

PAPER (पेपर)- 1

- This question paper has three (03) parts: **PART-I: Physics, PART-II: Chemistry and PART-III: Mathematics.**
- Each part has total of eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).
- Total number of questions in Paper-1 : Fifty four (54).
- Paper-1 Maximum Marks : One Hundred Eighty (180).

Instructions for Section-1 : Questions and Marking Scheme

SECTION-1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.
Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen.
Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct options.
Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **-2** In all other cases.
- **For Example** : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

Answering Section-1 Questions :



- To select the option(s), **using the mouse click** on the corresponding button(s) of the option(s).
- To deselect chosen option(s), click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.
- To change the option(s) of a previously answered question, if required, first click on the **Clear Response** button to clear all the chosen options and then select the new option(s).
- To mark a question **ONLY** for review (i.e. without answering it), click on the **Mark for Review & Next** button.
- To mark a question for review (after answering it), click on **Mark for Review & Next** button – answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next** button – the answered question will be evaluated.

Resonance Eduventures Ltd.

Registered & Corporate Office: CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005

Tel.No.: 0744-6607777, 3012100, 3012222, 6635555 | **Toll Free:** 1800 258 5555 | **Fax:** +91-022-39167222 | **08003 444 888**

Website: www.resonance.ac.in | **E-mail:** contact@resonance.ac.in | **CIN:** U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

This solution was download from Resonance JEE ADVANCED 2018 Solution portal

Instructions for Section-2 : Questions and Marking Scheme

SECTION-2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -0.30,,30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : **+3** If **ONLY** the correct numerical value is entered as answer.
Zero Marks : **0** In all other cases.

Answering Section-2 Questions :

- Using the attached computer mouse, click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value as answer in the space provided for answer.
- To change the answer, if required, first click on the **Clear Response** button to clear the entered answer and then enter the new numerical value.
- To mark a question **ONLY** for review (i.e. answering it), click on **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To mark a question for review (after answering it), click **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

Instructions for Section-3 : Questions and Marking Scheme

SECTION-3 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : **+3** If **ONLY** the correct option is chosen.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **-1** In all other cases.

Answering Section-3 Questions :





- To select an option, using the mouse click on the corresponding button of the option.
- To deselect the chosen answer, click on the button of the chosen option again or click on the **Clear Response button**.
- To change the chosen answer, click on the button of another option.
- To mark a question **ONLY** for review (i.e. without answering it), click on **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – the answered which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

Resonance Eduventures Ltd.

Registered & Corporate Office: CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005

Tel.No.: 0744-6607777, 3012100, 3012222, 6635555 | Toll Free: 1800 258 5555 | Fax: +91-022-39167222 | 08003 444 888

Website: www.resonance.ac.in | E-mail: contact@resonance.ac.in | CIN: U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

This solution was download from Resonance JEE ADVANCED 2018 Solution portal

PART : III MATHEMATICS

SECTION – 1 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen.

Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct options.

Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **-2** In all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

1. For a non-zero complex number z , let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) **FALSE** ?
- (A) $\text{Arg}(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$
- (B) The function $f : \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of \mathbb{R} , where $i = \sqrt{-1}$
- (C) For any two non-zero complex numbers z_1 and z_2 , $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π
- (D) For any three given distinct complex numbers z_1, z_2 and z_3 , the locus of the point z satisfying the condition $\arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$, lies on a straight line.

Ans. (ABD)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

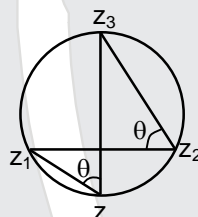
- Sol.** (A) $\text{Arg}(-1 - i) = -\frac{3\pi}{4}$
 (B) $f(t) = \text{Arg}(-1 + it)$

$$\begin{cases} \pi - \tan^{-1} t & t \geq 0 \\ -(\pi + \tan^{-1} t) & t < 0 \end{cases}$$

 If is discontinuous at $t = 0$
 (C) $\text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg} z_1 - \text{Arg} z_2 + 2n\pi$

$$\text{Arg}\left(\frac{z_1}{z}\right) = \text{Arg} z_1 - \text{Arg} z + 2n\pi$$

so the expression becomes $2n\pi$



- (D) $\text{Arg}\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$
 If is circle

2. In a triangle PQR, let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?
 (A) $\angle QPR = 45^\circ$
 (B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$
 (C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$
 (D) The area of the circumcircle of the triangle PQR is 100π

Ans. (BCD)

Sol. $\cos Q = \frac{100 + 300 - (PR)^2}{2 \cdot 10 \cdot 10\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{100 + 300 - (PR)^2}{2 \cdot 10 \cdot 10\sqrt{3}}$

$$300 = 400 - (PR)^2 \Rightarrow PR = 10$$

$$\Delta = \frac{1}{2} (PQ)(QR) \sin Q = \frac{1}{2} \cdot 10 \cdot 10\sqrt{3} \times \frac{1}{2} = 25\sqrt{3}$$

$$r = \frac{\Delta}{s} = \frac{25\sqrt{3} \times 2}{(20 + 10\sqrt{3})} = \frac{50\sqrt{3}}{20 + 10\sqrt{3}} = \frac{5\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 5(2\sqrt{3} - 3) = 10\sqrt{3} - 15$$

by sine rule $\frac{10\sqrt{3}}{\sin R} = \frac{10}{\sin Q} \Rightarrow \angle R = 30^\circ$

$$2(\text{circumradius}) = \frac{PR}{\sin Q} = \frac{10}{1/2} \Rightarrow \text{circumradius} = 10$$

Hence area of circumcircle = $\pi R^2 = 100\pi$

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 08003 444 888 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

3. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?
- (A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
- (B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
- (C) The acute angle between P_1 and P_2 is 60°
- (D) If P_3 is the plane passing through the point (4, 2, -2) and perpendicular to the line of intersection of P_1 and P_2 , then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

Ans. (CD)

Sol. Direction ratio of common line is $n_1 \times n_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(3) + \hat{k}(3) = 3(\hat{i} - \hat{j} + \hat{k})$$

(B) $\frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$

This is || to line of intersection

(C) $\cos \theta = \frac{x_1 x_2}{|x_1| |x_2|} = \frac{2+2-1}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

(D) $P_3 : x - y + z = \lambda$ satisfy (4, 2, -2)

$$4 - 2 - 2 = \lambda \Rightarrow x - y + z = 0$$

$$(2, 1, 1) \perp \Rightarrow \left| \frac{2-1+1}{\sqrt{3}} \right| \Rightarrow \frac{2}{\sqrt{3}}$$

4. For every twice differentiable function $f : \mathbb{R} \rightarrow [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?
- (A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)
- (B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$
- (C) $\lim_{x \rightarrow \infty} f(x) = 1$
- (D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

Ans. (ABD)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

Sol. $f^2(0) + (f'(0))^2 = 85 \quad f : \mathbb{R} \rightarrow [-2, 2]$

(A) This is true of every continuous function

(B) $f'(c) = \frac{f(-4) - f(0)}{-4 - 0}$

$$|f'(c)| = \left| \frac{f(-4) - f(0)}{4} \right|$$

$$-2 \leq f(-4) \leq 2$$

$$-2 \leq f(0) \leq 2$$

$$-4 \leq f(-4) - f(0) \leq 4$$

This $|f'(c)| \leq 1$

(C) $\lim_{x \rightarrow \infty} f(x) = 1$

Note $f(x)$ should have a bound ∞ which can be concluded by considering

$$f(x) = 2 \sin \left(\frac{\sqrt{85} x}{2} \right)$$

$$f'(x) = \sqrt{85} \cos \left(\frac{\sqrt{85} x}{2} \right)$$

$$f^2(0) + (f'(0))^2 = 85$$

and $\lim_{x \rightarrow \infty} f(x)$ does not exist

(D) Consider $H(x) = f^2(x) + (f'(x))^2$

$$H(0) = 85$$

By (B) choice there exists some x_0 such that $(f'(x_0))^2 \leq 1$ for some x_0 in $(-4, 0)$

$$\text{hence } H(x_0) = f^2(x_0) + (f'(x_0))^2 \leq 4 + 1$$

$$H(x_0) \leq 5$$

Hence let $p \in (-4, 0)$ for which $H(p) = 5$

(note that we have considered p as largest such negative number)

similarly let q be smallest positive number $\in (0, 4)$ such that $H(q) = 5$

Hence By Rolle's theorem is (p, q)

$H'(c) = 0$ for some $c \in (-4, 4)$ and since $H(x)$ is greater than 5 as we move from $x = p$ to $x = q$ and $f^2(x) \leq 4$

$$\Rightarrow (f'(x))^2 \geq 1 \text{ in } (p, q)$$

$$\text{Thus } H'(c) = 0 \Rightarrow f f' + f f'' = 0$$

$$\text{so } f + f'' = 0 \text{ and } f' \neq 0$$

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{f(x) - g(x)}) g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Ans. (BC)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

Sol. $f'(x) = e^{f(x)-g(x)} g'(x) : f(1) = g(2) = 1$

$$e^{-f(x)} = e^{-g(x)} + c$$

$$e^{-f(x)} \cdot f'(x) = e^{-g(x)} \cdot g'(x)$$

$$\int d(e^{-f(x)}) = \int d(e^{-g(x)})$$

$$e^{-f(x)} = e^{-g(x)} + c$$

$$x = 1 \quad \frac{1}{e} = e^{-g(1)} + c$$

$$x = 2 \quad e^{-f(2)} = \frac{1}{e} + c$$

$$\therefore g(1) > 1 - \ln 2$$

$$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$$

$$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$$

$$f(2) > 1 - \ln 2$$

$$e^{-1} - e^{-f(2)} = e^{-g(1)} - e^{-1} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2e^{-1}$$

$$e^{-g(1)} < 2e^{-1}$$

$$-g(1) < \ln 2 - 1$$

6. Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$

for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE?

(A) The curve $y = f(x)$ passes through the point (1, 2)

(B) The curve $y = f(x)$ passes through the point (2, -1)

(C) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

Ans. (BC)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

Sol. $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt \Rightarrow f(x) \cdot e^{-x} = (1 - 2x) \cdot e^{-x} + \int_0^x e^{-t} f(t) dt$

$\Rightarrow f'(x)e^{-x} - e^{-x} \cdot f(x) = -2 \cdot e^{-x} - (1 - 2x) \cdot e^{-x} + e^{-x} \cdot f(x)$

$\Rightarrow f'(x) - 2f(x) = (2x - 3)$

I.F. = e^{-2x}

$\therefore y \cdot e^{-2x} = \int (2x - 3) \cdot e^{-2x} dx$

$\Rightarrow y \cdot e^{-2x} = (2x - 3) \cdot \frac{e^{-2x}}{-2} - 2 \int \frac{e^{-2x}}{-2} dx \Rightarrow y \cdot e^{-2x} = -\frac{(2x - 3)e^{-2x}}{2} - \frac{e^{-2x}}{2} + c$

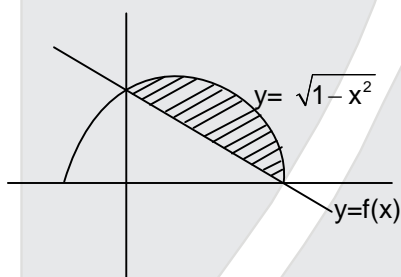
$\Rightarrow y \cdot e^{-2x} = \frac{-(2x - 3) - 1}{2} + c \cdot e^{2x} \Rightarrow y = (1 - x) + c \cdot e^{2x}$

$\Rightarrow y = (1 - x) + c \cdot e^{2x}$

put $x = 0$

$1 = 1 + c \Rightarrow c = 0$

$\therefore y = 1 - x$ which passes through point $(2, -1)$



Now

required area = $\frac{1}{4} \cdot \pi \cdot (1)^2 - \frac{1}{2} \cdot 1 \cdot 1 = \frac{\pi}{4} - \frac{1}{2}$

SECTION – 2 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -0.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	+3	If ONLY the correct numerical value is entered as answer.
Zero Marks	:	0	In all other cases.

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 08003 444 888 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

7. The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is _____.

Ans. (8)

Sol. $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\log_7 4}$
 $(\log_2 9)^{2 \log_2(\log_2 9)} \cdot (2) = 4 \cdot 2 = 8$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____.

Ans. (625)

Sol. Last two digits are 12, 32, 24, 52, 44
 Number of numbers = $5 \times 5 \times 5 \times 5 = 625$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11,, and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23,, Then, the number of elements in the set $X \cup Y$ is _____.

Ans. (3748)

Sol. $P = \{1, 6, 11, \dots\}$
 $Q = \{9, 16, 23, \dots\}$
 Common terms : 16, 51, 86
 $t_p = 16 + (p - 1)35 = 35p - 19 \leq 10086$
 $\Rightarrow p \leq 288.7$
 $\therefore n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
 $= 2018 + 2018 - 288$
 $= 3748$

10. The number of real solutions of the equation

$$\sin^{-1} \left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i \right) = \frac{\pi}{2} - \cos^{-1} \left(\sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i \right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is _____.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively).





Ans. (2)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

Sol. $\sin^{-1} \left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2} \right)^i \right) = \frac{\pi}{2} - \cos^{-1} \left(\sum_{i=1}^{\infty} \left(-\frac{x}{2} \right)^i - \sum_{i=1}^{\infty} (-x)^i \right)$

$$\left(\frac{x^2}{1-x} - x \frac{x}{1-\frac{x}{2}} \right) = \frac{x}{1+x} + \frac{\left(-\frac{x}{2} \right)}{1+\frac{x}{2}}$$

$$\frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{x}{1+x} - \frac{x}{2+x}$$

$$\frac{x^2}{1-x} - \frac{x}{1+x} = \frac{x^2}{2-x} - \frac{x}{2+x}$$

$$\frac{x(1+x) - (1-x)}{1-x^2} = \frac{2x + x^2 - 2 + x}{4-x^2} \text{ or } x = 0$$

$$\frac{x^2 + 2x - 1}{1-x^2} = \frac{x^2 + 3x - 2}{4-x^2}$$

$$\Rightarrow x^3 + 2x^2 + 5x - 2 = 0$$

Let $f(x) = x^3 + 2x^2 + 5x - 2$

$$f'(x) > 0$$

$$f(0) = -2 \text{ and } f(1/2) = 9/8 \text{ so one root in } \left(0, \frac{1}{2} \right)$$

$$\Rightarrow 2 \text{ roots}$$

11. For each positive integer n , let $y_n = \frac{1}{n} ((n+1)(n+2) \dots (n+n))^{1/n}$.

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . If $\lim_{n \rightarrow \infty} y_n = L$, then the value of $[L]$ is

_____.

Ans. (1)

Sol. $y_n = \left(\frac{n+1}{n} \frac{n+2}{n} \dots \frac{n+n}{n} \right)^{\frac{1}{n}}$

$$\log L = \lim_{x \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \left(1 + \frac{r}{n} \right)$$

$$= \int_0^1 \log(1+x) dx = \int_1^2 \log x dx = |x \log x - x|_1^2 = 2 \log 2 = \log \frac{4}{e}$$

$$\Rightarrow L = \frac{4}{e} \Rightarrow [L] = 1$$

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

12. Let \vec{a} and \vec{b} be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector \vec{c} is inclined at the same angle α to both \vec{a} and \vec{b} , then the value of $8 \cos^2 \alpha$ is _____.

Ans. (3)

Sol. $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$ & $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = \alpha$$

$$\vec{c} \cdot \vec{a} = \vec{c} \cdot \vec{b} = 2 \cos \alpha \quad \Rightarrow \quad x = y = 2 \cos \alpha$$

$$|\vec{c}|^2 = x^2 + y^2 + |\vec{a} \times \vec{b}|^2 = 2(4 \cos^2 \alpha) + 1 - 0$$

$$4 = 8 \cos^2 \alpha + 1 \quad \Rightarrow \quad 8 \cos^2 \alpha = 3$$

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3} a \cos x + 2b \sin x = c$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____.

Ans. (0.5)

Sol. $\sqrt{3} a \cos x + 2b \sin x = c \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sqrt{3} a \left(\frac{1-t^2}{1+t^2}\right) + 2b \left(\frac{2t}{1+t^2}\right) = c, \text{ where } t = \tan \frac{x}{2}$$

$$\sqrt{3} a(1-t^2) + 4bt = c(1+t^2)$$

$$t^2(c + \sqrt{3}a) - 4bt + c - \sqrt{3}a = 0$$

$$\frac{\alpha + \beta}{2} = \frac{\pi}{6}$$

$$\tan\left(\frac{\alpha + \beta}{2}\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{t_1 + t_2}{1 - t_1 t_2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{4b}{c + \sqrt{3}a - c + \sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$\frac{b}{a} = \frac{1}{2}$$

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

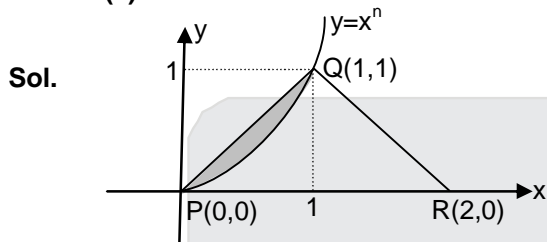
Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in

14. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0)$, $Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ ($n > 1$). If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR , then the value of n is

Ans. (4)



$$\int_0^1 (x - x^n) dx = \frac{3}{10} \left(\frac{1}{2} \times 2 \times 1 \right) \Rightarrow \left| \frac{x^2}{2} - \frac{x^{n+1}}{n+1} \right|_0^1 = \frac{3}{10} \Rightarrow \frac{1}{2} - \frac{1}{n+1} = \frac{3}{10}$$

$$\Rightarrow \frac{1}{n+1} = \frac{1}{2} - \frac{3}{10} = \frac{1}{5} \Rightarrow n = 4$$

SECTION – 3 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If **ONLY** the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -1 In all other cases.

PARAGRAPH "X"

Let S be the circle in the xy -plane defined by the equation $x^2 + y^2 = 4$.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x -axis and the y -axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1 . Let the tangents to S at E_1 and E_2 meet at E_3 , the tangents to S at F_1 and F_2 meet at F_3 , and the tangents to S at G_1 and G_2 meet at G_3 . Then, then, the points E_3 , F_3 , and G_3 lie on the curve
- (A) $x + y = 4$ (B) $(x - 4)^2 + (y - 4)^2 = 16$ (C) $(x - 4)(y - 4) = 4$ (D) $xy = 4$

Ans. (A)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

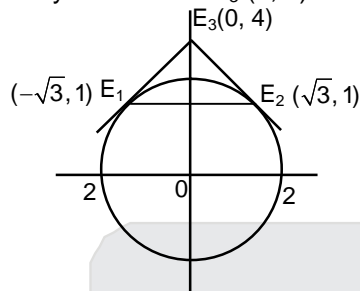
Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 08003 444 888 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

Sol. Tangent at E_1 and E_2 are $-\sqrt{3}x + y = 4$ and $\sqrt{3}x + y = 4$

They intersect at $E_3(0, 4)$



$F_1(1, \sqrt{3}), F_2(1, -\sqrt{3}), F_3(4, 0)$

$G_1(0, 2), G_2(2, 0), G_3(2, 2)$

E_3, F_3, G_3 lie on line $x + y = 4$

PARAGRAPH "X"

Let S be the circle in the xy -plane defined by the equation $x^2 + y^2 = 4$.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N . Then, the mid-point of the line segment MN must lie on the curve

(A) $(x + y)^2 = 3xy$

(B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$

(D) $x^2 + y^2 = x^2y^2$

Ans. (D)

Sol. Let $P(2 \cos \theta, 2 \sin \theta)$
Tangent is $x \cos \theta + y \sin \theta = 2$

$M\left(\frac{2}{\cos \theta}, 0\right), N\left(0, \frac{2}{\sin \theta}\right)$

$$x = \frac{1}{\cos \theta} \text{ and } y = \frac{1}{\sin \theta} \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 1 \Rightarrow x^2 + y^2 = x^2y^2$$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 , and **NONE** of the remaining students gets the seat previously allotted to him/her, is

(A) $\frac{3}{40}$

(B) $\frac{1}{8}$

(C) $\frac{7}{40}$

(D) $\frac{1}{5}$

Ans. (A)

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555 08003 444 888 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

Sol. Probability = $\frac{4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)}{5!} = \frac{9}{120} = \frac{3}{40}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

18. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

- (A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Ans. (C)

Sol. Total cases = $5!$

favorable ways = 14

$$\left. \begin{array}{l} \underline{1} \ \underline{3} \ \underline{5} \ \underline{2} \ \underline{4} \\ \underline{1} \ \underline{4} \ \underline{2} \ \underline{5} \ \underline{3} \end{array} \right\} \rightarrow 2$$

$$5 \rightarrow 2$$

$$\underline{2} \ \underline{4} \ \underline{1} \ \dots \ \dots \rightarrow 2$$

$$\underline{2} \ \underline{5} \ \underline{3} \ \underline{1} \ \underline{4} \rightarrow 1$$

$$4 \rightarrow 3$$

$$\left. \begin{array}{l} \underline{3} \ \underline{1} \ \underline{5} \ \underline{2} \ \underline{4} \\ \underline{3} \ \underline{1} \ \underline{4} \ \underline{2} \ \underline{5} \end{array} \right\} \rightarrow 2$$

$$\underline{3} \ \underline{5} \ \dots \ \dots \ \dots \} \rightarrow 2$$

$$= 14$$

$$\text{Probability} = \frac{14}{120}$$

Resonance Eduventures Ltd.

REG. & CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph.No. : +91-744-3012222, 6635555 | To Know more : sms RESO at 56677

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in | CIN : U80302RJ2007PLC024029

Toll Free : 1800 258 5555  08003 444 888  facebook.com/ResonanceEdu  twitter.com/ResonanceEdu  www.youtube.com/resowatch  blog.resonance.ac.in