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NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2018-19

Examination Date : 25-11-2018 Time: 2 Hrs. Max. Marks : 240

PAPER CODE : A421

Write the question paper code mentioned above on YOUR answer sheet (in the space provided), otherwise your answer sheet will NOT be assessed. Note that the same Q. P. Code appears on each page of the question paper.

INSTRUCTIONS TO CANDIDATES

1. Use of mobile phones, smartphones, ipads during examination is **STRICTLY PROHIBITED.**
2. In addition to this question paper, you are given answer sheet along with Candidate's copy.
3. On the answer sheet, make all the entries carefully in the space provided ONLY in **BLOCK** CAPITALS as well as by properly darkening the appropriate bubbles.

   Incomplete/Incorrect/carelessly filled information may disqualify your candidature.
4. On the answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.
5. The email ID and date birth entered in the answer sheet will be your login credentials for accessing performance report. Please take care while entering.
6. Question paper has 80 multiple choice questions. Each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

   Q. No.22 (a) (●) (c) (d)
7. A correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer.
8. Any rough work should be done only in the space provided.
9. Use of **non-programmable** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting your answer paper, take away the Candidate's copy for your reference.

    Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the answer sheet.

    Answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED.

    Scratching or overwriting may result in a wrong score.

    DO NOT WRITE ON THE BACK SIDE OF THE ANSWER SHEET.

Instructions to Candidates (continued) –

Read the following instructions after submitting the answer sheet.

12. Comments regarding this question paper, if any, may be filled in Google forms only at https://goo.gle/forms/afgXeVB2k0CyC7A53 till 27th November, 2018.
13. The answers/solutions to this question paper will be available on our website — www.iapt.org.in by 2nd December, 2018.
14. **CERTIFICATES and AWARDS –**
    Following certificates are awarded by the LAPT to students successful in NSEs
    (i) "Centre Top 10%" that will be sent to NSE centre by post.
    (ii) "Statewise Top 1%" that can be downloaded after Feb-15th, 2019 from iapt.org.in.
    (iii) "National Top 1%" . Certificate can be downloaded after Feb-15th, 2019 iapt.org.in
15. Result sheets can be downloaded from our website in the month of February. The "Centre Top 10%" certificates will be dispatched to the Prof-in-charge of the centre by February, 2019**.**
16. List of students (with centre number and roll number only) having score above MAS will be displayed on our website (www.iapt.org.in) by 22nd December, 2018. See the Eligibility Clause in the Student's brochure on our website.
17. Students eligible for the INO Examination on the basis of selection criteria mentioned in Student's brochure will be informed accordingly.
18. Students qualified for OCSC (Astronomy) –2019 will be awarded gold medals.

Physical constants you may need.....

Magnitude of charge on electron e = 1.60×10^{-19}C	Mass of electron m_e = 9.10×10^{-31}kg
Universal gas constant R = 8.31 J/mol K	Planck constant h = 6.62×10^{-34} Js
Stefan constant s = 5.67×10^{-8} W/m^{2}K^4	Boltazmann constant k = 1.38×10^{-23} J/K
Mass of proton m_p = 1.67×10^{-27} kg	Faraday constant = 96500 C/mol
Boiling point of nitrogen = 77.4 K	Boiling point of oxygen = 90.19 K
Boling point of hydrogen = 20.3 K	Boiling point of helium = 4.2K
Universal gravitational constant G = 6.67×10^{-11} Nm^2/Kg^2	Permittivity of free space e_0 = 8.85×10^{-12} C^2/Nm^2

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT

1. Sun is at a mean distance of about 27,000 light years from the centre of the Milky way galaxy and completes one revolution about the galactic centre in about 225 million years. The linear speed of Sun is

(a) 160 km s^{-1} (b) 230 km s^{-1} (c) 30 km s^{-1} (d) 80 km s^{-1}

Ans. **(b)**

Sol. $V = \frac{2\pi R}{T} = \frac{2\pi \times [27000 \times 3 \times 10^8 \times 365 \times 24 \times 3600]}{[225 \times 10^6 \times 365 \times 24 \times 3600]} = \frac{2\pi \times 27000 \times 300}{225}$

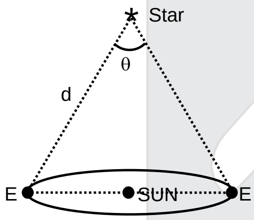
$V = 8\pi \times 9000$ m/s = $72\,\pi$. km/sec. $\cong$ 230 km/sec.

2. Light from the nearest star 'proxima centauri' takes 4.24 light years to reach earth. The stellar parallax of this star is about.

(a) 1.30 s (b) 0.77 s (c) 13.8 s (d) 0.24 s

Ans. **(a)**

Sol.



$\theta = \left(\frac{2R}{d}\right)$ radian

$\theta = \left(\frac{2R}{d}\right)\left(\frac{180}{\pi} \times 3600\right)$ sec

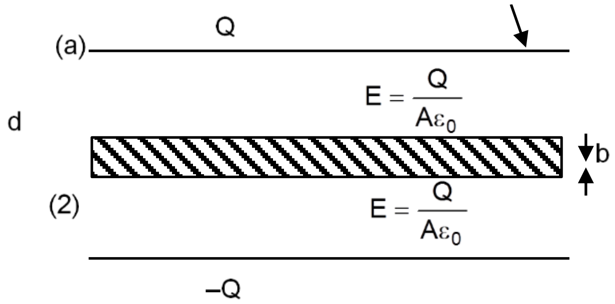
θ = 1.51 sec **Ans. (A)**

3. A block of conductor with its area equal to 'A' and thickness 'b' is placed between the plates of a parallel plate capacitor without touching either of the plates. If the area of the plates of the capacitor be 'A' each can 'd' be the separation between the plates then the capacitance of the system after the introduction of the block is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 A}{d\left(1+\frac{b}{a}\right)}$ (c) $\frac{\varepsilon_0 A}{d\left(1-\frac{b}{d}\right)}$ (d) $\frac{\varepsilon_0 A}{d\left[1+\left(\frac{b}{d}\right)^2\right]}$

Ans. **(c)**

Sol.



$$\mathbf{C_{eq}} = \frac{Q}{v_1 - v_2}$$

$$\frac{Q}{\dfrac{Q}{A\varepsilon_0 (d-b)}} = \frac{\varepsilon_0 A}{d-b}$$

4. The number of real solutions of the equation $|x - |x - |x - 4||| = x^2 - 4x$ is

(a) 0 (b) 1 (c) 2 (d) more than 2

Ans. **(c)**

Sol. $|x - |x - |x - 4||| = x^2 - 4x$

Case-I: when $x \geq 4$

$x - 4 = x^2 - 4x$

$x^2 - 5x + 4 = 0$

$x = 1, 4$ $\quad x \neq 1$

Case-II: when $x \leq 0$

$4 - 3x = x^2 - 4x$

$x^2 - x - 4 = 0$

$x = \dfrac{1 \pm \sqrt{17}}{2}$ $\quad x \neq \dfrac{1 + \sqrt{17}}{2}$

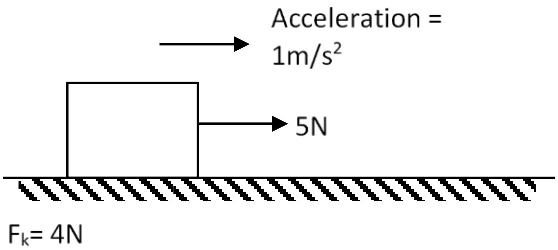
Total 2 solution **Ans. (c)**

5. A body of mass 1.0 kg is pulled along a rough horizontal surface by a horizontal force of 5N for 10 s starting from rest. If the kinetic friction is $\mu_k = 0.40$, the amount of heat generated is equal to (assuming $g = 10\ ms^{-2}$)

(a) 190 J (b) 200 J (c) 210 J (d) 205 J

Ans. **(b)**

Sol.



Displacement = $\dfrac{1}{2} \times 1 \times 10^2 = 50m$

W.D. by kinetic friction = –200J $\quad \therefore \quad$ (b)

6. Six dice are rolled simultaneously. The probability of getting at least four identical numbers is

(a) $\frac{2250}{6^6}$ (b) $\frac{2436}{6^6}$ (c) $\frac{2535}{6^6}$ (d) $\frac{2738}{6^6}$

Ans. **(b)**

Sol. Prob. = $^6C_1 \times \frac{6!}{6!} + {}^6C_1 \times {}^5C_1 \times \frac{6!}{5!} + {}^6C_1 \times {}^5C_1 \times \frac{6!}{4!\,2!} + {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times \frac{6!}{4!}$

= 6 + 180 + 450+ 1800

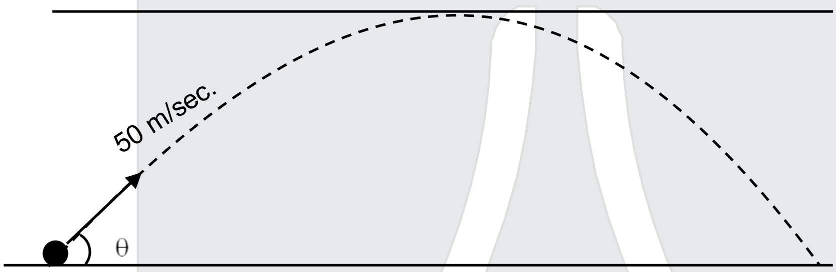
= 2436.

7. The ceiling of a long hall is 45 m high. The maximum horizontal distance that a ball thrown with a speed of 50 ms^{-1} can go without hitting the ceiling is nearly equal to (g = 10 ms^{-2})

(a) 250 m (b) 240 m (c) 230 m (d) 300 m

Ans. **(b)**

Sol.



$$H_{max} = \frac{(50)^2 \times \sin^2\theta}{20} = 45 \Rightarrow \quad \sin\theta = \frac{3}{5}$$

$$\therefore \quad R_{max} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times(50)^2}{10} \times \frac{3}{5} \times \frac{4}{5} = 240 \text{ m}$$

8. The tangents drawn from a certain point P to the parabola 2y = x^2 – 2 are also tangents to the parabola 4y = x^2 – 10x + 37. The sum of the coordinates of P is

(a) 10 (b) 6 (c) 0 (d) – 10

Ans. **(d)**

Sol. Let tangent to the parabola $x^2 = 2(y + 1)$ & $(x - 5)^2 = 4(y - 3)$

Are $x = my + \frac{1}{2m} + m$ …(1)

And $x = my + \frac{1}{m} - 3m + 5$ respectively … (2)

Which are identical

$\Rightarrow \quad \frac{1}{2m} + m = \frac{1}{m} - 3m + 4$

$\Rightarrow \quad 8m^2 - 10m - 1 = 0$

By (1) $X = m_1 y + \frac{1}{2m_1} + m_1$

$X = m_2 y + \frac{1}{2m_2} + m_2$

On solving $(m_1 - m_2)\, y + \frac{1}{2}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) + (m_1 - m_2) = 0$

$\Rightarrow$ $Y = \dfrac{1}{2m_1m_2} - 1$ …(1)

$X = \dfrac{m_1 + m_2}{2m_1m_2}$ …(2)

Hence sum of coordinates

$$X + Y = \frac{m_1 + m_2 + 1}{2m_1m_2} - 1 = \frac{\frac{5}{4} + 1}{2\left(-\frac{1}{8}\right)} - 1$$

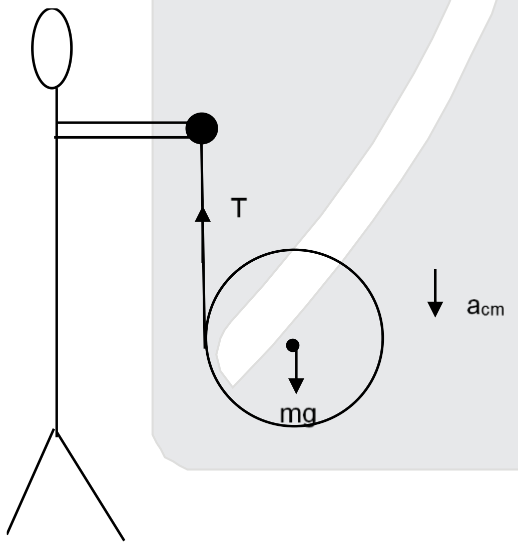
–9 –1 = –10

9. A yo-yo of mass 'M' and radius of the inner hub 'r' is completely wound with a string. It is allowed to start unwinding with zero downward initial velocity. The moment of inertia of the yo-yo about an axis passing through its centre of mass and normal to the discs is I. The acceleration with which the yo-yo falls when I = Mr^2 can be given by

(a) a = g (b) a = g/2 (c) a = 2g/3 (d) a = g/4

Ans. **(b)**

Sol.



$Mg - T = ma_{cm}$ …(1)

$TR = I\left(\dfrac{a_{cm}}{R}\right)$

$TR = MR^2\left(\dfrac{a_{cm}}{R}\right)$ …(2)

(1) + (2) $\Rightarrow$ $Mg = Ma_{cm} + Ma_{cm}$

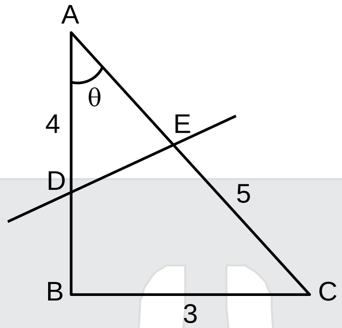
$\Rightarrow$ $a_{cm} = g/2$

10. What is the least possible length of a line segment that cuts a triangle with sides 3, 4, 5 in to two geometrical figures having equal area ?

(a) $\sqrt{12}$ (b) $\sqrt{6}$ (c) $\sqrt{5}$ (d) 2

Ans. **(d)**

Sol. For shortest length line will cuts the time with bigger two length and form isosceles triangle.
Let it cuts AB & AC at D & E respectively



$\Rightarrow \quad \frac{1}{2}(AD)^2 \times \sin\theta = 3 \quad \Rightarrow \quad \frac{1}{2}(AD)^2 \times \frac{3}{5} = 3$

AD = $\sqrt{10}$

$\cos\theta = \frac{(AD)^2 + (AD)^2 - (DE)^2}{2(AD)^2}$

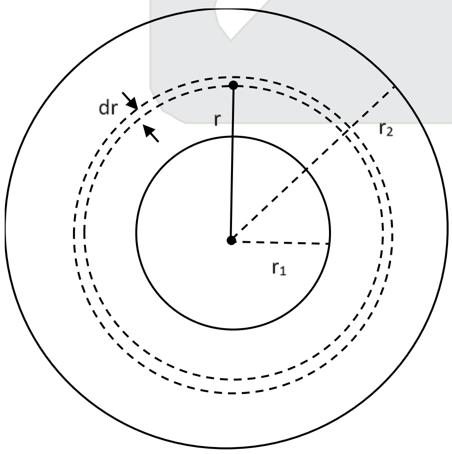
$\frac{4}{5} = \frac{10+10-(DE)^2}{2(10)} \quad \Rightarrow \quad DE = 2$

11. A plane spiral of N turns, having the radii of internal and external loops as r_1 and r_2 carries a current I. The magnetic induction at the centre of the spiral will be

(a) $\frac{\mu_0 NI}{(r_2 - r_1)} \ell n \frac{r_2}{r_1}$ (b) $\frac{\mu_0 NI}{2(r_2 - r_1)} \ell n \frac{r_2}{r_1}$ (c) $\frac{\mu_0 NI}{(r_2 - r_1)} \ell n \frac{r_1}{r_2}$ (d) $\frac{\mu_0 NI}{2(r_2 - r_1)} \ell n \frac{r_1}{r_2}$

Ans. **(b)**

Sol.



Consider a rings of radius 'r' and thickness "dr".

$dB_{rings} = \left(\frac{\mu_0 i}{2r}\right)$ (No. of turns) $= \frac{\mu_0 i}{2r} \times \frac{N}{(r_2 - r_2)} dr$

$B_{total} = \frac{\mu_0 IN}{2(r_2 - r_1)} \ell n\left(\frac{r_2}{r_1}\right)$

12. The number of nonzero real solutions of the equations $x^{x+y} = y^3$, $y^{x+y} = x^{12}$ is

(a) 0 (b) 1 (c) 2 (d) more than 2

Ans. **(d)**

Sol. $x^{x+y} = y^3 \quad \Rightarrow \quad y = x^{\frac{x+4}{3}}$

$y^{x+y} = x^{12} \quad \Rightarrow \quad y = x^{12/x+y}$

$x^{\frac{x+y}{3}} = x^{\frac{12}{x+y}}$

$\frac{x+y}{3} = \frac{12}{x+y}$ or $x = 1, y = 1.$

$(x + y)^2 = 36$

$x + y = \pm 6$

when $x + y = 6$

$x^6 = y^3$

$y = x^2$

$6 - x = x^2$

$x^2 + x - 6 = 0$

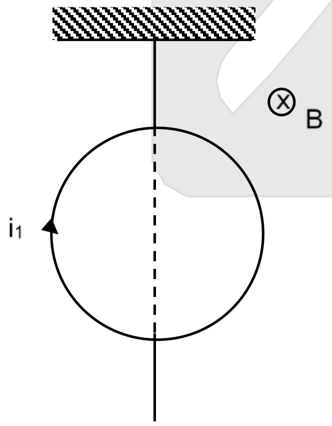
$x = 2, -3 \quad \Rightarrow$ More than 2 solutions

13. Two identical circular coils are carrying currents i_1 and i_2 are suspended from a torsion free cotton thread in placed in a region of uniform magnetic field B. Each time the coils are given a small angular displacement from their respective equilibrium positions. The time period of the small torsional oscillations were found to be T_1 and T_2. The ratio $\frac{T_1}{T_2}$ would be

(a) $\frac{i_1}{i_2}$ (b) $\frac{i_2}{i_1}$ (c) $\sqrt{\frac{i_1}{i_2}}$ (d) $\sqrt{\frac{i_2}{i_1}}$

Ans. **(d)**

Sol.



Restoring torque if given a small angular displacement = $MB\sin\theta$.

$= (i_1\pi R^2)B\theta$ (For small θ)

$(i_1\pi R^2)B\theta = \left(\frac{MR^2}{2}\right)\alpha \qquad \therefore \qquad \alpha = \left(\frac{2\pi B i_1}{M}\right)\theta = \omega^2\theta$

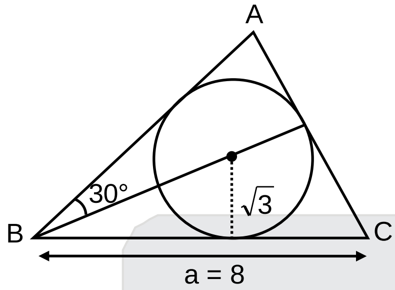
$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{M}{2\pi B i_1}} \qquad \frac{T_1}{T_2} = \sqrt{\frac{i_2}{i_1}} \qquad \therefore$ (d)

14. A triangle has a side of length 8 units, one of the angles of the triangle on this side is 60°. If the in radius of the triangle is $\sqrt{3}$ units, the perimeter of the triangle is

(a) $15\sqrt{3}$ (b) 24 (c) $12\sqrt{3}$ (d) 20

Ans. **(d)**

Sol.



r = (s – b) tan B/2   B = 60°

$\sqrt{3} = (s-a)\frac{1}{\sqrt{3}}$

S – b = 3.

$\therefore$ 2S = a + b + c

6 + 2b = 8 + b + c

b = c + 2

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\frac{1}{2} = \frac{64 + c^2 - b^2}{2 \times 8 \times c}$

$8c = 64 + c^2 - b^2$

$8c = 64 + c^2 - 4 - c^2 - 4c$

12c = 60

c = 5

b = 7

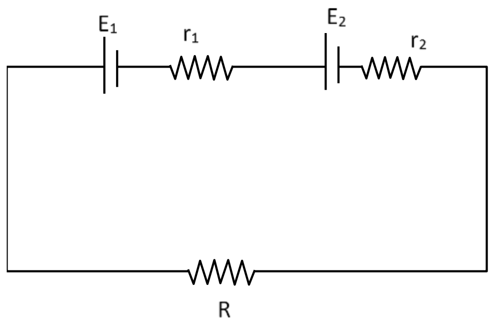
a + b + c = 8 + 7 + 5 = 20   Ans. (d)

15. Two cells with emfs E_1 and E_2 have internal resistances r_1 and r_2 respectively. The two cells are connected in series with an external resistance and the current through the external resistance is found to be 1.5 A. When the polarities of the cells are reversed this current is found be 0.5A. The ratio of the emfs of the cells is

(a) 2.5 (b) 1.5 (c) 2 (d) 4

Ans. **(c)**

Sol.



$E_1 + E_2 = 1.5(r_1 + r_2 + R)$

On reversing the polarity of a cell.

$E_1 - E_2 = 0.5(r_1 + r_2 + R)$

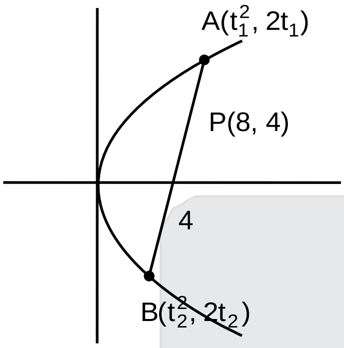
$\therefore \quad \frac{E_1 + E_2}{E_1 - E_2} = 3 \quad \therefore \quad E_1 : E_2 = 2 : 1 \quad \therefore \quad$ (c)

16. A points P(8, 4) divides a chord, lying completely in the first quadrant, of a parabola $y^2 = 4x$ in the ratio 1 : 4. The mid-point of the chord has coordinates

(a) (17.5, 8) (b) (18.5, 7) (c) (19.5, 6) (d) (20.5, 5)

Ans. **(b)**

Sol.



$\frac{4t_1^2 + t_2^2}{5} = 8$, $\frac{8t_1 + 2t_2}{5} = 4$

$4t_1^2 + t_2^2 = 40$; $4t_1 + t_2 = 10$

$4t_1^2 + (10 - 4t_1)^2 = 40$

$4t_1^2 + 100 + 16t_1^2 - 80t_1 = 40$

$20t_1^2 - 80t_1 + 60 = 0$

$t_1^2 - 4t_1 + 3 = 0$

$t_1 = 1, 3$

$t_1 = 1, t_2 = 6$

A(1, 2), B(36, 12)

mid-point (18.5, 7)

Ans. b

17. The de broglie wavelength associated with neutrons with thermal equilibrium with matter at temperatures 300K and at 400 K are in the ratio close to

(a) 1 : 1 (b) 1.15 : 1 (c) 1 : 2.3 (d) 1 : 2.8

Ans. **(b)**

Sol. $\lambda_{De-Broglie} = \frac{h}{mv}$ Here , $V \propto \sqrt{T}$

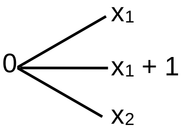
$\frac{\lambda_{300}}{\lambda_{400}} = \sqrt{\frac{400}{300}}$ = 1.15 $\therefore$ (b)

18. The sum of all real values of a for which the equation $x^3 - 7x + \alpha - 0$ has two real roots differeing by 1 is

(a) 0 (b) 6 (c) 12 (d) –12

Ans. **(a)**

Sol. $x^3 - 7x + \alpha = 0$



$2x_1 + x_2 + 1 = 0$ .....(1)

$x_1(x_1 + 1) + x_2(x_1 + 1) + x_1x_2 = -7$

$x_1^2 + x_1 - (1 + 2x_1)^2 = -7$ from (1)

$x_1^2 + x_1 - 1 - 4x_1 - 4x_1^2 = -7$

$3x_1^2 + 3x_1 - 6 = 0$

$x_1^2 + x_1 - 2 = 0$

$(x_1 + 2)(x_1 - 1) = 0$

$x_1 = -2, 1$

$\alpha = 6, -6$

Sum of value of $\alpha = 0$

19. Which of the following physical quantities has the unit volt-second

(a) Energy (b) Electric flux (c) Magnetic flux (d) Inductance

Ans. **(c)**

Sol. EMF induced = Rate of change of magnetic flux.

20. A die is rolled 5 times. The probability that there are at least two equal numbers among the outcomes obtained is

(a) $\frac{319}{324}$ (b) $\frac{49}{54}$ (c) $\frac{13}{18}$ (d) $\frac{4}{9}$

Ans. **(b)**

Sol. Required probability = 1 – No two equal numbers.

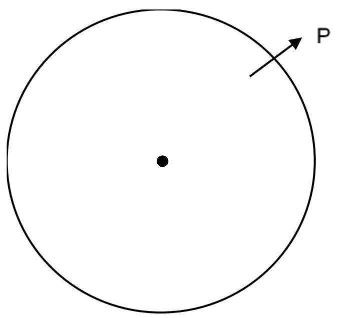
$$= 1 - \frac{{}^6C_5 \times 5!}{6^5} = \frac{49}{54}$$ **Ans. (b)**

21. There is a uniformly charged non-conducting solid sphere made of material of dielectric constant 1. If the electric potential at infinity is taken to be zero, then the potential at its surface is V. If we take the electric potential at its surface to be zero, then the potential at the center will be

(a) 3V/2 (b) V/2 (c) V (d) Zero

Ans. **(b)**

Sol.



If, $V_\infty = 0$, $V_{surface} = V$ and $V_{centre} = \frac{3V}{2}$

So, if $V_{surface} = 0 \Rightarrow V_{centre} = V/2$

$\therefore$ (b)

22. Suppose 5 cos x + 12 cos y = 13. The maximum possible value of 5 sin x + 12 sin y is

(a) $\sqrt{13}$ (b) $\sqrt{120}$ (c) $\sqrt{240}$ (d) 13

Ans. **(b)**

Sol. 5 cos x + 12 cos y = 13 .....(1)

5 sin x + 12 sin y = α .....(2)

$(1)^2 + (2)^2$

$25 + 144 + 120\cos(x-y) = 169 + \alpha^2$

$\alpha^2 = 120\cos(x-y)$

maximum value of $\alpha = \sqrt{120}$

Ans. b

23. If speed of light (C), acceleration due to gravity (g) and pressure (P) are taken to be fundamental units, then dimension of universal gravitational constant (G) is

(a) CgP^{-3} (b) $C^2g^3P^{-2}$ (c) $C^0g^2P^{-1}$ (d) $C^2g^2P^{-2}$

Ans. **(c)**

Sol. G : $M^{-1}L^3T^{-2}$

Let $G = c^a g^b p^c$

$[M^{-1}L^3T^{-2}] = [LT^{-1}]^a [LT^{-2}]^b [ML^{-1}T^{-2}]^c$

On solving a = 0, b = 2 and c = –1

$\therefore$ Ans. (c)

24. Let $f(x) = \begin{cases} \frac{\pi}{2}\sin x, & \text{for } 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$. Then

(a) no where continuous $(0, \pi)$

(b) continuous on $(0, \pi)$ except at x = $\frac{\pi}{2}$

(c) continuous on $(0, \pi)$, but nowhere differentiable

(d) differentiable at all points of $(0, \pi)$

Ans. **(d)**

Sol. $f(x) = \begin{cases} \frac{\pi}{2}\sin x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} \le x < \pi \end{cases}$

f(x) is continuous and differentiable for $x \in (0, \pi)$.

25. A wave propagating along X-axis is represented by y = a sin (At – Bx + C) where y is the displacement of the particle, a the amplitude of the wave and t is the time. If A, B and C are three constants then the dimensions of $\left(\frac{aBC}{A}\right)$ is the same as that of

(a) Length (b) Mass (c) Time (d) Velocity

Ans. **(c)**

Sol. $A = \omega = [T^{-1}]$

$B = K = [L^{-1}]$

$a = A = [L]$

C = dimensions

$\therefore \left[\frac{aBC}{A}\right] = \left[\frac{LL^{-1}}{T^{-1}}\right] = T$

26. The sides of a triangle are 8, 10, x where x is a positive integer. The number of possible values of for which triangle becomes acute is

(a) 6 (b) 5 (c) 4 (d) 3

Ans. (a)

Sol. $\cos A = \frac{8^2 + 10^2 - x^2}{2 \times 8 \times 10}$

$\Rightarrow$ $164 - x^2 > 0$

$x^2 < 164$

$\Rightarrow$ x = 1, 2, ..., 12.

$\cos B = \frac{x^2 + 8^2 - 10^2}{2 \times x \times 8}$

$\Rightarrow$ $x^2 + 8^2 - 10^2 > 0$

$x^2 > 36$

$x > 6$

$\Rightarrow$ x = 7, 8, 9, 10, 11, 12.

27. The speed (v in m/s) and time (t in second) for an object moving along a straight line are related as $t^2 - 2\sqrt{2}vt + 50 = 0$. The possible values of v is

(a) $v \geq 5$ m/s only (b) $v \geq 10$ m/s only (c) $v \geq 15$ m/s only (d) $v \geq 25$ m/s only

Ans. (a)

Sol. $t^2 - 2\sqrt{2}vt + 50 = 0$

$\Rightarrow$ $t = \frac{2\sqrt{2}v \pm \sqrt{8V^2 - 4 \times 50}}{2} = \sqrt{2}v \pm \sqrt{2V^2 - 50}$

For real roots : $2V^2 \geq 50$

$\Rightarrow$ $V^2 \geq 25$

$\Rightarrow$ $\boxed{V \geq 5 m/s}$

28. There are n teachers in a school and all possible 4 member committees are formed. Among these, exactly $\frac{1}{22}$th pat of the committees have 2 fixed members. The sum of the digits of n is

(a) 8 (b) 7 (c) 6 (d) 5

Ans. (b)

Sol. $\frac{1}{20}{}^{n}C_4 = {}^{n-2}C_2$

$\frac{n!}{4!(n-4)!} = 20\frac{(n-2)!}{2!(n-4)!}$

$\Rightarrow$ $n(n-1) = 240$

$\Rightarrow$ $(n - 16)(n + 15) = 0$ $\Rightarrow$ $n = 16$

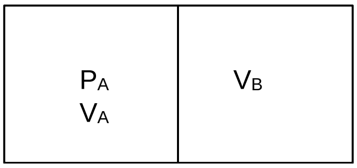
Ans. b

29. A chamber is enclosed in a thermally insulated cover and a partition wall separates it into two parts A and B. Part A is filled up with an ideal gas at pressure p_A and as a volume V_A. The other part (part B) is evacuated and has a volume V_B. Assume this part to be vacuum. The partition wall is now removed. When the equilibrium is set in. The pressure p in the entire chamber is

(a) $p = p_A$ (b) $p = \frac{p_A(V_A + V_B)}{V_B}$ (c) $p = \frac{p_A V_A}{V_A + V_B}$ (d) $p = \frac{p_A V_B}{V_A + V_B}$

Ans. (c)

Sol. Since No change in internal energy during the process.



$P_A(V_A) = P.(V_A + V_B)$

$\Rightarrow \quad \boxed{P = \dfrac{P_A V_A}{V_A + V_B}}$

30. Let $(1 + x - 3x^2)^{2018} = a_0 + a_1x + a_2x^2 + ... + a_{4036}x^{4036}$. The last digit of $a_0 + a_2 + a_4 + ... a_{4036}$ is

(a) 0 (b) 5 (c) 7 (d) 9

Ans. **(b)**

Sol. Put x = 1

$1 = a_0 + a_1 + a_2 + ... + a_{4036}$

Put x = -1

$3^{2018} = a_0 - a_1 + a_2 - a_3 + ... + a_{4036}$

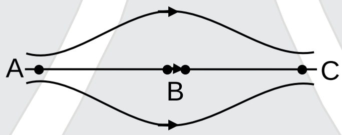
$2[a_0 + a_2 + a_4 + ... + a_{4036}] = (10 - 1)^{1009} + 1$

$2[a_0 + a_2 + a_4 + ... + a_{4036}] = {}^{1009}c_0\, 10^{1009} - {}^{1009}c_1\, 10^{1008} + ...+ {}^{1009}c_{1008}\, 10 - 1 + 1.$

$= 100\lambda + 10090$

Last digit = 5

31. The figure shows some of the field lines of an electric field. The figure suggests that



(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$ (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

Ans. **(c)**

Sol. Closer the field line, greater the strength of $\vec{E}$ field

$E_A = E_C > E_B$

32. The value of the integral $\int_0^2 x\cos(\pi\{x\})\,dx$, where {x} denotes the fractioal part of x, is

(a) 0 (b) $\dfrac{4}{\pi^2}$ (c) $\dfrac{-4}{\pi^2}$ (d) $\dfrac{-2}{\pi^2}$

Ans. **(c)**

Sol. $\int_0^2 x\cos(\pi\{x\})\,dx$

$= \int_0^1 x\cos\pi x dx + \int_1^2 x\cos(x-1)\pi\,dx$

$= \int_0^1 x\cos\pi x\,dx - \int_1^2 x\cos\pi x\,dx$

$= -\dfrac{2}{\pi^2} - \dfrac{2}{\pi^2} = -\dfrac{4}{\pi^2}$

Ans. c

33. The moment of the force **F** = 4**i** + 5**j** – 6**k** acting at the point (2, 0, –3) and about the axis passing through a point (2, – 2, – 2) is given by

(a) –7**i** – 4**j** – 8**k** (b) –7**i** – 8**j** – 4**k** (c) –4**i** – **j** – 8**k** (d) –8**i** – 4**j** – 7**k**

Ans. **(a)**

Sol. $\vec{r} = (2\hat{i} - 3\hat{k}) - (2\hat{i} - 2\hat{j} - 2\hat{k}) = 2\hat{j} - \hat{k}$

$\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} = (2\hat{j} - \hat{k}) \times (4\hat{i} + 5\hat{j} - 6\hat{k})$

$= -8\hat{k} - 12\hat{i} - 4\hat{j} - 5(-\hat{i})$

$= -7\hat{i} - 4\hat{j} - 8\hat{k}$

34. If $\alpha\ \beta\ \gamma$ are the roots of $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = 0$, then $\dfrac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2}$ equals

(a) $\dfrac{1}{7}$ (b) 7 (c) $\dfrac{1}{6}$ (d) 6

Ans. **(b)**

Sol. $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = 0$

$\Rightarrow$ $x^3 + 6 - 7x$

$\Rightarrow$ (x – 1) (x – 2) (x + 3) = 0

$\Rightarrow$ α, β, γ are 1, 2, –3

$\Rightarrow$ $\dfrac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2} = \dfrac{1 + 16 + 81}{1 + 4 + 9} = \dfrac{98}{14} = 7$

35. If all nuclear reactions in the Sun now were to suddenly stop for ever, then

(a) Distances between planets and sun would decrease
(b) Angular momentum of planets would increase
(c) Inner planets will be engulfed by the sun
(d) Speed of rotation of the sun about its own axis would increase

Ans. **(d)**

Sol. As reactions will stop, sun will contract, as a result moment of inertia will decrease and hence ω will increase.

36. Three well known stars (a) Procyon (b) Antares and (c) Vega are respectively in the constellation

(a) Orion, Sagittarius and Scorpius (b) Orion, Taurus and Ursa major
(c) Canis minor, Scorpius and Lyra (d) Scorpius, Canis minor and Leo

Ans. **(c)**

Sol. Procyon belongs to canis minor constellation
Antares belongs to scorpius constellation
vega belongs to lyra constellation

37. One gram of Radium, with atomic weight 226, emits 4×10^{10} particles per second. The half-life of Radium is

(a) 4.6×10^{10} s (b) 4.6×10^{9} s (c) 4.6×10^{12} s (d) 4.6×10^{14} s

Ans. **(a)**

Sol. Moles = $\frac{1}{226}$

Atoms = $\frac{1}{226} \times N_A$

$A = A_0\lambda$

$4 \times 10^{10} = \frac{1}{226} \times N_A \lambda$

$\lambda = \frac{4 \times 10^{10} \times 226}{6.022 \times 10^{23}} = \frac{904 \times 10^{10}}{6.022 \times 10^{23}} = 1.45 \times 10^{-11}$

$t = \frac{\ln 2}{\lambda} = 4.7 \times 10^{10}$ s

38. Let $\langle a_n \rangle_{n \geq 0}$ be a geometric progression with common ratio r, |r| < 1. Let $s_1 = \sum_{k=0}^{\infty} a_k$, $s_2 = \sum_{k=0}^{\infty} a_{2k}$ and $s_3 = \sum_{k=0}^{\infty} a_{3k}$. Suppose $\frac{s_1}{s_2} = \frac{5}{4}$. Then $\frac{s_2}{s_3}$ equals

(a) $\frac{5}{4}$ (b) $\frac{25}{24}$ (c) $\frac{21}{20}$ (d) $\frac{9}{10}$

Ans. **(c)**

Sol. $S_1 = a_0 + a_1 + a_2 + ... \infty = \frac{a_0}{1-r}$

$S_2 = a_0 + a_2 + a_4 + ... = \frac{a_0}{1-r^2}$

$S_3 = a_0 + a_3 + a_6 + ... = \frac{a_0}{1-r^3}$

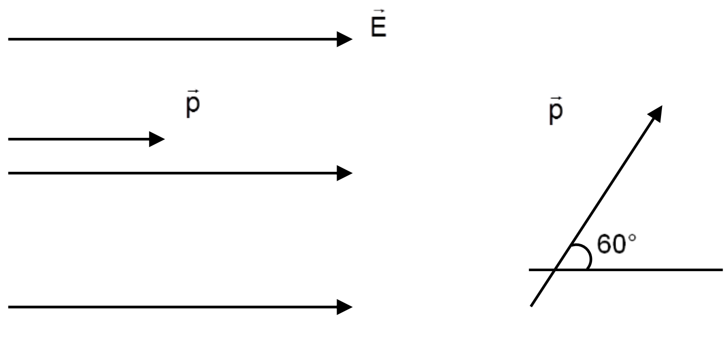
$\frac{S_1}{S_2} = \frac{5}{4} \quad \Rightarrow \quad 1 + r = \frac{5}{4} \quad \Rightarrow \quad r = \frac{1}{4} = \frac{S_2}{S_3} = \frac{1+r+r^2}{1+r} = \frac{21}{20}$

39. An electric dipole of moment p is lying on a plane in a uniform electric field E_0 with the dipole axis along the field. The dipole on the plane is rotated by an angle 60° keeping its center of mass fixed. The potential energy of the dipole in its new position will be

(a) $-pE_0$ (b) $-(pE_0)/2$ (c) $-(pE_0)/3$ (d) $-(pE_0)/4$

Ans. **(b)**

Sol.



U = $-\vec{p}\cdot\vec{E}$

$= -pE\frac{1}{2} = \frac{-pE}{2}$

40. Let $I_1 = \int_0^1 \frac{dx}{1+\sqrt[3]{x}}$ and $I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}}$. Then $4I_1 + 3I_2$ equals

(a) 3 (b) 4 (c) 6 (d) 7

Ans. **(b)**

Sol. $I_1 = \int_0^1 \frac{dx}{1+x^{1/3}}$ & $I_2 = \int_0^1 \frac{dx}{1+x^{1/4}}$

$I_1 = \int_0^1 \frac{dx}{1+x^{1/3}}$ Put $x = t^3$

$dx = 3t^2dt$

$= \int_0^1 \frac{3t^2dt}{1+t}$

$= \int_0^1 \left(3(t-1) + \frac{3}{t+1}\right)dt$

$= 3\left(\frac{t^2}{2} - t + \ln(t+1)\right)\Big|_0^1$

$= 3\left(\frac{1}{2} - 1 + \ln 2\right)$

$= 3\left(\ln 2 - \frac{1}{2}\right)$

$I_2 = \int_0^1 \frac{dx}{1+x^{1/4}}$ Put $x = t^4$

$dx = 4t^3\,dt$

$= \int_0^1 \frac{4t^3dt}{1+t}$

$= \int_0^1 \left(t^2 - t + 1 - \frac{1}{t+1}\right)dt \quad = 4\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \ln(t+1)\right]_0^1$

$= 4\left[\frac{1}{3} - \frac{1}{2} + 1 - \ln 2\right] \quad = 4\left[\frac{5}{6} - \ln 2\right]$

$\Rightarrow \quad 4I_1 + 3I_2 = 4$ **Ans. (b)**

41. The wave length of H_α line from hydrogen discharge tube in a laboratory is 656 nm. The corresponding radiation received from two galaxies A and B have wavelengths of 648 nm and 688 nm respectively. Then

(a) A is approaching the earth with a speed of 2.4×10^4 kms^{-1}

(b) B is approaching the earth with a speed of 1×10^4 kms^{-1}

(c) A is receding from the earth with a speed of 3.6×10^4 kms^{-1}

(d) B is receding the earth with a speed of 1.5×10^4 kms^{-1}

Ans. **(d)**

Sol. Since the wavelength received from A is smaller than the actual (656 nm) thus A is approaching earth. Similarly, B is receeding away from us according to Doppler shift of light.

For A : $\frac{\Delta\lambda}{\lambda} = \frac{v_A}{C}$

$\therefore \quad v_A = \frac{8}{656} \times 3 \times 10^8 \text{ m/s}$

$= 0.0366 \times 10^8$ m/s $= 3.66 \times 10^3$ km/s

For B : $\frac{\Delta\lambda}{\lambda} = \frac{v_B}{c}$

$\therefore \quad v_B = \frac{32}{656} \times 3 \times 10^8$

$= 1.46 \times 10^4$ km/s $\approx 1.5 \times 10^4$ km/s

42. The correct sequence of the objects in the ascending order of distance from the sun, is

(a) Kupier belt, Uranus, asteroid belt and Oort cloud

(b) Uranus, asteroid belt, Oort cloud and Kupier belt

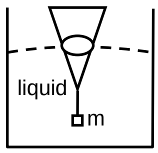
(c) Oort cloud, asteroid belt, Uranus and Kupier belt

(d) asteroid belt, Uranus, Kupier belt, and Oort cloud

Ans. **(d)**

Sol. Asteroid belt lies between Mars and Jupiter and is closest to sun. Then comes uranus. Kupier belt is about 30 a.u. distance away from the sun. Oort cloud is considered at the edge of the solar system and is farthest.

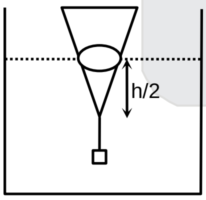
43. A cone of height h is floating in a liquid upside down with a mass m attached to it as shown in the figure. Water reaches a height of h/2 at equilibrium. The cone is now given a small downward push and is found to oscillate about its mean position. If friction is ignored the frequency of this oscillation is



(a) $\frac{1}{2\pi}\sqrt{\frac{g}{h}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2g}{h}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6g}{h}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{9g}{h}}$

Ans. **(c)**

Sol.



$\frac{h}{R} = \frac{\frac{h}{2}}{r'}$, $\quad r' = R/2$

mg of cone $= \frac{h}{2}\left(\frac{R}{2}\right)^2 \pi \frac{1}{3} \rho_\ell g$

Restoring force = Extra buoyant force.

$F_B = \pi r^2 \text{ x } \rho_\ell g = \pi \frac{R^2}{4} \text{x} \rho_\ell g$

$m\omega^2 = \frac{\pi R^2}{4} \rho_\ell g$

$\omega^2 = \frac{6g}{h}$ $\quad \omega = \sqrt{\frac{6g}{h}}$ $\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{6g}}$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{6g}{h}}$ **Ans. (C)**

44. The number of solutions of $1 - \sin^4x - 2\cos^4x = 0$ in the interval $[0, 2\pi]$ is

(a) 6 (b) 4 (c) 2 (d) 0

Ans. (a)

Sol. $1 - \sin^4x - 2\cos^4x = 0$

$2\cos^4x = (1 - \sin^2x)(1 + \sin^2x)$

$\cos^2x\,(2\cos^2x - 1 - \sin^2x) = 0$

$\cos^2x = 0$ or $2\cos^2x = 1 + \sin^2x$

$3\cos^2x = 2$

$\Rightarrow \cos^2x = 0, \frac{2}{3}$

Hence number of solutions = 6 **Ans. (a)**

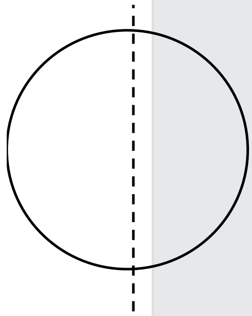
45. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

(a) Angular momentum (b) Rotational kinetic energy

(c) Moment of inertia (d) Angular velocity

Ans. (a)

Sol.



$\therefore$ As angular momentum remain conserved.

$I_1\omega_1 = I_2\omega_2$

$I_2 \uparrow, \quad \omega_2 \downarrow$

46. If n is the least positive integer such that $\binom{n-1}{5} + \binom{n-1}{7} < \binom{n}{7}$, the sum of digits of n is

(a) 6 (b) 5 (c) 4 (d) 3

Ans. (c)

Sol. $\binom{n-1}{5} + \binom{n-1}{7} < \binom{n}{7}$

$\Rightarrow {}^{n-1}C_5 + {}^{n-1}C_7 + {}^{n}C_7$

$\Rightarrow {}^{n-1}C_5 + {}^{n-1}C_6 + {}^{n-1}C_7 < {}^{n-1}C_6 + {}^{n}C_7$

$\Rightarrow {}^{n}C_6 + {}^{n-1}C_7 < {}^{n-1}C_6 + {}^{n}C_7$

$\Rightarrow \frac{\underline{|n}}{\underline{|6}\,\underline{|n-6}} + \frac{\underline{|n-1}}{\underline{|7}\,\underline{|n-8}} < \frac{\underline{|n-1}}{\underline{|6}\,\underline{|n-7}} + \frac{\underline{|n}}{\underline{|7}\,\underline{|n-7}}$

$\Rightarrow \frac{n}{(n-6)(n-7)} + \frac{1}{7} < \frac{1}{n-7} + \frac{n}{7.(n-7)}$

$\Rightarrow 7n + (n-6)(n-7) < 7(n-6) + n(n-6)$

$\Rightarrow n^2 - 6n + 42 < n^2 + n - 42 \quad \Rightarrow \quad 84 < 7n$

$\Rightarrow \boxed{n > 12}$

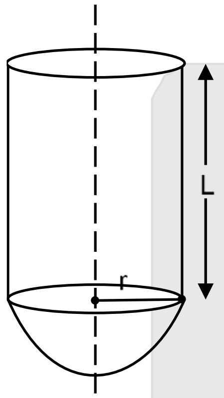
$\Rightarrow$ Least value of n = 13 **Ans. (c)**

47. The flat surface of a solid hemisphere of radius r is cemented to one flat surface of a cylinder (of identical material) of radius r and length L. If the total mass is M, moment of inertial of the combination about the axis of the cylinder will be

(a) $Mr^2 \dfrac{\frac{L}{2}+\frac{4r}{15}}{L+\frac{2r}{3}}$ (b) $Mr^2 \dfrac{\frac{L}{3}+\frac{4r}{5}}{L+\frac{2r}{3}}$ (c) $Mr^2 \dfrac{\frac{L}{3}+\frac{4r}{5}}{\frac{L}{2}+\frac{r}{3}}$ (d) $Mr^2 \dfrac{\frac{L}{6}+\frac{2r}{5}}{L+\frac{4r}{3}}$

Ans. **(a)**

Sol.



Mass = M

$$I = \frac{(M)\pi r^2 h(r^2)}{2\pi r^2 h + \frac{2}{3}\pi r^3} + \frac{2M\left(\frac{2}{3}\pi r^3\right)r^2}{5\left(\frac{2}{3}\pi r^3 + \pi r^2 h\right)}$$

$$\frac{Mr^2\pi}{\frac{2}{3}\pi r^3 + \pi r^2 h}\left(\frac{L}{2} + \frac{4}{3}\frac{r}{(5)}\right) = \frac{3Mr^2}{2r+3L}\left(\frac{L}{2}+\frac{4r}{15}\right) = \frac{Mr^2}{\frac{2r}{3}+L}\left(\frac{L}{2}+\frac{4r}{15}\right)$$

48. The limit $\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}$

(a) does not exist (b) is $\frac{1}{2}$ (c) is 2 (d) is ln 2

Ans. **(b)**

Sol. $\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}$

$$= \lim_{x\to\infty} \frac{x+\sqrt{x+\sqrt{x}} - x}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{3/2}}}}+1} = \frac{1}{1+1} = \frac{1}{2}$$ **Ans. (b)**

49. An electron is moving with uniform velocity along a line in the plane of the paper. It is now subjected to a uniform magnetic field B perpendicular to the plane of the paper and going into it. The electron will move in a circular path in the plane of the paper in

(a) clockwise direction with time period proportional to B

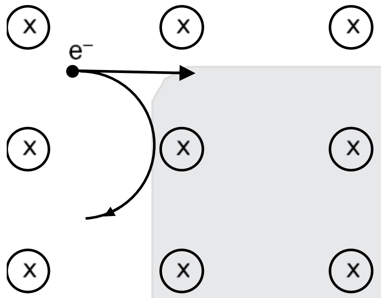
(b) anticlockwise direction with time period inversely proportional to B

(c) clockwise direction with time period inversely proportional to B.

(d) anticlockwise direction with time period proportional to B

Ans. **(c)**

Sol.



$$R = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi m}{qB}$$

Option (c)

50. Let $s_n = 1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + \ldots + n\left(1+\frac{1}{n}\right)^n$. Then $\sum_{n=1}^{\infty} \frac{1}{2^{2\sqrt{s_n}}}$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) 1

Ans. **(b)**

Sol. $S_n = 1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + \ldots\ldots + n\left(1+\frac{1}{n}\right)^n$ .....(1)

$\left(1+\frac{1}{n}\right)S_n = 1\left(1+\frac{1}{n}\right) + 2\left(1+\frac{1}{n}\right) + \ldots.. + (n-1)\left(1+\frac{1}{n}\right)^n + n\left(1+\frac{1}{n}\right)^{n+1}$ .....(2)

Form (1) – (2)

$S_n(-1/n) = 1 + \left(1+\frac{1}{n}\right) + \left(1+\frac{1}{n}\right)^2 + \ldots.. + \left(1+\frac{1}{n}\right)^n - n\left(1+\frac{1}{n}\right)^{n+1}$

$$= \frac{\left(1+\frac{1}{n}\right)^{n+1} - 1}{\frac{1}{n}} - n\left(1+\frac{1}{n}\right)^{n+1}$$

$= S_n = n^2$

$\Rightarrow \sqrt{S_n} = n$

$$\sum_{n=1}^{\infty} \frac{1}{2^{2\sqrt{S_n}}} = \sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1/4}{1-1/4} = \frac{1}{3}$$ Ans. (b)

51. A loudspeaker emits sound at a maximum audible level of 130 dB when measured directly from a distance of 1 metre. If the safe limit of audible sound to our ears is 90 dB, a listener must stand directly at a minimum distance of :

(a) 1.44 m (b) e^2 m (c) 100 m (d) 2.09 m

Ans. **(c)**

Sol. L = 130 dB

$$B = 10\log\left(\frac{I}{I_0}\right)\text{dB}$$

$$130 = 10\log\left(\frac{K_1}{I_0}\right) \qquad \ldots(i)$$

$$90 = 10\log\left(\frac{\frac{K}{x^2}}{I_0}\right) \qquad \ldots(ii)$$

$$130 - 90 = 10\log\frac{K}{I_0}\times\frac{I_0(x^2)}{K}$$

$4 = \log(x^2)$

$10^4 = x^2$

$x = 100$

52. The diameter of radio telescope, working at a wavelength of λ = 1 cm, with the same resolution as optical telescope of diameter D = 10 cm is :

(a) 2m (b) 2 km (c) 20 km (d) 200 km

Ans. **(b)**

Sol. For radio Telescope

$$\text{R.P.} = \frac{D_r}{1.22\lambda} = \frac{D_r}{1.22\times10^{-2}}$$

For optical Telescope

$$\text{R.P.} = \frac{D}{1.22\lambda_v} \approx \frac{0.1}{1.22\times5.5\times10^{-7}}$$

(λ_v is taken average of the visible spectrum range)

Both being equal ; $D_r = \dfrac{10^{-3}}{5.5\times10^{-7}} = \dfrac{10^4}{5.5} = 1.82\text{Km} \sim 2$ km

53. In a binary system, the apparent magnitude of the primary star is 1.0 and that of the secondary star is 2.0. The maximum combined magnitude of this system is

(a) 3 (b) 1.5 (c) 1 (d) 0.64

Ans. **(d)**

Sol. $m_f = -2.5\log_{10}(10^{-0.4m_1} + 10^{-0.8m_2})$

$m_1 = 1$, $m_2 = 2$

$\Rightarrow \quad m_f = 0.64$

54. Suppose the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 (b < a) at the point $\left(ae, -\frac{b^2}{a}\right)$ makes an angle of 30° with x-axis. The $\frac{b^2}{a^2}$ equals

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans. **(c)**

Sol. Equation of tangent at $\left(ae, -\frac{b^2}{a}\right)$ is

$\frac{ex}{a}-\frac{y}{a}=1 \Rightarrow$ y = ex – a

Given slope of tangent is = tan30°

$\Rightarrow$ e = tan 30 = $\frac{1}{\sqrt{3}}$ $\Rightarrow$ $e^2 = \frac{1}{3}$

$\Rightarrow$ $1 - e^2 = \frac{2}{3}$ $\Rightarrow$ $\frac{b^2}{a^2}=\frac{2}{3}$ **Ans. (c)**

55. A piece of strong magnet is suspended from a helical spring made of a non magnetic material and oscillates in a vertical plane with a time period of T on the surface of the earth. If this is taken to the moon then it will oscillate

(a) with a time period T_1 > T as the value of 'g' is smaller on the moon

(b) with a time period T_1 < T as the value of 'g' is smaller on the moon

(c) with a time period T_1 > T as three is no magnetic field on the moon

(d) with the same time period as the spring and the suspended body are the same on the moon

Ans. **(d)**

Sol. Since the spring is non magnetic, thus there will be no effect on it due to magnet. Also, it is a spring mass system, thus there will be no impact on the time period $\left(T = 2\pi\sqrt{\frac{m}{K}}\right)$ of oscillation on the moon.

56. The number of triples (a, b, c) of natural numbers satisfying the equation $\frac{5}{12}=\frac{1}{a}+\frac{1}{ab}+\frac{1}{abc}$ is

(a) 7 (b) 8 (c) 9 (d) 12

Ans. **(a)**

Sol. $\frac{5}{12}=\frac{1}{a}+\frac{1}{ab}+\frac{1}{abc}$

$\frac{5}{12}=\frac{1+c+bc}{abc}$

Case-**I** : abc = 12 & (b + 1) c = 4

$\Rightarrow$ b =1, c = 2, a = 4

b = 3, c = 1, a = 4 (a, b, c) = {(4, 1, 2) or (4, 3, 1)}

Case-**II** : abc = 24 & (b + 1) c = 9

$\Rightarrow$ b = 2, c = 3, a = 4

b = 8, c = 1, a = 3 (a, b, c) = {(4, 2, 3) or (3, 8, 1)}

Case-III : abc = 36 & (b + 1)c = 14

(a, b, c) = {(3, 6, 2)}

Case-IV : abc = 48 & (b + 1)c = 19

(a, b, c) = No solutions

Case-V : abc = 60 & (b + 1) c = 24

(a, b, c) = {(5, 1, 12), (3, 5, 4)}

Case-VI : abc = 72 & (b + 1)c = 29 no solution

Case-VII : abc = 84 & (b + 1)c = 34 no solution

Case-VIII : abc = 96 & (b + 1)c = 39 no solution

Hence 7 solutions only **Ans. a**

57. A 1.5 times magnified real image of an object is obtained when it is placed 16 cm away from a thin convex lens. Now a thin concave lens is placed in contact with the convex lens keeping the object undisturbed and an image of same magnification is formed by the combination. The focal length of the concave lens is

(a) 8 cm (b) 10 cm (c) 12 cm (d) 16 cm

Ans. **(c)**

Sol. $\frac{V}{U} = \frac{3}{2}$

V = $\frac{3}{2} \times 16$ 24 cm

$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$

$\frac{1}{24} + \frac{1}{16} = \frac{1}{f}$

f = $\frac{(16)24}{40}$ = 9.6 cm

After combining with concave lens

$\frac{-1}{24} + \frac{1}{16} = \frac{1}{f}$

$\Rightarrow \quad \frac{-16+24}{24 \times 16} = \frac{1}{f}$

f = $\frac{24 \times 16}{8} = 48$

$\frac{1}{48} = \frac{1}{9.6} + \frac{1}{f_1}$

0.0208 = 0.1041 + $\frac{1}{f_1}$

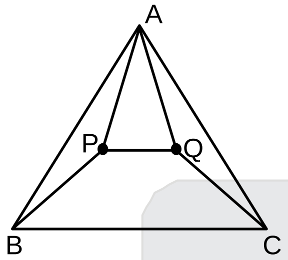
f_1 = –12 cm

58. Let ABC be an equilateral triangle with side x. Two points P and Q are inside ABC such that PQ is parallel to BC and AP = AQ = PB = QC = $\sqrt{3}+1$ and PQ = $\sqrt{2}$. Then x equals

(a) $4\sqrt{2}+2\sqrt{6}$ (b) $2\sqrt{2}+\sqrt{6}$ (c) $2\sqrt{3}+\sqrt{6}$ (d) $2\sqrt{6}+\sqrt{3}$

Ans. **(b)**

Sol.



In ΔAPQ AP = AQ = $\sqrt{3}$ + 1 & PQ = $\sqrt{2}$

$$\cos\angle PAQ = \frac{(\sqrt{3}+1)^2+(\sqrt{3}+1)^2-(\sqrt{2})^2}{2(\sqrt{3}+1)^2}$$

$$= 1-\frac{1}{(\sqrt{3}+1)^2}$$

$$= 1-\frac{(\sqrt{3}-1)^2}{4}$$

$$= 1-\frac{(4-2\sqrt{3})}{4} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $\angle PAQ = 30°$ $\Rightarrow$ $\angle BAP = \angle QAC = 15°$

Now In ΔAPB $\frac{AB}{\sin 150°} = \frac{AP}{\sin 15}$

$$\Rightarrow \quad x = (\sqrt{3}+1).\frac{\sin 150°}{\sin 15} = \frac{(\sqrt{3}+1)(1/2)}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\sqrt{2}(\sqrt{3}+1)}{\sqrt{3}-1}$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)^2}{2} = \sqrt{2}(2+\sqrt{3})$$ **Ans. (b)**

59. Critical velocity, drift velocity, escape velocity and rms velocity are the different types of velocities that we come across in the same order while discussing

(a) viscosity, electron motion in solids, gravitation, surface tension respectively

(b) motion of gas molecules, viscosity, gravitation, electron motion in solids respectively

(c) sound propagation, gravitation, motion of gas molecules, colour of light respectively.

(d) viscosity, electron motion in solids, gravitation, motion of gas molecules respectively

Ans. **(d)**

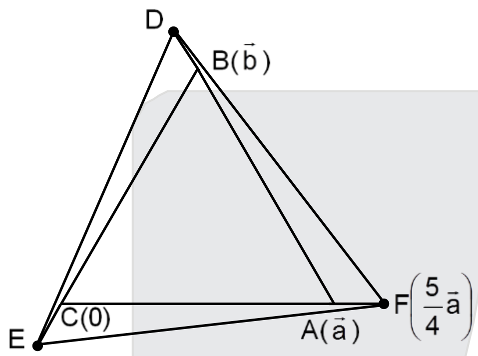
Sol. We define, critical velocity for viscous flow of fluid. Drift velocity is associated with the electron motion in solids. Escape velocity is discussed in the context of gravitation & r.m.s. velocity for ideal gas molecules in Brownian motion.

60. In a triangle ABC, AB is extended to D such that AB : BD = 4 : 1; BC is extended to E such that BC : CE = 4 : 1; and CA is extended to F such that CA : AF = 4 : 1. The ratio of the area of triangle DEF to that of ABC is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{15}{8}$ (d) $\frac{31}{16}$

Ans. **(d)**

Sol.



Let P.V of A and B are $\vec{a}$ & $\vec{b}$ w.r.t. c($\vec{0}$)

Hence area of ΔABC = $\frac{1}{2}|\vec{a}\times\vec{b}|$

Clearly P.V. of F$\left(\frac{5}{4}\vec{a}\right)$ and E$\left(-\frac{1}{4}\vec{b}\right)$ and P.V. of D $\left(\frac{5\vec{b}-\vec{a}}{4}\right)$

$\Rightarrow$ Area of ΔDEF = $\frac{1}{2}\left|\left(\frac{5\vec{a}+\vec{b}}{4}\right)\times\left(\frac{6\vec{b}-\vec{a}}{4}\right)\right| = \frac{1}{32}|31(\vec{a}\times\vec{b})|$

$\Rightarrow$ $\frac{\Delta DEF}{\Delta ABC} = \frac{\frac{31}{32}|\vec{a}\times\vec{b}|}{\frac{1}{2}|\vec{a}\times\vec{b}|} = \frac{31}{16}$

61. Imagine a planet of same mass as that of the earth but having a radius twice of that of the earth. A simple pendulum located at some point on its equator failed to show any oscillation when given a small displacement from its equilibrium position. The time taken by this planet to spin once about its own axis is

(a) nearly 2 hours (b) nearly 4 hours (c) nearly 6 hours (d) nearly 8 hours

Ans. **(b)**

Sol. $g' = \frac{GM}{(2R)^2} = \frac{g}{4}$

$\frac{g}{4} = w_{new}^2(2R)$

$w_{new} = \sqrt{\frac{g}{8R}}$

$\therefore$ T $\approx$ 4 h

62. Let ABCD be a rectangle. Let E be a point on the diagonal AC at a distance 16 from the side AB and let DE = 15. Then the area of the rectangle ABCD to the nearest integer is

(a) 468 (b) 469 (c) 470 (d) 471

Ans. **(Bonus or d)**

Sol. Let AD = b, AB = a & AF = ℓ

ΔAEF and ΔACB are similar

$\Rightarrow \quad \frac{16}{b} = \frac{\ell}{a} \quad \Rightarrow \quad \ell = \frac{16a}{b}$

In ΔDEG

$\Rightarrow \quad (b-16)^2 + \left(\frac{16a}{b}\right)^2 = 15^2 \quad \Rightarrow \quad b^2(b-16)^2 + 256a^2 = 225b^2$

$\Rightarrow \quad 256a^2 = -b^4 + 32b^3 - 31b^2 \quad \Rightarrow \quad a = \frac{1}{16}\sqrt{32b^3 - b^4 - 31b^2}$

$\Rightarrow \quad a = \frac{b}{16}\sqrt{32b - b^2 - 31} \quad \Rightarrow \quad ab = \frac{b^2}{16}\sqrt{225-(16-b)^2}$

Δ = Area = $\frac{b^2}{16}\sqrt{225-(16-b)^2}$

$\frac{d\Delta}{db} = 0 \Rightarrow \quad b = \frac{80+\sqrt{5656}}{6} = 25.8677305$

Hence area = 41.8212176 × 1.2972517 = 472.46

63. An alloy of two metals is formed by taking their equal masses and it was found to float on mercury (density 13.6 g cm^{-3}) with 52.7% above the mercury surface. When an alloy is formed by taking equal volumes of these two metals it was found to float on mercury with 51.5% of its volume below the surface of mercury. The densities of the two metals in g cm^{-3} are closest to

(a) 6 and 8 (b) 5 and 9 (c) 4.5 and 9.5 (d) 4 and 10

Ans. **(b)**

Sol. $\rho_A = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$ same mass

$\rho_B = \frac{\rho_1+\rho_2}{2}$ same volume

$\Rightarrow \quad \frac{2\rho_1\rho_2}{\rho_1+\rho_2} = 0.473.\ \rho_{Hg} \Rightarrow \rho_1\rho_2 = 0.473\times13.6\times7$

$\Rightarrow \rho_1\rho_2 = 45.02$

$\frac{\rho_1+\rho_2}{2} = 0.515\ \rho_{Hg} \Rightarrow \rho_1+\rho_2 = 1.030\times13.6$

$\Rightarrow \rho_1+\rho_2 = 14.008$

$(\rho_2-\rho_1)^2 = (\rho_2+\rho_1)^2 - 4\rho_1\rho_2$

$= (14)^2 - 4 \times 45$

$(\rho_1-\rho_2)^2 = 196 - 180$

$\rho_1-\rho_2 = 4$ …(1)

$\boxed{\begin{array}{l}\rho_1 = 9\\ \rho_2 = 5\end{array}}$

64. If n is the number of functions f : {a, b, c, d} → {a, b, c, d} such that no more than two elements in the domain of f have the same image, then

(a) $n \le 100$ (b) $100 < n \le 150$ (c) $150 < n \le 200$ (d) $n > 200$

Ans. **(d)**

Sol. f : {a, b, c, d} → {a, b, c, d}

Total no. of functions f are = $^4P_4 + {}^4C_2 \cdot 4.3.2 + 3 \cdot {}^4C_2 \cdot 2$

= 24 + 144 + 36 = 204

Ans. d

65. On the rechargeable batteries of 1.5 V often used for digital cameras one can find 2300 mAh or 2800 mAh or something similar is written. This is connected to the

(a) power that the battery can provide (b) current that can be drawn from the battery

(c) total charge that the battery can supply (d) time for which the battery can be used.

Ans. **(c)**

Sol. The unit mAh correspond to unit of charge supplied by the battery in 1hour.

mAh = 1mA × 1h = 10^{-3} A × 3600 s = 3.6 c

66. The planet in which sun appears to rise in the west is

(a) Venus (b) Uranus (c) Saturn (d) Mercury

Ans. **(a)**

Sol. Sun rises west to east in Venus.

67. Apart from the earth, Aurora phenomena are observed on which of the following planet(s)

(a) Venus (b) Mars (c) Mercury (d) Jupiter

Ans. **(a,d)**

Sol. Information based.

68. The sum of the last three digits in the expansion of 5^{2018} is

(a) 8 (b) 9 (c) 13 (d) 14

Ans. **(c)**

Sol. $5^1 = 005$

$5^2 = 025$

$5^3 = 125$

$5^4 = 625$

$5^5 = 3125$

$5^6 = 15625$

$5^7 = 78125$

Clearly in 5^n ($n \ge 3$) last three digits are 125 or 625 according to n is odd or even respectively.

⇒ 5^{2018} has last three digits are 625

Ans. c

69. If the wavelength of the incident light changes from 400 nm to 300 nm the stopping potential for photoelectrons emitted from the surface of a material becomes

(a) 0.56 V lower (b) 1.04 V higher (c) 0.34 V lower (d) 0.56 V higher

Ans. **(b)**

Sol. Energy of photon in case-1 :

$$\frac{hc}{\lambda_1} = \frac{1240}{400} eV \approx 3.1eV$$

Energy of photon in case-2

$$\frac{hc}{\lambda_2} = \frac{1240}{300} eV \approx 4.1eV$$

∴ Diff. energy = 4.1 – 3.1 ≈ 1.03eV

70. Find integer closest to the integral $\int_0^6 x^{|\sqrt{x}|} dx$, where {x} denotes the largest integer not exceeding x.

(a) 58 (b) 59 (c) 60 (d) 61

Ans. **(Bonus or b)**

Sol. $\int_0^6 x^{\{\sqrt{x}\}} = \int_0^1 1dx + \int_1^4 x\,dx + \int_4^6 x^2 dx$

$= 1 + \frac{15}{2} + \frac{15^2}{3} = \frac{355}{6} = 59.1\overline{6} \approx 59$

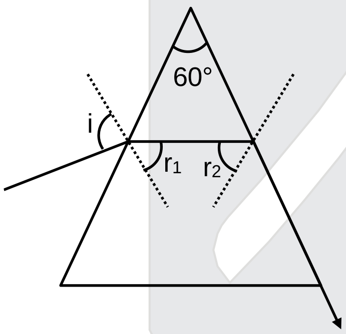
(Here | $\sqrt{x}$ | is same as { $\sqrt{x}$ } otherwise question is wrong)

71. A ray of light enters a glass prism of refractive index 1.55. The cross section of the prism is an equilateral triangle. The emergent ray comes out of the other refracting surface at the grazing angle. The angle of incidence on the first surface is about

(a) 30.7° (b) 28.2° (c) 37.6° (d) 41.2°

Ans. **(a)**

Sol.



1.55 sin r_2 = sin 90° × (1)

sin r_2 = $\frac{1}{1.55}$

r_2 = 40.16°

r_1 = 19.84°

(1) sin i = 1.55 sin (19.84°)

sin i ≈ 0.526

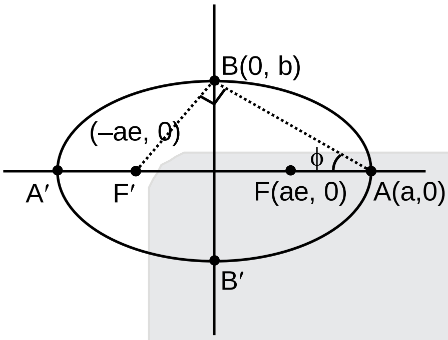
i ≈ 31.7°

72. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) be an ellipse with major axis AA' and minor axis BB'. Let F and F' be the foci of the ellipse, with F between A and F'. Suppose ABF' forms a right-angled triangle. Let e denote the eccentricity of the ellipse. If ϕ denotes $\angle$FAB,, then $\tan^2(\phi)$ is equal to

(a) $\sqrt{e}$ (b) e (c) e^2 (d) 1 + e

Ans. **(b)**

Sol.



Clearly F(ae, 0), F'(–ae, 0) & B(0, b)

$\Rightarrow \quad \tan\phi = \frac{b}{a}$

$\Rightarrow \quad \tan^2\varphi = \frac{b^2}{a^2}$ ......(1)

F'B $\perp$ AB

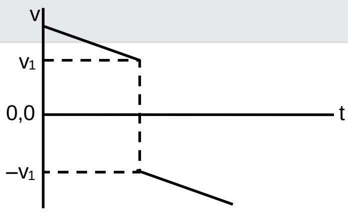
$\Rightarrow \quad \frac{b}{ae}.\left(\frac{-b}{a}\right) = -1$

$\Rightarrow \quad \frac{b^2}{a^2} = e$ ......(2)

by (1) and (2) $\tan^2\phi = e$

Ans. b

73. The following graph shows a velocity versus time graph for a ball. Which explanation best fits the motion of the ball as shown by the graph?



(a) The ball falls from a height, is caught, and is thrown down with a greater velocity

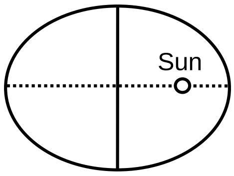
(b) The ball rises to a height, hits the ceiling, and falls down

(c) The ball falls from a height, hits the floor, and bounces up.

(d) The ball rises to a height, is caught, and then is thrown down with the same velocity.

Ans. **(b)**

74. According to Kepler's first law, planets go round the Sun in elliptic orbits. If orbit of the earth of eccentricity e around Sun is divided into two halves by the minor axis, the difference in times spent in the two halves of the orbit is



(a) $2e/\pi$ year (b) e/π year (c) e/(1-e) year (d) $2e^2/(1-e^2)$ year

Ans. **(a)**

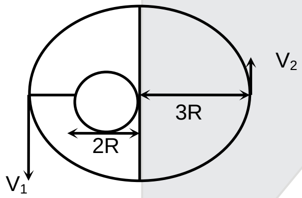
Sol. $\frac{dA}{dT} = \frac{\pi ab}{T}$

$A_1 = \frac{\pi ab}{2} + \frac{1}{2}(2b)ae = ab\left(\frac{\pi}{2} + e\right)$

$A_2 = ab\left(\frac{\pi}{2} - e\right)$

$\Delta T = \frac{A_1 - A_2}{\frac{\pi ab}{T}} = \frac{2abe}{\frac{\pi ab}{T}} = \frac{2eT}{\pi} = \frac{2e}{\pi}$ year

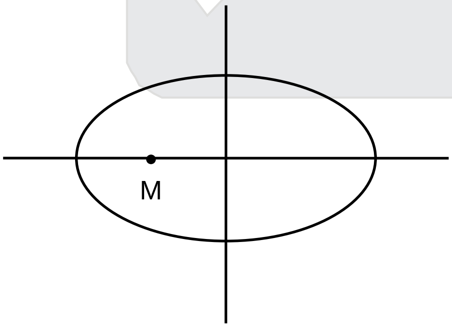
75. A planet goes around a star of mass M and radius R in an orbit of semi major axis 3R, with the distances as shown. What is the velocity V_1 at the point closest to the star:



(a) $(GM/2R)^{1/2}$ (b) $(2GM/3R)^{1/2}$ (c) $(4GM/3R)^{1/2}$ (d) $(GM/6R)^{1/2}$

Ans. **(b)**

Sol.



$mV_2 4R = mV_1 2R$

$\frac{1}{2}mV_1^2 - \frac{GMm}{2R} = -\frac{GMm}{6R}$

$\frac{1}{2}mV_1^2 = \frac{3GMm}{6R} - \frac{GMm}{6R} = \frac{GMm}{3R}$

$v_1 = \sqrt{\frac{2GM}{3R}}$

76. What are the eccentricity and length of semi minor axis in the orbit in Q.34?

(a) 0.30, 2.50R (b) 0.33, 2.00R (c) 0.33, 2.83R (d) 0.25, 2.75R

Ans. **(c)**

Sol. $e\ (3R) = R \Rightarrow \boxed{e = \frac{1}{3}}$

$a^2 = a^2e^2 + b^2$

$\Rightarrow b = a\sqrt{1-e^2} = \frac{2\sqrt{2}a}{3} = 2\sqrt{2}R$

77. If the earth of mass M is assumed to be a sphere of 6400 Km, with what velocity must a projectile be fired from the earth's surface in order that its subsequent path may be an ellipse with major axis 80, 000 Km? [Take the product GM = 4.0×10^{14} m^3s^2]

(a) 10.70 Km/s (b) 11.20 Km/s (c) 9.50 Km/s (d) 11.70 Km/s

Ans. **(a)**

Sol. $\frac{1}{2}mv^2 - \frac{GMm}{R_e} = -\frac{GMm}{2a}$

$v^2 = \frac{2GM}{R_e} - \frac{GM}{a} = GM\left(\frac{2a - R_e}{aR_e}\right)$ = 115

v = 10.72 km/s

78. Consider the cubic curve $y = 2x^3 - 12x^2 + 18x + 5$. Let A and C be its extremum points. The tangents at A and C to the curve intersect it again at two other points B and D respectively. The area of the quadrilateral ABCD is

(a) 12 (b) 24 (c) 36 (d) 48

Ans. **(b)**

Sol. $y = 2x^3 - 12x^2 + 18x + 5$

$\frac{dy}{dx} = 6x^2 - 24x + 18$

$\frac{dy}{dx} = 0 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x = 3, 1$

Points A(1, 13) & C (3, 5)

tangent at A(1, 13) is $y - 13 = 0$

tangent at C(3, 5) is $y - 5 = 0$

y = 13 meets the curve again at B (+4, 13)

y = 5 meets the curve again at D (0, 5)

Area of ABCD $= \frac{1}{2}\begin{vmatrix} 1 & 4 & 3 & 0 & 1 \\ 13 & 13 & 5 & 5 & 13 \end{vmatrix}$

$= \frac{1}{2}|(13 + 20 + 15 + 0 - 52 - 39 - 0 - 5)|$

$= \frac{1}{2}|(48 - 96)| = 24$ **Ans. (b)**

79. A crater on the surface of the moon has a diameter of 80 km. If the distance to earth and moon is 3.78×10^5 km then the visual angle in degree is

(a) 0.012 (b) 0.021 (c) 0.019 (d) 0.026

Ans. **(a)**

Sol. $\delta = \left(\frac{80}{3.78 \times 10^5}\right)\left(\frac{180 \times 7}{22}\right) = 0.0121$ Degree

80. A K-type star in the main sequence has a luminosity of 0.40 times the luminosity of sun. This star is observed to have a flux of 6.23×10^{-4} Wm^{-2}. The distance (in parsec) to this star is about (ignore atmospheric effects, luminosity of sun is 3.8×10^{26} Wm^{-2} and 1 parsec is 3.08×10^{16} km)

(a) 45 pc (b) 4.5 pc (c) 450 pc (d) 0.45 pc

Ans. **(bonus)**

Sol. $\frac{(0.40)(3.8\times10^{26})}{4\pi r^2} = 6.23 \times 10^{-4}$

$r^2 = \frac{(0.40)(3.8\times10^{26})}{4\pi(6.23\times10^{-4})} = 0.0194 \times 10^{30}$

$r^2 = 1.94 \times 10^{28}\ m^2 \Rightarrow \quad r = 1.4 \times 10^{14}$ m

$r = 1.4 \times 10^{11}$ km $= 0.454 \times 10^{-5}$ Per sec.

$= 4.54 \times 10^{-6}$ per sec $= 4.5\ \mu$ per sec.

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