(Q), (R), (S), (T) and (U).
- FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :
 Full Marks : **+3** If **ONLY** the option corresponding to the correct combination is chosen.
 Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
 Negative Marks : **-1** In all other cases.

खंड 3 (अधिकतम अंक: 12)





- इस खंड में **दो (02)** सूची-सुमेलन (List-Match) सेट्स (sets) हैं।
- प्रत्येक सूची-सुमेलन सेट (set) में **दो (02)** एकाधिक विकल्प प्रश्न (Multiple Choice Question) हैं।
- प्रत्येक सूची-सुमेलन सेट में दो सूचियाँ हैं : **सूची-I** और **सूची-II**
- सूची-I** में चार प्रविष्टियाँ (I),(II),(III) और (IV) हैं एवं **सूची-II** में छः प्रविष्टियाँ (P),(Q),(R),(S),(T) और (U) हैं।
- प्रत्येक एकाधिक विकल्प प्रश्न में सूची-I और सूची-II पर आधारित चार विकल्पों में से केवल एक विकल्प ही एकाधिक विकल्प प्रश्न की शर्त को पूरा करता है।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्नयोजना के अनुसार होगा—
 पूर्ण अंक : **+3** यदि सिर्फ सही विकल्प ही चुना गया है।
 शून्य अंक : **0** यदि कोई भी विकल्प नहीं चुना गया है (अर्थात् प्रश्न अनुत्तरित है)।
 ऋण अंक : **-1** अन्य सभी परिस्थितियों में

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MATCH TYPE

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in increasing order

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

List-I contains sets X, Y, Z and W List-II contains some information regarding these set.

List - I

(I) X

(II) Y

(III) Z

(IV) W

List - II

$$(P) \supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$$

(Q) an arithmetic progression

(R) NOT an arithmetic progression

$$(S) \supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$$

$$(T) \supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$$

$$(U) \supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$$

अनुच्छेद में दी गई जानकारी के आधार पर सूचियों का उचित मिलान करके प्रश्न का उत्तर दें।

माना कि $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ दो फलन (function) हैं जो $x > 0$ में परिभाषित हैं। निम्नलिखित समुच्चय (sets) जिनके तत्वों को बढ़ते हुए क्रम में लिखा गया है, इस प्रकार परिभाषित हैं।

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

सूची-I (List-I) में X, Y, Z और W समुच्चय हैं। सूची II (List-II) में इन समुच्चयों के बारे में कुछ सूचनाएं हैं।

List - I

(I) X

(II) Y

(III) Z

(IV) W

List - II

$$(P) \supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$$

(Q) समान्तर श्रेणी (an arithmetic progression)

(R) समान्तर श्रेणी नहीं है NOT an arithmetic progression

$$(S) \supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$$

$$(T) \supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$$





$$(U) \supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$$

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1. Which of the following is the only correct combination ?

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- (1) IV – (Q), (T)
- (2) III – (R), (U)
- (3) III – (P), (Q), (U)
- (4) IV – (P), (R), (S)

Ans. (4)

2. Which of the following is only CORRECT combination ?

निम्न में से कौनसा मिलान सही है—

- (1) I – (Q), (U)
- (2) I – (P), (R)
- (3) II – (Q), (T)
- (4) II – (R), (S)

Ans. (3)

Sol. (1 to 2)

$$f(x) = 0 \Rightarrow \sin(\pi \cos x) = 0$$

$$\Rightarrow \pi \cos x = n\pi \Rightarrow \cos x = n$$

$$\Rightarrow \cos x = -1, 0, 1 \Rightarrow X = \{n\pi, (2n+1)\frac{\pi}{2}\} = \{n\frac{\pi}{2}, n \in \mathbb{I}\}$$

$$f'(x) = 0 \Rightarrow \cos(\pi \cos x) (-\pi \sin x) = 0$$

$$\Rightarrow \pi \cos x = (2n+1)\frac{\pi}{2} \text{ or } x = n\pi$$

$$\Rightarrow \cos x = n + \frac{1}{2} \text{ or } x = n\pi$$

$$\Rightarrow \cos x = \pm \frac{1}{2} \text{ or } x = n\pi$$

$$\Rightarrow Y = \left\{ 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n\pi, n \in \mathbb{I} \right\}$$

$$= \left\{ \dots, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \dots \right\} \text{ which is an arithmetic progression}$$

$$g(x) = 0 \Rightarrow \cos(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \sin x = \frac{2n+1}{4} = \pm \frac{1}{4}, \pm \frac{3}{4}$$

$$\Rightarrow Z = \left\{ n\pi \pm \sin^{-1} \frac{1}{4}, n\pi \pm \sin^{-1} \frac{3}{4}, n \in \mathbb{I} \right\}$$

$$g'(x) = 0 \Rightarrow -\sin(2\pi \sin x) (2\pi \cos x) = 0$$

$$\Rightarrow 2\pi \sin x = n\pi \text{ or } x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \sin x = \frac{n}{2} = 0, \pm \frac{1}{2}, \pm 1 \text{ or } x = (2n+1)\frac{\pi}{2}$$





$$\Rightarrow W = \left\{ n\pi, (2n+1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in \mathbb{I} \right\}$$

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MATCH TYPE

Let the circle $C_1: x^2 + y^2 = 9$ and $C_2: (x - 3)^2 + (y - 4)^2 = 16$ intersect at the points X and Y. Suppose that another circle $C_3: (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions.

- (i) centre of C_3 is collinear with the centres of C_1 & C_2 ,
- (ii) C_1 & C_2 both lie inside C_3 and
- (iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W and let a common tangent of C_1 & C_3 be a tangent to the parabola $x^2 = 8\alpha y$

There are some expressions given in the list-I, whose values are given in list-II below :

	List-I		List-II
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{length of } ZW}{\text{length of } XY}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of } \triangle MZN}{\text{Area of } \triangle ZMW}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

स्तम्भ मिलान कीजिए-

माना वृत्त $C_1: x^2 + y^2 = 9$ तथा वृत्त $C_2: (x - 3)^2 + (y - 4)^2 = 16$ बिन्दु X तथा Y पर प्रतिच्छेद करते हैं। माना एक वृत्त $C_3: (x - h)^2 + (y - k)^2 = r^2$ निम्न प्रतिबन्धों को सन्तुष्ट करता है-

- (i) C_3 का केन्द्र, C_1 और C_2 के केन्द्रों के साथ संरेख है।
- (ii) C_1 तथा C_2 दोनों, C_3 के अन्दर उपस्थित है।
- (iii) C_3 , C_1 को बिन्दु M तथा C_2 को बिन्दु N पर स्पर्श करता है।

यदि X तथा Y से गुजरने वाली रेखा C_3 को Z तथा W पर प्रतिच्छेद करती है और C_1 तथा C_3 की उभयनिष्ठ स्पर्श रेखा परवलय $x^2 = 8\alpha y$ की भी स्पर्श रेखा है।

	सूची-I		सूची-II
(I)	$2h + k$	(P)	6
(II)	$\frac{ZW \text{ की लम्बाई}}{XY \text{ की लम्बाई}}$	(Q)	$\sqrt{6}$
(III)	$\frac{MZN \text{ का क्षेत्रफल}}{ZMW \text{ का क्षेत्रफल}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

3. Which of the following is the only correct combination?

निम्न में से कौनसा मिलान सही है-

- (1) (II) - (T)
- (2) (I) - (S)
- (3) (II) - (Q)
- (4) (I) - (U)





Ans. (3)

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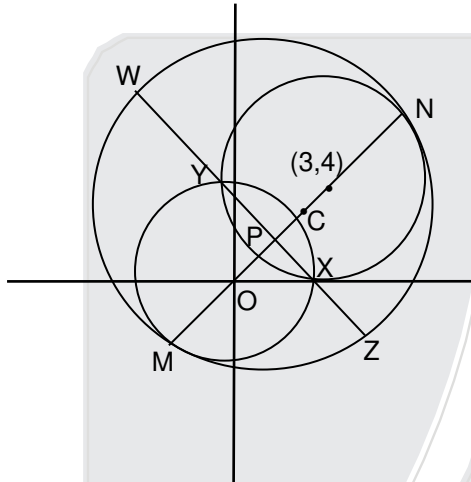
4. Which of the following is the only Incorrect combination?

निम्न में से कौनसा मिलान गलत है-

- (1) (IV) - (S)
- (2) (I) - (P)
- (3) (III) - (R)
- (4) (IV) - (U)

Ans. (1)

Sol. 3 to 4



(I) $2r = MN = 3 + \sqrt{3^2 + 4^2} + 4 = 12 \Rightarrow r = 6$

centre C of circle C_3 lies on $y = \frac{4}{3}x$

Let C $\left(h, \frac{4}{3}h \right)$

$OC = MC - OM = \frac{12}{2} - 3 = 3$

$\therefore \sqrt{h^2 + \frac{16}{9}h^2} = 3 \Rightarrow \frac{5h}{3} = 3 \Rightarrow h = \frac{9}{5}$

$k = \frac{4}{3}h = \frac{12}{5}$

$\therefore 2h + k = \frac{18}{5} + \frac{12}{5} = 6$

(II) Equation of line ZW

$C_1 = C_2$

$\Rightarrow 3x + 4y = 9$

Distance of ZW from (0, 0)

$\frac{|-9|}{\sqrt{3^2 + 4^2}} = \frac{9}{5}$

Length of XY = $2 \sqrt{3^2 - \left(\frac{9}{5}\right)^2} = \frac{24}{5}$

Distance of ZW from C

$\frac{\left| 3 \times \frac{9}{5} + 4 \times \frac{12}{5} - 9 \right|}{\sqrt{3^2 + 4^2}} = \frac{6}{5}$

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$$\text{Length of } ZW = 2 \sqrt{6^2 - \frac{6^2}{5^2}} = \frac{24\sqrt{6}}{5}$$

$$\therefore \frac{\text{length of } ZW}{\text{length of } XY} = \sqrt{6}$$

$$(III) \text{ Area of } \Delta MZN = \frac{1}{2} \cdot NM \left(\frac{1}{2} ZW \right) = \frac{72\sqrt{6}}{5}$$

$$\text{Area of } \Delta ZMW = \frac{1}{2} \cdot ZW (OM + OP) = \frac{1}{2} \cdot \frac{24\sqrt{6}}{5} \cdot \left(3 + \frac{9}{5} \right) = \frac{288\sqrt{6}}{25}$$

$$\therefore \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{5}{4}$$

$$(IV) \text{ Slope of tangent to } C_1 \text{ at } M = \frac{-1}{4/3} = -\frac{3}{4}$$

$$\therefore \text{Equation of tangent } y = mx - 3\sqrt{1+m^2}$$

$$y = -\frac{3}{4}x - 3\sqrt{1+\frac{9}{16}}$$

$$y = \frac{-3x}{4} - \frac{15}{4} \Rightarrow x = -\frac{4y}{3} - 5 \dots(i)$$

tangent to $x^2 = 4(2\alpha)y$ is

$$x = m'y + \frac{2\alpha}{m'} \dots(ii)$$

Compare (i) and (ii)





$$m' = -\frac{4}{3} \text{ and } \frac{2\alpha}{m'} = -5 \Rightarrow \alpha = \frac{10}{3}$$

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Harish Yadav
Classroom student
since class XI

AIR 70

Jatin Munjal
Short Term
Classroom Contact Program
(JEE Main to JEE Advanced
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AIR 46

Sukhmanjit Mann
Classroom student
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AIR 45

Utkarsh Agarwal
Classroom student
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AIR 4

Pawan Goyal
Classroom student
since class VIII

AIR 39

Shashank Roy
Classroom student
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AIR 34

Sayantana Pal
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1 Student in
AIR Top - 5

5 Students in
AIR Top - 50

7 Students in
AIR Top - 100

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