PAPER (पेपर)- 1

- **This question paper has three (03) parts: PART-I: Physics, PART-II: Chemistry and PART-III: Mathematics.**
- **Each part has total of eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).**
- **Total number of questions in Paper-1 : Fifty four (54).**
- **Paper-1 Maximum Marks : One Hundred Eighty (180).**

Instructions for Section-1 : Questions and Marking Scheme

SECTION-1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–2** In all other cases.

- **For Example :** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

Answering Section-1 Questions :

- To select the option(s), **using the mouse click** on the corresponding button(s) of the option(s).
- To deselect chosen option(s), click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.
- To change the option(s) of a previously answered question, if required, first click on the **Clear Response** button to clear all the chosen options and then select the new option(s).
- To mark a question ONLY for review (i.e. without answering it), click on the **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next** button – the answered question will be evaluated.

Instructions for Section-2 : Questions and Marking Scheme

SECTION-2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –0.30,,30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

Zero Marks : **0** In all other cases.

Answering Section-2 Questions :

- Using the attached computer mouse, click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value as answer in the space provided for answer.
- To change the answer, if required, first click on the **Clear Response** button to clear the entered answer and then enter the new numerical value.
- To mark a question ONLY for review (i.e. answering it), click on **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To mark a question for review (after answering it), click **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

Instructions for Section-3 : Questions and Marking Scheme

SECTION-3 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct option is chosen.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

Answering Section-3 Questions :

- To select an option, using the mouse click on the corresponding button of the option.
- To deselect the chosen answer, click on the button of the chosen option again or click on the **Clear Response button**.
- To change the chosen answer, click on the button of another option.
- To mark a question ONLY for review (i.e. without answering it), click on **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – the answered which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

PART-I: PHYSICS

SECTION 1 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–2** In all other cases.

For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$ (C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

Ans. **(B,C)**

Sol. $U = \frac{kr^2}{2} \Rightarrow \vec{F} = -k\vec{r}$

$\frac{mv^2}{R} = +kR \Rightarrow v = \sqrt{\frac{k}{m}}R$

Angular momentum $L = mvR = \sqrt{mk}\,R^2$

2. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ \text{Ns}^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\tau| = \frac{1}{3}$Nm

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$m

Ans. **(A,C)**

Sol. m = 1kg

$m\vec{a} = \vec{F} = t\hat{i} + \hat{j}$

(1) $\vec{a} = t\hat{i} + \hat{j}$

$\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j} = \frac{1}{2}\hat{i} + \hat{j}$

$\vec{s} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

$\vec{s}(t = 1) = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$= \left[\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right] \times \left[\hat{i} + \hat{j}\right]$

$= \frac{1}{6}(\hat{i} \times \hat{j}) + \frac{1}{2}(\hat{j} \times \hat{i})$

$= \frac{1}{6}\hat{k} - \frac{1}{2}\hat{k} = \frac{-2}{6}\hat{k} = -\frac{1}{3}\hat{k}$

$|\tau| = \frac{1}{3}$ at t = 1 sec

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

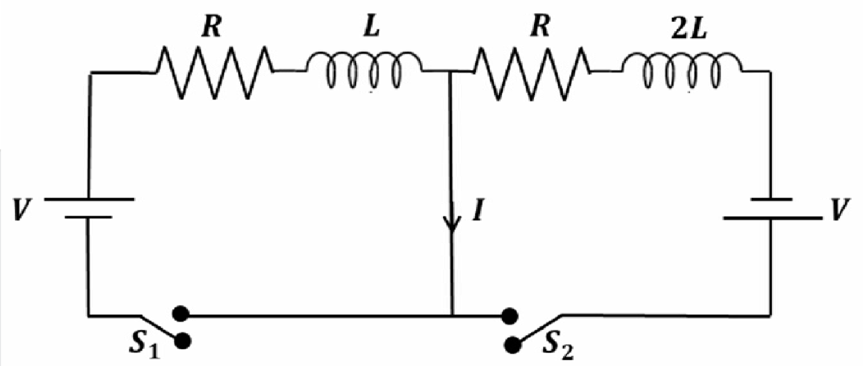
(D) h is proportional to contact angle θ

Ans. **(A,C)**

Sol. $h = \frac{2\sigma\cos\theta}{\rho g r}$

When lift is going up with constant acceleration $h = \frac{2\sigma\cos\theta}{\rho(g + a)r}$

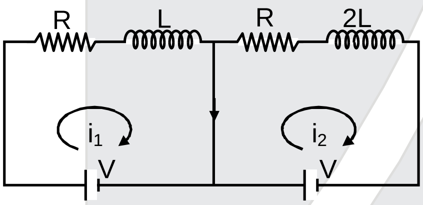
4. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statements is (are) true?



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

Ans. **(B,D)**

Sol.



Δi (current in middle wire) = $i_1 - i_2$

$$= \frac{V}{R}\left(1-e^{-\frac{Rt}{L}}\right) - \frac{V}{R}\left(1-e^{-\frac{Rt}{2L}}\right)$$

$$= \frac{V}{R}\left(e^{-\frac{Rt}{2L}} - e^{-\frac{Rt}{L}}\right)$$

For i_{max} : $\frac{d}{dt}(\Delta i) = 0$

$\Rightarrow \quad \frac{V}{R}\left(-\frac{R}{L}e^{-\frac{Rt}{L}} + \frac{R}{2L}e^{-\frac{Rt}{2L}}\right) = 0 \qquad \Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{1}{2}e^{-\frac{Rt}{2L}}$

$\Rightarrow \quad e^{-\frac{Rt}{2L}} = \frac{1}{2} \qquad \& \qquad t = \frac{2L}{R}\ell n2$

$\Rightarrow \quad$ and $i_{max} = \frac{V}{4R}$

5. Two infinitely long straight wires lie in the xy-plane along the lines x = ±R. The wire located at x = +R carries a constant current I_1 and the wire located at x = – R carries a constant current I_2. A circular loop of radius R is suspended with its centre at (0, 0, $\sqrt{3}R$) and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

(A) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin (0, 0, 0)

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

Sol. **(A,B,D)**

Magnetic field due to ring at origin

$$= \frac{\mu_0 \times I \times R^2}{2 \times 8R^3}(-\hat{K}) = \frac{\mu_0 I}{16R}(-\hat{K})$$

Magnetic field at origin due to wires

$$= \left(\frac{\mu_0 I_1}{2\pi R} - \frac{\mu_0 I_2}{2\pi R}\right)\hat{K}$$

Here I_1 and I_2 will be substituted with sign

If $I_1 = I_2$ then $\vec{B}_0 = \frac{\mu_0 I}{16R}(-\hat{K})$

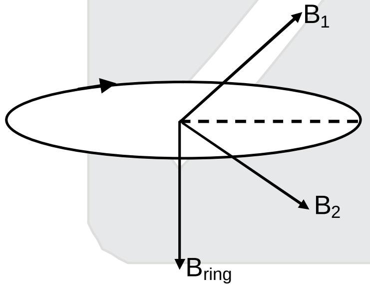
It can be zero

If $I_1 < 0$, $I_2 > 0$

then $\vec{B}_0 = -\left[\frac{\mu_0 (I_1 + I_2)}{2\pi R} + \frac{\mu_0 I}{16R}\right]\hat{K}$

It can not be zero

(D)

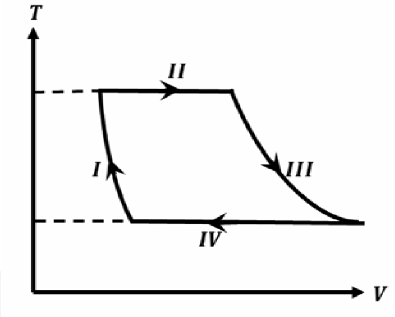


$B_1 = B_2$

magnetic field along z-axis is only due to ring

$B = \frac{\mu_0 i}{2R}$ in –z direction

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true?



(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and III are **not** isobaric

Ans. **(B,C,D)**

Sol. Process II is isothermal expansion
$\Rightarrow$ heat is positive
Process IV is isothermal compression
$\Rightarrow$ heat is negative

SECTION 2 (Maximum Marks: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = \alpha\hat{i}$ and $\vec{B} = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$, where α is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A} + \vec{B}| = \sqrt{3}\,|\vec{A} - \vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in second, is ________.

Ans. **2.00**

Sol. $\vec{A} = a\hat{i}$ and $\vec{B} = a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}$

$|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$

$\sqrt{(a + a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3}\ \sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$

$\Rightarrow\ 2\cos\frac{\omega t}{2} = \pm\sqrt{3} \times 2\sin\frac{\omega t}{2}$

$\tan\frac{\omega t}{2} = \pm\frac{1}{\sqrt{3}}$

$\frac{\omega t}{2} = n\pi \pm \frac{\pi}{6}$

$\frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$

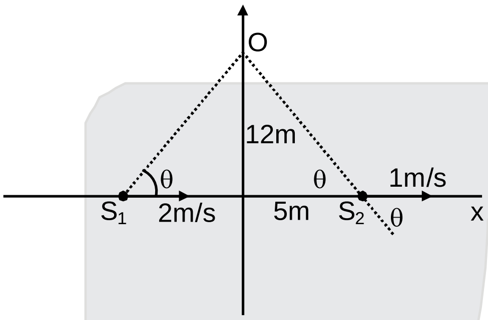
$t = (12n \pm 2)$s

= 2s, 10s, 14s ....

8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is __________.

Ans. **5.00**

Sol.



$$f_1 = f\left(\frac{v}{v - 2\cos\theta}\right) \qquad f_2 = f\left(\frac{v}{v + \cos\theta}\right)$$

Beat frequency = $f_1 - f_2$

$$= \frac{fvx3\cos\theta}{(v - 2\cos\theta)(v + \cos\theta)} \approx \frac{2f\cos\theta}{v} = 5$$

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take g = 10 ms^{-2}.

Ans. **0.75**

Sol. $a_{cm} = \dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}} = \dfrac{\dfrac{\sqrt{3}g}{2}}{1+\dfrac{K^2}{R^2}}$

$$a_{ring} = \frac{\dfrac{\sqrt{3}g}{2}}{1+1} = \frac{\sqrt{3}g}{4}$$

$$a_{disc} = \frac{\dfrac{\sqrt{3}g}{2}}{1+\dfrac{1}{2}} = \frac{g}{\sqrt{3}}$$

$$t_{ring} = \sqrt{\frac{2.\dfrac{2h}{\sqrt{3}}}{\dfrac{\sqrt{3}g}{4}}} = \sqrt{\frac{4h}{\sqrt{3}}\frac{4}{\sqrt{3}g}} = \sqrt{\frac{16h}{3g}}$$

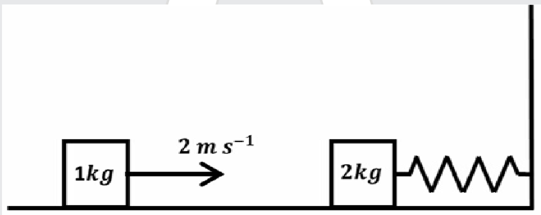
$$t_{disc} = \sqrt{\frac{2.\frac{2h}{\sqrt{3}}}{\frac{g}{\sqrt{3}}}} = \sqrt{\frac{4h}{\sqrt{3}}\frac{\sqrt{3}}{g}} = \sqrt{\frac{4h}{g}}$$

$$\Delta t = \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \sqrt{\frac{h}{g}}\left(\sqrt{\frac{16}{3}} - \sqrt{4}\right) = \sqrt{\frac{h}{g}}\left(\frac{4}{\sqrt{3}} - 2\right)$$

$$= 2\sqrt{\frac{h}{g}}\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 2\sqrt{\frac{h}{3}} = 1$$

$$= \sqrt{\frac{h}{3}} = \frac{1}{2} \qquad \Rightarrow \qquad h = \frac{3}{4} = 0.75\text{m}$$

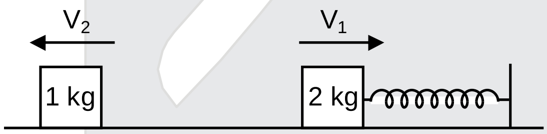
10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _________.



Ans. 2.09

Sol. **Just Before Collision**

Just After Collision



$V_1 + V_2 = 2$ .....(1)

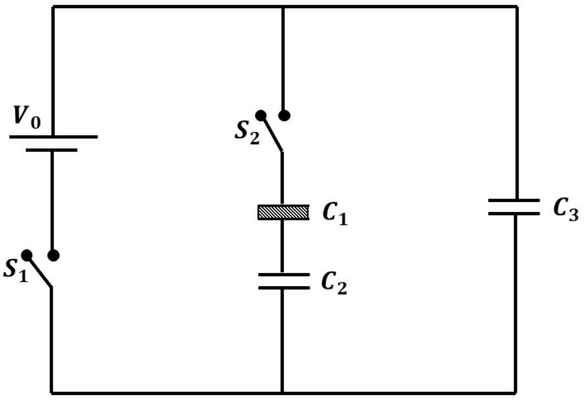
$2V_1 - V_2 = 4$ .....(2)

$3V_1 = 4 \Rightarrow V_1 = \frac{4}{3}$ $\qquad V_2 = 2 - \frac{4}{3} = \frac{2}{3}$ m/s

$$\Delta t = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{2}{2}} = \pi \text{ sec.}$$

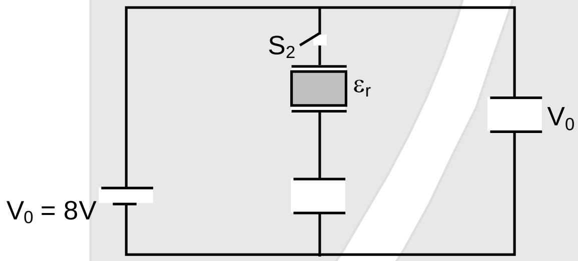
$$\text{Distance} = \pi V_2 = \left(\frac{2}{3}\right)\left(\frac{22}{7}\right)\text{m} = \frac{44}{21}\text{m} = 2.09$$

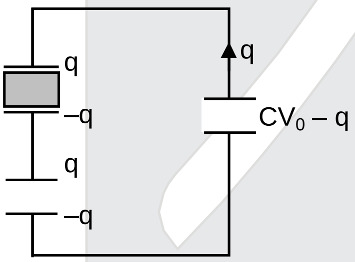
11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) V_0 = 8V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ε_r =____________.



Ans. 1.50

Sol.





$$\frac{CV_0 - q}{C} - \frac{q}{\varepsilon_r C} - \frac{q}{C} = 0 \qquad \frac{CV_0 - q}{C} = 5$$

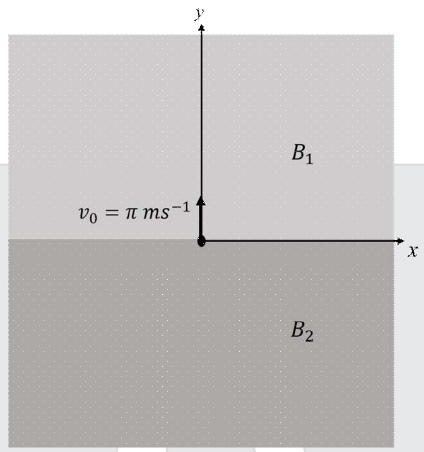
$$5 = \frac{q}{C}\left(1 + \frac{1}{\varepsilon_r}\right) \qquad 8C - q = 5C$$

$$= 3\left(1 + \frac{1}{\varepsilon_r}\right) \qquad \Rightarrow q = 3C$$

$$\frac{5}{3} = 1 + \frac{1}{\varepsilon_r} \Rightarrow \frac{1}{\varepsilon_r} = \frac{2}{3}$$

$$\varepsilon_r = \frac{3}{2} = 1.5$$

12. In the xy-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is __________.



Ans. **2.00**

Sol. If average speed is considered along x-axis

$R_1 = \frac{mv_0}{qB_1}$, $R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1}$

$R_1 > R_2$

distance along x-axis $\Delta x = 2(R_1 + R_2) = \frac{5mv_0}{2qB_1}$

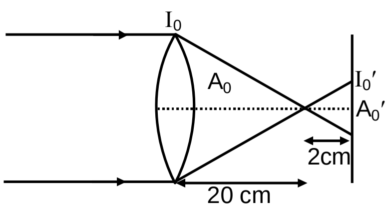
Total time = $\frac{\pi m}{qB_1} + \frac{\pi m}{qB_2} = \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$

Magnitude of average speed = $\frac{\frac{5mv_0}{2qB_1}}{\frac{5\pi m}{4qB_1}} = 2\,m/s$

13. Sunlight of intensity 1.3 kWm^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kWm^{-2}, at a distance 22 cm from the lens on the other side is __________.

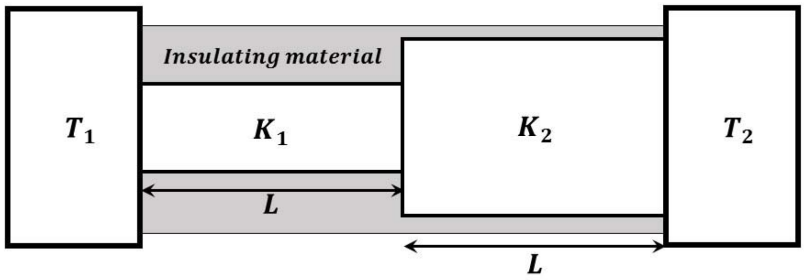
Ans. **130.00**

Sol.



$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100} \Rightarrow A_0' = \frac{A_0}{100} \Rightarrow I_0' = \frac{I_0 A_0}{\frac{A_0}{100}} = 100 I_0 = 130\,kW/m^2$

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300K and T_2 = 100K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200K, then K_1/K_2 =__________.



Ans. **4.00**

Sol. since rate of heat flow is same, we can say

$$\frac{300-200}{R_1} = \frac{200-100}{R_2}$$

$$R_1 = R_2 \qquad \Rightarrow \frac{L_1}{K_1A_1} = \frac{L_2}{K_2A_2}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{A_2}{A_1} = 4$$

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : −1 In all other cases.

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [ε_0] and [μ_0] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. The relation between [E] and [B] is

(A) [E] = [B] [L] [T] (B) $[E] = [B] [L]^{-1} [T]$ (C) $[E] = [B] [L] [T]^{-1}$ (D) $[E] = [B] [L]^{-1} [T]^{-1}$

Ans. **(C)**

Sol. In terms of dimension

qE = qvB

E = vB

$[E] = [B] [LT^{-1}]$

16. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(A) $[\mu_0] = [\varepsilon_0] [L]^2 [T]^{-2}$ (B) $[\mu_0] = [\varepsilon_0] [L]^{-2} [T]^2$ (C) $[\mu_0] = [\varepsilon_0]^{-1} [L]^2 [T]^{-2}$ (D) $[\mu_0] = [\varepsilon_0]^{-1} [L]^{-2} [T]^2$

Ans. **(D)**

Sol. $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$C^2 = \frac{1}{\mu_0 \epsilon_0}$

$\mu_0 = \epsilon_0 . C^2$

$[\mu_0] = [\epsilon_0]^{-1} L^{-2} T^2$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp \left(\frac{\Delta y}{y}\right)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x << 1$, $\Delta y/y << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

17. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a << 1$), then what is the error Δr in determining r ?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

Ans. **(B)**

Sol. $\frac{dr}{da} = \frac{(1+a)(-1)-(1-a)}{(1+a)^2}$

$\Rightarrow \quad dr = \frac{-2dr}{(1+a)^2}$

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For |x| << 1, $\ell n(1 + x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

Ans. **(C)**

Sol. $N = N_0 e^{-\lambda t}$

$\ell nN = \ell nN_0 - \lambda t$

different w.r.t. λ

$\frac{1}{N}\frac{dN}{d\lambda} = 0 - t$

$d\lambda = \frac{dN}{Nt} = \frac{40}{2000 \times 1} = 0.02$