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NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2016-17

Examination Date : 27-11-2016 Time: 2 Hrs. Max. Marks : 240

PAPER CODE : A421 HBCSE Olympiad (STAGE - 1)

Write the question paper code mentioned above on YOUR answer sheet (in the space provided), otherwise your answer sheet will NOT be assessed. Note that the same Q. P. Code appears on each page of the question paper.

INSTRUCTION TO CANDIDATES

1. Use of mobile phones, smart phones, ipads during examination is STRICTLY PROHIBITED.

2. In addition to this question paper, you are given answer sheet along with Candidate's copy.

3. On the answer sheet, fill up all the entries carefully in the space provided, **ONLY In BLOCK CAPITALS**. Use only **BLUE or BACK BALL PEN** for making entries and marking answer.

Incomplete / incorrect / carelessly filled information may disqualify your candidature.

4. On the answer sheet, use only BLUE or BLACK BALL POINT PEN for making entries and filling the bubbles.

5. The email ID and date of birth entered in the answer sheet will be your login credentials for accessing performance report. Please take care while entering.

6. Question paper has 80 multiple choice questions. Each question has four alternatives, out of which only one is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

Q. No. 22 (a) (●) (c) (d)

7. A correct answer carries 3 marks whereas and 1 mark will be deducted for each wrong answer.

8. Any rough work should be done only in the space provided.

9. Use of a non-programmable calculator is allowed.

10. No candidate should leave the examination hall before the completion of the examination.

11. After submitting your answer paper, take away the Candidate's copy for your reference.

Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the answer sheet. Answer sheet are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED.

Scratching or overwriting may result in wrong score.

DO NOT WRITE ANYTHING ON THE BACK OF ANSWER SHEET.

Read the following instructions after submitting the answer sheet.

12. Comment regarding this question paper, if any, may be sent by email only to iapt.nse@gmail.com till **29th November 2016.**

13. The answers/solutions to this question paper will be available on our website www.iapt.org.in by **2nd December, 2016.**

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16. List of students (with center number and roll number only) having score above MAS will be display on our website (www.iapt.org.in) by **22nd December, 2016**. See the **Eligibility Clause** in the Student's brochure on our website.

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CORPORATE OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005
Ph.No. : +91-744-3012222, 6635555 | **Toll Free :** 1800 258 5555
Reg. Office : J-2, Jawahar Nagar, Main Road, Kota (Raj.)-324005 | **Ph. No.:** +91-744-3192222 | FAX No. : +91-022-39167222
Website : www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN:** U80302RJ2007PLC024029

1. Two identical stars with mass M orbit around their centre of mass in circular orbit. If radius of the orbit is R and the stars are always diametrically opposite. Consider the following statements:

(i) Their binding force is equal to $\frac{GM^2}{4R^2}$

(ii) If the stars are heavier and closer, their orbital speed is greater.

(iii) The period of the orbit is T = $\pi\sqrt{\frac{R^3}{GM}}$

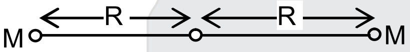
(iv) The minimum energy required to separate the two stars to infinity is equal to $\frac{GM^2}{4R}$.

Select correct statement's

(A) Only (i) and (ii) (B) Only (i), (ii) and (iv) (C) Only (i), (iii) and (iv) (D) Only (i) and (iii)

Ans. **(B)**

Sol.



(i) $F = \frac{-GM^2}{(2R)^2} = \frac{-GM^2}{4R^2}$

(ii) $\frac{GM^2}{4R^2} = \frac{mv^2}{R}$

$v = \sqrt{\frac{GM}{4R}}$

(iii) $T = \frac{2\pi R}{\sqrt{\frac{GM}{4R}}} = \frac{4\pi R^{3/2}}{\sqrt{GM}}$

$= 4\pi\sqrt{\frac{R^3}{GM}}$

(iv) $E_i = \frac{1}{2}MV^2 \times 2 - \frac{GM^2}{2R} = \frac{MGM}{4R} - \frac{GM^2}{2R} = \frac{-GM^2}{4R}$

$E_i + \Delta E_{min} = 0$

$\Delta E_{min} = -E_i$

2. The number of natural numbers n = $\le$ 50 such that $\sqrt{n+\sqrt[3]{n+\sqrt[3]{n+..........}}}$ is a natural number is :

(A) zero (B) 2 (C) 50 (D) 5

Ans. **(B)**

Sol. $x = \sqrt[3]{n+x}$

$x^3 - x - n = 0$

put x = 1 , n = 0

put x = 2 , n = 6

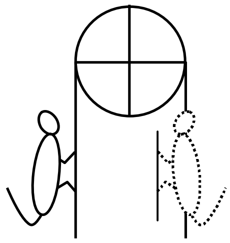
put x = 3 n = 24

put x = 4 n = 60 x

$\therefore$ n = 6 or n = 24

Hence (B) option

3. A monkey is holding onto one end of a rope which passes over a frictionless pulley and at the other end is a plane mirror which has a mass equal to the mass of the monkey. At equilibrium the monkey is able to see her image in the mirror. How does the monkey see her image in the mirror as she climbs up the rope ?



(A) The image of the monkey moves with double speed of that of the monkey.
(B) The image of the monkey moves with half the speed of that of the monkey.
(C) The image of the monkey moves as fast as the monkey.
(D) The monkey will not be able to see her image.

Ans. **(C)**

Sol. Acceleration of monkey and mirror are equal then he sees his image always.

4. If i = $\sqrt{-1}$ then $i^{2\ell}$ is a
(A) purely imaginary number (B) natural number
(C) real number (D) complex with non-zero real and imaginary parts

Ans. **(C)**

Sol. $x = i^{2i}$

$\ln x = 2i (\ln i)$

$= 2i \ln (e^{i(\pi/2)})$

$\ln x = 2i \ln e^{i(2n\pi+\pi/2)}$

$\therefore \quad \ln x = 2i\,(i(2n\pi + \frac{\pi}{2}))$

$x = e^{-\pi}$

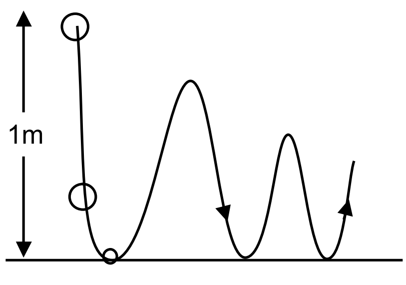
$\therefore \quad i^{2i}$ is real number.

Hence (C)

5. A steel ball is dropped from a height of 1 m on to a hard non-conducting surface. Every time it bounces it reaches 80% of its previous height. All the losses in the energy are accounted only for increasing the temperature. Nearly how much is the rise in temperature of the ball just after the third bounce ? (g + 10 m/s^2), specific heat capacity of material of the ball = 500 J/(kg.K))
(A) 0.005 ºC (B) 0.01 ºC (C) 0.015 ºC (D) 0.02 ºC

Ans. **(B)**

Sol.



$\Delta H = mg \times 1 \times 0.2 + mg \times 1 \times 0.8 \times 0.2 + mg \times 0.8 \times 0.8 \times 0.2$ .....(1)

$\Delta H = m \times 500\, \Delta T$ .....(2)

$2 + 1.6 + 1.28 = 500\, \Delta T$

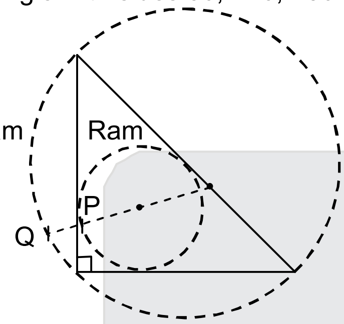
$\Delta T = 0.01°C$

6. In a track event, a circumcircle and an incircle were drawn for a triangle having sides 50 m, 120 m and 130 m respectively, Ram and Sham were asked to walk on the in circle and the circumcircle respectively. They started walking with same speed in the same direction (sense of rotation) from a point where they were closest. After how may rounds each, will they be closest again ?

(A) Ram 4 and Sham 13 (B) Ram 13 and Sham 4
(C) Ram 5 and Sham 15 (D) Ram 5 and Sham 12

Ans. **(B)**

Sol. Triangle with sides 50, 120, 130 will be right angled.



Circumradius = 65 inradius = $\frac{\pi}{5}$

Perimeter = $2\pi(65)$ $= \frac{\frac{1}{2}\times 50\times 120}{150} = 20$

Perimeter = 2π (20)

Initially Ram & Sham should be at 'P' & 'Q' respectively.

Let often time t, they are again closest.

$\therefore$ $(2\pi(65))\, N_1 = (2\pi(20))\, N_2$

$\therefore$ $13N_1 = 4N_2 \begin{cases} N_1 : \text{Number of rotation by Sham} \\ N_2 : \text{Number of rotation by Ram} \end{cases}$

$\therefore$ $N_1 = 4$ & $N_2 = 13$ Hence option (B)

7. The angle between the two complex numbers a i^1 and b = 1 is

(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

Ans. **(B)**

Sol. Q $= i^i$

ln a $= \ln i^i$

$= i \ln i$

$= i \ln\, e^{i(2n\pi+\pi/2)}$

$= -\,(2n\pi + \frac{\pi}{2})$

$\therefore$ $a = e^{-(2n\pi+\pi/2)}$

$\therefore$ a is a perfect real number.

which is positive else.

$\therefore$ Angle b/w a & b = 0

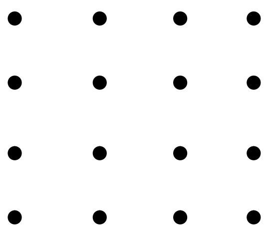
Hence option (B) is correct.

8. The number of rectangles that can be formed by joining the points of 4 × 4 grid of equispaced points is

(A) 16 (B) 36 (C) 40 (D) 42

Ans. **(D)**

Sol.



The number of rectangles = $4_{C_2} \times 4_{C_2}$ (Horizontal rectangles) + [(3 + 3)(tilted rectangles)]

= 42

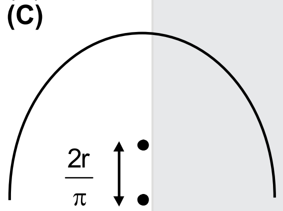
Hence option (D) is correct.

9. A train of mass m is moving on a circular track of radius r with constant speed v. The length of the train exactly equal to half the circumference of the circular track. Magnitude of is linear momentum is.

(A) mv/π (B) 0.5 mv (C) 2mv/π (D) mv

Ans. **(C)**

Sol.



$P_{cm} = MV_{cm}$

$= m\left(\frac{2r}{\pi}\right)\omega$

$= \frac{m2r}{\pi}\frac{v}{r} = \frac{2mv}{\pi}$

10. The number of integers a, b, c for which $2a^2 + b^2 - 8c = 7$ is

(A) 2 (B) infinite (C) 0 (D) 4

Ans. **(C)**

Sol. $2a^2 + b^2 - 8c = 7$

$b \in$ odd integer

$2a^2 + b^2 = 8c + 7$

$2a^2 + (2k + 1)^2 = 8c + 7$

$2a^2 + 4k^2 + 4k = 8c + 6$

$a^2 + 2k^2 + 2k = 4c + 3$

To hold the equation $a \in$ odd integer

$(2\ell + 1)^2 + 2k^2 + 2k = 4c + 3$

$4\ell^2 + 4\ell + 2k^2 + 2k = 4c + 2$

$2\ell^2 + 2\ell + k^2 + k = 2c + 1$

$\Rightarrow$ even = odd

There for equation has no solution Hence option (C) correct.

11. In SI units we use length, mass and time as fundamental quantities. Another intelligent world may not know these. However (universal gravitational constant), c (speed of light in vacuum) and (Planck's constant) are really universal and can be related to almost all the known interactions. In terms of these fundamental constants, the dimensions of time are

(A) $\left[G^{\frac{1}{2}}c^{-\frac{5}{2}}h^{\frac{1}{2}}\right]$ (B) $\left[G^{1}c^{-2}h^{\frac{1}{2}}\right]$ (C) $\left[G^{2}c^{-\frac{1}{2}}h^{\frac{1}{2}}\right]$ (D) $\left[G^{\frac{1}{2}}c^{-\frac{3}{2}}h^{\frac{1}{2}}\right]$

Ans. **(A)**

Sol. $[T] = [G]^a[C]^b[h]^C$ ...(i)

$F = \frac{Gm^2}{R^2} \Rightarrow mLT^{-2} = \frac{GM^2}{L^2}$

$[G] = [M^{-1}L^3T^{-2}]$

$[C] = [LT^{-1}]$

$E = hf$

$ML^2T^{-2} = \frac{h}{T} \Rightarrow h = ML^2T^{-1}$

$[T]^1 = [M^{-1}L^3T^{-2}]^a\ [LT^{-1}]^b\ [ML^2T^{-1}]^C$

$0 = -a + c \Rightarrow a = c$ …(ii)

$0 = 3a + b + 2c$ …(iii)

$a = c = \frac{1}{2}$

$b = -\frac{5}{2}$

$1 = -2a - b - c$ …(iv)

$[T] = G^{1/2}\ C^{-5/2}\ h^{1/2}$

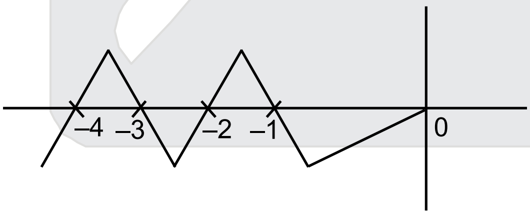
12. If p(x) = x(x + 1) (x + 2) ..... (x + 2001) – c then the maximum multiplicity of the roots of p(x) can be

(A) 1 (B) 2 (C) 3 (D) 2001

Ans. **(B)**

Sol. If we draw,

y = (x + 1) (x +2) ….. (x + 2001)



If a root is repeated & its frequency is 2,

$f(\alpha) = 0, f'(\alpha) = 0$

Which is possible & that depends open the value of 'c'.

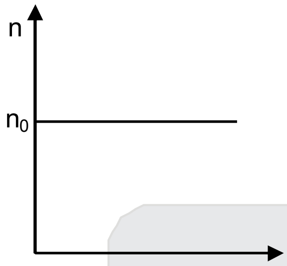
But for three repeated roots,

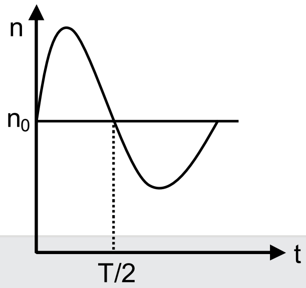
$f(\alpha) = 0, f'(\alpha) = 0, f''(\alpha) = 0$

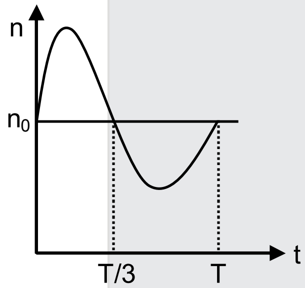
Which is not possible as wherever $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} > 0$ or < 0 from the graph.

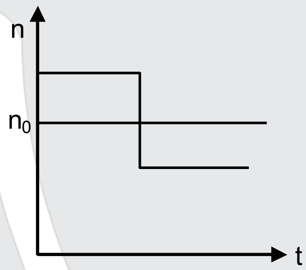
Hence : Option (B).

13. A train is running on a circular track of radius R with a constant speed. The driver is blowing siren of a constant frequency (n_0) throughout the circular motion of period T. There is a listener on the diameter of the track at a distance R/2 from centre of the circle. At t = 0, the train siren is farthest from the listener. In the following graphs the frequency, as recorded by the listener, is plotted against time. Which of them is closes to the correct pattern ?

(A)


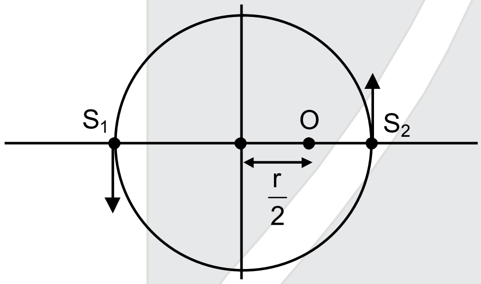
(B)


(C)


(D)


Ans. **(B)**

Sol.



At position S_1 and S_2 ; frequency will be exact equal to n_0 , It happen after T/2

14. If {x} denotes the fractional part of a real number then $\int_0^{\sqrt{2}} \{x^2\} dx$ =

(A) $2\frac{\sqrt{2}}{3}$ (B) $\frac{1}{3}$ (C) $1-\frac{\sqrt{2}}{3}$ (D) $1+\frac{\sqrt{2}}{3}$

Ans. **(C)**

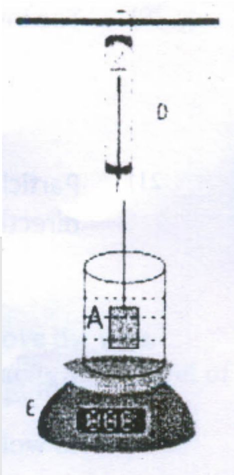
Sol. $\int_0^{\sqrt{2}} \{x^2\}\, dx = \int_0^1 x^2 dx + \int_1^{\sqrt{2}} (x^2 - 1)\, dx$

$= \frac{1}{3} + \left| \frac{x^3}{3} - x \right|_1^{\sqrt{2}}$

$= \frac{1}{3} + \left(\frac{2\sqrt{2}}{3} - \sqrt{2} - \frac{1}{3} + 1 \right)$

$= 1 - \frac{\sqrt{2}}{3}$ Hence option (C)

15. Adjacent figure shows a block A, held by a spring balance D and submerged into a liquid in a beaker. The beaker is kept on a weighing balance E. Mass of the beaker plus the liquid is 2.5 kg. Balance D reads 2.5 kg and E reads 7.5 kg. Volume of the block is 0.003 m^3. Consider the following statements.

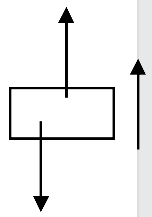


(i) The density of the liquid is 5000/3 kg/m^3
(ii) The mass of block A is 7.5 kg
(iii) The buoyant force is 5 kg wt.
(iv) If half the volume of the block is pulled out of the liquid, E would read 5 kg
Select correct options(s)
(A) (i), (iii) and (iv) (B) (i), (ii) and (iv) (C) (i) and (iv) (D) (i), (ii), (iii) and (iv)

Ans. **(D)**

Sol. On system.

$kx = 2.5g$



$N = 7.5g$

$2.5g + mg$

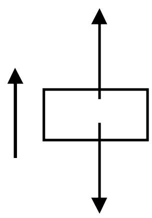
Now $2.5\text{ g} + 7.5\text{ g} = 2.5\text{ g} + mg$

$m = 7.5$ kg **Ans.**

F.B.D. of block

2.5g

F_b



7.5 g

$F_V + 2.5\text{ g} = 7.5\text{ g}$

$F_V = 5g$ **Ans.**

Now $0.003\,(\rho)10 = F_b = 5g$

$$\rho = \frac{5000}{3}$$

After half submerged it will be 5 kgf.

16. AM-HM inequality for positive real numbers a, b, c states that $\frac{a+b+c}{3} \geq \frac{3abc}{ab+cb+ca}$. If a, b are positive irrational numbers then.

(A) $\frac{9ab}{2a+b} \leq a+b$ (B) $\frac{9ab}{2a+b} \leq 1$ (C) $\frac{9ab}{a+2b} \leq 2a+b$ (D) $\frac{18ab}{2a+b} \leq a+2b$

Ans. **(C)**

Sol. Let the three number be a, b, b,

$\therefore$ AM $\geq$ HM

or $\frac{a+b+b}{3} \geq \frac{3ab^2}{ab+b^2+ab}$

or $\frac{a+2b}{3} \geq \frac{3ab}{2a+b}$ or $(2a + b) \geq \frac{9ab}{(a+2b)}$ Hence option (C)

17. The optical effects (phenomena) involved when we see a rainbow could be associated with

(i) internal reflection

(ii) dispersion

(iii) total internal reflection

(iv) deviation

Select the correct options

(A) (ii), (iii) and (iv) (B) (i), (ii) and (iv) (C) (i) and (iv) (D) (iii) and (iv)

Ans. **(B)**

Sol. TIR is not print

18. Which of the following statements are true about periodic functions defined on the set of real numbers

A : Sum of two functions with finite period is always a periodic function with finite period

B : The period of a function that is sum of two periodic functions with finite period is least common multiple of the period of two functions

(A) A and B are correct (B) A is correct but B is incorrect

(C) A is false but B is correct (D) A and B are false

Ans. **(D)**

Sol. If Period = fundamental period

A : let $f(x) = (\sin x) + 2$

$g(x) = -\sin x$,

Here f(x) &g(x) both are periodic but fundamental period of f(x) + g(x) is not defined.

B : Let $f(x) = \sin^2 x$

$g(x) = \cos^4 x$

here,

f(x) & g(x) both are periodic with period π but f(x) + g(x) is periodic with $\frac{\pi}{2}$ which is not LCM of π & π.

Hence both statement are false hence option (D).

19. Unaware about the fact that analong ammeters and voltmeters can also have zero error, a student recorded following readings while determining resistance by using Ohm's law

Obs. no.	Voltage / V	Current / mA
1	1.0	40
2	3.0	80
3	5.0	120
4	7.0	160
5	9.0	200

If the ammeter has no zero error, the zero error in the voltmeter is.

(A) –1V (B) –1.5 V (C) 0.5 V (D) 1V

Ans. **(A)**

Sol. V = 2n –1

i = 40n

Then using

V = iR

2n – 1 = 40 nR

This hold for all values of n.

Then zero error = – 1

20. The inverse function of the function sin x + cos x is

(A) $\sin^{-1} x + \cos^{-1} x$ (B) $\frac{1}{\sin x + \cos x}$ (C) $\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$ (D) $\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{\pi}{4}$

Ans. **(D)**

Sol. f(x) = sinx + cosx

Though inverse exists only when f(x) is monotonic.

Therfore assuming that we have to find $f^{-1}(x)$ only for the interval in which f(x) is monotonic.

$\therefore$ y = sin x + cos x

$= \sqrt{2}\sin(x + \frac{\pi}{4})$

or $x = \sin^{-1}\left(\frac{y}{\sqrt{2}}\right) - \frac{\pi}{4}$

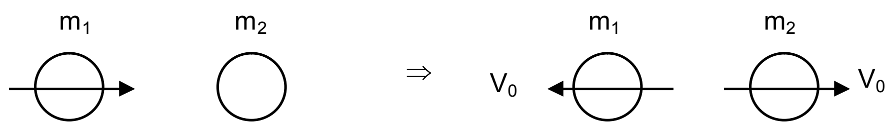
$\therefore$ $f^{-1}(x) = \sin^{-1}\left(\frac{\pi}{\sqrt{2}}\right) - \frac{\pi}{4}$ Hence option (D)

21. Particle A collides elastically (perfect) with another particle B which was at rest. They disperse in opposite directions with same speeds. Ratio of their masses must respectively be

(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 2 : 3

Ans. **(B)**

Sol.



$m_1v = -m_1v_0 + m_2v_0$ …(i)

$v = 2v_0$

$2m_1 = -m_1 + m_2$

$3m_1 = m_2 \Rightarrow \frac{m_1}{m_2} = \frac{1}{3}$

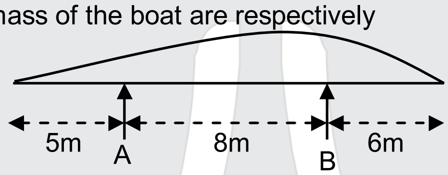
22. $\lim_{x\to-\frac{\pi}{4}}\left(\dfrac{\sin x+\cos x}{a+\dfrac{\pi}{4}}\right) =$

(A) ∞ (B) $-\infty$ (C) $\dfrac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Ans. **(D)**

Sol. $\lim_{x\to\frac{-\pi}{4}} \dfrac{\sin x+\cos x}{\left(x+\dfrac{\pi}{4}\right)} = \sqrt{2}$ Hence option (D)

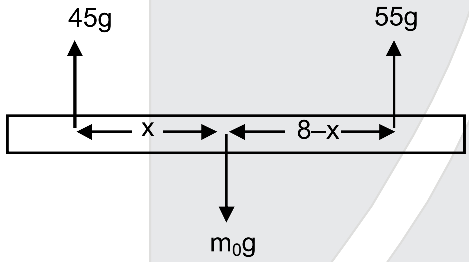
23. A long rowing boat put upside down, shown in the adjacent figure, has to be weighed using only a single bathroom scale. The boat will sag if it is supported only in the middle, and so the scales must be put first at position a with a wooden support at B, and then at position B with the wooden support at A. The readings on the scale are 45 kg and 55 kg respectively. The distance of centre of mass (from point A) and mass of the boat are respectively



(A) 4.4 m, 120 kg (B) 4.4 m, 100 kg (C) 4.2 m, 100 kg (D) 4.2 m, 120 kg

Ans. **(B)**

Sol.



$45g + 55g = m_0 g \Rightarrow m_0 = 100$

$45gx = (8-x)\,55g$

$x = 4.4$ m

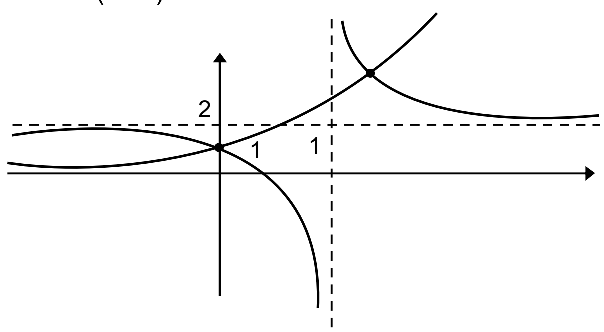
24. The number of real solutions of the equation $(x-1)(3^x-2) = 1$ is

(A) 0 (B) 1 (C) 2 (D) More than 2

Ans. **(C)**

Sol. $(x-1)(3^x-2) = 1$

or $3^x = \dfrac{1}{(x-1)} + 2$



The two graphs intersect each other of two distinct point.

$\therefore$ Number of solution = 2

But we have to check at x = 1 separately, which does not satisfy the equation.

$\therefore$ no. of solutions = 2 Hence option (C)

25. A car is fitted with a rear view mirror of focal length 20 cm. Another car, 2.8 m behind the first car is 15 $m.s^{-1}$ faster then the first car and approaching. The relative speed of image of the second car, with respect to first car at this instant, is

(A) 1/15 $m.s^{-1}$ (B) 1/10 $m.s^{-1}$ (C) 1/5 $m.s^{-1}$ (D) 2/15 $m.s^{-1}$

Ans. (A)

Sol. $\frac{V_I}{V_O} = \frac{-V^2}{u^2}$

$\frac{V_I}{15} = -\left(\frac{V}{u}\right)^2$ .....(1)

$\frac{1}{V} + \frac{1}{-2.8} = \frac{1}{0.2}$

$\frac{1}{V} = \frac{1}{2.8} + \frac{1}{0.2} = \frac{3}{0.56}$

$\frac{V_I}{15} = -\left[\frac{0.56}{3 \times 2.8}\right]^2 = -\left[\frac{0.56}{3 \times 0.28 \times 10}\right]^2$

$\frac{V_I}{15} = \frac{1}{15^2}$

$V_I = \frac{1}{15}$ m/s

26. $\log_{\sqrt{2}} 16 + \log_{27} 9 + \log_{\frac{1}{3}} 3$

(A) Is defined but cannot be found (B) Is not defined

(C) Is defined and equals $-\frac{1}{3}$ (D) Is defined and equals $\frac{23}{3}$

Ans. (D)

Sol. $\log_{\sqrt{2}} 16 + \log_{27} 9 + \log_{\frac{1}{3}} 3$

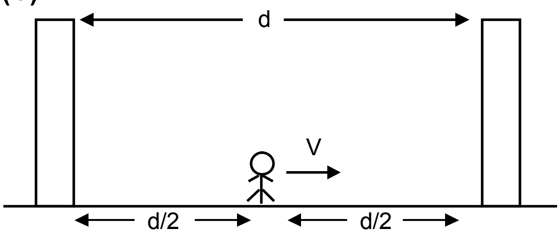
$= 8 + \frac{2}{3} - 1 = \frac{23}{3}$ Hence option (D)

27. An electric buggy is stationed exactly midway between two vertical walls parallel to each other. A man standing adjacent to buggy blows whistle momentarily. Instantly the buggy starts running towards one of the walls with a velocity 35 m/s. The driver of the buggy records first two echoes of the whistle with a delay of exactly one second. Speed of sound in air at that temperature is 350 m/s. Distance between the walls must be

(A) 433.125 m (B) 866.25 m (C) 1732.5 m (D) 3465 m

Ans. (C)

Sol.



$t = \frac{\frac{d}{2} + \left(\frac{d}{2} - vt\right)}{c}$; $t + 1 = \frac{\frac{d}{2} + \left(\frac{d}{2} + v(t+1)\right)}{c}$

v = 35 m/s

c = 350 m/s

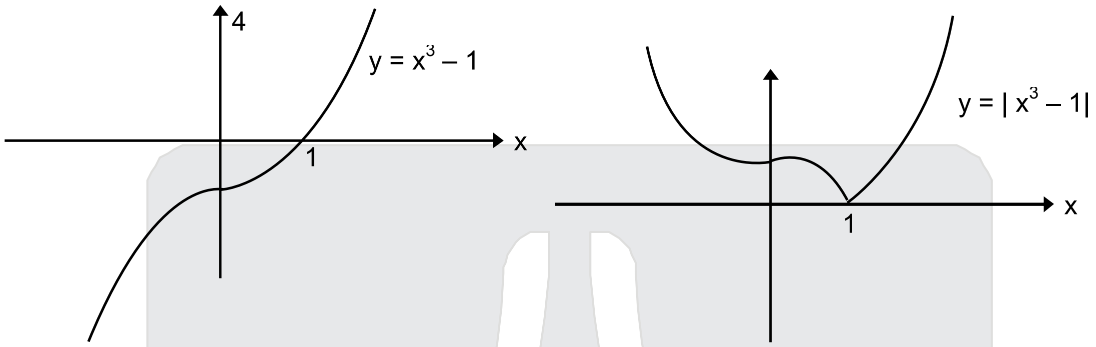
solving d = 1732.5 m

28. The number of points at which $|x^3 - 1|$ is not differentiable is

(A) 3 (B) 2 (C) 1 (D) 0

Ans. **(C)**

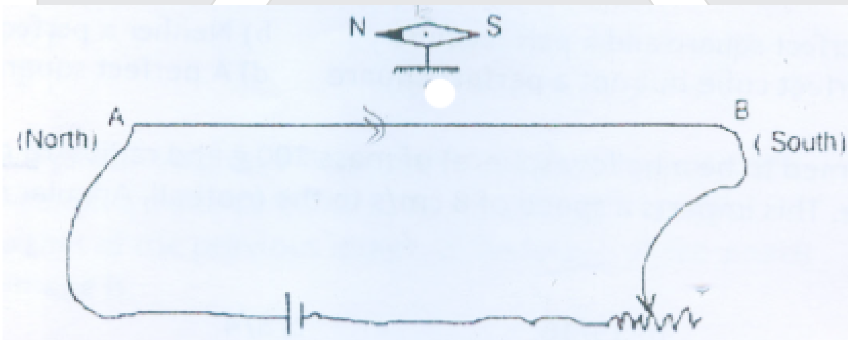
Sol. $f(x) = |x^3 - 1|$



Hence Non differentiable at $x = 1$

Therefore option (C)

29. The circuit given below has a long straight wire AB placed horizontal along North-South direction. A small magnetic needle can be held anywhere near this wire. Choose the correct statement.



(A) North Pole of the magnetic needle will deflect towards East, if the compass is just above the wire

(B) North pole of the magnetic needle will deflect towards West, if the compass is at exactly same level of the wire.

(C) North pole of the magnetic needle will deflect towards East, if the compass is just below the wire.

(D) Magnetic needle will not deflect, if kept just below the wire

Ans. **(C)**

30. If $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ are unit vectors such that $\overline{AB}\cdot\overline{BC} = \frac{1}{\sqrt{2}}$ then

(A) Points A, B, C, D are concyclic

(B) Quadrilateral ABCD has area $\frac{1}{2\sqrt{2}}$

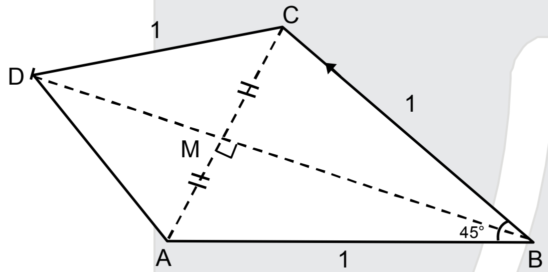
(C) Quadrilateral ABCD has half of the maximal area for quadrilateral with same perimeter

(D) The area determined by the vectors is $\frac{1}{\sqrt{2}}$

Ans. **(D)**

Sol. $\because$ $\overline{AB}\cdot\overline{BC} = \frac{1}{\sqrt{2}}$

$\therefore$ Angle b/w $\overrightarrow{AB}$ & $\overrightarrow{BC}$ = 45°



$\because$ ΔABC is isosceles

$\therefore$ If AM = MC,

$\angle$ AMB = $\angle$CMB = 90°

$\because$ CD = DA

$\therefore$ D lies on extended BM

$\left|\overline{AC}\right| = \sqrt{1+1-\sqrt{2}} = \sqrt{2-\sqrt{2}}$

$\therefore$ Area of ABCD = $2\times\left(\frac{1}{2}\times|\overrightarrow{AC}|\times|\overrightarrow{MB}|\right) = \sqrt{2-\sqrt{2}} \times \cos\left(22\frac{1}{2}\right)$

$= \sqrt{2-\sqrt{2}} \times \frac{\sqrt{2+\sqrt{2}}}{2} = \frac{1}{\sqrt{2}}$ Hence option (D)

31. INSAT series of satellites are launched by India for telecommunication. Such satellites appear stationary at a particular point in the sky when observed from the earth. Consider the following statements :

(i) The satellite always experiences gravitation of the earth

(ii) The satellite does not need any fuel for its motion.

(iii) The satellite does not experience net force.

(iv) Such satellites have to be positioned vertically above the equator.

(A) Only (ii), (iii) & (iv) are correct

(B) Only (i), (ii) & (iv) are correct

(C) Only (i) & (iii) are correct

(D) Only (i) & (ii) are correct

Ans. **(B)**

32. The number $3^6\left(3^{10}+6^5\right)+2^3\left(2^{12}+6^7\right)$ is

(A) A perfect square and a perfect cube

(B) Neither a perfect square nor a perfect cube

(C) A perfect cube but not a perfect square

(D) A perfect square but not a perfect cube

Ans. **(C)**

Sol. $= 3^8 (3^{10} + 6^5) + 2^3 (2^{12} + 6^7)$

$= 3^{18} + 2^{15} + (3^{13} \times 2^5) + (2^{10} \times 3^7)$

$= (3^6)^3 + (2^5)^3 + 3(3^6)(2^5)[3^6 + 2^5] = (3^6 + 2^5)^3 = (761)^3$

Hence a perfect cube but not a perfect square.

Hence option (C) is correct.

33. Evaporation of (sweat) water is an essential mechanism in human beings for maintaining normal body temperature. For human beings, heat of vaporization of water at a body temperature of 37 °C is nearly 2.3 MJ/kg and specific heat capacity s 3.5 kJ/(kg.K). On consuming a certain prescribed diet, the body temperature of an athlete of mass 82 kg is expected to increase by 2 °C in order to prevent this, be drinks N bottles of mineral water (250 ml water in each) at 37 ºC. Assume that the entire amount of this water is given out as sweat, which vaporizes N is nearly (density of water = 1000 kg. m^{-3})

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. $Q = mS\Delta T$

$= 82 \times 3.5 \times 10^3 \times 2$

$Q = 574 \times 10^3 J$

From evaporation

$Q = N \times 250 \times 10^{-3} \times 2.3 \times 10^6$

$= 575 \times 10^3 \times N$

So N = 1

34. The number n = 1 + 12 + 60 + 160 + 240 + 192 + 64 is

(A) A perfect square and a perfect cube
(B) Neither a perfect square nor a perfect cube
(C) A perfect cube but not a perfect square
(D) A perfect square but not a perfect cube

Ans. **(A)**

Sol. 1 + 12 + 60 + 160 + 240 + 192 +64

$^6C_0 + {^6C_1}(2) + {^6C_2}(2)^2 + {^6C_3}(2)^3 + {^6C_4}(2)^4 + {^6C_5}(2)^5 + {^6C_6}(2)^6$

$= (1 + 2)^6 = 3^6$

Hence this is a perfect square as well as a perfect cube

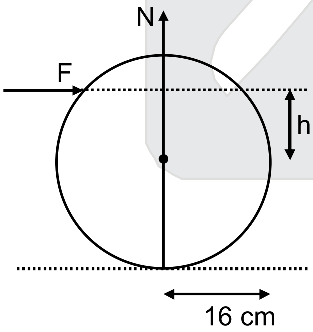
Hence option (A)

35. A football (assumed to be a hollow sphere) of mass 200 g and radius 16 cm, is given horizontal kick 4 cm above its centre. This imparts a speed of 8 cm/s to the football. Angular speed acquired by the football in radian/s is

(A) 9/16 (B) 15/16 (C) 3/4 (D) 16/15

Ans. **(No option match)**

Sol.



m = 200g

R = 16 cm

$J = mV_{cm}$ .....(1)

$J \times h = I_{cm}w$ .....(2)

$mv_{cm}h = \left(\frac{2}{3}mR^2\right)\omega$

$\omega = \frac{3v_{cm}h}{2R^2} = \frac{3 \times 8 \times 4}{2 \times 16 \times 16} = \frac{3}{16} \text{rad/s}$

36. For sets A,B we have (here X^C denote complement of set X) $(A \times B)^C =$

(A) $A^C \times B^C$ (B) $B^C \times A^C$

(C) $A^C \times B \cup B^C \times A \cup A^C \times B^C$ (D) $A^C \times B \cup A \times B^C \cup A^C \times B^C$

Ans. **(D)**

Sol. Let $A \subset X$, $B \subset Y$

$((A^c \times B) \cup (A^c \times B^c)) \cap (A \times B) = \phi$

and $(A \times B^c) \cap (A \times B) = \ell$

therefore

$(A \times B)^c = (A^c \times B) \cup (A \times B^c) \cup (A^c \times B^c)$

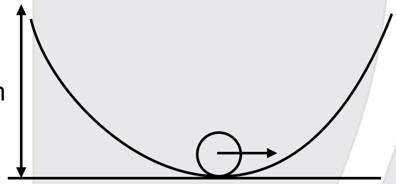
Hence option (D)

37. A small marble (assumed to be a uniform solid sphere) is released on one end of a parabolic mirror from a vertical height of 70 cm. First half part of this mirror is rough on which the marble is released. Other half of the mirror is smooth. Throughout its motion the marble never slips. To what vertical height will it rise on the smooth surface ?

(A) 98 cm (B) 70 cm (C) 63 cm (D) 50 cm

Ans. **(D)**

Sol.



On rough surface $\Rightarrow$ mgh = (1 + y) $\frac{1}{2}mV_{cm}^2$ .....(1)

On smooth surface $\frac{1}{2}mv_{cm}^2 = mgh^1$ .....(2)

h = (1 + y)h′

70 = (1 + 2/5)h′

$\Rightarrow$ h′ = 50 cm

38. The number $3^{12} + 2^9 + 3\left(3 \times 6^4 + 6^5\right) + 2^6$ is

(A) A perfect square and a perfect cube (B) A perfect cube but not a perfect square

(C) A perfect square but not a perfect cube (D) Neither a perfect square nor a perfect cube

Ans. **(C)**

Sol. $3^{12} + 2^9 + 3(3 \times 6^4 + 6^5) + 2^6$

$3^{12} + 2^9 + 3 \times 3 \times (6)^4 + 3\,(6)^5 + 2^6$

$3^{12} + 2^9 + (3^6 \times 2^4) + (3^6 \times 2^5) + (2^6)$

$= 3^6\,(3^6 + 2^4 + 2^5) + (2^6 + 2^9)$

$= 3^6\,(3^6 + (3 \times 2^4)) + (2^6 \times 9)$

$= 3^{12} + (3^7 \times 2^4) + (3^2 \times 2^6)$

$= (3^6 + (3 \times 2^3))^2 = 3^2\,[3^5 + 2^3]^2 = 3^2\,(251)^2 = (753)^2$

Hence a perfect square

$\therefore$ option (C) is correct.

39. Radius and moment of inertia of a smooth pulley are 0.1 m and 1 kg.m^2 respectively. A tangential force f = 40 t – $10t^2$ sets the pulley into rotation. Direction of its rotation reverses after some time. The time duration after which the direction will reverse is

(A) 6s (B) 8s (C) 4s (D) 12 s

Ans. **(A)**

Sol. $\tau = I\alpha$

$0.1(f) = 1.\alpha$

$\alpha = 4t - t^2$

$\frac{d\omega}{dt} = 4t - t^2$

$\int d\omega = \int (4t - t^2)dt$

$\omega = 2t^2 - \frac{t^3}{3}$

$\omega = 0$ when t = 0 or t = 6 sec.

40. $\log_{10} 0.01 + \log_{0.1} 10 + \log_{10} 0.001 + \log_{0.1} 0.001 =$

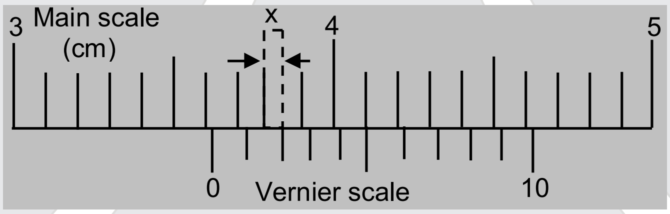
(A) $\log_{10.2} 10.012$ (B) $\log_{10} 0.000001 + 3$ (C) $-4 + \log_2 8$ (D) None of the above

Ans. **(B)**

Sol. $\log_{10} (0.01) + \log_{0.1} 10 + \log_{10} (0.001) + \log_{0.1} (0.001)$

$= (-2) + (-1) + (-3) + (3) = -3$ Hence option (B)

41. The figure shows a particular position of a Vernier callipers. The value of x in cm is



(A) 0.03 (B) 0.15 (C) 3.83 (D) 0.02

Ans. **(A)**

Sol. x = 3(MSD – VSD)

= 3 × LC

= 3 × 0.01 cm

x = 0.03 cm

42. If A and B are two sets then the set A × B is given by

(A) $\{a \times b \mid a \in A, b \in B\}$ (B) $\{(a,b) \mid a \in B, b \in A\}$

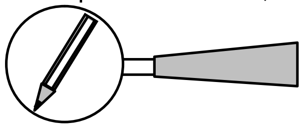
(C) $\{(a,b) \mid a \in A, b \in B\}$ (D) $\{ab \mid a \in A, b \in B\}$

Ans. **(C)**

Sol. $A \times B = \{(a, b) ; a \in A, b \in B\}$

Hence option (C) is correct

43. A lens is held directly above a pencil lying on a floor and forms an image of it. On moving the lens vertically through a distance equal to its focal length, it again forms image of same size as that of the previous image. If the length of the pencil is 5.0 cm, the length of the image is



(A) 10.0 cm (B) 15.0 cm (C) 20.0 cm (D) 12.5 cm

Ans. **(A)**

Sol. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ $\frac{I}{o}=\frac{f}{f-f/2}$ = 2

$\frac{u}{v}=\frac{u}{f}+1$ h_I = 10cm

m = $\frac{v}{u}=\frac{f}{f+u}$ = $\frac{f}{f+(-x)}$

$\frac{f}{f+(-x)}=\frac{-f}{f+(-x-f)}$

$\frac{f}{f-x}=\frac{f}{x}$

x = $\frac{f}{2}$

44. If A = {2,3}, B = {4, 5} then A × B =

(A) {8, 15} (B) {8, 10 12, 15} (C) {(2,4), (3,5)} (D) {(2,4), (2,5), (3,4)(3,5) }

Ans. **(D)**

Sol. A = {2, 3} , B = {4, 5}

A × B = {(2, 4), (2, 5), (3, 4), (3, 5)}

hence option (D) is correct.

45. A balloon less dense than air is tied at the floor of a truck with a massless, inextensible and flexible string. The truck is observed to be taking a left turn. Select correct statement.

(A) The string will incline towards right (outward, w.r.t person in the truck)

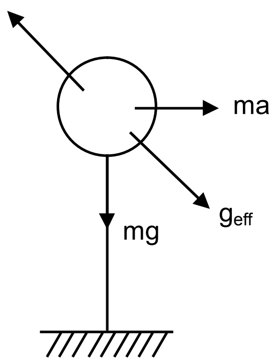
(B) The string will incline towards left (inward, w.r.t. person in the truck)

(C) The string will still be vertical as the balloon is less dense than air

(D) Buoyant force on the balloon is equal to weight of the balloon.

Ans. **(B)**

Sol.



Ballon will float opposite to g_{eff}

46. A function F from A to B is
(A) Relation F with (a,b), (c,b) $\in F \Rightarrow a = c$
(B) $F \subset A \times B$
(C) Relation F with (a,b), (a,c) $\in F \Rightarrow b = c$
(D) Relation F with (a,b), (b,c) $\in F \Rightarrow (a,c) \in F$

Ans. **(C)**

Sol. As, function from A to B is a subset of Cartesian product A × B in such a way that for each input taken from A, there should be unique output in B.
$\therefore$ function (f) $\subseteq$ A × B & f = A × B whenever
n (B) = 1
Hence option (B) is wrong because there is no equality
option (C) is correct.

47. An ice cube with a steel ball bearing trapped inside it is floating above water in a glass. What will happen to the water level in the glass after the ice melts completely ?
(A) Rise
(B) Fall
(C) will not change
(D) Answer depends upon actual position of the steel ball.

Ans. **(B)**

Sol. Initially the steel ball is floated with ice so the liquid displaced is having same weight as that of ball & finally the ball sinks & liquid displaced has weight less than that of ball. so water level fall.

48. Which of the following is a mathematically acceptable statement ?
(A) It is an even number
(B) 13th December is Saturday
(C) Common donkey belongs to class orthopoda
(D) Alexander was a great king

Ans. **(C)**

Sol. Statement are those sentences which have fixed truth value it should be either 'T' or 'F'
Hence option (C) is correct.

49. A block of mass 5 kg is to be dragged along a rough horizontal surface having μ_s = 0.5 and μ_k = 0.3. The horizontal force applied for dragging it is 20 N. (g = 10 m/s^2). Select correct statement/s.

(A) Frictional force acting on the block is 20 N.

(B) Block will be displaced.

(C) Block will move with acceleration 1 m/s^2.

(D) Block will initially move and then stop

Ans. **(A)**

Sol. $(f_s)_{max} = \mu_s N = 25\text{ N} > f_{ext}$

$f_s = f_{ext} = 20$ N

50. The negation of the statement : f(x) is continuous for all real numbers x. is

(A) f(x) is not continuous for all real numbers x

(B) f(x) is not continuous for any real numbers x

(C) f(x) is not continuous for every real numbers x

(D) f(x) is not continuous for some real numbers x

Ans. **(D)**

Sol. ' f(x) is continuous for all real numbers x '
negation will be
There exists some real number x for which f(x) is not continuous
Hence option (D) is correct

51. A bullet moving with a speed of 72 m/s comes to a halt in a fixed wooden block on travelling 9 cm inside it. If the wooden block (of the same type of wood) were to be 8 cm thick, the bullet would come out of the block with a speed.
(A) 9 $m.s^{-1}$ (B) 8 $m.s^{-1}$ (C) 24 $m.s^{-1}$ (D) 64 $m.s^{-1}$

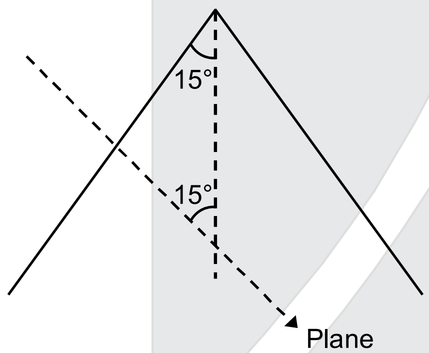
Ans. **(C)**

Sol. $v^2 - u^2 = 2as$
$0^2 - 72^2 = 2 \times a \times 9$ cm
$v^2 - 72^2 = 2 \times a \times 8$ cm
$$\frac{-v^2 + 72^2}{+72^2} = \frac{8}{9}$$
$72^2 = 9v^2$
$v = \frac{72}{3} = 24$ m/s

52. Let ℓ be a vertical line and m a line that makes an angle of $\frac{\pi}{6}$ with ℓ. Consider the cone generated by rotating m around the axis ℓ. If plane L makes an angle of 15° with line ℓ then the intersection of the plane and the cone is
(A) A parabola (B) A pair of straight line
(C) An ellipse (D) A hyperbola

Ans. **(D)**

Sol.



From the above figure we can easily observe that plane is parallel to slant height of the cone.
Hence the conic will be hyperbola.
Hence option (D) is correct

53. A piece of brass (an alloy of copper and zinc) weighs 12.9 g in air. When completely immersed in water, it weighs 11.3 g. What is the mass of copper contained in the alloy? The density of copper and zinc are 8.9 g/cm^3 and 7.1 g/cm^3 respectively.
(A) 6.89 g (B) 4.54 g (C) 8.93 g (D) 7.61 g

Ans. **(D)**

Sol. $\delta_{cu} = 8.9$ g/cc $\quad \delta_{Brass} = 7.1$ g/cc
$m_{cu} + m_B = 12.9$ g …(i)
$$m_\omega\left(1 - \frac{\delta_\ell}{\delta_{cu}}\right) + m_B\left(1 - \frac{\delta_\ell}{\delta_B}\right) = 11.3g \quad \text{…(ii)}$$
$$12.9 - \frac{m_{cu}}{8.9} - \frac{m_B}{7.1} = 11.3$$
$7.1\ m_{cu} + 8.9\ m_B = 1.6 \times 8.9 \times 7.1$ …(iii)
From (i) & (iii)
$1.8\ m_{cu} = 8.9 \times 12.9 - 1.6 \times 8.9 \times 7.1$
$= 114.81 - 101.104$
$1.8\ m_{cu} = 13.70$
$m_{cm} = \frac{13.7}{1.8} = 7.61$ g

54. The coefficients of x in the expansion of $(1 + x)^5$ correspond to the

(A) 5^{th} row of Pascal's triangle

(B) 6^{th} row of Pascal's triangle

(C) 7^{th} row of Pascal's triangle

(D) 4^{th} row of the Pascal's triangle

Ans. **(B)**

Sol. Pascal triangle :

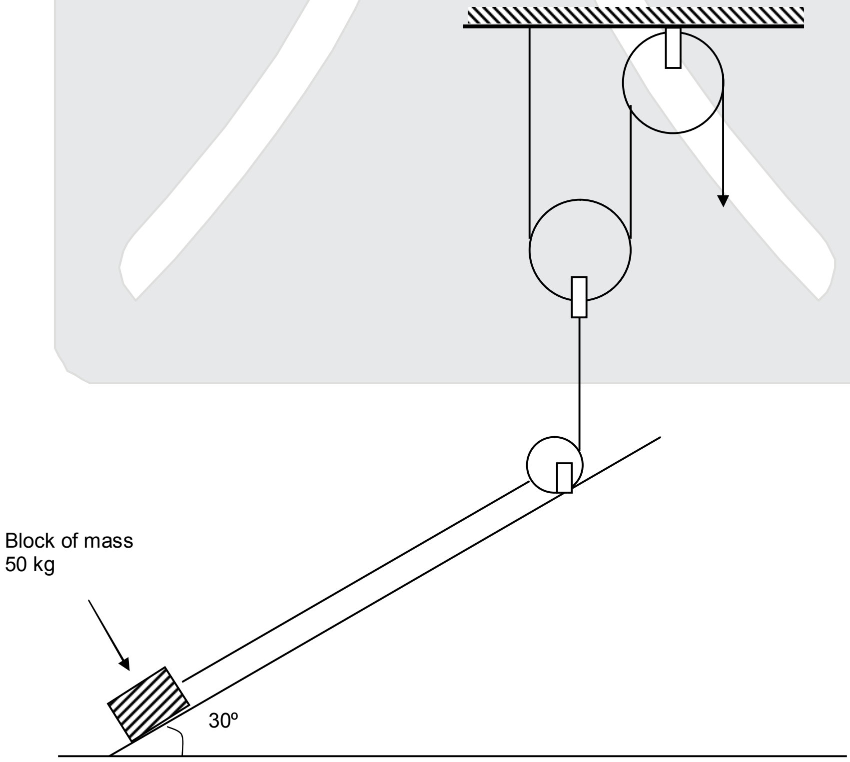
```
                1
            1       1
        1       2       1
    1       3       3       1
1       4       6       4       1
```

1	5	10	10	5	1

6^{th} now corresponding to $(1 + x)^5$

Hence option (B) is correct

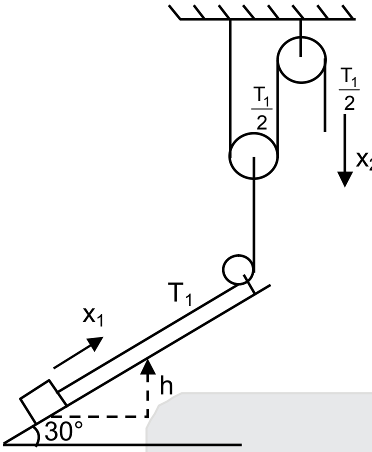
55. Linked question (55-56): The adjacent figure shows a ramp (30°) holding a block of mass 50 kg. The block attached to a movable pulley A with an inextensible massless string . The movable pulley is in turn held with the help of another fixed pulley B. The block kept on the ramp is to be raised through a vertical height of 10 cm. By what distance the string should be lowered down vertically, below E?



(A) 20 cm (B) 5 cm (C) 40 cm (D) $10(3^{1/2})$ cm

Ans. **(C)**

Sol.



h = 10 cm

$\sin 30° = \frac{h}{x_1}$

$x_1 = 20$ cm

$2x_1 = x_2$

$x_2 = 40$ cm

56. Refer to figure in question 55. Pulleys in the figure are massles and frictionless. Neglecting friction between block and the ramp, the force that should be applied vertically downwards, at E, to slide the block along the ramp without acceleration is (g = 10 m-s^{-1})

(A) 65 N (B) 125 N (C) 175 N (D) 250 N

Ans. **(B)**

Sol. Tension is rope connecting block

T = mg sin 30°

$= 500 \times \frac{1}{2} = 250$N

Force at E = 125N

57. If $\cos^2 x = \frac{1}{3}$ then cosec x =

(A) $\sqrt{3}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}}$

Ans. **(D)**

Sol. $\cos^2 x = \frac{1}{3}$

$\sin^2 x = \frac{2}{3}$

$\text{cosec}^2 x = \frac{3}{2}$

$\therefore$ cosec x = $\sqrt{\frac{3}{2}}$

$\therefore$ option (D) is correct

58. Which of digits 1,3,4,5,7,0 cannot appear at the ten's place of powers of 3 ?

(A) 1,3,5 only (B) 1,3,4,7 only (C) 1,3,5,7 only (D) 0,1,3,5 only

Ans. **(C)**

Sol. Let $n \in$ even i.e., n = 2k

$3^n = 3^{2k} = 9^k = (10 - 1)^k$

$(10 - 1)^k = 100\,I + {}^kC_{k-1}\,(10)(-1)^{k-1} + {}^kC_k(-1)^{-k}$

Let $k \in$ even; $100\,I + (-1)[10k - 1]$

x x x x x.......... x x 0 0

– (10k –1)

x x x x .... x x 1

if k = 2, Lest two placer = 81

if k = 4, Lest two placer = 61

if k = 6, Lest two placer = 41

if k = 8, Lest two placer = 21

if k = 10, Lest two placer = 01

Let $k \in$ odd

$(10 - 1)^k = 100\,I + (10k - 1)$

for u = 1, 3, 5, 7, 9

we get last two digit as

09, 29, 49, 69, 89 respectively

Let $n \in$ odd i.e., n = 2k + 1

$3^n = 3 \times 3^{2k} = 3(10 - 1)^{k,}$ Again we

Set the same digit at 10^{th} place

Hence out of 1, 3, 4, 5, 7, 0, (1, 3, 5 and 7) can not appear at ten's place

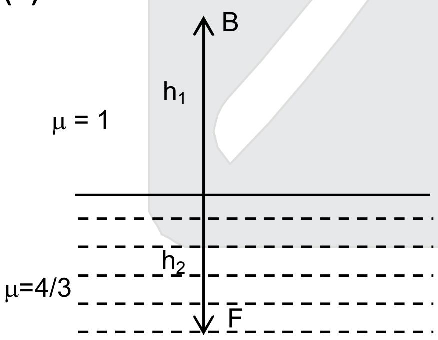
Hence option (C) is correct

59. A fish is swimming in still water. At a given instant there is a bird flying vertically above the fish. For the bird the fish appears to be 15 m below the surface of water while for the fish the bird appears to be 20 m above the surface. Refractive index of water is 4/3. Actual distance between the bird and the fish is

(A) 35 m (B) 40 m (C) 30 m (D) 25 m

Ans. **(A)**

Sol.



$$\frac{h_2}{\frac{4/3}{1}} = 15$$

$3h_2 = 60$

$h_2 = 20$ N

$$\frac{h_1}{\frac{1}{4/3}} = 20$$

$$\frac{4}{3}h_1 = 20$$

$h_1 = 15$M

$h_1 + h_2 = 35$ N

60. the smallest integer n such that $\sqrt{n+1} - \sqrt{n} \le 0.01$ is

(A) 2499 (B) 2500 (C) 2501 (D) 2502

Ans. (B)

Sol. $\sqrt{n+1} - \sqrt{n} \le 0.01$

Let f(x) = $\sqrt{x}$

f'(x) = $\frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} \le 0.01$

$\frac{1}{4x} \le 10^{-4}$

or $x \ge \frac{10^4}{4}$

or $x \ge 2500$

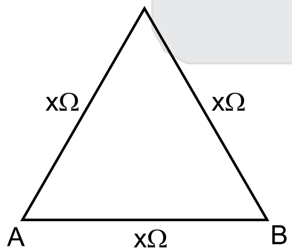
Hence option (B) is correct

61. A uniform wire of resistance per unit length 1 Ω/m is bent in the form of an equilateral triangle. If effective resistance between adjacent vertices is 2.4 Ω, length of each side of the triangle is

(A) 1.8 m (B) 2.4 m (C) 3.6 m (D) 7.2 m

Ans. (C)

Sol.



$R_{AS} = \frac{2x \cdot x}{2x + x}$

$2.4\Omega = \frac{2x}{3}$

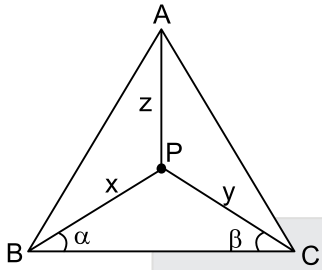
x = 3.6

62. ΔABC is equilateral with each side being of unit length and P is an interior point then the maximum product of the length AP.BP and CP is

(A) $\frac{1}{5\sqrt{3}}$ (B) $\frac{1}{4\sqrt{3}}$ (C) $\frac{1}{3\sqrt{3}}$ (D) $\frac{1}{6}$

Ans. **(C)**

Sol.



$\because$ x cos α + y sin β = 1

$\therefore$ xy = $\frac{x(1-x\cos\alpha)}{\sin\beta}$

To be mex. $\frac{df}{dx}$ = 0

$\therefore$ 1 – 2x cosα = 0

or x cosα = $\frac{1}{2}$

Therefore x cos α = y cos β = $\frac{1}{2}$

Hence 'P' should lie on perpendicular bisector of BC to make (x, y) maximum.
Similarly, for xz to be maximum, 'p' should be on perpendicular bisector of AB.
Hence 'P' must be at circumcenter

$\therefore$ (PA)(PB)(PC) = R^3

= $\left(\frac{abc}{4\Delta}\right)^3$ = $\frac{1}{3\sqrt{3}}$

Hence option (C) is correct

63. The resultant of the forces P and Q is R if Q is doubled then R gets doubled. If Q is reversed even then R gets doubled. Then

(A) $P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{2}$ (B) $P:Q:R=\sqrt{2}:\sqrt{2}:\sqrt{3}$

(C) $P:Q:R=\sqrt{3}:\sqrt{3}:\sqrt{2}$ (D) $P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{3}$

Ans. **(A)**

Sol. Let $|\vec{P}|$ = P, $|\vec{Q}|$ = Q, $|\vec{R}|$ = R

$R^2 = P^2 + Q^2 + 2PQ \cos Q$ .... (1)

$4R^2 = P^2 + 4Q^2 + 4PQ \cos Q$ .... (2)

$4R^2 = P^2 + Q^2 - 2PQ \cos Q$ .... (3)

$\therefore R^2 - P^2 - Q^2 = \left(\frac{4R^2 - P^2 - 4Q^2}{2}\right) = P^2 + Q^2 - 4R^2$

$\therefore P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$

Hence option (A) is correct

64. The unit digit of $23^{2015} \times 7^{2016} \times 13^{2017}$ is

(A) 1 (B) 3 (C) 7 (D) 9

Ans. **(A)**

Sol. unit digit of

$(23)^{2015} \times (7)^{2016} \times (13)^{2017}$

= unit's place of

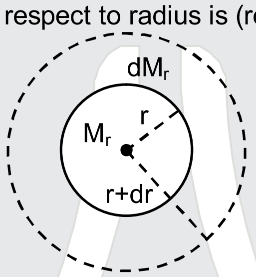
$7 \times 1 \times 3$

= 1

Hence option (A) is correct

65. Linked questions (65-69)

A star can be considered as a spherical ball of hot gas of radius R. Inside the star, the density of the gas is ρ_r at a radius r and mass of the gas within this region is M_r. The correct differential equation for variation of mass with respect to radius is (refer to the adjacent figure)



(A) $\frac{dM_r}{dr} = \frac{4}{3}\pi\rho_r r^3$ (B) $\frac{dM_r}{dr} = 4\pi\rho_r r^2$ (C) $\frac{dM_r}{dr} = \frac{2}{3}\pi\rho_r r^2$ (D) $\frac{dM_r}{dr} = \frac{1}{3}\pi\rho_r r^2$

Ans. **(B)**

Sol. $dM_r = \rho_r 4\pi r^2 dr$

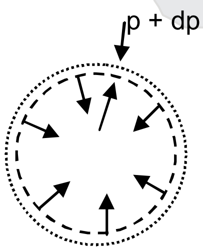
$\frac{dM_r}{d_r} = \rho_r 4\pi r^2$

66. A star in its prime age is said to be under equilibrium due to gravitational pull and outward radiation pressure (p). Consider the shell of thickness dr as in the figure of question (65). If the pressure on this shell id dp then the correct equation is (G is universal gravitational constant)

(A) $\frac{dp}{dr} = -\frac{GM_r}{r^2}\rho_r$ (B) $\frac{dp}{dr} = \frac{GM_r}{r^2}\rho_r$ (C) $\frac{dp}{dr} = -\frac{2}{3}\frac{GM_r}{r^2}\rho_r$ (D) $\frac{dp}{dr} = \frac{2}{3}\frac{GM_r}{r^2}\rho_r$

Ans. **(A)**

Sol.



$\left(\frac{GM_r}{r^2}\right) = \rho_r 4\pi r^2 dr$

$(P)(4\pi r^2) - (p + dp)(4\pi r^2)$

$= \frac{GM_r}{r^2} = \Sigma_r 4\pi r^2 dr$

$-dp = \frac{GM_r \rho_r}{r^2} dr$

$\frac{dp}{dr} = -\frac{GM_r}{r^2}\rho_r$

67. In astronomy order of magnitude estimation plays an important role. the derivative $\frac{dp}{dr}$ can be taken difference ratio $\frac{\Delta P}{\Delta t}$. Consider the star has a radius R, pressure at its centre is P_e and pressure at outer layer is zero if the average mass is $\frac{M_Q}{2}$ and average radius $\frac{R_0}{2}$ then the expression for P_C is

(A) $P_C = \frac{3}{2}\frac{GM_0^2}{\pi R_0^4}$ (B) $P_C = \frac{3}{4}\frac{GM_0^2}{\pi R_0^4}$ (C) $P_C = \frac{2}{3}\frac{GM_0^2}{\pi R_0^4}$ (D) $P_C = \frac{3}{2}\frac{GM_0^2}{R_0^4}$

Ans. **(A)**

Sol. $-\int_{P_c}^{0} dp = \frac{4}{3}\pi G\rho_r^2 \int_0^R r dr$

$P_C = \frac{4}{3}\pi G\rho_r^2 \frac{R^2}{2}$

$P_C = \frac{GM_0^2}{R_0^4} \times \left(\frac{3}{2\pi}\right)$

68. The value of mass and radius of sun are given by $M_0 = 2 \times 10^{30}$ kg and $R_0 = 7 \times 10^5$ km respectively. The pressure at the centre is about ($G = 6.67 \times 10^{-11}\ m^3 . kg^{-1}. s^{-2}$)

(A) $2 \times 10^{14}\ N.m^{-2}$ (B) $2 \times 10^{15}\ N.m^{-2}$ (C) $5 \times 10^{14}\ N.m^{-2}$ (D) $7 \times 10^{15}\ N.m^{-2}$

Ans. **(C)**

Sol. $P_c = \frac{3}{2}\frac{GM_0^2}{\pi R_s^4}$

$= \frac{3 \times 6.67 \times 10^{-11} \times 4 \times 10^{60}}{2 \times 3.14 \times (7 \times 10^8)^4}$

$= \frac{3 \times 6.67 \times 4 \times 10^{49}}{2 \times 3.14 \times 49 \times 49 \times 10^{32}} \times 10^{17}$

$= \frac{3 \times 4 \times 6.67}{2 \times (3.14) \times (49)^2} \times 10^{17}$

$= 0.00490 \times 10^{17}$

$P_c = 4.9 \times 10^{14} = 5 \times 10^{14}\ N/m^2$

69. Assuming that the gas inside the sun behaves very much like the perfect gas, the temperature at the centre of the sun is nearly (the number density of gas particles $= \frac{2\rho}{M_H}$), Boltzmann constant $k_B = 1.4 \times 10^{-23}\ J.K^{-1}$ and mass of proton $M_H = 1.67 \times 10^{-27}$ kg)

(A) 3×10^7 K (B) 2×10^7 K (C) 4×10^7 K (D) 6×10^7 K

Ans. **(B)**

Sol. PV = nKT

$P = \frac{2PKT}{M_H}$

$5.2 \times 10^{14} = \frac{2 \times 1.40 \times 10^{-23}}{1.67 \times 10^{-27}} \times PT$

$5.2 \times 10^{14} = \frac{2.8}{1.67} \times 10^4 PT$

$PT = 3.1 \times 10^{10} \Rightarrow T = \frac{3.1 \times 10^{10}}{1.4 \times 10^3} = 2.2 \times 10^7$

70. At the earth's equator a satellite is observed passing directly overhead moving west to east in the sky. Exactly 12 hours later, satellite is again observed directly overhead. the altitude of the satellite is (Radius of the earth = 6400 km)

(A) 1.82×10^7 m (B) 1.39×10^7 m (C) 3.59×10^7 m (D) 6.4×10^7 m

Ans. **(B)**

Sol. $12 \text{ hour} = \frac{2\pi}{(\omega_2 - \omega_1)}$

$(\omega_2 - \omega_1) = \frac{2\pi}{12}$

$\omega_{satellite} = \frac{2\pi}{12} + \frac{2\pi}{24}$

$= 2\pi\left(\frac{2+1}{24}\right)$

$\omega_{satellite} = \frac{2\pi}{24} \times 3$

$T_{satellite} = \frac{2\pi}{2\pi \times 3} \times 24 = \frac{24}{3}\text{hr} = \frac{T_{Earth}}{3}$

$T^2 \propto R^2$

$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} \Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2}$

$= (24 \text{ hrs})\left(\frac{R_2}{R_1}\right)^{3/2}$

$\Rightarrow \left(\frac{R_1}{R_2}\right)^{3/2} = 3 \Rightarrow \frac{R_1}{R_2} = 3^{3/2} \Rightarrow R_2 = \frac{R_1}{(3)^{3/2}} = 6400 \text{ km}$

71. **Passage 71 to 73**

Two stars, with masses M_1 and M_2 are in circular orbit around their common centre of mass. The star with mass M_1 has an orbit of radius R_1 and the star with mass M_2 has an orbit of radius R_2. The correct relation is

(A) $\frac{R_1}{R_2} = \frac{M_2}{M_1}$ (B) $\frac{R_1}{R_2} = \frac{M_1}{M_2}$ (C) $\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$ (D) $\frac{R_1}{R_2} = \sqrt{\frac{M_1}{M_2}}$

Ans. **(A)**

Sol. $\frac{R_1}{R_2} = \frac{M_2}{M_1}$

72. The time period of each of the star is

(A) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_2}{GM_2}$ (B) $T^2 = \frac{4\pi^2(R_1+R_2)^3}{G(M_1+M_2)}$

(C) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_2}{GM_1}$ (D) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_1}{GM_1}$

Ans. **(B)**

Sol. $T = \frac{2\pi R^{3/2}}{\sqrt{G(M_1+M_2)}} = \frac{2\pi(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}$

$T^2 = \frac{4\pi^2(R_1+R_2)^3}{G(M_1+M_2)}$

73. The two stars in certain binary system move in circular orbits. The first star alpha has an orbital speed of 36.0 km-s^{-1}. The second star, beat has an orbital speed of 12.0 km.s^{-1}. The orbital period of first star is 137 days. The mass of the two stars are about

(A) 2.1×10^{30} and 6.8×10^{30} kg (B) 1.3×10^{30} and 3.9×10^{30} kg
(C) 3.5×10^{30} and 9.2×10^{30} kg (D) 0.8×10^{30} and 6.8×10^{30} kg

Ans. **(B)**

Sol. V_1 = 36 km/s V_2 = 12 km/s

T_1 = 137 days

$\omega_1 = \omega_2$

$$\Rightarrow \quad \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{M_2}{M_1}$$

$$\frac{V_1}{V_2} = \frac{36}{12} = \frac{3}{1} = \frac{M_2}{M_1} \qquad \text{...(i)}$$

(B) option matches

74. Passage question 74-76

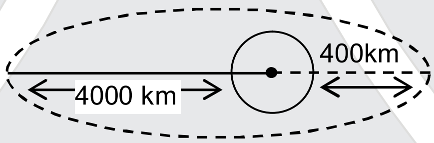
Consider a spacecraft in an elliptical orbit around the earth. At the low point or perigee of its orbit it is 400 km above the earth's surface. At the high point or apogee it is 4000 km above the earth's surface. The period of the space craft's orbit is (g = 9.8 ms^{-2} and R = 6400 km)

(A) 0.29 hr (B) 1.82 hrs (C) 2.21 hrs (D) 3.56 hrs

Ans. **(C)**

Sol. $T = \dfrac{2\pi R^{3/2}}{\sqrt{GM}}$

$137 \times 24 \times 3600 = 2\pi R^3$



4400 + 12800 = 17200km = length of major axis

length of semi major axis = $\dfrac{17200}{2}$ = 8600 km

$T^2 \propto a^3$

$$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$

$$\frac{24\text{hr.}}{T_2} = \left(\frac{6.6 \times 6400}{8600}\right)^{3/2}$$

$$T_2 = 24 \times \left(\frac{86}{6.6 \times 64}\right)^{3/2} \approx 2.20 \text{ hrs.}$$

75. The ratio of speed of the spacecraft at perigee to its speed at apogee is almost equal to

(A) 10 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 10

Ans. **(B)**

Sol. $mv_1r_1 = mv_2r_2$

$\Rightarrow \quad (V_1)(6400 + 4000) = (V_2)(6400 + 400)$

$\Rightarrow \quad (V_a)(10400) = v_p(6800)$

$$\Rightarrow \quad \frac{v_p}{v_a} = \frac{10400}{6800} = \frac{6}{4} = \frac{3}{2}$$

76. The speed of the satellite at perigee is

(A) 8576 m-s^{-1} (B) 57.307 m-s^{-1} (C) 5876 m-s^{-1} (D) 7856 m-s^{-1}

Ans. **(A)**

Sol. $$\frac{1}{2}mv_p^2 - \frac{GMm}{6800km} = \frac{1}{2}m\left(\frac{2v_p}{3}\right)^2 - \frac{GMm}{10400}$$

$$\frac{1}{2}mv_p^2\left(1-\frac{4}{9}\right) = GMm\left(\frac{1}{6800}-\frac{1}{10400}\right)$$

$$\frac{1}{2}mv_p^2\left(\frac{5}{9}\right) = GMm\left(\frac{10400-6800}{10400\times6800}\right)$$

$$\frac{5}{18}v_p^2 = \frac{GM\times36}{68\times104}\times\frac{1}{10^5} \quad\Rightarrow\quad v_p = \sqrt{\frac{GM\times36\times18}{68\times5\times104\times10^5}} = 8576\,m/s$$

77. Astronomers believe that a large percentage of the mass of the universe is dark matter. In one recent study the transverse velocity of the large Magellanic cloud (LCM) was measured to be 200 km-s^{-1}. the LMC is belleved to orbit the centre of our galaxy at about 17 × 10^4 ly (1.6 × 10^{21} m). Assuming a circular orbit percentage of dark matter in our galaxy is about (independent estimate of visible matter is 2 × 10^{41} kg)

(A) 77 % (B) 82 % (C) 70 % (D) 80 %

Ans. **(D)**

Sol. $$\sqrt{\frac{G(M_D+m_v)}{R}} = v$$

M_D : mass of dark matter

m_v : Visible metter

on solving M_D = 7.6 × 10^{41} which is approx

80% of total mass

78. The escape speed from jupiter is approximately 59.5 km-s^{-1} and its radius is about 12 times that of earth. From this we may estimate the mean density of jupiter to be about (Radius of earth = cape speed from the earth is 11.2 km-s^{-1})

(A) 5 times that of earth (B) 0.2 times that of the earth

(C) 2.5 times that of the earth (D) 0.4 times that of the earth

Ans. **(B)**

Sol. $$\frac{11.2}{59.5} = \frac{\sqrt{\dfrac{2G\frac{4}{3}\pi R_\varepsilon^3\rho_\varepsilon}{R_\varepsilon}}}{\sqrt{\dfrac{2G\frac{4}{3}\pi R_j^3\delta_j}{R_j}}} = \frac{R_\varepsilon}{R_j}\sqrt{\frac{\delta_\varepsilon}{\delta_j}} \quad ; \quad \text{on Solving } \frac{\delta_\varepsilon}{\delta_j} = 5$$

79. The orbit of planet mercury has the largest eccentricity of about 0.2 in the solar system. If the maximum distance of mercury from the centre of the sun is about 69 million km, its minimum distance from sun is about

(A) 13.8 million km (B) 57.7 million km (C) 46 million km (D) 18 million km

Ans. **(C)**

Sol. a (He) = 6g

a (i e) = r_{min}

solve r_{min} = 46 million km

80. As observed from a place in Australia the pole star

(A) appears in the southern direction (B) appears at about 30º above the horizon

(C) much brighter than that seen from India (D) can never be seen

Ans. **(D)**

Sol. We can not see pole star from Australia

because Australia is in southern hemisphere