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INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2017-18

Examination Date : 26-11-2017 **Time: 2 Hrs.**

PAPER CODE : A421 **HBCSE Olympiad (STAGE - 1)**

Write the question paper code mentioned above on YOUR answer sheet (in the space provided), otherwise your answer sheet will NOT be assessed. Note that the same Q. P. Code appears on each page of the question paper.

INSTRUCTION TO CANDIDATES

1. Use of mobile phones, smart phones, ipads during examination is STRICTLY PROHIBITED.
2. In addition to this question paper, you are given answer sheet along with Candidate's copy.
3. On the answer sheet, fill up all the entries carefully in the space provided, **ONLY In BLOCK CAPITALS**. Use only **BLUE or BACK BALL PEN** for making entries and marking answer.
   Incomplete / incorrect / carelessly filled information may disqualify your candidature.
4. On the answer sheet, use only BLUE or BLACK BALL POINT PEN for making entries and filling the bubbles.
5. The question paper contain 80 multiple-choice question. Each question has 4 options, out of which only one is correct. Choose the correct alternative and fill the appropriate bubble, as shown

6. A correct answer carries 3 marks and 1 mark will be deducted for each wrong answer.
7. Any rough work should be done only in the space provided.
8. Periodic Table is provided at the end of the question paper.
9. Use of a nonprogrammable calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting your answer paper, take away the Candidate's copy for your reference.
    Please DO NOT make any mar other than filling the appropriate bubbles properly in the space provided on the answer sheet. Answer sheet are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED.
    Scratching or overwriting may result in wrong score.
    DO NOT WRITE ANYTHING ON THE BACK OF ANSWER SHEET.

Read the following instructions after submitting the answer sheet.

12. Comments regarding this question paper, if any, may be filled in Google forms only at https://google/forms/9GP03NRgUVuhWJn52 till 28th November, 2017.
13. The answers/solutions to this question paper will be available on our website — www.iapt.org.in by 2nd December, 2017.
14. **Certificates & Awards**
    Following certificates are awarded by the IAPT to students successful in NSEs
    (i) Certificates to "Centre Top" 10% students
    (ii) Merit certificates to "State wise Top" 1% students.
    (iii) Merit certificate and a prize in term to "National wise" Top 1% students.
15. Result sheets can be downloaded from our website in the month of February. The "Centre Top 10%" certificates will be dispatched to the Prof-in-charge of the centre by February, 2017.
16. List of students (with centre number and roll number only) having score above MAS will be displayed on our website (www.iapt.org.in) by 22nd December, 2017. See the Eligibility Clause in the Student's brochure on our website.
17. Students eligible for the INO Examination on the basis of selection criteria mentioned in Student's brochure will be informed accordingly.

1. An aircraft is moving with velocity ν through air of density ρ and has wings of width L. The lift force per unit wingspan depends on L, ν and ρ as

(A) $L\,\nu^2\,\rho^2$ (B) $L^2\,\nu^2\,\rho$ (C) $L\,\nu^2\,\rho$ (D) $L^2\,\nu^2\,\rho^2$

Ans. **(C)**

Sol. $\frac{f}{\ell} = v^x \rho^y L^z$

$ML^0T^{-2} = M^y L^{x-3y+z} T^{-x}$

$x = z, \quad y = 1$

$x - 3y + z = 0$

$z - 3y + 1 = 0$

$3y = 3 \;\Rightarrow\; y = 1$

$v^2 \rho L$

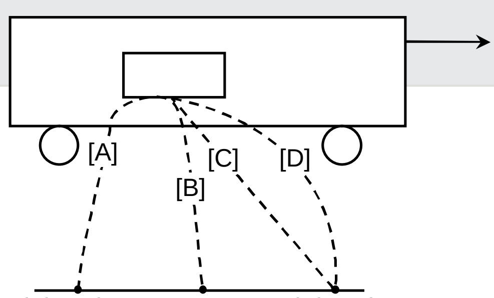
2. Let L be the limit $\lim_{x\to 0} = \frac{(1+3x)^2 - (1+2x)^3}{(1+3x)^{1/3} - (1+2x)^{1/2}}$. The L equals

(A) 6 (B) –6 (C) $\frac{1}{6}$ (D) $\frac{-1}{6}$

Ans. **(A)**

Sol. $\lim_{x\to 0} \frac{1+9x^2+6x-1-8x^3-6x-12x^2}{(1+x-x^2+.......)-\left(1+x-\frac{x^2}{2}+........\right)} = \lim_{x\to 0} \frac{-8x^3-3x^2}{-\frac{x^2}{2}+........} = 6$

3. A stone is dropped from the window of a train compartment moving in a horizontal direction as shown below. As observed by a person standing on the ground, which path would the stone most closely follow after leaving the window ?



(A) path A (B) path B (C) path C (D) path D

Ans. **(D)**

Sol.

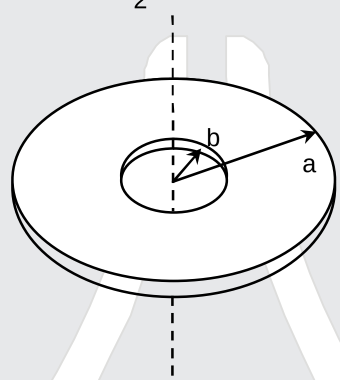
4. If the circumcentre of of a triangle lies on its incircle, then the ratio $\frac{r}{R}$, where r is the inradius and R is the circumradius of the triangle, is closest to

(A) 0.2 (B) 0.3 (C) 0.4 (D) 0. 5

Ans. **(C)**

Sol. OI = r $\Rightarrow OI^2 = r^2$ $\Rightarrow R^2 – 2Rr = r^2$ $\Rightarrow \frac{r}{R} = \sqrt{2} – 1 \approx 0.0414$

5. A flat circular ring has mass M, outer radius a and inner radius b (see figure). The measured values of these quantities are M = 0.191 ± 0.003 kg, a = 110 ± 1 mm and b = 15 ± 1 mm, the moment of inertia of the ring about an axis passing through the centre, and normal to the plane of the ring along with its error is: (Given $I = \frac{M}{2}(a^2 + b^2)$)



(A) $(1.18 \pm 0.04) \times 10^{-3}$ kg m^2
(B) $(1.18 \pm 0.02) \times 10^{-3}$ kg m^2
(C) $(1.18 \pm 0.05) \times 10^{-3}$ kg m^2
(D) $(1.18 \pm 0.08) \times 10^{-3}$ kg m^2

Ans. **(A)**

Sol. $I = \frac{M}{2}(a^2 + b^2)$

$$I \pm \Delta I = \frac{M \pm \Delta M}{2}\left[(a \pm \Delta a)^2 + (b \pm \Delta b)^2\right]$$

$$E \pm \Delta I = \frac{M}{2}\left(1 \pm \frac{\Delta M}{m}\right)\left[a^2 \pm 2\Delta aa + b^2 \pm 2\Delta bb\right]$$

$$\pm \Delta I = \pm M\Delta a.a \pm M\Delta b.b \pm \frac{\Delta M}{2}\left(a^2 + b^2\right)$$

$$\Delta I = 0.191 \times 0.11 \times 10^{-3} + 0.191 \times 0.015 \times 10^{-3} + \frac{0.003}{2}(0.11^2 + 0.015^2)$$

$$= 0.02 \times 10^{-3} + 0.003 \times 10^{-3} + 10^{-3} \times 1.5\ (0.01 + 0.0002)$$

$$\approx 0.04 \times 10^{-3}$$

6. There are certain number of red balls and white balls in an Urn. The probability of picking up a white ball is $\frac{1}{6}$ more than that of a red ball. If 6 white balls are added to the Urn the above difference is doubled. If instead, 6 white balls are removed, the probability of picking up a white ball is :

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{5}{9}$

Ans. **(C)**

Sol. Red balls = 5λ

white balls = 7λ

$$\frac{7\lambda+6}{12\lambda+6} = 2\left(\frac{5\lambda}{12\lambda+6}\right) \Rightarrow \lambda = 2$$

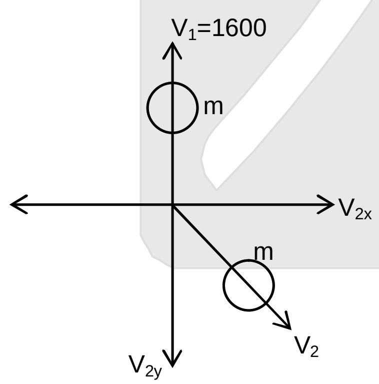
$$\Rightarrow \text{reqd prob} = \frac{14-6}{24-6} = \frac{8}{18} = \frac{4}{9}$$

7. A chlorine molecule with an initial velocity of 600 ms^{-1} absorbs a photon of wavelength 350 nm and is then dissociated into two chlorine atoms. One of the atoms is detected moving perpendicular to the initial direction of the molecule and having a velocity of 1600 ms^{-1}. The binding energy of the molecule is. [Neglect the momentum of the absorbed photon. The relative atomic mass of chlorine is 35]

(A) 3.36×10^{-17} J (B) 3.36×10^{-19} J (C) 3.36×10^{-21} J (D) 3.36×10^{-28} J

Ans. **(B)**

Sol. $2m \rightarrow V_0 = 600$ m/s



energy of photon $E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{350\times10^{-9}} = 0.056 \times \alpha10^{-17}$J

$V_2y = 1600$ m/s

$2m \times 600 = m \times V2_m$

$V_{2x} = 1200$

$$\text{Loss of K.E.} = \frac{1}{2} \times 2m\,V_0^2 - \left[\frac{1}{2}\times m\times V_1^2 + \frac{1}{2}m\left(V_{2x}^2+V_{2y}^2\right)\right]$$

$$= m\left[V_0^2 - \frac{\left(V_1^2 + V_{2x}^2 + V_{xy}^2\right)}{2}\right]$$

$= 35 \times 1.67 \times 10^{-27}$ [600^2]

Aliter :

$V_0 = 600$ m/s

$V_1 = V_{2y} = 1600$ m/s

$2mV_1 = m.V_{2x}$

$V_{2x} = 1200$ m/s

$$\Delta K = \left(\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2\right) - \frac{1}{2} \times 2m \times V_0^2$$

$= m[(1600)^2 + (1600^2 + 1200^2) - 2 \times 600^2]$

$$= \frac{35 \times 1.67 \times 10^{-25}}{2}[256 \times 2 + 144 - 72] \times 10^4$$

$= 17067 \times 10^{-23}$

$= +1.7 \times 10^{-19}$J

$$\text{energy of photon} = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{350 \times 10^{-9}} = \frac{3 \times 6.62 \times 10^{-17}}{350} = \frac{3 \times 662}{350} \times 10^{-19}\text{J}$$

$= 3 \times 1.89 \times 10^{-19}$J

$= 5.7 \times 10^{-19}$J

B.E. $= 5.7 \times 10^{-19} - 1.7 \times 10^{-19} = 4 \times 10^{-19}$J

8. Let z be a complex number such that |z + 2| + |z – 3| = 5. Then the locus of z is :

(A) and ellipse (B) a straight line segment
(C) a circle (D) a hyperbola

Ans. (B)

Sol. A(–2), B(3), P(z)

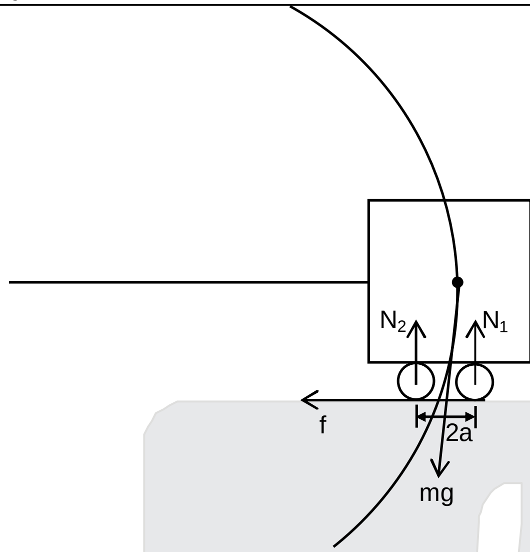
|z + 2| + |z – 3| = 5 ⇒ PA + PB = AB ⇒ locus is line segment AB

9. A car of mass m travelling at speed v moves on a horizontal track so that the centre of mass describes a circle of radius r. if 2a is the separation of the inner and outer wheels and h is the height of the centre of mass above the ground, the limiting speed beyond which the car will overturn is given by :

(A) $v^2 = \frac{gra}{2h}$ (B) $v^2 = \frac{2gra}{3h}$ (C) $v^2 = \frac{5gra}{3h}$ (D) $v^2 = \frac{gra}{h}$

Ans. (D)

Sol.



In the case of limiting

speed, $N_2 = 0$

$N_1 = mg$ …(i)

$f = \frac{mV_m^2}{r}$ …(ii)

Torque about C.M.

$f.h. = N_1 a$ …(iii)

(i), (ii) & (iii) $\Rightarrow$

$\frac{mV_m^2}{r}.h = mg.a$

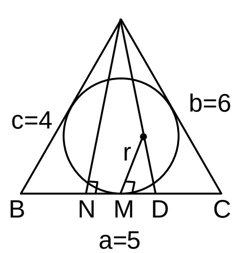
$V_m^2 = \frac{ga.r}{h}$

10. In a triangle ABC, the side length are given by BC = 5, CA = 6, AB = 4. Let I be its incentre ; AI be produced to meet BC in D. Then the ratio $\frac{ID}{AI}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) 2

Ans. **(A)**

Sol.



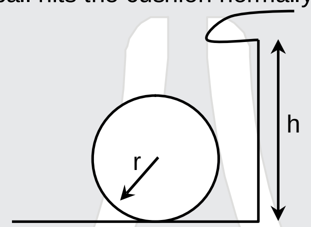
$\Delta MID \sim \Delta NAD$

$$\Rightarrow \frac{ID}{AD} = \frac{IM}{AN}$$

$$\Rightarrow \frac{ID}{AD} = \frac{r}{R}$$

$$\Rightarrow \frac{AI + ID}{ID} = \frac{h}{r} \Rightarrow \frac{AI}{ID} = \frac{h}{r} - 1 = \frac{2S}{a} - 1 = \frac{a+b+c}{a} - 1 = \frac{b+c}{a} = \frac{10}{5} = 2$$

11. The ratio of the height h of a cushion on a snooker table to the radius r of a ball (solid sphere as shown in figure), such that when the ball hits the cushion with a pure rolling motion it rebounds with a pure rolling motion. (Assume that the force exerted on the ball by the cushion is horizontal during the impact and that the ball hits the cushion normally).



(A) $\frac{7}{5}$ (B) $\frac{5}{7}$ (C) $\frac{5}{3}$ (D) $\frac{3}{5}$

Ans. **(A)**

Sol. Taking torque about o,

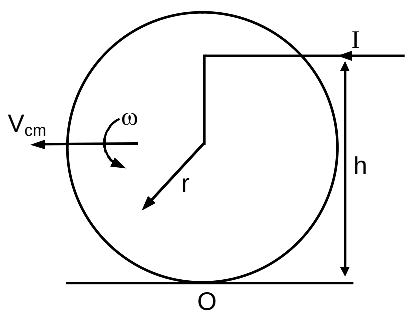
$Ih = 2 \times (\frac{2}{5} mr^2 + mr^2) \times \omega$ ....(i)

$I = 2 \times mV_{cm}$ ...(ii)

$V_{cm} = w_r$ ...(iii)

Solving (i), (ii) & (iii)

$\frac{h}{r} = \frac{7}{5} \Rightarrow$ (A)



12. If the parabola $y^3 = 8x$ and the ellipse $\frac{(x-2)^2}{p^2}+\frac{y^2}{q^2}=1$, $0 < p < q$ are tangent to each other then :

(A) $q=2\sqrt{2-\sqrt{4-p}}$ (B) $q=2\sqrt{2+\sqrt{4+p}}$

(C) $q=2\sqrt{2-\sqrt{4-p^2}}$ (D) $q=2\sqrt{2+\sqrt{4-p^2}}$

Ans. **(D)**

Sol. $\frac{(x-2)^2}{p^2} + \frac{8x}{q^2} = 1 \Rightarrow q^2(x^2 + 4 - 4x) + 8p^2x = p^2q^2$

$\Rightarrow q^2x^2 + 4(2p^2 - q^2)x + q^2(4 - p^2) = 0$

$D = 0 \Rightarrow 4(2p^2 - q^2)^2 = q^4(4 - p^2)$

$\Rightarrow q^4 - 16q^2 + 16p^2 = 0$

$\Rightarrow q^2 = \frac{16 \pm \sqrt{256-64p^2}}{2}$

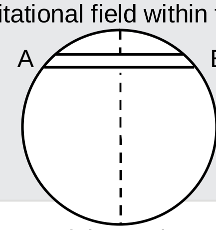
$\Rightarrow q^2 = 8 \pm 4\sqrt{4-p^2}$

As root > 0 $\Rightarrow q^2 - 2p^2 > 0$

so $8 - 4\sqrt{4-p^2}$ is

rejected $\Rightarrow q^2 = 8 + 4\sqrt{4-p^2}$

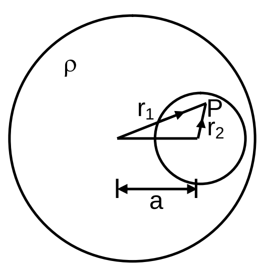
13. A sphere of uniform density ρ has within it a spherical cavity whose centre is at a distance a from the centre of the sphere. The gravitational field within the cavity is :



(A) non-uniform and zero at the center of the cavity
(B) zero at every point inside the cavity
(C) uniform and not zero
(D) same magnitude but has different direction at different points

Ans. **(C)**

Sol. Gravitational field at P



$= \frac{4\pi G}{3}\rho\vec{r}_1 - \frac{4\pi G}{3}\rho\vec{r}_2$

$= \frac{4\pi G}{3}\rho(\vec{r}_1 - \vec{r}_2)$

$= \frac{4\pi G}{3}\rho\vec{a}$

14. Four young kids want to exchange their toys, so that each one of them gets a new toy. The number of ways they can exchange the toys is :

(A) 3 (B) 7 (C) 9 (D) 24

Ans. **(C)**

Sol. Dearrangment of 4 objects

$= n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...\right)$

n = 4

$= 4!\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + ...\right)$

$= 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$

$= 24\left(\frac{12 - 4 + 1}{24}\right)$

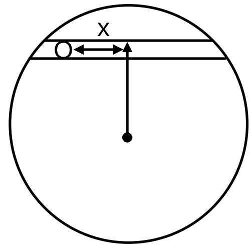
= 9 Ans.

15. A hole is bored in a straight line though the Earth from A to B a ball-bearing is dropped at end A. Assuming that frictional and air resistance are negligible, and that the Earth may be taken to be uniform-density sphere of radius 6400 km, how long does it take the ball bearing to arrive at B ? [Neglect any effects due to the rotation of the Earth, and assume the acceleration due to gravity at the Earth's surface to be $9.8 ms^{-2}$.]

(A) 2.5×10^3 s (B) 3.5×10^3 s (C) 3.0×10^3 s (D) 3.2×10^3 s

Ans. **(A)**

Sol. $F_r = -\frac{GMm}{R^3}x$



$\Rightarrow \quad T = 2\pi\sqrt{\frac{R^3}{GM}}$

$= 2\pi\sqrt{\frac{R}{g}}$ = 84.6 × 60 sec

$t_{A.B.} = \frac{T}{2}$ = 2538 sec

16. The value of the integral $\int_0^{\tan^{-1}3} [\tan x] dx$ is :

(A) $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ and $\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ and $\frac{5\pi}{18}$ (D) $\frac{5\pi}{18}$ and π

Ans. **(A)**

Sol. $\int_0^{\tan^{-1}3} [\tan x] dx$

$\int_0^{\tan^{-1}1} 0\, dx + \int_{\tan^{-1}1}^{\tan^{-1}2} 1\, dx + \int_{\tan^{-1}2}^{\tan^{-1}3} 2\, dx$

$0 + \tan^{-1}2 - \tan^{-1}1 + 2[\tan^{-1}3 - \tan^{-1}2]$

$= \tan^{-1}\left(\frac{9}{13}\right)$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) < \tan^{-1}\left(\frac{9}{13}\right) < \tan^{-1}1$

$\frac{\pi}{6} < \tan^{-1}\left(\frac{9}{13}\right) < \frac{\pi}{4}$

17. In 1910 AD, on its sixth trip around the Sun after that of 1456 AD, Halley's comet was observed to pass near the Sun at a distance of 9.0×10^{10}m. The maximum distance from the Sun (I astronomical unit is 1.50×10^{11}m)

(A) 5.3×10^{10} m (B) 5.3×10^{14} m (C) 5.3×10^{11} m (D) 5.3×10^{12} m

Ans. **(D)**

Sol. $T = \frac{1910 - 1456}{6} = \frac{454}{6} = \frac{227}{3}$ yes

Te = 1 yr.

$T = 2\pi\sqrt{\frac{a^3}{GM}}$

$T_e = 2\pi\sqrt{\frac{R^3}{GM}}$

$\frac{T}{T_e} = \sqrt{\frac{a^3}{R^3}}$

$R^3\left(\frac{227}{3}\right)^2 = a^3$

$a^3 = (1.5\times10^{11})^3\times\left(\frac{227}{3}\right)^2$

$a^3 = (1.5\times10^{11})^3\times\left(\frac{227}{3}\right)^2$

max distance = 2a – minimum distance

$= 2 \times 1.5 \times 10^{11} \times \left(\frac{227}{3}\right)^{2/3} - 9 \times 10^{10}$

$= 54.3 \times 10^{11} - 9 \times 10^{10}$

$= 534 \times 10^{10}$ m

$= 5.3 \times 10^{12}$ m

18. The planet Jupiter has a radius equal to 11.2 times the Earth's radius, a mass equal to 318 times the Earth's mass and a period of rotation about its axis of 10.2 hours. The minimum velocity with which a rocket would need to leave the Jovian urface in order to escape entirely from the gravitational attraction of Jupiter is, (take the escape velocity from the Earth to be 11.2 km s^{-1})

(A) 64.4 kms^{-} (B) 59.7 kms^{-1} (C) 49.7 kms^{-1} (D) 63.7 kms^{-1}

Ans. **(B)**

Sol. $v_e = \sqrt{2\frac{GM}{R}}$

Energy due to rotation negligible

$v'_e = \sqrt{2\frac{GM'}{R'}}$

$\frac{v'_e}{v_e} = \sqrt{\frac{M'}{M}\cdot\frac{R}{R'}} = \sqrt{\frac{318}{11.2}}$

$v'_e = 59.7\text{km / sec}$

19. A [polynomial of degree 14 takes the value zero at each of the first 7 odd primes and also at their reciprocals. Then the ratio $\frac{P(2)}{P\left(\frac{1}{2}\right)}$ is :

(A) 4^7 (B) -4^7 (C) 2^7 (D) 0

Ans. **(A)**

Sol. $P(x) = \lambda(x-3)(x-5)(x-7)(x-11)(x-13)(x-17)(x-19)\left(x-\frac{1}{3}\right)\left(x-\frac{1}{5}\right)$

$\left(x-\frac{1}{7}\right)\left(x-\frac{1}{11}\right)\left(x-\frac{1}{13}\right)\left(x-\frac{1}{17}\right)\left(x-\frac{1}{19}\right)$

$$P\left(\frac{1}{x}\right) = \lambda\left(\frac{1}{x}-3\right)\left(\frac{1}{x}-5\right)\left(\frac{1}{x}-7\right)\left(\frac{1}{x}-11\right)\left(\frac{1}{x}-13\right)\left(\frac{1}{x}-17\right)\left(\frac{1}{x}-19\right)\left(\frac{1}{x}-\frac{1}{3}\right)$$
$$\left(\frac{1}{x}-\frac{1}{5}\right)\left(\frac{1}{x}-\frac{1}{7}\right)\left(\frac{1}{x}-\frac{1}{11}\right)\left(\frac{1}{x}-\frac{1}{13}\right)\left(\frac{1}{x}-\frac{1}{17}\right)\left(\frac{1}{x}-\frac{1}{19}\right)$$

$$\therefore \frac{P(x)}{P\left(\frac{1}{x}\right)} = x^{14}$$

$$\therefore \frac{P(2)}{P\left(\frac{1}{2}\right)} = 4^{7}$$

20. A reasonable guess for the dimensions of a thundercloud might be a 1-km cube. If all the charge is separated to opposite faces of the cloud, it behaves like a parallel-plate capacitor. If the breakdown electric field in air is 3×10^6 V m^{-1}, approximate charge that flows down a lightning bolt?

(A) 30C (B) 20C (C) 15C (D) 40C

Sol. **(A)**

$$Q = CV = CEd = \frac{\varepsilon.A}{d}.E.d = \varepsilon_0 AE$$

$= 8.85 \times 10^{-12} \times (10^3)^2 \times 3 \times 10^6 = 26.5$ C

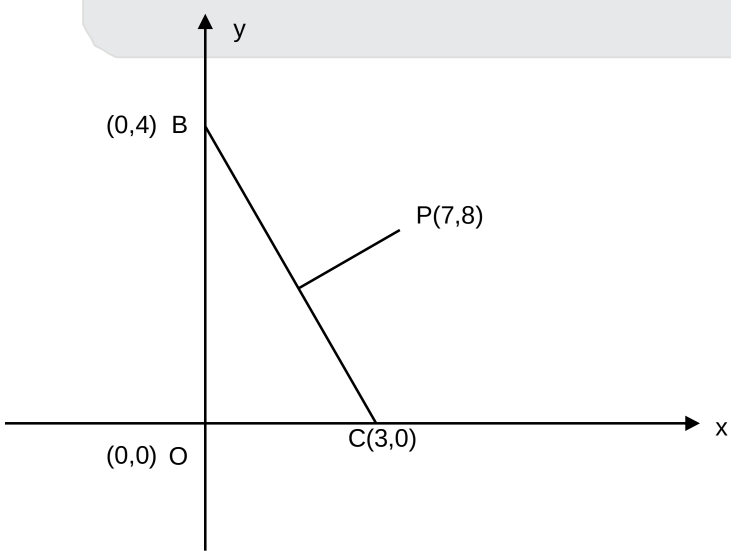
21. In a triangle ABC, right angled at A, AB = 4, AC = 3, suppose P is a point lying outside triangle ABC, and lying the angular region formed by the rays AB and AC (such that PA intersects BC in its interior). Suppose the perpendicular distance from P to AB and AC are 7 and 8 respectively. Then the perpendicular distance from P to BC is

(A) 4 (B) 5

(C) 8 (D) not determinable using the data

Ans. **(C)**

Sol.



BC is $\frac{x}{3}+\frac{y}{4}=1$

$\Rightarrow$ 4x + 3y = 12

PM = $\frac{28+24-12}{5}=8$

22. A light bulb filament is constructed from 2 cm of tungsten wire of diameter 50 μm and is enclosed in an evacuated glass bulb. What temperature does the filament reach when it is operated at a power of 1 W ?(Assume the emissivity of the tungsten surface to be 0.04).

(A) 2.44×10^3K (B) T = 1.99×10^3K (C) 2.54×10^3K (D) 2.34×10^3K

Ans. **(B)**

Sol. $P = e\sigma AT^4 = e.\sigma.(2\pi r\ell).T^4$

$1 = 0.4 \times 5.67 \times 10^{-8} \times 2\pi \times \frac{50\times10^{-6}}{2} \times 2 \times 10^{-2} \times T^4 = 14 \times 10^{12}$

$T^4 = 0.4 \times 5.67 \times 2\pi \times 50$

$\Rightarrow$ $T = 1.94 \times 10^3$ k

23. Let ∈ {1,2,3........9}. The number of such n, for which there is at least one permutation of the digits 1,2....n which is divisible by 11 is

(A) 3 (B) 4 (C) 5 (D) 6

Ans. **(C)**

Sol. n can be 3(132) or 4(1243) or 7(2136475) or 8(13862475) or 9(142738695)

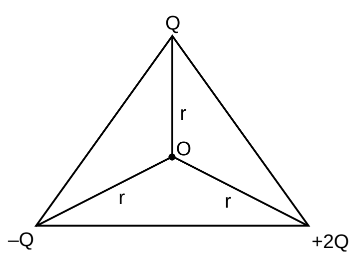
24. Three point charges –Q, 2Q and –Q are placed at the vertices of an equilateral triangle. Of the following quantities, the one whose magnitude is zero is.

(A) the electric field the centroid (B) the potential at the centroid

(C) the potential gradient at the centroid (D) dipole moment of the system

Ans. **(B)**

Sol.



$v = -\frac{kQ}{r} - \frac{kQ}{r} + \frac{2kQ}{r} = 0$

25. A capacitor is charged using a battery. The work done by the battery is W energy stored in the capacitor is U. Then which of the following is correct?

(A) U = W (B) $U = \frac{W}{2}$ (C) $U = \frac{W}{3}$ (D) $U = \frac{W}{4}$

Ans. **(B)**

Sol. $w = cv^2$

$U = \frac{1}{2}cv^2$

26. The sum $\sum_{k=1}^{672} \frac{1}{(3k-2)(3k+1)} = \frac{1}{14} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{2014.2017}$ is expressed as a rational number $\frac{p}{q}$, where p and q are mutually coprime positive integers. Then the value of p + q lies between.

(A) 2000 and 4000 (B) 4000 and 6000

(C) 6000 and 8000 (D) 8000 and 10,000

Ans. **(A)**

Sol. $\sum_{k=1}^{672} \frac{1}{(3k-2)(3k+1)} = \frac{1}{3} \sum_{k=1}^{672} \frac{(3k+1)-(3k-2)}{(3k-2)(3k+1)}$

$= \frac{1}{3}\sum_{k=1}^{672}\left(\frac{1}{3k-2} - \frac{1}{3k+1}\right)$

$= \frac{1}{3}\left[\left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+.......+\left(\frac{1}{2014}-\frac{1}{2017}\right)\right]$

$= \frac{1}{3}\left[1-\frac{1}{2017}\right]$

$= \frac{1}{3}\left[\frac{2016}{2017}\right] = \frac{672}{2017} = \frac{p}{q}$

p + q = 672 + 2017 = 2689

27. A uniform electric field exists along positive x-axis. A particle of mass m and charge Q is projected with a velocity v_0 along y-axis from origin. The x-coordinate of the particle at which its kinetic energy doubles is

(A) $x = \frac{m_0^2}{2EQ}$ (B) $x = \frac{2mv_0^2}{EQ}$ (C) $x = \frac{mv_0^2}{3EQ}$ $x = \frac{mv_0^2}{EQ}$ (D) $x = \frac{mv_0^2}{EQ}$

Ans. **(A)**

Sol. $QEx = \Delta k = mv_0^2 - \frac{1}{2}mv_0^2$

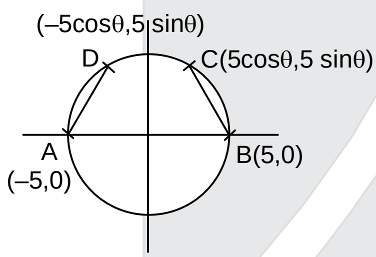
$x = \frac{mv_0^2}{2QE}$

28. Suppose AB is a distance of a circle of radius 5, and C and D are two points on the circle lying on the same side of AB such that AD = BC = 6. Then the length of CD is closest to

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(B)**

Sol.



BC = 6

$BC^2 = 36$

$(5\cos\theta - 5)^2 + (5\sin\theta)^2 = 36$

$\cos\theta = \frac{7}{25}$ $\sin\theta = \frac{24}{25}$

$C\left(\frac{7}{5}, \frac{24}{5}\right)$, $B\left(-\frac{7}{5}, \frac{24}{5}\right)$

$BC = \frac{14}{5} = 2.8$

29. The resistivity of a conductor increases with temperature as

(A) relaxation time increases with temperature

(B) relaxation time decreases with temperature

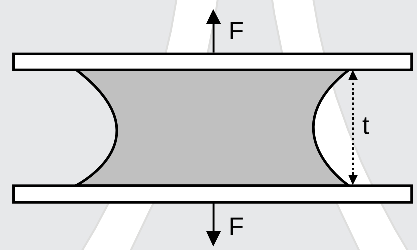
(C) number density of electrons decreases with temperature

(D) number density of electrons increases with temperature

Ans. **(B)**

Sol.

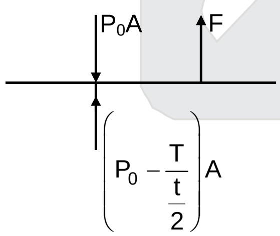
30. A drop of water is trapped between two glass plates. Area of contact between each plate and water is A and T is the surface tension of water. Separation between the plates is t. Then the force F required to separate the plates is



(A) $\frac{TA}{4t}$ (B) $\frac{2TA}{t}$ (C) $\frac{TA}{2t}$ (D) $\frac{4TA}{t}$

Ans. **(B)**

Sol.



$$FA = \frac{2T}{t}A$$

$$F = \frac{2TA}{t}$$

31. The product of two positive integers is 42000. What is the maximum possible value their ged ?

(A) 10 (B) 20 (C) 70 (D) 140

Ans. **(B)**

Sol. $42000 = 2^4.3.5^3.7$

x.y = 42000 Let $x = 2^{a_1}.3^{b_1}.5^{c_1}.\ 7^{d_1}$

$\therefore\ 2^{a_1+a_2}.\ 3^{b_1+b_2}.\ 5^{c_1+c_2}.\ 7^{d_1+d_2}$ $y = 2^{a_2}.3^{b_2}.5^{c_2}.\ 7^{d_2}$

$= 2^4.\ 3.5^3.7$

$\Rightarrow a_1 + a_2 = 4$ $b_1 + b_2 = 1$

$c_1 + c_2 = 3$ $d_1 + d_2 = 1$

$g.c.d = 2^2.5^1.3^0.7^0 = 20$

32. 64 number of small identical droplets of liquid coalesce to from a single droplet. Radius of each small drop is r. Surface tension of the liquid is T and density is ρ. Then the rise in the temperature is (assume that excess energy dissipates only as heat)

(A) $\frac{4T}{9r\rho C}$ (B) $\frac{3T}{2r\rho C}$ (C) $\frac{9T}{4r\rho C}$ (D) $\frac{T}{r\rho C}$

Ans. **(C)**

Sol. $64\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 4r$

$\Delta A = 64(4\pi r^{2+}) - 4\pi R^2$

$= 4\pi(64r^2 - 16r^2)$

$= 4\pi(48r^2)$

$\Delta\theta = \frac{9T}{4r\rho c}$

33. Consider an n by n chess board having n^2 squares. Two squares are said to be neighbours if they share a common edge. Let k_1 ,k_2, k_3,..........$k_n{}^2$ be the number of neighbours of all the n^2 squares, and α be the average of these n^2 numbers. Then the number of possible value of n for which a lies in the internal [6,7] is

(A) 5 (B) 6 (C) 7 (D) 8

Ans. **[Bonus]**

Sol.

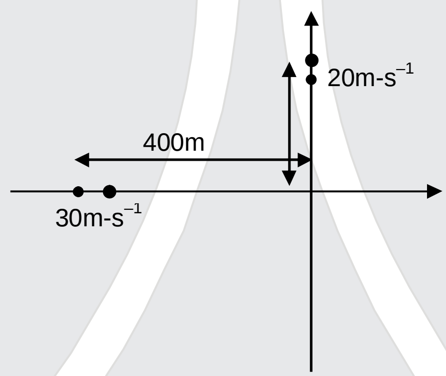
34. Two closed pipes are emitting their fundamental notes 250 Hz and 255 Hz. Ratio of their lengths is

(A) $\frac{49}{50}$ (B) $\frac{50}{49}$ (C) $\frac{49}{51}$ (D) $\frac{51}{50}$

Ans. **(D)**

Sol. $\frac{\ell_1}{\ell_2} = \frac{f_2}{f_1} = \frac{255}{250} = \frac{51}{50}$

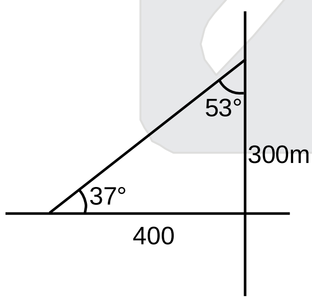
35. A source of sound emitting 459 Hz is travelling along positive x-axis with a speed of 30 m-s^{-1}. An observer is moving along the negative y-axis with a speed of 20 m-s^{2} as shown in the figure. The sound emitted by source when it is at 400m away from origin is received by the observer when he is 300m away from origin. The frequency recorded by the observer is (speed of sound = 330ms^{-1})



(A) 535.5 Hz (B) 513 Hz (C) 486.8 Hz (D) 504.9 Hz

Ans. **(B)**

Sol.



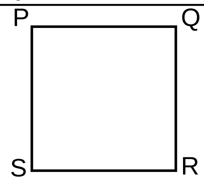
$$f = \left(\frac{330 + 20\cos 53}{330 - 30\cos 37}\right)459 = \left(\frac{342}{306}\right)459 \approx 513\text{Hz}$$

36. A scooter and a bicycle travel along the perimeter of a square PQRS of side length 45km. They both start at the vertex P and go through Q,R,S,P,Q,R,S......... in that order several times. The scooter travels at a constant speed of 25 kMPH and the bicyle at 15 kMPH. After some time, they meet a vertex of the square PQRS for the first time . This vertex is.

(A) P (B) Q (C) R (D) S

Ans. **(C)**

Sol.

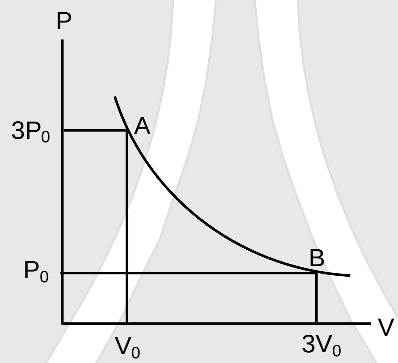


Time required for meeting = $\frac{180}{10} = 18\text{hrs}$

Distance travelled by scooter = 18 × 25 = 450 km = 10 times side length.

So they meet at R.

37. A certain ideal gas undergoes expansion and its PV diagram is a shown in the figure. The ratio of rms speed of molecules at state A and B is

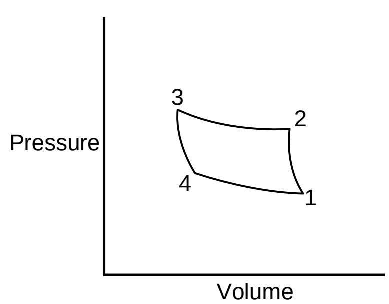


(A) 1/2 (B) 2 (C) 1/3 (D) 1

Ans. **(D)**

Sol. $\frac{V_A}{V_B} = \sqrt{\frac{3P_0V_0}{P_0(3V_0)}} = 1$

38. An ideal gas undergoes two isothermal and two adiabatic processes as shown in the figure. The temperature during isothermal process from 3 to 2 is 2T and that during process from 1 to 4 is T. If V_1, V_2, V_3,V_4 are the volumes of the gas at state 1,2,3 and 4 (refer figure) then which of the following is correct.



(A) $\frac{v_3}{v_4} = \frac{v_1}{v_2}$ (B) $\frac{v_3}{v_2} = \frac{v_4}{v_1}$ (C) $\frac{v_4}{v_3} = \frac{2v_2}{v_1}$ (D) $\frac{v_4}{v_3} = \frac{v_2}{2v_1}$

Ans. **(B)**

Sol. $(2T)V_2^{r-1} = TV_1^{r-1}$

$(2T)V_3^{r-1} = TV_4^{r-1} \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \Rightarrow \frac{V_3}{V_2} = \frac{V_4}{V_1}$

39. Let P(x) be a polynomial of degree 4 with real coefficients, If P(1) = 1, $P^{i}(1) = 1$, $P^{ii}(1) = 4$, $P^{iii}(1) = 24$, $P^{iv}(1)=72$, then the value of P (2) is

(A) 9 (B) 11 (C) 13 (D) 15

Ans. **(B)**

Sol. $P(x) = ax^4 + bx^3 + cx^2 + dx + e, \quad a \neq 0$

$P^{i}(x) = 4ax^3 + 3bx^2 + 2cx + d \Rightarrow P^{I}(x) = 1 \Rightarrow d = -3$

$P^{ii}(x) = 12ax^2 + 6bx + 2C \Rightarrow P^{II}(x) = 4 \Rightarrow C = 8$

$P^{iii}(x) = 24ax + 6b \Rightarrow P^{III}(1) = 24 = 24 \times 3 + 6b \Rightarrow b = -8$

$P^{iv}(x) = 24a \Rightarrow P^{II}(1) = 72 = 24a \Rightarrow a = 3$

$P(1) = a + b + c + d + e = 1 \Rightarrow e = 1$

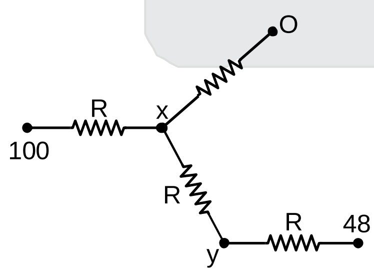
$\therefore P(x) = 3x^4 - 8x^3 + 8x^2 - 3x + 1 \qquad \therefore P(2) = 11$

40. Four rods of identical dimensions (cross sectional area A and length L) arranged as shown in the figure. The thermal conductivity of each rod is indicated in the figure. Temperature at junction B is

(A) 72°C (B) 40°C (C) 60°C (D) 48°C

Ans. **(D)**

Sol.



$$\frac{x-100}{R} + \frac{x-0}{R} + \frac{x-48}{2R} = 0$$

$2x - 200 + 2x + x - 48 = 0$

$5x = 248$

$x = \frac{248}{5}$

$\frac{248}{5} - y = y - 48 \Rightarrow 2y = \frac{248}{5} + 48$

y = 48.8

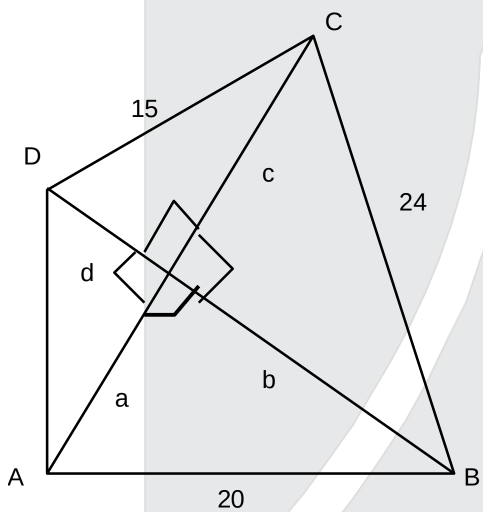
Bert answer is D.

41. In the quadrilateral ABCD, diagonal AC is perpendicular to diagonal BD. If AB = 20, BC = 24, CD = 15, then the length of side AD is

(A) 6 (B) $\frac{24}{5}\sqrt{2}$ (C) $4\sqrt{3}$ (D) 7

Ans. **(D)**

Sol.



$a^2 + b^2 = 400$

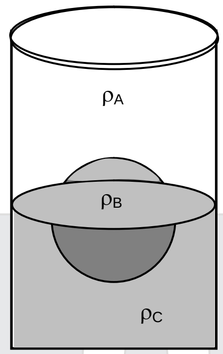
$b^2 + c^2 = 576$

$\Rightarrow c^2 - a^2 = 176$

also $c^2 + d^2 = 225$

$\Rightarrow d^2 + a^2 = 225 - 176 = 49 \Rightarrow AD = 7$

42. In the figure, B is a spherical object density. ρ_B floating in a beaker containing immiscible liquids A and C of densities ρ_A and ρ_C respectively. If one third of volume of B is in liquid A, the correct relation between the density is



(A) $\rho_B = \frac{\rho_A + 2\rho_C}{3}$ (B) $\rho_B = \frac{2\rho_A + \rho_C}{3}$ (C) $\rho_B = \frac{\rho_A + \rho_C}{3}$ (D) $\rho_B = \frac{\rho_A + \rho_C}{2}$

Ans. **(A)**

Sol. $\rho_B V = \rho_A \frac{V}{3} + 2\rho_C \frac{V}{3} \Rightarrow 3\rho_B = \rho_A + 2\rho_C$

$$\rho_B = \frac{\rho_A + 2\rho_C}{3}$$

43. The limit $\lim_{x\to 0+} \frac{\tan\sqrt{x} - \sin\sqrt{x}}{\tan^{-1}x - \sin^{-1}x}$ is

(A) 0 (B) 1 (C) ∞ (D) $-\infty$

Ans. **(D)**

Sol. $\lim_{x\to 0^+} \frac{\tan\sqrt{x} - \sin\sqrt{x}}{\tan^{-1}x - \sin^{-1}x}$

$$\Rightarrow \lim_{x\to 0^+} \frac{\left(\sqrt{x} + \frac{(\sqrt{x})^3}{3}\right) - \left(\sqrt{x} - \frac{(\sqrt{x})^3}{3!}\right)}{\left(x - \frac{x^3}{3}\right) - \left(x + \frac{x^3}{6}\right)}$$

$$\Rightarrow \lim_{x\to 0^+} \frac{(x)^{3/2}\left(\frac{1}{3} + \frac{1}{3!}\right)}{x^3\left(-\frac{1}{3} - \frac{1}{6}\right)}$$

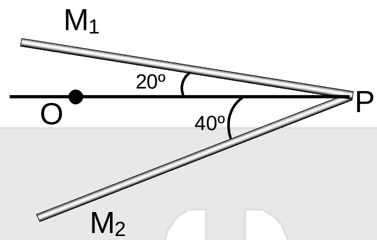
$$\Rightarrow \lim_{x\to 0^+} -(x^{-3/2}) \text{ ....} -\infty$$

44. Approximate angular speed of sun relative to earth is $\frac{\pi}{12}$ rad – hr^{-1} . Then ratio of area of shadows of a circular ring measured 2 hour after the sunrise and 4 hour after the sunrise is

(A) 1 (B) 2 (C) 3 (D) 1/2

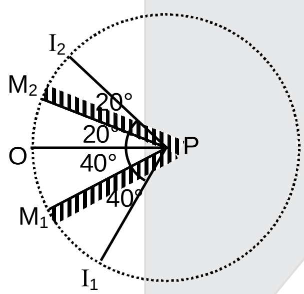
Ans. **(C)**

45. A point object O is placed between two plane mirror as shown in the figure. Due to multiple reflections several images are formed. I_1 and I_2 are nearest images formed by mirror M_1 and M_2 respectively. Then the angle between PI_1 and PI_2 is



(A) $\frac{\pi}{3}$ rad (B) $\frac{2\pi}{3}$ rad (C) $\frac{\pi}{2}$ rad (D) $\frac{\pi}{6}$ rad

Sol.



46. In triangle ABC, $\angle B = 2\angle C$, and BC = 2AB. The triangle is

(A) a scalene and acute angled triangle (B) an isosceles acute-angled triangle

(C) an obtuse-angled triangle (D) a right-angled triangle

Ans. **(D)**

Sol. $\angle B = 2\angle C \Rightarrow \frac{B}{C} = 2$

$\angle A + \angle B + \angle C = \pi$

a = 2c, $A + 2C + C = \pi$

sin A = 2sinC $3C = \pi - A$

$\sin 3C = \sin A$

$\sin 3C = 2 \sin C$

$3\sin C - 4\sin^3 C - 2\sin C = 0$

$\sin C(1 - 4\sin^2 C) = 0$

$\sin C \neq 0$, $\sin 2C = \frac{1}{2} = \left(\frac{1}{2}\right)^2$

$C = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$C = \frac{\pi}{6} \therefore B = \frac{\pi}{3} \therefore \angle A = \frac{\pi}{2} \left(C \neq \frac{5\pi}{6}\right)$

47. The value of the integral $\int_2^5 \frac{\left[\sqrt{x}\right]}{\sqrt{[x]}} dx$ lies between

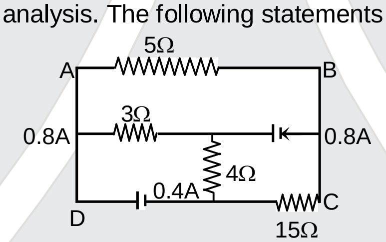
(A) 2.00 and 2.25 (B) 2.25 and 2.50 (C) 2.50 and 2.75 (D) 2.75 and 3.00

Ans. **(B)**

Sol. $\int_2^3 \frac{1}{\sqrt{2}} dx + \int_3^4 \frac{1}{\sqrt{3}} dx + \int_4^5 \frac{2}{\sqrt{4}} dx \quad \Rightarrow \left(\frac{3-2}{\sqrt{2}}\right) + \left(\frac{4-3}{\sqrt{3}}\right) + \left(\frac{5-4}{1}\right)$

$\Rightarrow 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \approx 1 + 0.70 + 0.57 \approx 2.27$

48. In the adjoining circuit, currents in some of the branches are shown. Currents in other braches can be determined using circuit analysis. The following statements are made



(i) Current in 3Ω is 1.2 A

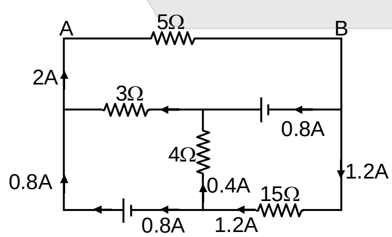
(ii) Voltage across AB is 10V

(A) statements (i) and (ii) are correct (B) statement (i) is correct and (ii) is incorrect

(C) statement (i) is correct and (ii) is incorrect (D) statement (i) and (ii) incorrect

Ans. **(A)**

Sol.



As it can be seen in

figure current in

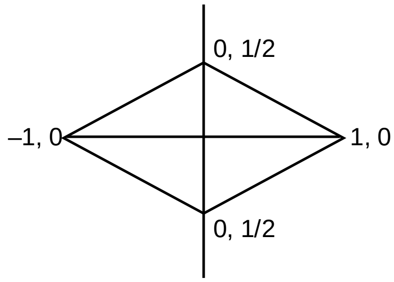
5Ω resistor = 2A ⇒ V_{AB} = 10V Current in 3Ω = 1.2A

∴ Both the options are correct.

49. The locus the point (x,y) which satisfies the condition |x| + 2|y| = 1 is

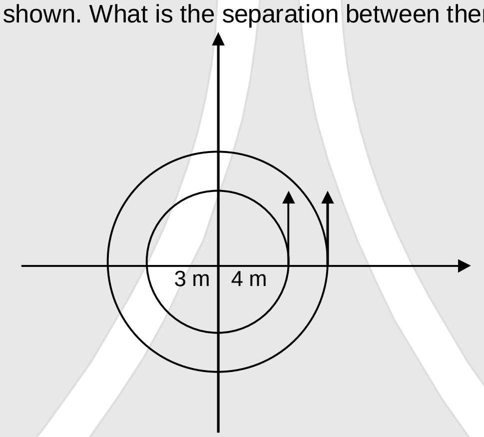
(A) a rectangle, which is not a square
(B) a rhombus
(C) an isosceles trapezium
(D) a polygon of 8 sides

Ans. **(B)**



Sol.

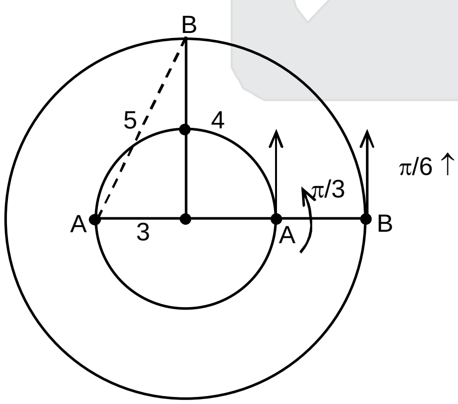
50. Two particles A and B are moving along concentric circles of radii 3m and 4m respectively. Angular speed of A is $\pi/3$ rad-min^{-1} and angular speed of B is $\pi/6$ rad-min+ . At t = 0 they cross x-axis simultaneously as shown. What is the separation between them after 3 minutes ?



(A) 7 m
(B) 5m
(C) 1m
(D) 0.5 m

Ans. **(B)**

Sol.



Angle traveled by A = $\pi/3 \times 3 = \Pi$

angle traveled by B = $\pi/6 \times 3 = \pi/2$

distance = $\sqrt{3^2 + 4^2}$ = 5m **Ans. B**

51. The lengths of the medians of triangle ABC are 3,4,5. The area of triangle ABC is

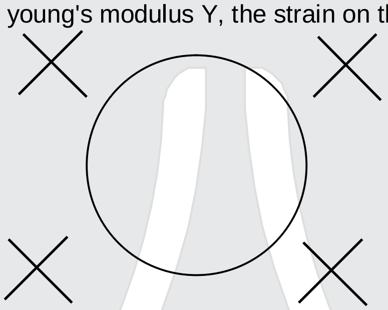
(A) $\frac{8}{5}$ (B) 4 (C) 8 (D) 12

Ans. **(C)**

Sol. Area of triangle where side are 3,4,5 is $\frac{1}{2} \times 3 \times 4 = 6$ square unit

$\Rightarrow$ Area of triangle where median length are 3,4,5 is $\frac{4}{3} \times 6 = 8$ square units

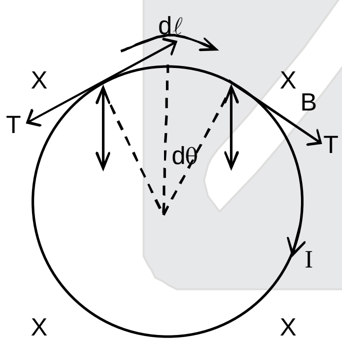
52. A circular loop carrying current I is placed in a uniform magnetic field B. If radius of the circle is R and material of the loop has young's modulus Y, the strain on the loop is



(A) $\frac{BIR}{3AY}$ (B) $\frac{2BIR}{AY}$ (C) $\frac{BIR}{2AY}$ (D) $\frac{BIR}{AY}$

Ans. **(D)**

Sol.



$\frac{T}{AY}$ = strain

F = IdLB = 2Tsin $\frac{d\theta}{2}$

$I(Rd\theta)$ B = 2T $\frac{d\theta}{2}$

T = IRB

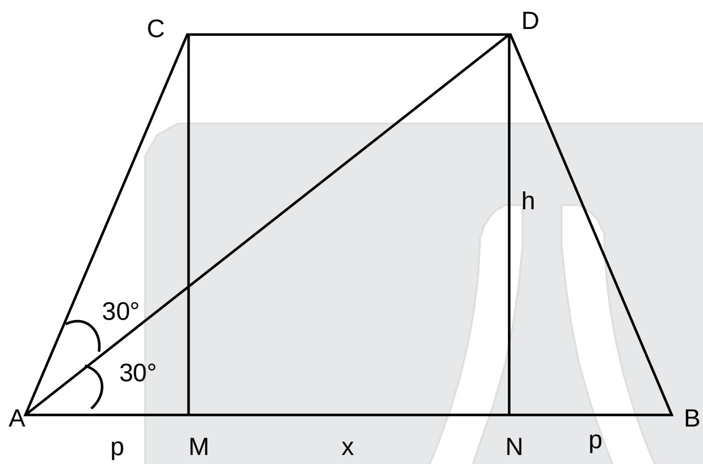
$\therefore$ Strain = $\frac{IRB}{AY}$

53. A rod of is suspended horizontally at a certain height above the ground, the two ends C and D is observed from two points A and B on the ground which are such that the four points A, B, C, D are in the same vertical plane. If the angles of elevation of C and D from A are 60°, and AB = a then the length of the rod CD can be (some other correct option may be missing)

(A) $\frac{\sqrt{3}}{2}a$ or $\frac{2}{\sqrt{3}}a$ (B) $\sqrt{3}$ a or $\frac{a}{\sqrt{3}}$ (C) 2a or $\frac{a}{2}$ (D) $\sqrt{2}$a or $\frac{a}{\sqrt{2}}$

Ans. **(C)**

Sol.



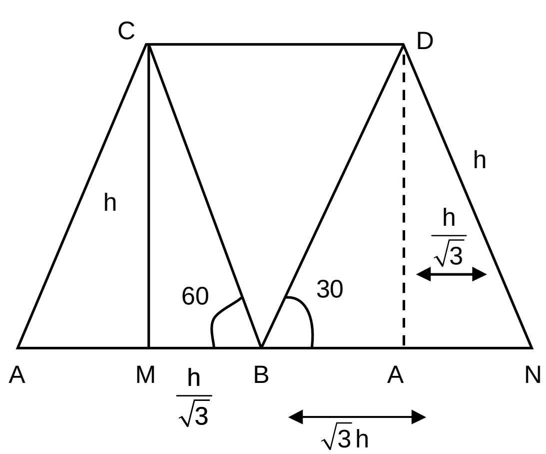
tan 30° = $\frac{h}{p+x}$ and tan 60 = $\frac{h}{p}$

$\Rightarrow \frac{p+x}{\sqrt{3}} = \sqrt{3}p$

$\Rightarrow$ x = 2p

AB = 2p + x = 2x = 2CD $\Rightarrow$ CD = $\frac{a}{2}$

Or AB = $\sqrt{3}\ h - \frac{h}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$



$a = \frac{2h}{\sqrt{3}}$

$CD = \frac{h}{\sqrt{3}} + \sqrt{3}h$

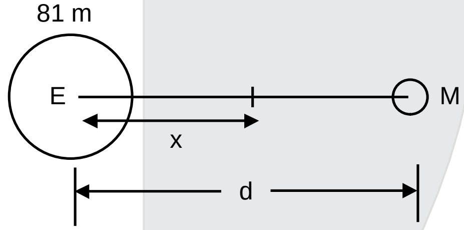
$= \frac{4h}{\sqrt{3}} \quad = 2a$

54. Let the earth and moon distance is d. the mass of earth is 81 times that of the moon. A body located at a distance x from the earth on a line joining the centres of earth and moon does not experience gravitational force when x is

(A) $\frac{d}{10}$ (B) $\frac{9d}{10}$ (C) $\frac{5d}{10}$ (D) $\frac{9d}{11}$

Ans. **(B)**

Sol.



$$\frac{G \times 81M}{x^2} = \frac{GM}{(d-x)^2}$$

$$\Rightarrow \quad \frac{x}{9} = (d - x)$$

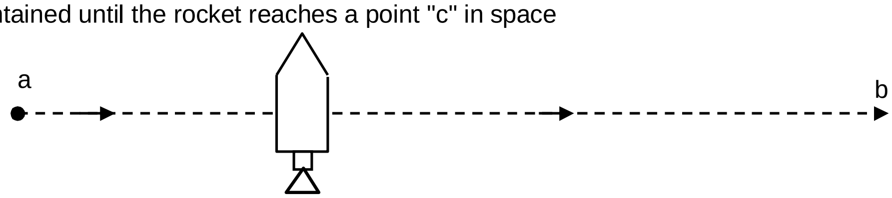
$$x + \frac{x}{9} = d$$

$$\frac{10x}{9} = d$$

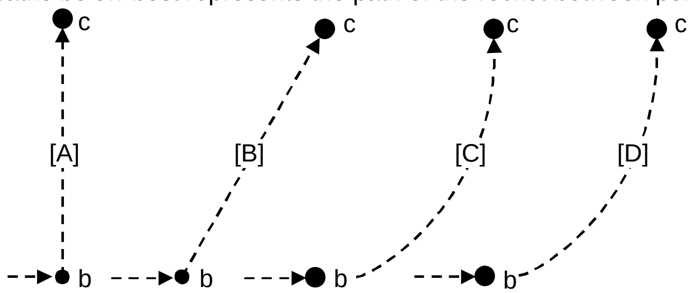
$$x = \frac{9d}{10}$$

Passage question (55 through 58)

A rocket drifts sideways in outer space from point "a" to point "b" as shown below. The rocket is subject to no outside forces. Starting at position "b", the rocket's engine is turned on and produces a constant thrust (force on the rocket) at right angles to the line "ab". The constant thrust is maintained until the rocket reaches a point "c" in space



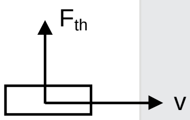
55. Which of the paths below best represents the path of the rocket between points "b" and "c" ?



(A) (B) (C) (D)

Ans. **(B)**

Sol.

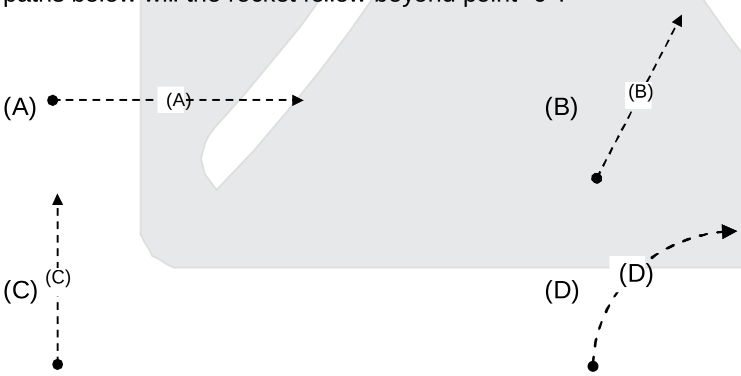


Path mill be circular Right options is B.

56. As the rocket moves from the position "b" "c" its speed is:
(A) constant
(B) continuously increasing
(C) continuously decreasing
(D) increasing for a while and constant thereafter.

Ans. **(B)**

57. At point "c" the rocket's engine is turned off and the thrust immediately drops to zero. Which of the paths below will the rocket follow beyond point "c"?



Ans. **(B)**

Sol. Beyond point C it mill move in straight line, velocity making some only form initial velocity but option

58. Beyond position "c" the speed of the rocket is:
(A) constant
(B) continuously increasing
(C) continuously decreasing
(D) increasing for a while and constant thereafter

Ans. **(A)**

Sol. v → content

59. The number of 4-letter palindromes which contains at least one vowel (a palindrome is a word which reads the same from left to right and from right to left, eg., DEED) lies between

(A) 200 and 210 (B) 210 and 220 (C) 220 and 230 (D) 230 and 240

Ans. **(D)**

Sol. Number of words = $\underbrace{{}^5C_1 \times {}^{21}C_1 \times 2!}_{(1v\text{ and }1c)} + \underbrace{{}^5C_1 \times 1}_{(2\text{alike } v's)} + \underbrace{{}^5C_2 \times 2!}_{(2\text{ ditl. } v's)} = 235$

60. The number of positive integers n such that both the equations $x^2 – 20x\ n^2 = 0$ and $x^2 – nx + 10 = 0$ have real roots is

(A) 4 (B) 8 (C) 12 (D) 16

Ans. **(A)**

Sol. $x^2 – 20x + n^2 = 0$ and $x^2 – nx + 10 = 0$

For real roots $D_1 \ge 0$ and $D_2 \ge 0$

$\Rightarrow n^2 \le 100$ and $n^2 \ge 40$

$\Rightarrow n \in [–10, –\sqrt{40}\,] \cup [\sqrt{40}, 10]$

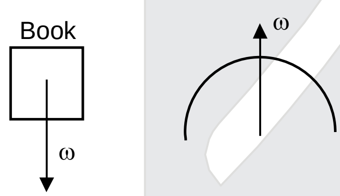
$n \in \{–10, –9, –8, –7, 7, 8, 9, 10\}$ i.e., only positive integer 4 values

61. A book lies at rest on table. The table is at rest on the surface of the earth. By Newton's third low reaction force to the weight of the book is

(A) The gravitational forces of the earth on the book
(B) The normal forces exerted by table on the book
(C) The normal forces exerted by table on the book
(D) The gravitational forces of the book on the earth

Ans. **(D)**

Sol.

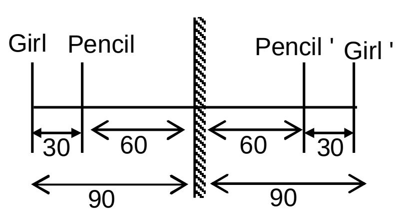


Book attract earth with some force as that its weight.

62. A student has myopia. Her near point and far point of vision are at 25 cm and 120 cm, respectively. Standing 90 cm in front of a plane mirror she holds up a pencil 30 cm in front of her eye. Then

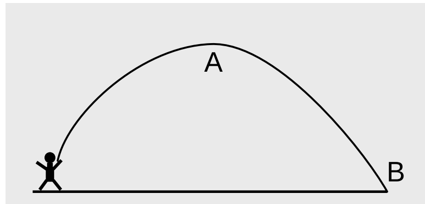
(A) She can see both the pencil and its image clearly.
(B) She can see the pencil clearly but not its image.
(C) She can see the image of the pencil clearly but not the pencil itself.
(D) She cannot see either the pencil or its image clearly.

Ans. **(B)**



Sol. Girl can see the pencil clearly.

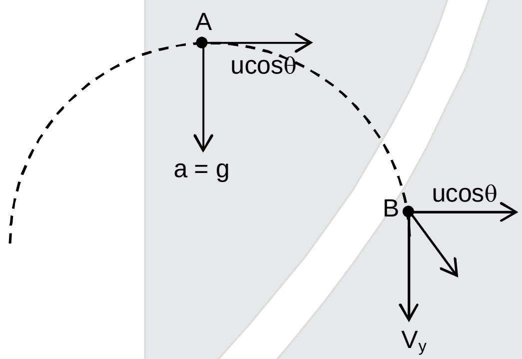
63. A ball is thrown into the air and it moves along the path shown in the figure. Ignore air resistance. At the position A the ball is at the highest point in its path, whereas position B is just before the ball hits ground. Which of the following statements is true ?



(A) the speed of the ball at A is zero and acceleration of the ball at B is the same as at A
(B) The speed of the ball at A is the same as the speed at B and the acceleration at B is higher than at A
(C) The speed at A is lower than the speed at B and acceleration at A is higher tan the acceleration at B
(D) The speed at A is lower than speed at B and the acceleration at A is the same as the acceleration at B

Ans **(D)**

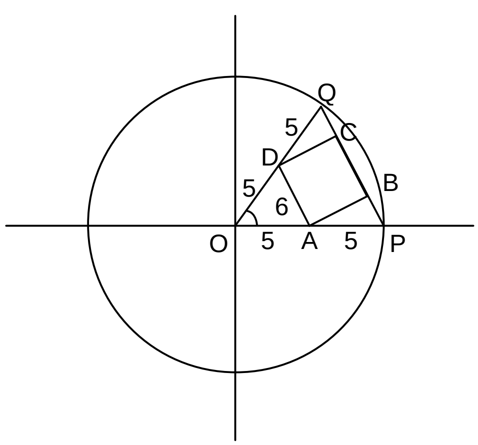
Sol. At highest point A



64. Consider a circle, centre O, radius 10. Let P and Q be two points on the circumference such that $\angle POQ = 60^{o}$. Let A be a point on the radius OP ; B and C be points one are PQ ; D be a point on the radius OQ such that ABCD is a square. Then the side length of the square is closest to
(A) 5 (B) 6 (C) 7 (D) 8

Ans. **(A)**

Sol.



∵ OPQ is an equilateral Triangle

∴ OAD also equilateral triangle

∴ side of square is 5.

65. When astronaut observes Earth from moon he will see
(A) Earth rising in the west and setting in the east
(B) Earth neither setting nor rising but stays at one position through out
(C) Earth rising in the east and setting in the west
(D) Earth will have a complex motion, some time rising in the east and sometime in the west

Ans. **(B)**

66. Which of the following best describes what the microwave background radiation is ?
(A) The particles that move throughout the Universe that were created during the Big Bang and thrown out in all directions.
(B) Radiation that is present everywhere in the Universe that came from the time when light first was
(C) The shockwave or echo that travels throughout the Universe that marks the event we call the Big Bang.
(D) The total light we observe when we look at the blackbody curve for all the wavelengths of radiation given off by all the stars in the Universe.

Ans. **(B)**

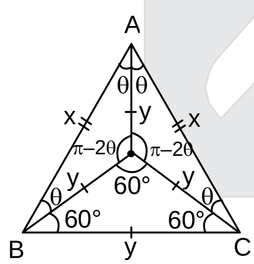
67. 3D images of moon were obtained by
(A) Luna mission (B) Apollo mission
(C) Hubble Space Telescope (D) Chandrayaan mission

Ans. **(D)**

68. Suppose D is an interior point of triangle ABC such that AB = AC and AD = BD = CD = BC. Then the ratio $\frac{BC}{AB}$ is closest to
(A) 0.4 (B) 0.5 (C) 0.6 (D) 0.7

Ans. **(B)**

Sol.



$\pi - 2\theta + \pi - 2\theta + 60 = 360°$

$\Rightarrow$ q = 15°

Apply sine rule

$$\frac{x}{\sin(\pi-2\theta)} = \frac{y}{\sin\theta} \Rightarrow \frac{y}{x} = \frac{\sin\theta}{2\sin\theta\cos\theta} = \frac{1}{2\cos\theta} = \frac{1}{2\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}$$

$$\frac{y}{x} \simeq 0.51$$

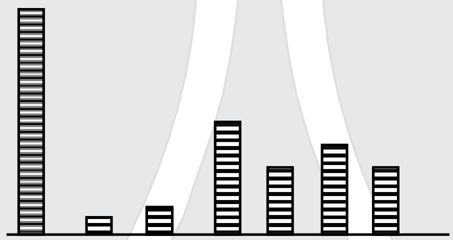
69. Imagine that the Earth's spin were to be reversed, so that the planet rotated about its axis from East to West, at the same rate at which it now rotates from West to East. If Earth's orbital motion about the Sun were unchanged, which of these statements is true ?
(A) The duration of the sidereal day would not change.
(B) The sidereal day would be 8 minutes longer than it is now.
(C) The duration of the Solar day would not change.
(D) The solar day would be 8 minutes longer than it is now.

Ans. (A)

70. Cepheid variable stars have fluctuating masses. Some astronomers attempt to use these stars to
(A) Compare to dying stars to determine time of stellar death
(B) Mark distances throughout a galaxy
(C) Map solar system
(D) Both B and C

Ans. (B)

71. The bar graph below show some characteristic X of all the planets in our solar system. What is the characteristic ?



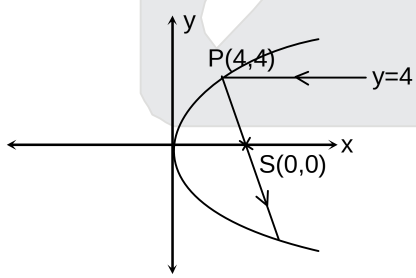
(A) Mass
(B) Density
(C) Eccentricity of the orbit
(D) Period of oscillation

Ans. (C)

72. A mirror is in the shape of the parabola $y^2 = 4x$. The ray $y = 4$, when incident on the parabola is reflected along
(A) $3x - 4y + 4 = 0$
(B) $3x - 4y - 4 = 0$
(C) $4x - 3y + 4 = 0$
(D) $4x - 3y - 4 = 0$

Ans. (D)

Sol.



Equation of reflected ray is

$$y - 4 = \left(\frac{0-4}{1-4}\right)(x - 4)$$

$$\Rightarrow 4x - 3y - 4 = 0$$

Linked questions (73-74)

73. An absorption line that is found at 5000Å in the lab is found at 5050Å when analysing the spectrum of a particular galaxy. The velocity of the galaxy is.
(A) 3000 km/s^{-1} (B) 2500 km/s (C) 3500 km/s (D) 4000 km/s

Ans. (A)

Sol. $V_r = C\left(1 - \frac{\lambda}{\lambda_r}\right)$

$V_r = 3 \times 10^8 \left(1 - \frac{5000}{5050}\right)$

$= 3 \times 10^8 \left(\frac{50}{5050}\right)$

$= \frac{3 \times 10^8}{101} = 3 \times 10^6 \text{m / s} = 3000 \text{ km/s}$

74. In the above problem using Hubble's constant to be 71 km (Mpc-s)$^{-1}$ the distance of that galaxy from us is obtained as
(A) 52.25 Mpc (B) 35.25 Mpc (C) 40.25 Mpc (D) 42.25 Mpc

Ans. (D)

Sol. Hubble's const. = $\frac{\text{speed of recession of galaey}}{\text{distance from observer}}$

distance = $\frac{3000 \text{km / sec}}{71 \text{km}(M - PC - S)^{-1}}$

= 42.25 MPC

75. A star is seen to be rising on eastern horizon at 23 : 00 hrs. At what time the same star will rise 20 days later ?
(A) 00 : 00 hrs (B) 21 : 40 hrs (C) 23 : 20 hrs (D) 23 : 00 hrs

Ans. (B)

Sol. Each day star rises 4 minutes earlier

as earths rotation time is 23 hr 56 minutes

20 × 4 = 80 minutes

1 hr 20 minutes ⇒ 21: 40 hrs

76. Two stars A and B are assigned apparent magnitude of +3.5 and –1.5 respectively. If observed from the earth.
(A) Star A is 5 times brighter than B
(B) Star B is 5 times brighter than A
(C) Star A is 2^5 times brighter than B
(D) Star B is 100 times brighter than B

Ans. (D)

77. How many circles can be drawn in the plane such that any two of them touch each other at the same fixed point.

(A) Two (B) Three (C) Four (D) Infinitely many

Ans. (D)

Sol. Obvious

78. If all nuclear reaction in the sun now were to suddenly stop for ever, then

(A) Distances between planets and sun would decrease
(B) Angular momentum of planets would increase
(C) Inner planets will be engulfed by the sun
(D) Speed of rotation of the sun would increase

Ans. (D)

79. A certain number of boys and girls can be seated in a row such that no two girls are together in 1440 ways. If one more boy joins them, the number of ways in which they can be seated in a row such that no two girls are together increases.

(A) 4-fold (B) 6-fold (C) 8-fold (D) 10-fold

Ans. (D)

Sol. Let x-boys and y-girls

$x!\ {}^{x+1}C_y\ y! = 1440$

$\Rightarrow$ which is true when x = 4 and y = 3

Now when one more boy joins then, then number of ways = $5! \times {}^6C_3 \times 3!$

$= 10 \times 1440$

80. Two dice whose faces are numbered form 1 to 6 are rolled. The probability that the sum of the numbers that show up is a prime number is approximately.

(A) /12 (B) 5/12 (C) 7/12 (D) 23/36

Ans. (B)

Sol. Total ways = 6 × 6 = 36

= {(1, 1), (2, 1), (1, 2),(2, 3), (3, 2), (1, 4), (4, 1), (2, 5), (5, 2), (3, 4),(4, 3), (6, 1),(1, 6),(5, 6), (6, 5)}

$\therefore$ probability = $\frac{15}{36} = \frac{5}{12}$