®

INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2019-20

Examination Date : 24-11-2019 **Time: 2 Hrs.** **Max. Marks : 240**

Q. PAPER CODE : 42 **HBCSE Olympiad (STAGE - 1)**

Write the question paper code mentioned above on YOUR answer sheet (in the space provided), otherwise your answer sheet will NOT be assessed. Note that the same Question Paper. Code appears on each page of the question paper.

INSTRUCTION TO CANDIDATES

1. Use of mobile phones, smart watches and ipads during examination is **STRICTLY PROHIBITED.**
2. In addition to this question paper, you are given answer sheet along with Candidate's copy.
3. On the answer sheet, make all the entries carefully in the space provided ONLY in **BLOCK** CAPITALS as well as by properly darkening the appropriate bubbles.
   Incomplete/Incorrect/carelessly filled information may disqualify your candidature.
4. On the answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.
5. Your **ten-digit roll number and date of birth** entered on the OMR Answer Sheet shall remain your login credentials means login id and password respectively for accessing your performance / result in NSE-2019.
6. Question paper contain 80 multiple-choice question. Each question has 4 options, out of which only one is correct. Choose the correct alternative and fill the appropriate bubble, as shown

Q. No. 22 (a) ● (c) (d)

7. A correct answer carries 3 marks and 1 mark will be deducted for each wrong answer.
8. Any rough work should be done only in the space provided.
9. Use of **non-programmable scientific** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting your answer paper, take away the Candidate's copy for your reference.

Please DO NOT make any mar other than filling the appropriate bubbles properly in the space provided on the answer sheet. Answer sheet are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED.

Scratching or overwriting may result in wrong score.

DO NOT WRITE ANYTHING ON THE BACK OF ANSWER SHEET.

Instructions to candidates :

You may read the following instructions after submitting the answer sheet.

12. **Comments/Inquiries/Grievences regarding this quesiton paper, if any, can be shared on the Inquiry/Grievence column on www.iaptexam.in on the specified format till 27th November, 2019.**
13. **The answers/solutions to this question paper will be available on the website : www.iapt.org.in by 2nd December, 2019.**
14. **CERTIFICATES and AWARDS –**
    Following certificates are awarded by IAPT to students successful in the NSEA-2019.
    (i) "CENTRE TOP 10%"
    (ii) "STATE TOP 1%"
    (iii) "NATIONAL TOP 1%" .
    (iv) "GOLD MEDAL & MERIT CERTIFICATE" to all students who attend OCSC-2020 at HBCSE.
15. All these certificates (except GOLD Medal) will be sent/dispatched to the centre in-charge by February 1, 2020 along with the result sheet of the centre.
16. List of students (with centre number and roll number only) having score above MAS will be displayed on the website : www.iapt.org.in by **December 20nd, 2019**. **See the Minimum Admissible score Clause** on the Student's brochure on the web**.**
17. List of Students eligible for the Indian National Chemistry Olympiad (INAO-2020) shall be displayed on www.iapt.org.in by December 28,2019. These students have to register/enroll themselves on the website : Olympiads.hbcse.tifr.in of HBCSE Mumbai within the specified period.

Atempt All Eighty Question
ONLY ONE OUT OF FOUR OPTIONS IS CORRECT

1. If the product of two roots of the equation $x^4 - 11x^3 + kx^2 + 269x - 2001 = 0$ is -69, the value of k is

(a) 10 (b) 1 (c) -5 (d) -10

Ans. **(d)**

Sol. $x^4 - 11x^3 + k x^2 + 269 x - 2001 = 0$

$\alpha\beta = -69$ given

$\alpha\beta\gamma\delta = -2001$

$\Rightarrow \quad \gamma\delta = \dfrac{-2001}{-69} = \dfrac{667}{23} = 29$ ....(1)

$\alpha + \beta + \gamma + \delta = 11$ ……(2)

$k = \alpha\beta + \gamma\delta + (\alpha + \beta)(\gamma + \delta)$

$k = -69 + 29 + (\alpha + \beta)(\gamma + \delta)$

$k = -40 + (\alpha + \beta)(\gamma + \delta)$ .....(3)

also, $\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) = -269 \quad \Rightarrow -69(\gamma + \delta) + 29(\alpha + \beta) = -269$

$\Rightarrow -69(\gamma + \delta) + 29\{11 - (\gamma + \delta)\} = -269 \Rightarrow -98(\gamma + \delta) = -269 - 319$

$\Rightarrow \gamma + \delta = \dfrac{588}{98} = 6 \quad \Rightarrow \alpha + \beta = 5 \quad \Rightarrow k = -40 + (5)(6) = -10$

2. By a chord of the curve $y = x^3$ we mean any line joining two points on it. The number of chords which have slope -1 is

(a) Infinite (b) 0 (c) 1 (d) 2

Ans. **(b)**

Sol. Let (α, α^3) & (β, β^3) are two points on curve

$$\Rightarrow \text{slope } \frac{\beta^3 - \alpha^3}{\beta - \alpha} = -1 \quad \Rightarrow \frac{(\beta - \alpha)(\beta^2 + \alpha\beta + \alpha^2)}{(\beta - \alpha)} = -1$$

$\beta \neq \alpha \Rightarrow \beta^2 + \alpha\beta + \alpha^2 = -1$

Which is not possible for any α & β

3. The universe is estimated to be between ten and twenty billion years old. This estimate is based on the value of the constant(s)

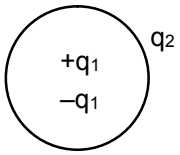
(a) Speed of light (b) Fine structure constant

(c) Planck constant (d) Hubble constant

Ans. **(d)**

Sol. **Informative**

4. Consider the charge configuration and a spherical Gaussian surface as shown in the figure. While calculating the flux of the electric field over any part of the spherical surface the electric field will be due to



(a) q_2 (b) Only the positive charges

(c) All the charges (d) $+q_1$ and $-q_1$

Ans. **(c)**

Sol. All the charges [$\bar{E}$ is due to all the charges which are outside as well as inside]

5. The expression for electric potential at any point due to an electric dipole $(\bar{p})$ is (with usual notation)

(a) $k\frac{\bar{p}\times\bar{r}}{r^2}$ (b) $k\frac{\bar{p}\times\bar{r}}{r^3}$ (c) $k\frac{\bar{p}\,.\,\bar{r}}{r^2}$ (d) $k\frac{\bar{p}\,.\,\bar{r}}{r^3}$

Ans. **(c)**

Sol. Formula based

6. A function f(n) is defined by f (n) = $\frac{4^n-4^{-n}}{4^n+4^{-n}}$ for every integer n. If p and q are integers such that p > q, the sign of f(p) –f(q) is

(a) Positive (b) Negative (c) Indeterminate (d) Zero

Ans. **(a)**

Sol. $f(n)=\frac{4^n-4^{-n}}{4^n+4^{-n}} \quad \forall n\in I$

$f(n)=\frac{4^{2n}-1}{4^{2n}+1}$

$f'(n)=\frac{(4^{2n}+1)(4^{2n}.\ell n4.2)-(4^{2n}-1)(4^{2n}.2\ell n4)}{(4^{2n}+1)^2}=\frac{4^{2n}.4\ell n4}{(4^{2n}+1)^2}>0\ \forall n\in I$

$\Rightarrow$ f(n) is an increasing function as p > q

f(p) > f(q) $\Rightarrow$ f(p) – f(q) > 0

7. The first term of a descending A.P. series of 4 distinct positive integers with greatest possible last term and sum 2004 is

(a) 552 (b) 536 (c) 512 (d) 504

Ans. **(d)**

Sol. Let terms of AP are

$a, a - d, a - 2d, a - 3d$ where $d > 0$

Now $a + a - d + a - 2d + a - 3d = 2004$

$\Rightarrow \quad 4a - 6d = 2004$

$\Rightarrow \quad 2a - 3d = 1002$

$a + (a - 3d) = 1002$

$(a - 3d)$ is maximum and $a > a - 3d \quad \Rightarrow \quad a - 3d < 501$

also $d \in I$ and $a \in I$.

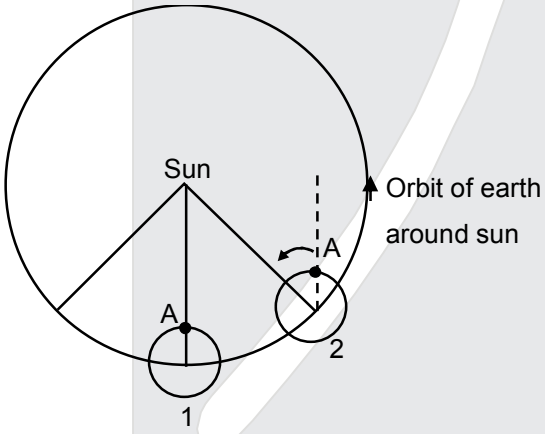
$\Rightarrow \quad a = 504$ k& $a - 3d = 498$ & $d = 2$

8. If the Earth is made to rotate in the opposite sense (clockwise rather than counterclockwise), the length of a solar day will be

(a) 23 hr 56 min (b) 24 hr (c) 23hr 52 min (d) 24hr 4 min

Ans. **(a)**

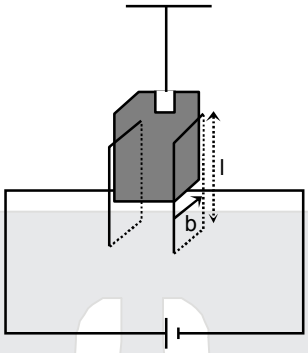
Sol.



In one rotation time of earth about its axis, it moves from 1 to 2 but position of sun is not in front of point A. So it needs to rotate slightly more to make sun in front of point A. So a solar day (24 hr) is slightly greater then rotation time period of earth.

When earth rotates in opposite direction then it needs to rotate slightly less to make point A in front of sun. So now solar day will be less then rotation time period of earth so 23 hr 56 min

9. A dielectric slab of constant $\varepsilon_r = 2$ is inserted into a parallel plate capacitor as shown in the figure. The spring attached to the slab can set the slab into oscillation of frequency ω. At equilibrium, end of the slab is pulled till the entire slab is inside the capacitor and allowed to oscillate, the current flowing through the circuit is

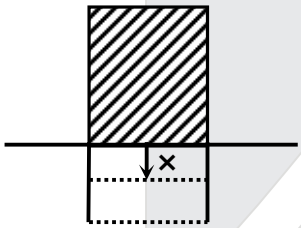


(a) $-\frac{b\in_0 V\omega\ell}{2d}\sin(\omega t)$ (b) $+\frac{b\in_0 V\omega\ell}{2d}\cos(\omega t)$

(c) $-\frac{b\in_0 V\omega\ell}{2d}\sin^2(\omega t)$ (d) $-\frac{b\in_0 V\omega\ell}{2d}\cos^2(\omega t)$

Ans. **(a)**

Sol.



$$C = C = \frac{\in_0\left[2\left(\frac{\ell}{2}+x\right)b+1\left(\frac{\ell}{2}-x\right)b\right]}{d}$$

$$x = \frac{\ell}{2}\cos\omega t$$

$$c = \frac{b\in_0}{d}\left[\frac{3\ell}{2}+\frac{\ell}{2}\cos\omega t\right]$$

$$q = \frac{b\in_0 v}{d}\frac{\ell}{2}[3+\cos\omega t]$$

$$i = \frac{dq}{dt}$$

$$= -\frac{\omega b\in_0 v\ell}{2d}\sin\omega t$$

10. If a, b,c are in G.P. and $\log_c a$, $\log_b c$ and $\log_a b$ are in A.P., the common difference of the A.P. is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{3}{2}$

Ans. **(d)**

Sol. a , b = ar , C = ar^2 put $\log_a r = x$

$\log_{ar^2} a, \log_{ar} ar^2, \log ar$ are in A.P

$$\frac{1}{1+2x}, \frac{1+2x}{1+x}, \frac{1+x}{1} \text{ are in A.P.}$$

$$\Rightarrow \quad 2\left(\frac{1+2x}{1+x}\right) = \frac{1}{1+2x} + 1+x$$

$$\frac{2(1+2x)}{(1+x)} = \frac{1+(1+x)(1+2x)}{(1+2x)}$$

$2(2x+1)^2 = (1+x)\{2x^2+3x+2\}$

$8x^2 + 8x + 2 = 2x^2 + 3x + 2 + 2x^3 + 3x^2 + 2x$

$2x^3 - 3x^2 - 3x = 0$

$X = 0$ or $2x^2 - 3x - 3 = 0 \Rightarrow \frac{x^2}{(1+x)} = \left(\frac{3}{2}\right)$

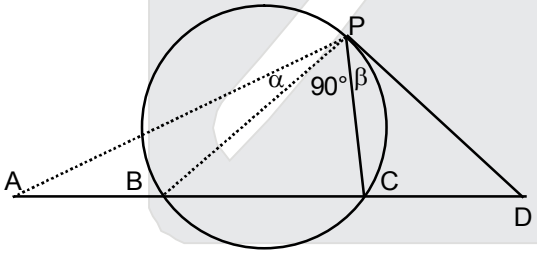
since $\frac{(1+x)^2-(2x+1)}{(1+x)} = \frac{x^2}{1+x} = \frac{3}{2}$

11. ABCD is a line segment trisected by the points B and C while P is any point on the circle whose diameter is BC. If angles APB and CPD are α and β respectively, the value of $\tan\alpha \times \tan\beta$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{2}$

Ans. **(b)**

Sol.



Applying m – n rule in ΔAPC & ΔBPD

$2\cot\theta = \cot\alpha - \cot 90°$ …(i)

$2\cot(90+\theta) = \cot 90 - \cot\beta$ …(ii)

Multiplying (i) & (ii)

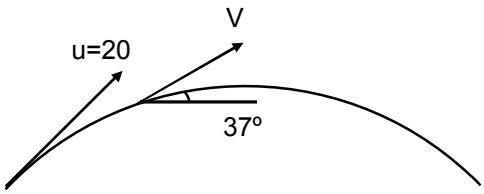
$\tan\alpha \tan\beta = \frac{1}{4}$

12. A particle is projected with a velocity 20 ms^{-1}, at angle of 60° to the horizontal. Then radius of curvature of its trajectory at a point where its velocity makes an angle of 37° with the horizontal is close to ($g = 10ms^{-2}$)

(a) 16 m (b) 19.53 m (c) 15.52 m (d) 25 m

Ans. **(b)**

Sol.



$V\cos 37^\circ = 20\cos 60 = 10$

$V = \frac{10\times 5}{4} = 12.5$

$Roc = \frac{(12.5)^2}{10\cos 37} = \frac{156.25}{10\times 4/5} = \frac{625}{24} = 19.53$

13. A body is thrown up in a lift with a velocity u relative to moving lift and its time of flight is t. The acceleration with which the lift is moving will be

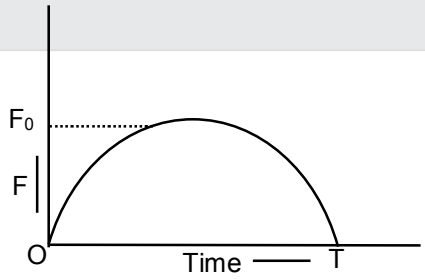
(a) $\frac{u^2}{t} - g$ (b) $\frac{2u}{t} - g$ (c) $\frac{2u}{t}$ (d) none

Ans. **(b)**

Sol. $t = \frac{2u}{g+a} \Rightarrow g + a = \frac{2u}{t}$

$\Rightarrow a = \frac{2u}{t} - g$

14. A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u just after the force stops acting. F, as a function of time, is shown in the graph. The value of u is



(a) $u = \frac{\pi F_0^2}{2m}$ (b) $u = \frac{\pi T^2}{8m}$ (c) $u = \frac{\pi F_0 T}{4m}$ (d) $u = \frac{F_0 T}{2m}$

Ans. **(c)**

Sol. $\frac{1}{2}\pi F_0 \frac{T}{2} = mu$

so $u = \frac{\pi F_0 T}{4m}$

15. If a and b are the roots of $x^2 - 3x + p = 0$ while c and d are the roots of $x^2 - 12x + q = 0$. Also if a, b, c, d are in GP then the value of $\frac{(q+p)}{(q-p)}$ is

(a) $\frac{17}{15}$ (b) $\frac{19}{16}$ (c) $\frac{35}{32}$ (d) $\frac{9}{8}$

Ans. **(a)**

Sol. $x^2 - 3x + p = 0$ have roots a & b where b = ar

$x^2 - 12x + q = 0$ have roots c & d where $c = ar^2$, $d = ar^3$

$a + ar = 3$ $ar^2 + ar^3 = 12$

$a^2r = p$ $a^2r^5 = q$

$r^3(3) = 12 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

case-I : If r = 2, then a = 1

p = 2 & q = 32

$$\frac{q+p}{q-p} = \frac{34}{30} = \frac{17}{15}$$

case-II : If r = –2, then a = –3

p = –18 q = –9 × 32

$$\frac{q+p}{q-p} = \frac{-9\times 32-18}{-932+18} = \frac{-9(32+2)}{-9(32-2)} = \frac{34}{30}$$

16. The date on which the Earth is at a minimum distance from the Sun is around

(a) 3 January (b) 4 July (c) 22 June (d) 21 September

Ans. **(a)**

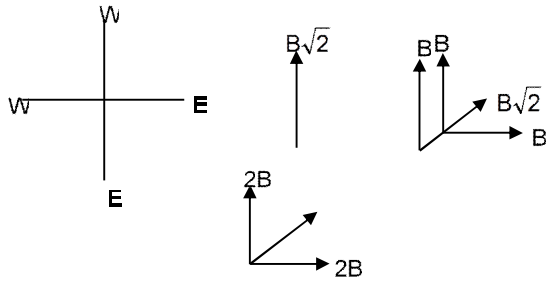
Sol. **Informative**

17. The plane of the circular coil is held in the east–west direction. A steady current passed through the coil produces a magnetic field (B) equal to $\sqrt{2}$ times the horizontal component of the earth's magnetic field (B_H) at the place. Now the plane of the coil is rotated carefully through an angle 45° about the vertical axis through its diameter. What is the deflection of the needle placed at the centre of the coil with respect to B_H?

(a) $\tan^{-1}\frac{1}{2}$ (b) $\tan^{-1}\frac{1}{3}$ (c) $\tan^{-1}2$ (d) $\tan^{-1}3$

Ans. **(a)**

Sol.



$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\frac{1}{2}$$

18. If the ratio of the sum of first n terms of two different AP series is (7n + 1) : (4n + 27), the ratio of their 10^{th} term is

(a) $\frac{135}{102}$ (b) $\frac{134}{103}$ (c) $\frac{78}{69}$ (d) $\frac{103}{89}$

Ans. **(b)**

Sol. $\frac{S_n}{S_n'} = \frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2A+(n-1)D)} = \frac{7n+1}{4n+27} \Rightarrow \frac{a+\left(n-\frac{1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$

For 10^{th} term $\frac{n-1}{2} = 9 \Rightarrow n = 19$

$$\frac{t_{10}}{T_{10}} = \frac{7\times 19+1}{4x19+27} = \frac{134}{103}$$

19. The solution to the set of simultaneous equations with three variables x, y, z

$x + y + z = 5$; $x^2 + y^2 + z^2 = 5$ and $xy + yz + zx = 10$ are

(a) all real (b) one real, other two complex

(c) two real, one complex (d) all complex

Ans. **(d)**

Sol. Let $x = \lambda$, $y + z = 5 - \lambda$, $y^2 + z^2 = 5 - \lambda^2$

$yz = l^2 - 5\lambda + 10$

if $t^2 - (5 - \lambda) + \lambda^2 - 5\lambda + 10 = 0$

then y & z are roots of its equation

whose discriminant is negative if $\lambda \in R$

20. An asteroid's closest approach to the Sun (perihelion) is 2 AU, and farthest distance from the Sun (aphelion) is 4 AU. The period of revolution of the asteroid and the eccentricity of the orbit are respectively.

(a) 4.2 Year, 0.40 (b) 4.8 Year, 0.50

(c) 5.2 Year, 0.33 (d) 6.0 Year, 0.20

Ans. **(c)**

Sol. $1 = \frac{2\pi}{\sqrt{Gm}}(1)^{3/2}$

$T = \frac{2\pi}{\sqrt{Gm}}(3)^{3/2}$ $\qquad T = 3 \times \sqrt{3}$

$= 3 \times 1.73$

$\cong 5.29$

$2 = a(1 - e)$ $\qquad 2 = \frac{1+e}{1-e}$

$4 = a(1 + e)$ $\qquad 2 - 2e = 1 + e$

$\Rightarrow e = 0.33$

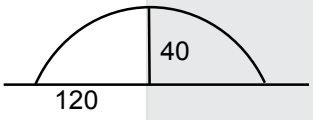
21. A and B are two locations 120 m apart with a 40 m tall wall midway between them. The minimum velocity and the corresponding angle at which a ball be projected from A so as just to clear the wall and strike at B are :

(a) 28.63 ms^{-1} and 53° (b) 28.63 ms^{-1} and 37°

(c) 35.06 ms^{-1} and 53° (d) 28.28 ms^{-1} and 37°

Ans. **(c)**

Sol.



$R = \frac{u^2 \sin 2\theta}{g} = 120$

$\frac{u^2 \sin^2\theta}{2g} = 40$

$\frac{\tan\theta}{4} = \frac{1}{3}$

$\tan\theta = \frac{4}{3}$

$\theta = 53°$

$\frac{u^2}{20}\frac{16}{25} = 40 \Rightarrow u = \frac{5}{4} \times 20\sqrt{2} = 25\sqrt{2} = 35.06$

22. The argument [arg(z)] of the colmplex number $z = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$

Ans. **(c)**

Sol. $Z = e^{i\left(2\pi/3\right)}$

23. The eccentricity of the ellipse represented by $4x^2 + 9y^2 - 16x = 20$ is

(a) $\frac{3}{5}$ (b) $\frac{\sqrt{5}}{3}$ (c) $\frac{\sqrt{3}}{5}$ (d) $\frac{1}{2}$

Ans. **(b)**

Sol. $4x^2 + 9y^2 - 16x = 20$

$\Rightarrow 4(x^2 - 4) + 9y^2 = 20$

$\Rightarrow 4(x-2)^2 + 9y^2 = 36$

$\Rightarrow \frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$

$\therefore b^2 = a^2(1 - e^2) \Rightarrow 4 = 9(1 - e^2) \Rightarrow e^2 = 1 - \frac{4}{9} = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

24. Which of the following is true for retrograde motion (apparent motion of planets in a direction opposite to the normal) ?

(a) caused by epicycles

(b) undergone only by superior planets

(c) undergone only by inferior planets

(d) an effect due to the projection of planet's orbit onto the sky

Ans. **(a)**

Sol. Informative

25. A proton and a deuteron are projected into a uniform magnetic field with velocities 50×10^6 ms^{-1} and 43.3×10^6 ms^{-1} at angles 30° and 60° respectively with respect to the direction of the magnetic field. Compare the respective pitch of their helical paths.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 4 : 1

Ans. **(c)**

Sol. $\frac{P_1}{P_2} = \frac{u'\cos\theta' \frac{2\pi m'}{q'B}}{u''\cos\theta'' \frac{2\pi m''}{q''B}}$ $q' = q''$

$\frac{50 \times 10^6 \frac{\sqrt{3}}{2}}{43.3 \times 10^6 \frac{1}{2}} \cdot \frac{1}{2}$

$\frac{50 \times 1.7}{43.2 \times 2} = 1 : 1$

26. In a time of 4.606 days a radioactive sample loses $\left(\frac{4}{5}\right)^{th}$ of the amount present initially. The mean life of the sample is :

(a) 2.2 day (b) 2.86 day (c) 2.46 day (d) 2.95 day

Ans. **(b)**

Sol. $\frac{1}{5} = e^{-\lambda(4.606)}$

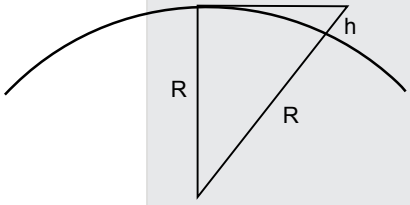
$$\frac{\ell n5}{4.606} = \lambda \Rightarrow < T > = \frac{4.606}{\ell n5} = 2.86$$

27. Assuming radius of the earth to be 6400 Km, the distance to the horizon visible from a 180 m tall building is close to :

(a) 56 km (b) 48 km (c) 90 km (d) 64 km

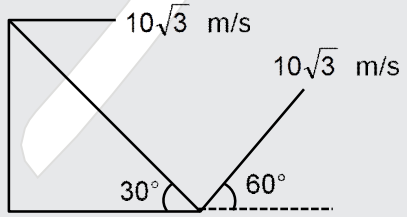
Ans. **(b)**

Sol.



$$\sqrt{2\times 6400} \times \frac{180}{1000} = 80 \times \frac{6}{10} = 48 \text{ km}$$

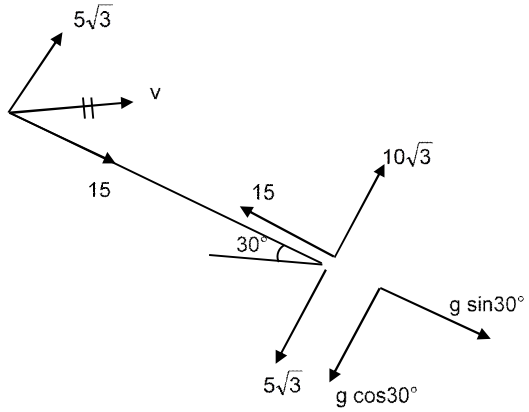
28. A ball is projected with a speed of $10\sqrt{3}$ ms^{-1} at angle of 60° from the foot of a wedge as shown in the figure. The wedge also starts moving horizontally with a speed of $10\sqrt{3}$ ms^{-1}. The time after which the ball hits the inclined surface of the wedge is :



(a) $\sqrt{2}$ s (b) 2s (c) $\sqrt{3}$ s (d) 3s

Ans. **(b)**

Sol.



$$T = \frac{2(10\sqrt{3} - 10\sqrt{3}/2)}{g\sqrt{3}/2} = 2 \text{ sec}$$

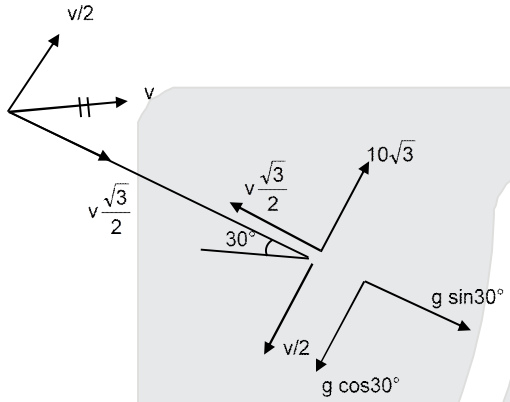
29. In the previous question (number 28), what whould be the velocity of the wedge so that the ball hits the incline perpendicular to it :

(a) $8\sqrt{3}$ ms^{-1} in the same direction (b) $5\sqrt{3}$ ms^{-1} in the opposite direction

(c) $16\sqrt{3}$ ms^{-1} in the same direction (d) $15\sqrt{3}$ ms^{-1} in the opposite direction

Ans. **(a)**

Sol.



$$T = \frac{2\left(10\sqrt{3}-\frac{v}{2}\right)}{5\sqrt{3}} = \frac{v\sqrt{3}/2}{5}$$

$$2\left(10\sqrt{3}-\frac{v}{2}\right)=\frac{3v}{2}$$

$$10\sqrt{3}-\frac{v}{2}=\frac{3v}{4}$$

$$10\sqrt{3}=\frac{5v}{4}$$

$$v = 8\sqrt{3}$$

30. If 'p' is the perpendicular distance from the origin to the straight line $x\cos\theta - y\sin\theta = k\cos 2\theta$ and 'q' is the corresponding distance to the straight line $x\sec\theta - y\operatorname{cosec}\theta = k$, then the value of $p^2 + 4q^2$ is:

(a) $2k^2$ (b) k^2 (c) $4k^2$ (d) $3k^2$

Ans. **(b)**

Sol. $P + \left|\frac{0 + 0 - K\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = K\cos 2\theta$

$\Rightarrow P^2 = k^2\cos^2 2\theta$ ---------- (1)

$q = \left|\frac{0 + 0 - K}{\sqrt{\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}}}\right| = K\sin\theta\cos\theta$

$\therefore \quad 4q^2 = 4K^2\sin^2\theta\cos^2\theta$ ----------(2)

$\therefore \quad p^2 + 4q^2 = K^2(\cos^2 2\theta + 4\sin^2\theta\cos^2\theta)$

$= K^2((\cos^2\theta - \sin^2\theta)^2 + 4\sin^2\theta\cos^2\theta) = K^2(\sin^2\theta + \cos^2\theta)^2 = K^2$

31. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$ then $\sin\frac{\theta}{2}$ is equal to :

(a) 1 (b) $\frac{1}{2}|\vec{a} - \vec{b}|$ (c) 0 (d) $\frac{1}{2}|\vec{a} + \vec{b}|$

Ans. **(b)**

Sol. $|\vec{a}| = 1 \quad |\vec{b}| = 1$

Let $(\vec{a} - \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$= 1+1 - 2|\vec{a}||\vec{b}|\cos\theta$

$= 2(1 - \cos\theta) = 2\left(2\sin^2\frac{\theta}{2}\right) = \left(2\sin\frac{\theta}{2}\right)^2$

$\Rightarrow \quad \left(2\sin\frac{\theta}{2}\right) = |\vec{a} - \vec{b}|$

$\Rightarrow \quad \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$

32. Which of the following statement is true for Saturn and Jupiter ?

(a) both rotate faster than the Earth

(b) both rotate slower than the Earth

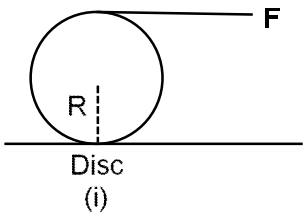
(c) only one rotates rapidly while the other rotates very slowly compared to Earth

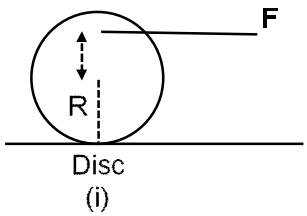
(d) their periods of rotation are linked to their period of revolution

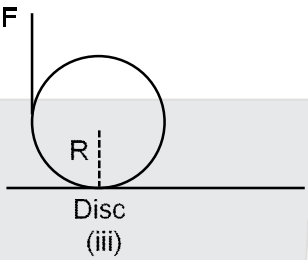
Ans. **(a)**

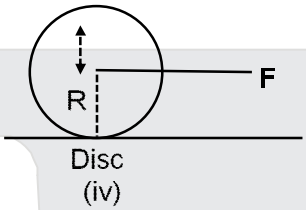
Sol. Informative

33. In which of the following cases friction cannot be zero for the disc having pure rolling ?





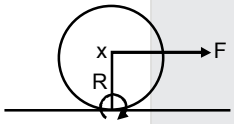




(a) (i), (iii) and (iv) (b) (i) and (ii) only (c) (i), (ii), (iii) and (iv) (d) (i), (ii) and (iii)

Ans. **(a)**

Sol.



$$\frac{F(R+x)}{\frac{3}{2}mR^2} = \frac{fR+Fx}{\frac{MR^2}{2}}$$

$F(R+x) = 3fR + 3Fx$

$f = 0$

$R + x = 3x \Rightarrow x = \frac{R}{2}$

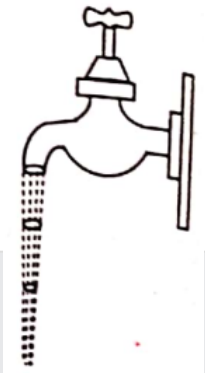
34. In the case of a diatomic gas, the ratio of the heat used in doing work for expansion of the gas to the total heat given to it at constant pressure is :

(a) $\frac{2}{5}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

Ans. **(c)**

Sol. $\frac{P\Delta V}{\left(\frac{f}{2}+1\right)P\Delta V} = \frac{2}{f+2} = \frac{2}{7}$

35. Water coming out of the mouth of a tap and falling vertically in streamline flow forms a tapering column, i.e., the area of cross–section of the liquid column goes on decreasing as it moves down, the most accurate explanation for this is :



(a) As the water moves down, its speed increases and hence its pressure decreases, it is then compressed by the atmosphere

(b) Falling water tries to reach a terminal velocitity and hence reduces the area of cross–section to balance upward and downward forces

(c) The mass of water flowing past any cross–section must remain constant. Also, water is almost incompressible. Hence, the rate of volume flow must remain constant.

(d) The surface tension causes the exposed surface area of the liquid to decrease continuously.

Ans. **(c)**

Sol. $A\nu$ = constant

ν is increasing

So A is decreasing

36. If x, y, z are positive integers with x + y + z = 10, the maximum value of xy + yz + zx + xyz is :

(a) 69 (b) 59 (c) 64 (d) 61

Ans. **(a)**

Sol. $1 + x + y + z + xy + yz + zx + xyz = (1 + x)(1 + y)(1 + z)$

$$\frac{(1+x)+(1+y)+(1+z)}{3} \geq \{(1 + x)(1 + y)(1 + z)\}^{1/3}$$

$$\frac{13}{3} \geq \{(1 + x)(1 + y)(1 + z)\}^{1/3}$$

$$(1 + x)(1 + y)(1 + z) \leq \left(\frac{13}{3}\right)^3$$

possible only if

$\Rightarrow$ x = 3, y = 3 and z = 4

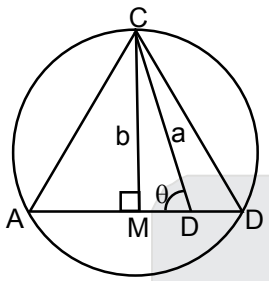
$xy + yz + zx + xy + |_{max} = 80 - 11 = 69$

37. If a and b are positive real numbers and AB is a line segment in a plane. The possible number of distinct points C in the plane for which the triangle ABC will have the lengths of medians and altitudes through C as a and b respectively is :

(a) 1 (b) 2 (c) 4 (d) Infinitely many

Ans. **(c)**

Sol.



$\sin\theta = \frac{b}{a}$

since $\angle CDM$ is fixed

Hence four such points are there

38. The pair of planets, that is never visible at midnight is :

(a) Mars and Neptune (b) Venus and Neptune

(c) Neptune and Mercury (d) Mercury and Venus

Ans. **(d)**

Sol. Informative

39. A drop of water is broken into two droplets of equal size. For this process, the correct statement is

(a) The sum of temperature of the two droplets together is equal to the original temperature of the drop

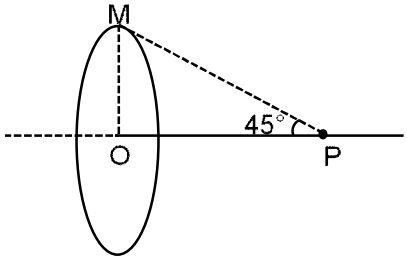
(b) The sum of masses of the two droplets is equal to the original mass of the drop.

(c) The sum of the radii of two droplets is equal to the radius of the original drop.

(d) The sum of the surface areas of the two droplets is equal to the surface area of the original drop.

Ans. **(b)**

40. A circular coil carrying a definite current 'i' produces a magnetic field B_0 = 2.83 T at the center 'O' of the coil. The magnetic field produced by the same current at a point 'P' on the axis of the coil, where the angle OPM as shown in figure is 45°, is :



(a) 1.23 T (b) 1.00 T (c) 1.78 T (d) 2.83 T

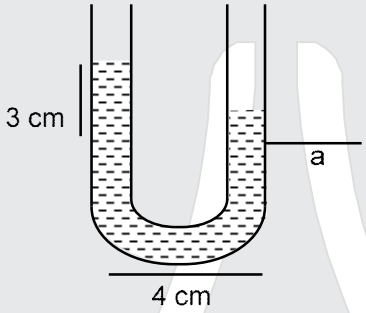
Ans. **(b)**

Sol. $\frac{\mu_0 iR^2}{2(R+x^2)^{3/2}} = B$ $\qquad$ $\frac{\mu_0 i}{2R} = 2.83$

$R = x$

$$B = \frac{\mu_0 iR^2}{2(2)^{3/2}R^3} = \frac{\mu_0 i}{4\sqrt{2}R} = \frac{2.83.2R}{4\sqrt{2}R} = \frac{2.83}{2\times 1.4} \cong 1$$

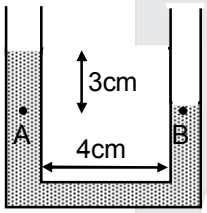
41. A narrow U–tube of uniform cross–section, having a liquid ofdensity 'ρ', is made to move with an acceleration 'a' along the horizontal direction as shown in the figure. What should be the value of 'a' so that the difference in the level in the two arms of the tube is 3 cm (g = 10 ms^{-2}) ?



(a) 6.5 ms^{-2} (b) 5.5 ms^{-2} (c) 13.3 ms^{-2} (d) 7.5 ms^{-2}

Ans. **(d)**

Sol.



$P_A - P_B = \rho g \times 3 \times 10^{-2}$

$P_A - P_B = \rho a \times 4 \times 10^{-2}$

$\rho g \times 3 \times 10^{-2} = \rho a \times 4 \times 10^{-2}$

$a = 7.5\ m/s^2$

42. The value of $4 \sin 50° - \sqrt{3} \tan 50°$ is :

(a) –2 (b) –1 (c) 1 (d) $\sqrt{3}$

Ans. **(c)**

Sol. $4\sin 50 - \sqrt{3}\left(\frac{\sin 50}{\cos 50}\right) = \frac{2\sin 100 - \sqrt{3}\sin 50}{\cos 50} = \frac{2\cos 10 - \sqrt{3}\cos(30+10)}{\cos 50}$

$$= \frac{2\cos 10 - \sqrt{3}\left\{\frac{\sqrt{3}}{2}\cos 10 - \frac{1}{2}\sin 10\right\}}{\cos 50} = \frac{\frac{\cos 10}{2} + \frac{\sqrt{3}}{2}\sin 10}{\cos 50} = \frac{\cos 50}{\cos 50} = 1$$

43. The value of Z = cos10° cos30° cos50° cos70° is :

(a) $\frac{3}{8}$ (b) $\frac{5}{16}$ (c) $\frac{5}{8}$ (d) $\frac{3}{16}$

Ans. **(d)**

Sol. z = cos 10 cos(60 – 10) cos(60 +10) cos 30°

$= \frac{1}{4}\cos^2 30$

$= \frac{1}{4}\times\frac{3}{4} = \frac{3}{16}$

44. The number of times a 5^{th} magnitude star is brighter than an 8^{th} magnitude star is :

(a) 15.85 (b) 3 (c) 20 (d) 6.4

Ans. **(a)**

Sol. 2.512 × 2.512 × 2.512 = 15.85

$\frac{\text{Brightness of } 7^{th} \text{ magnitude star}}{8^{th}} = 2.512$

$\frac{B_{6^{th}}}{B_{7^{th}}} = 2.512$

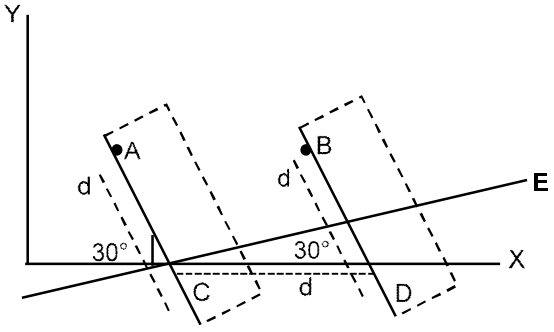
$\frac{B_{5^{th}}}{B_{6^{th}}} = 2.512$

45. A uniform gravitational field (E) exists in a certain region of space. Consider two parallel planes perpendicular to the field E as shown in the figure. Three statements are given below:

(i) Work done by the force in moving a particle of mass m from point A to B is $\frac{1}{2}$mEd

(ii) Work done in moving a particle from point B to D is zero

(iii) Work done in moving a particle from A to D is $\frac{\sqrt{3}}{2}$mEd



Choose the correct statement (s)

(a) (i), (ii) and (iii)

(b) (i) and (ii) only

(c) (i) only

(d) (ii) and (iii) only

Ans. **(d)**

Sol. $W_{\text{by F}}\big|_{A\to B} = -W_{\text{by E}}\big|_{A\to B} = -(-mEd\cos 30°) = \dfrac{mEd\sqrt{3}}{2}$

$W_{\text{by E}}\big|_{B\to D} = 0$

$W_{\text{by F}}\big|_{A\to D} = W_{\text{by F}}\big|_{A\to B} = \dfrac{mEd\sqrt{3}}{2}$

46. A bullet of mass m moving horizontally with velocity v strikes a wooden block of mass M suspended with a string of length 1. The bullet gets embedded into the block as a result the block rises up to a height h. The initial speed of wooden block and the embedded bullet system is :

(a) $\sqrt{2gh}$ (b) $\dfrac{M+m}{m}\sqrt{2gh}$ (c) $\dfrac{m}{M+m}2gh$ (d) $\dfrac{M+m}{M}\sqrt{2gh}$

Ans. **(a)**

Sol. $mv = (M + m)\, v'$ $\left(\vec{P}_f = \vec{P}_i\right)$

$\dfrac{1}{2}(M + m)\, v'^2 = (M + m)gh$ $(ME_i = ME_f)$

$\dfrac{1}{2}\dfrac{m^2v^2}{(M+m)^2} = gh$

$v = \dfrac{M+m}{m}\sqrt{2gh}$

But since they have asked the initial speed of wooden block and the embedded bullet system, therefore the answer should be $\sqrt{2gh}$

47. The number of diagonals in a regular polygon of 100 sides is :

(a) 4950 (b) 4850 (c) 4750 (d) 4650

Ans. **(b)**

Sol. $^{100}C_2 - 100 = \dfrac{100\times 99}{2} - 100$

$= 4950 - 100 = 4850$

48. Let a, b, c and p, q, r be all positive real numbers such that a, b, c are in G.P. and $a^p = b^q = c^r$. Then:

(a) p, q, r are in G.P. (b) p, q, r are in A.P.

(c) p, q, r are in H.P. (d) p^2, q^2, r^2 are in G.P.

Ans. **(c)**

Sol. $a = x$

$b = xy$

$c = xy^2$

$x^p = (xy)^q = (xy^2)^r = \lambda$

$p = \log_x \lambda \quad \Rightarrow \quad \dfrac{1}{p} = \log_\lambda x$

$q = \log_{xy} \lambda \quad \Rightarrow \quad \dfrac{1}{q} = \log_\lambda x + \log_\lambda y = \lambda$

$r = \log_{xy^2} \lambda \quad \Rightarrow \quad \dfrac{1}{r} = \log_\lambda x + 2\log_\lambda y$

p, q, r are in H.P.

49. If you look overhead at 6 p.m. (standard sunset time) and notice that the moon is directly overhead, what phase is it in ?

(a) Last quarter (b) First quarter

(c) Full moon (d) 12^{th} day from new moon

Ans. **(b)**

50. The potential energy of a particle in a certain field has the form $U = \frac{a}{r^2} - \frac{b}{r}$, where a and b are positive constants and r is the distance from the centre of the field. Find the value of r_0 corresponding to the equilibrium position of the particle. Is the equilibrium a stable or unstable :

(a) $\frac{2a}{b}$ stable equilibrium (b) $\frac{a}{b}$ stable equilibrium

(c) $\frac{2a}{b}$ unstable equilibrium (d) $\frac{a}{b}$ unstable equilibrium

Ans. **(a)**

Sol. $U = \frac{a}{r^2} - \frac{b}{r}$

$F = -\frac{du}{dr} = -\left(-\frac{2a}{r^3} + \frac{b}{r^2}\right)$

$F = 0 \quad \Rightarrow \frac{2a}{r^3} = \frac{b}{r^2} \Rightarrow r = \frac{2a}{b}$

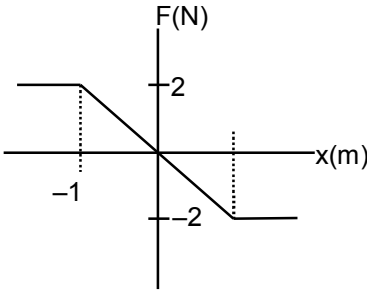
$\frac{dF}{dr} = -\left(\frac{6a}{r^4} - \frac{2b}{r^3}\right)$

at $r = \frac{2a}{b}$

$\frac{dF}{dr} = -\frac{1}{r^3}\left(\frac{6a}{2\frac{a}{b}} - 2b\right) = -\frac{b^3}{8a^3}(b) < 0$

So $\frac{2a}{b}$ is stable equilibrium.

51. The force acting on a particle is shown as a function of the position in a one dimensional system. The incorrect statement is



(a) If total energy of the system is 1 J, motion is SHM.

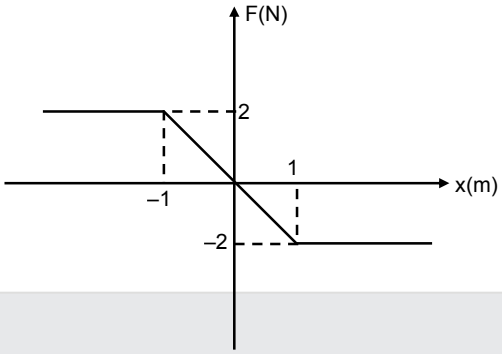
(b) If total energy of the system is 2 J, motion is periodic.

(c) Work done on the particle as it moves from x = 3m to 2 m is 2J.

(d) Work done on the particle as it moves from x = 3 m to 2 m is – 2J.

Ans. **(d)**

Sol.



Work by force from
x = 0 to x = 1 m
is –1J so if kinetic energy
of particle at origin is
1J or less it will be bounded b/w (x = –1 & x = 1)
& in this region F $\propto$ (– x) hence SHM
if energy is more then 1J motion will be periodic
W_{byF} from x = 3 to x = 2
= –2(2–3) = 2J

52. Since none of the six trigonometric functions are one-to-one, they are restricted in order to have their unambiguous inverse functions. This range of the inverse trigonometric functions is called the range of the principal values. Range of principal value of $\cot^{-1}(x)$ is

(a) $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ (b) $-\pi$ to $+\pi$ (c) 0 to π (d) $-\frac{\pi}{2}$ to 0 or 0 to $+\frac{\pi}{2}$

Ans. **(c)**

Sol. Obvious

53. On which of the following planets would the sun rise in the west?

(a) Saturn (b) Mercury (c) Venus (d) Jupiter

Ans. **(c)**

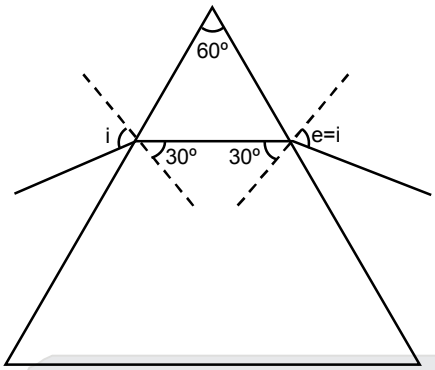
Sol. Informative

54. A prism of refracting angle 60° is made of a material of refractive index 1.732. The angle of minimum deviation produced by this prism is close to

(a) 30° (b) 38° (c) 60° (d) 45°

Ans. **(c)**

Sol.



$\sin i = 1.732 \sin 30^\circ$

$i = 60^\circ$

$\delta_{min} = 2 \times 60 - 60^\circ = 60^\circ$

55. Two particles undergo simple harmonic motion given by $y_1 = a \sin 50\ \pi t$ and $y_2 = a \sin 40\ \pi t$ respectively. Initially the particles are in phase. After what minimum time will the particles be in phase again?

(a) 0.1 s (b) 0.2 s (c) 0.25 s (d) 0.15 s

Ans. **(b)**

Sol. $\omega_1 t - \omega_2 t = 2\pi$

$50\pi t - 40\pi t = 2\pi$

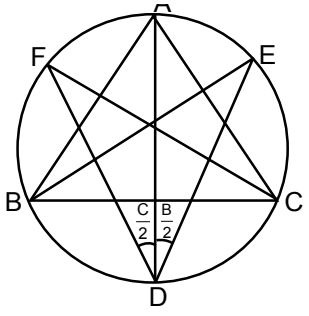
$t = 0.2$ s

56. Bisectors of vertex angles A, B and C of a triangle ABC intersect its circum-circle at the points D, E and F respectively. The angle EDF is equal to

(a) 90°–A/2 (b) 45°+A/2 (c) 90°–B/2 (d) A

Ans. **(a)**

Sol. $\angle EDF = \frac{\angle B + \angle C}{2} = \frac{\pi}{2} - \frac{\angle A}{2}$

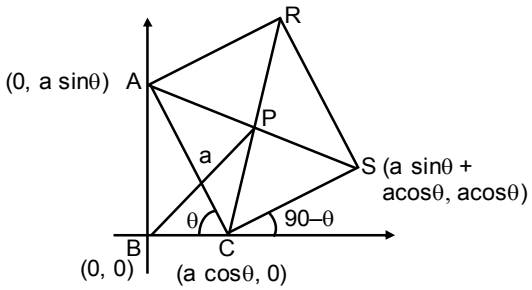


57. ΔABC is right angled at B.A square is constructed on the side AC opposite to that of angle B. P is the centre of this square. The angle PBC is equal to

(a) 30° (b) 45° (c) 60° (d) None of the above

Ans. **(b)**

Sol. $P \equiv \left\{\frac{a\cos\theta + a\sin\theta}{2}, \frac{a\cos\theta + a\sin\theta}{2}\right\}$



Slope of PB = 1 hence angle PBC is 45°

58. A line through the three stars in Orion's belt appears to point towards which one of the following star?

(a) Vega (b) Plaris (c) Rigel (d) Sirius

Ans. **(d)**

Sol. **Informative**

59. In an experiment on photoelectric effect, the maximum speed of photoelectron (v_{max}) is measured for different frequencies (f) of incident radiation. On a graph of v_{max} versus f, the slope of the curve at any point gives

(a) Planck's constant (h)

(b) Planck's constant divided by electron charge (h/e)

(c) de-Broglie wavelength of the photoelectron

(d) Wavelength of incident radiation

Ans. **Bonus**

Sol. $hf = \phi + \frac{1}{2} m v_{max}^2$

$v_{max}^2 = \frac{2h}{m} f - \frac{2\phi}{m}$

Bonus (No option is matching)

expected question is

$eV_s = h\nu - \phi$

$V_s = \frac{h}{e}\nu - \frac{\phi}{e}$

60. Let y be the solution of the differential equation $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$ satisfying y(1) = 1. Then satisfies

(a) $y = x^{y-1}$ (b) $y = x^y$ (c) $y = x^{y+1}$ (d) $y = x^{y+2}$

Ans. **(b)**

Sol. $dy = y\left(\frac{ydx}{x} + \ell nx\,dy\right)$

$\frac{dy}{y} = d(y\,\ell nx)$

$\Rightarrow \quad \ell ny = y\,\ell nx + C$

$y = x^y + \lambda$

$\therefore\ y(1) = 1$

$\therefore$ $\boxed{y = x^y}$

61. The circle on the celestial sphere along which declination of a star is measured is called

(a) Diurnal circle (b) Great circle

(c) Hour circle (d) Meridian

Ans. **(c)**

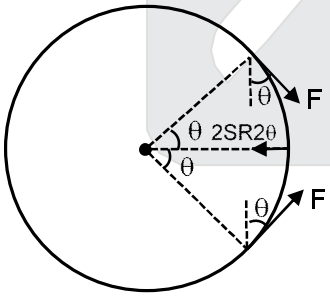
Sol. **Informative**

62. A circular wire frame of radius R is dipped in a soap solution of surface tension S. When it is taken out, a thin soap film is formed inside the frame. If cross sectional area of wire is A, then the stress developed in the wire due to suface tension is

(a) $\frac{\pi SR}{A}$ (b) $\frac{SR}{A}$ (c) $\frac{2SR}{A}$ (d) $\frac{SR}{2A}$

Ans. **(c)**

Sol.



($\theta \to$ small)

F = restoring force developed in wire

$2F\sin\theta = 2\,SR\,2\theta$

$F\theta = SR2\theta$

$F = 2SR$

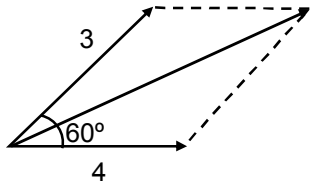
$\text{Stress} = \frac{F}{A} = \frac{2SR}{A}$

63. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about

(a) 7 (b) 6 (c) 5 (d) 3.5

Ans. **(b)**

Sol.



$A = \sqrt{3^2 + 4^2 + 2\times 3\times 4\times \cos 60^\circ}$

$= \sqrt{37} \approx 6$

64. The value of $\lim\limits_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}$ is

(a) 1 (b) $\frac{1}{e^2}$ (c) $\frac{1}{2e}$ (d) $\frac{1}{e}$

Ans. **(d)**

Sol. $\lim\limits_{n\to\infty}\left\{\frac{(n!)^{1/n}}{n}\right\}$

$x = \lim\limits_{n\to\infty}\left\{\left(\frac{n!}{n}\right)^{1/n}\right\}$

$\ell n(x) = \lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=1}^{n}\ell n\left(\frac{r}{n}\right) = \int_0^1(\ell nx)dx = x\,\ell nx\,\Big|_0^1 - \int_0^1 x.\frac{1}{x}dx = 0-(1)$

$x = e^{-1} = \frac{1}{e}$

65. The value of the integral $\int_1^5\left[|x-3|+|1-x|\right]dx$ is equal to

(a) 4 (b) 8 (c) 12 (d) 16

Ans. **(c)**

Sol. $\int_1^5(|x-3|+|x-1|)dx = \int_1^3(3-x+x-1)dx + \int_3^5(x-3+x-1)dx$

$2(3-1) + \frac{2}{2}(25-9) - 4(5-3)$

$4 + 16 - 8 = 12$

66. The definition of a nautical mile is

(a) one arc minute of latitude along any line of longitude

(b) one arc minute of longitude along any line of latitude

(c) one arc second of latitude along any line of longitude

(d) one arc second of longtitude along any line of latitude

Ans. **(a)**

Sol. Informative

67. If the earth were to become a black hole, the minimum radius to which it should be compressed is about (mass remains constant, radius of earth = 6400 km)

(a) 1.8 m (b) 9.0 mm (c) 3.6 cm (d) 1.8 km

Ans. **(b)**

Sol. $\sqrt{\frac{2GM}{r}} = 11200 \text{ m/s}$

$\sqrt{\frac{2GM}{r'}} = 3 \times 10^8 \text{ m/s}$

$\sqrt{\frac{r'}{r}} = \frac{11200}{3\times10^8}$

$r' = \left(\frac{11200}{3\times10^8}\right)^2 6400 \text{ km} \cong 9 \text{ mm}$

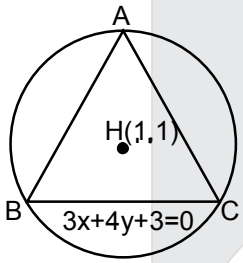
68. The in-centre of an equilateral is (1, 1) and the equation of one of the sides is $3x + 4y + 3 = 0$. Then the equation of the circum-circle of the triangle is

(a) $x^2 + y^2 - 2x - 2y - 2 = 0$ (b) $x^2 + y^2 - 2x - 2y - 14 = 0$

(c) $x^2 + y^2 - 2x - 2y + 2 = 0$ (d) $x^2 + y^2 - 2x - 2y + 14 = 0$

Ans. **(b)**

Sol.



$\frac{x-1}{3} = \frac{y-1}{4} = -2\left(\frac{10}{25}\right) = \frac{-4}{5}$

$x = 1 - \frac{12}{5} = -\frac{7}{5}$

$y = 1 - \frac{16}{5} = -\frac{11}{5}$

$(x-1)^2 + (y-1)^2 = \left(1+\frac{7}{5}\right)^2 + \left(1+\frac{11}{5}\right)^2 = \frac{144}{25} + \frac{256}{25} = \frac{400}{25}$

$x^2 + y^2 - 2x - 2y - 14 = 0$

69. How many odd numbers greater than 700000 can be formed using the digits 4, 5, 6, 7, 8, 9, 0 if repetitions are allowed?

(a) 216 (b) 4920 (c) 9261 (d) 21609

Ans. **BONUS or (d)**

Sol. 7 or 8 or 9 5 or 7 or 9

–	–	–	–	–	–
3	7	7	7	7	3

$9 \times 7^4 = 9 \times 49 \times 49 = 21609$

70. The False statement about planetary configurations is,

(a) A superior planet can have an elongation between 0 and 180 degree

(b) Quadrature is when the elongation of a superior planet is 90 degree

(c) Quadrature is when the elongation of an inferior planet is 90 degree

(d) An inferior planet has zero elongation when in conjunction

Ans. **(c)**

Sol. **Informative**

71. When an electron in a hydrogen like atom is excited from a lower orbit to a higher orbit its

(a) kinetic energy increases and potential energy decreases.

(b) both kinetic energy and potential energy increases.

(c) both kinetic energy and potential energy decreases.

(d) kinetic energy decreases and potential energy increases

Ans. **(d)**

Sol. $T.E_n = -\frac{13.6}{n^2}$

$P.E_n = -\frac{27.2}{n^2}$

$K.E_n = \frac{13.6}{n^2}$

at electron moves from lower to higher orbit K.E. decreases and P.E. increases.

72. The number of divisors of 480 of the form 8n + 4 ($n \geq 0$), where n is an integer is

(a) 3 (b) 4 (c) 8 (d) 10

Ans. **(b)**

Sol. 4(2n + 1)

$480 = 2^5 \times 3 \times 5$

4 or 4 × 3 or 4 × 5 or 4 × 15 = 4

73. The orbital period of Jupiter is 4333 mean solar days andJupiter's mass is 1/1048 times the Sun's mass. The orbital period of a small body, of negligible mass, moving in an elliptical orbit round the Sun with the same major axis as that of Jupiter is closest to

(a) 4329 days (b) 4333 days (c) 4335 days (d) 4339 days

Ans. **(b)**

Sol. $T^2 \propto a^3$

$\therefore$ same

74. Let S_k be the sum of an infinite G.P. whose first term is k and common ratio is $\frac{k}{(k+1)}$ (k > 0). Then the value of $\sum_{k=1}^{k=\infty} \frac{(-1)}{S^k}$ is equal to

(a) $\log_e 4$ (b) $\log_e 2 - 1$ (c) $1 - \log_e 2$ (d) $1 - \log_e 4$

Ans. **Bonus or (d)**

Sol. $\sum_{k=1}^{\infty}\frac{(-1)^k}{k(k+1)} = -\frac{1}{1.2}+\frac{1}{2.3}-\frac{1}{3.4}+\frac{1}{4.5}-.....$

$\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....., \ \ln(1-x) = -x-\frac{x^2}{2}+\frac{x^3}{3}$

$\ln(1+1) = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+.....$

$\ln 2 = \frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+.....$

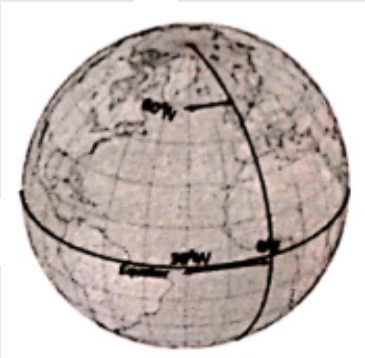
$\ln 2 = 1-\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+.....$

$2\ln 2 = 1+\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+.....$

$= 1 - 2\ln 2$

$= 1 - \ln 4$

75. Which of the following is the closest to the distance in kilometers of 30° extent of longitude on the surface of the earth along the equator? How much is the corresponding distance along a small circle (parallel to the equator) at 60° N latitude?



(a) 3500 Km, 3000 Km (b) 3000 Km, 2000 Km (c) 2500 Km, 1750Km (d) 3300 Km, 1650 Km

Ans. **(d)**

Sol. $d_1 = \frac{\pi}{6}$ 6400 [along equator] $d_2 = \frac{\pi}{6}$ 6400 . 1260cos60°

= 3350 km = 1675 km

76. Any location on the surface of Earth, as a sphere, is determined by two coordinates– Latitude and Longitude. Latitude of the North Pole is 90 degree. Its corresponding longitude is

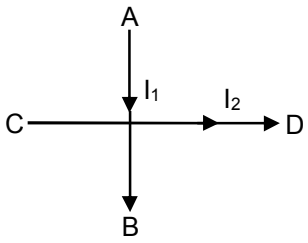
(a) 0 degree (b) 90 degree

(c) 180 degree (d) None of these / indeterminate

Ans. **(d)**

Sol.

77. Two long wires AB and CD carrying currents I_1 And I_2 are perpendicular to each other as shown in the figure (wires are insulated from each other)

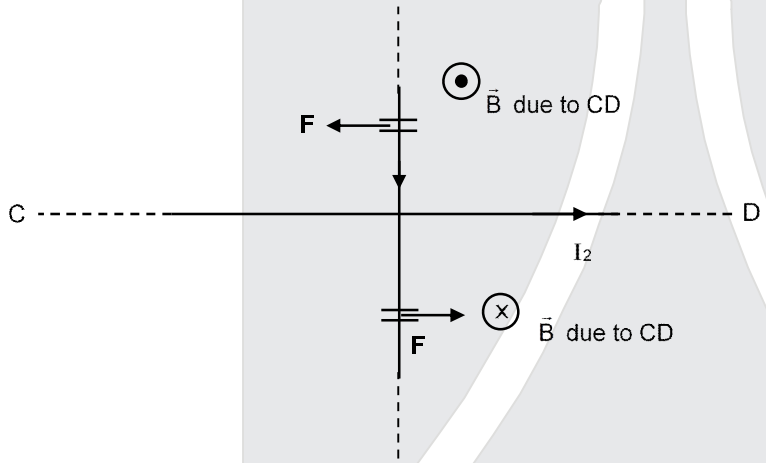


(a) Force on wire AB is towards left (b) Force on wire AB is towards right

(c) Torque on wire AB is clockwise (d) Torque on wire AB is anticlockwise.

Ans. **(d)**

Sol.



net force will be zero and net torque will be anticlockwise

78. A right circular cone of the height h and radius r is suspended in liquid of density (ρ). The density of cone is 2ρ and its circular face is at a depth $\frac{h}{3}$. The force exerted by liquid on the curved surface of cone is

(a) zero (b) $\frac{\pi r^2 h\rho g}{3}$ (c) $\pi r^2 h\rho g$ (d) $2\pi r^2 h\rho g$

Ans. **(d)**

Sol. $F_{\text{by liq on curved surface}} - \frac{\rho gh}{3}\pi r^2 = \rho \times \frac{1}{3}\pi r^2 hg$

$F_{\text{by liq on curved surface}} = \frac{2}{3}\rho gh\pi r^2$

79. A straight line through the point of intersection of the line x + 2y = 4 and 2x + y = 4 meets the coordinate axes at A and B. The locus of the midpoint of AB is

(a) 3 (x + y) = 2xy (b) 2(x + y) = 3xy (c) 2(x + y) = xy (d) (x+ y) = 3 xy

Ans. **(b)**

Sol. x + 2y = 4 and 2x + y = 4

on solving 3x = 4 $\Rightarrow$ x = 4/3, y = 4/3 $\Rightarrow$ $\left(\frac{4}{3},\frac{4}{3}\right)$

Equation AB is $y - \frac{4}{3} = m\left(x - \frac{4}{3}\right)$

A $\left(\frac{4}{3} - \frac{4}{3m}, 0\right)$, B $\left(0, \frac{4}{3} - \frac{4m}{3}\right)$

Let mid-point of AB is P(h, k)

$h = \frac{\frac{4}{3}\left(1 - \frac{1}{m}\right) + 0}{2}, k = \frac{\frac{4}{3}(1-m) + 0}{2}$

$\frac{3h}{2} = 1 - \frac{1}{m}$ and $\frac{3k}{2} = 1 - m$

$m = \left(1 - \frac{3k}{2}\right)$

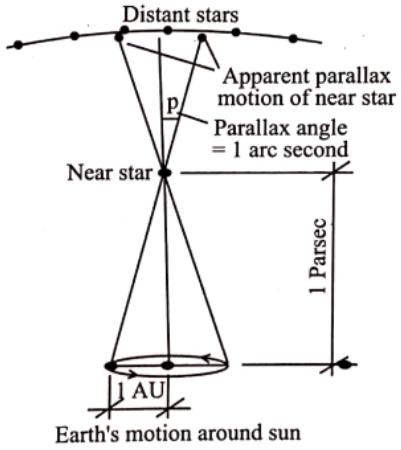
So $\frac{3h}{2} = 1 - \frac{2}{2-3k}$

$\frac{3h}{2} = \frac{2-3k-2}{2-3k}$

$\Rightarrow$ 6h – 9hk = – 6k

$\Rightarrow$ 2(x + y) = 3xy locus

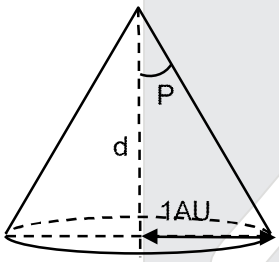
80. One Astronomical Unit (AU) is defined as the mean distance between the Sun and the Earth: 1 AU ~ 150 million kilometer. Even this unit is also small for measuring large stellar distances which are quoted in 'light year'. A star one parsec away from the Earth produces a parallax of noe second of arc when viewed from the Earth six months apart in its orbit around the Sun, as shown in the accompanying figure. There by one parsec is close to



(a) 205625 AU (b) 206265 AU (c) 22620 AU (d) 226265 AU

Ans. **(b)**

Sol.



$dP = 1AU$

$d\frac{1}{3600}\times\frac{\pi}{180} = 1AU$

$d = \frac{3600\times180}{\pi}\Delta U$

$= 206265$ AU