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# INDIAN OLYMPIAD QUALIFIER (IOQ) 2021-2022

# INDIAN OLYMPIAD QUALIFIER IN MATHEMATICS (IOQM), 2022

## QUESTION PAPER (PART-A)
## WITH SOLUTION



**Resonance Eduventures Ltd.**
**Reg. Office & Corp. Office :** CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005
**Ph. No.:** +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222
**To Know more :** sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029
**Toll Free : 1800 258 5555** | **7340010333** | facebook.com/ResonanceEdu | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

**Time : 9:00 - 10:15 hrs** **March 06, 2022** **Total marks : 40**

# INSTRUCTIONS

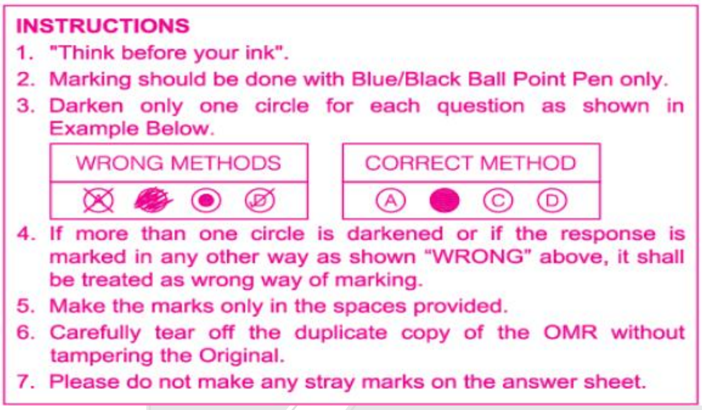
**1.** Use of mobile phones, smartphones, ipads, calculators, programmable wrist watches is **STRICTLY PROHIBITED.** Only ordinary pens and pencils are allowed inside the examination hall.

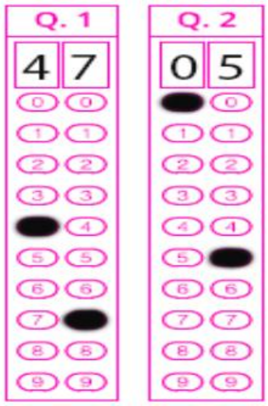
**2.** The correction is done by machines through scanning. On the OMR Sheet, darken bubbles completely with a **black or blue ball pen**. Please **DO NOT use a pencil or a gel pen**. Darken the bubbles completely, only after you are sure of your answer; else, erasing may lead to the OMR sheet getting damaged and the machine may not be able to read the answer. .

**3.** The name, email address, and date of birth entered on the OMR sheet will be your login credentials for accessing your score.

**4.** Incompletely, incorrectly or carelessly filled information may disqualify your candidature.

**5.** Each question has a one or two digit number as answer. The first diagram below shows improper and proper way of darkening the bubbles with detailed instructions. The second diagram shows how to mark a 2-digit number and a 1-digit number.




**6.** The answer you write on OMR sheet is irrelevant. The darkened bubble will be considered as your final answer.

**7.** Questions 1 to 2 carry 2 marks each; questions 3 to 9 carry 3 marks each; questions 10 to 12 carry 5 marks each.

**8.** All questions are compulsory.

**9.** There are no negative marks.

**10.** Do all rough work in the space provided below for it. You also have blank pages at the end of the question paper to continue with rough work.

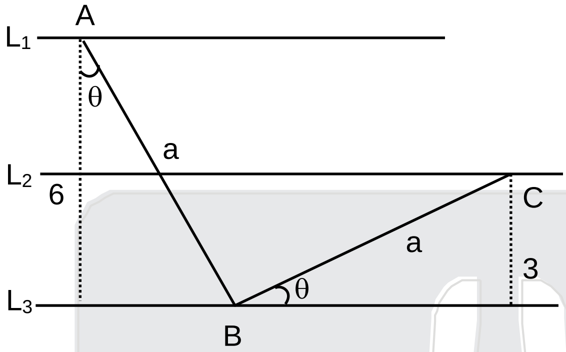
**11.** After the exam, you may take away the Candidate's copy of the OMR sheet.

**12.** Preserve your copy of OMR sheet till the end of current Olympiad season. You will need it later for verification purposes.

**13.** You may take away the question paper after the examination.

**1.** Three parallel lines $L_1$, $L_2$, $L_3$ are drawn in the plane such that the perpendicular distance between $L_1$ and $L_2$ is 3 and the perpendicular distance between $L_2$ and $L_3$ is also 3. A square ABCD is constructed such that A lies on $L_1$, B lies on $L_3$ and C lies on $L_2$. Find the area of the square

**Sol.** **45**




$\sin\theta = \frac{3}{a}$ and $\cos\theta = \frac{6}{a}$

$\frac{9}{a^2} + \frac{36}{a^2} = 1 \Rightarrow a^2 = 45$

**2.** Ria writes down the number 1, 2,……,101 in red and blue pens. The largest blue number is equal to the number of numbers written in blue and smallest red number is equal to half the number of numbers written in red. How many numbers did Ria write with red pen ?

**Ans.** **68**

**Sol.** $\underbrace{BBB.....B}_{x}\ \underbrace{\underbrace{R}_{x+1}RR....R}_{2x+1} = 101$

$x + 2(x + 1) = 101 \Rightarrow x = 33$ (blue)

Red = 68

**3.** Consider the set T of all triangles whose sides are distinct prime numbers which are also in arithmetic progression. Let $\Delta \in T$ be the triangle with the least perimeter. If $a^o$ is the largest angle of $\Delta$ and if L is its perimeter, determine the value of $\frac{a}{L}$

**Ans.** **8**

**Sol.** Side must be 3, 5, 7, L = 15

$\cos a^o = \frac{9+25-49}{2\times3\times5} = -\frac{1}{2}$

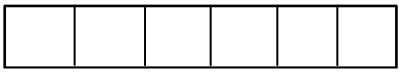
$a^o = 120^o$

$\frac{a}{L} = \frac{120}{15} = 8$ Ans.

**4.** Consider the set of all 6-digit numbers consisting of only 3 digits, a, b, c , where a, b, c are distinct. Suppose the sum of the these numbers is 593999406. What is the largest remainder when the three digit number abc is divided by 100 ?

**Ans.** **98**

**Sol**



with a, b, c distinct digit

sum of all '6' digit number = 593999406

Let a, b, c $\neq$ 0

a, b, c each comes $3^5$

sum of number = $3^5 \times (a + b + c) \times 111111$

= 243 + 111111 × (a + b + c)

= 26999973 × (a + b + c) = 593999406

a + b + c = 22

3 digit abc number sets

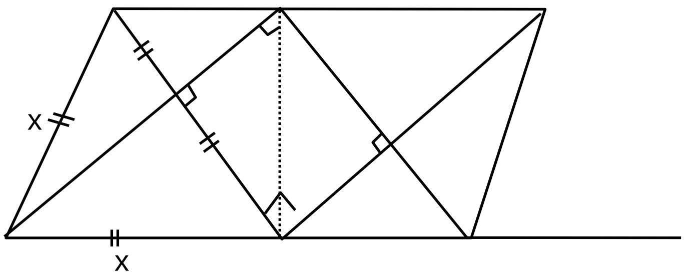
= 985, 958 ………. 598 largest remainder of $\frac{abc}{100} = \frac{598}{100}$

n = 98

**5.** In parallelogram ABCD the longer side is twice the shorter side. Let XYZW be the quadrilateral formed by the internal bisectors of the angles of ABCD. If the area of XYZW is 10, find the area of ABCD.

**Ans.** **40**

**Sol.**




As angle bisector and $\perp$ are same. The $\Delta$ is isosceles so all the eight triangle will be of equal area

Then Required Area = $\left(\frac{2}{3} \times 10\right) \frac{B}{2} \times 10 = 40$

**6.** Let x, y, z be positive real number such that $x^2 + y^2 = 49$, $y^2 + yz + z^2 = 36$ and $x^2 + \sqrt{3}xz + z^2 = 25$. If the value of $2xy + \sqrt{3}yz + zx$ can be written as $p\sqrt{q}$ where p,q are integers and q is not divisible by square of any prime number, find p + q.

**Ans.** **30**

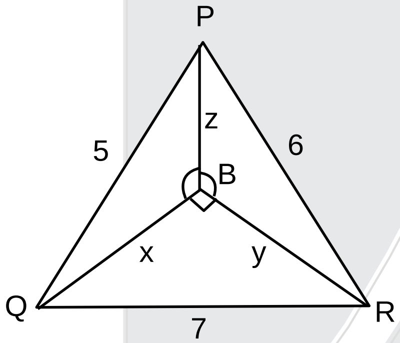
**Sol.** $x^2 + y^2 = 49$

$$\frac{x^2 + y^2 - 49}{2xy} = 0$$

cosA = 0, A = 90º

sin in cosB = $\frac{y^2 + z^2 - 36}{2yz} = -\frac{1}{2}, B = 120^o$

cosC = $\frac{z^2 + x^2 - 25}{2xz} = -\frac{\sqrt{3}}{2}, C = 150^o$




Area of PQR = $\sqrt{9 \times 4 \times 3 \times 2}$ = $6\sqrt{6}$

$2xy + \sqrt{3}yz + zx = 4$ Area of PQR

$= 4 \times 6\sqrt{6} = 24\sqrt{6}$

24 + 6 = 30

**7.** Find the number of maps f : {1, 2, 3} $\rightarrow$ {1, 2, 3,4, 5} such that $f(i) \le f(j)$ whenever i < j

**Ans.** **35**

**Sol.** $f(i) \le f(j)$ $\quad\quad$ $i < j$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad\quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

$y_1 + y_2 + y_3 + y_4 + y_5 = 3$

$^{3+4}C_4 = {}^7C_4 = 35$

**8.** For any real number t, let [t] denote the largest integer $\le t$ . Suppose that N is the greatest integer such that $\left[\sqrt{\left[\sqrt{\left[\sqrt{N}\right]}\right]}\right] = 4$.Find the sum of digit of N.

**Ans.** **24**

**Sol.** $\left[\sqrt{\left[\sqrt{N}\right]}\right] = 4$

$4 \le \sqrt{\left[\sqrt{\left[\sqrt{N}\right]}\right]} < 5$

$16 \le \left[\sqrt{\left[\sqrt{N}\right]}\right] < 25$

$16 \le \sqrt{\left[\sqrt{N}\right]} < 25$

$256 \le \left[\sqrt{N}\right] < 625$

$256 \le \sqrt{N} < 625$

$(256)^2 \le N < (625)^2$

$N_{max.} = 390624$

sum of digit = 24 Ans.

**9.** Let $P_0 = (3,1)$ and define $P_{n+1} = (x_n, y_n)$ for $n \ge 0$ by

$x_{n+1} = -\frac{3x_n - y_n}{2}, y_{n+1} = -\frac{x_n + y_n}{2}$

Find the area of the quadrilateral formed by the points $P_{96}$, $P_{97}$, $P_{98}$, $P_{99}$.

**Ans.** 8

**Sol.** $P \equiv (3,1)$

$P_1 \equiv (-4,-2)$

$P_2 \equiv (5,3)$

$P_3 \equiv (-6,-4)$

from shifting

area $= \frac{1}{2}\begin{vmatrix} 3 & 5 & -4 & -6 & 3 \\ 1 & 3 & -2 & -4 & 1 \end{vmatrix}$

$\frac{1}{2}(9 - 5 - 10 + 12 + 16 - 12 - 6 + 12)$

= 8 Ans.

**10.** Suppose that P is the polynomial of least degree with integer coefficients such that $P\left(\sqrt{7}+\sqrt{5}\right) = 2\left(\sqrt{7}-\sqrt{5}\right)$. Find P(2).

**Ans.** **40**

**Sol.** Let $\sqrt{7}+\sqrt{5} = x$

then $P(x) = \left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)$ since $2 = \left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)$

$= \left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)^2$

$= \left(\sqrt{7}+\sqrt{5}\right)\left(24-\left(\sqrt{7}+\sqrt{5}\right)^2\right)$

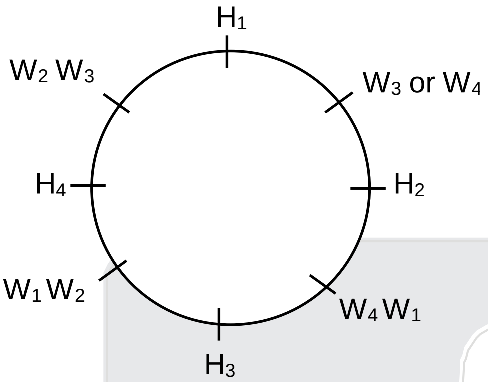
$P(x) = x(24 - x^2)$

$P(2) = 2(24 - 4)$

**11.** In how many ways can four married couples sit in a merry go round with identical seats such that men and women occupy alternate seats and no husband seats next to his wife ?

**Ans.** **12**

**Sol.**

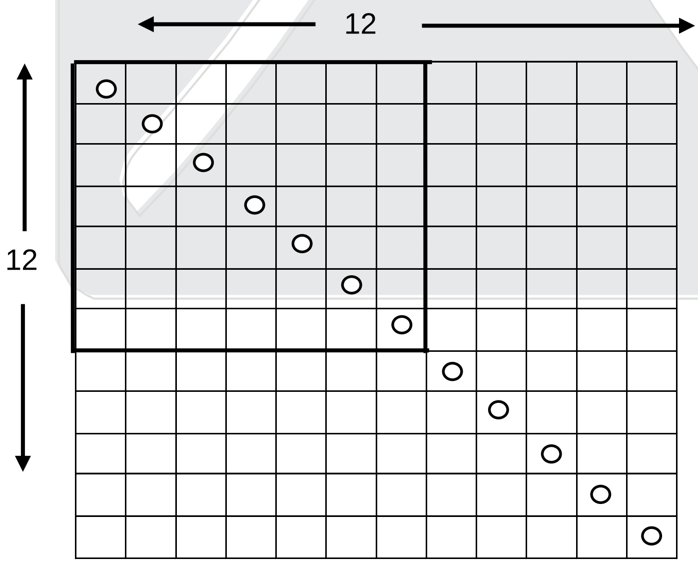



Husband ⇒ 3!
wife ⇒ 2
3! × 2 = 12

**12.** A 12 ×1 2 board is divided into 144 unit squares by drawing lines parallel to the sides. Two rooks placed on two unit squares are said to be non attacking if they are not in the same column or same row. Find the least number N such that if N rooks are placed on the unit square, one rook per square, we can always find 7 rooks such that no that no two are attacking each other.

**Ans.** **73**

**Sol.**




Clearly maximum 12 rooks
can be placed in non attacking
as each rooks will cover one row and one column
7 rooks in non-attacking position mean 7 horizontal and 7 vertical gets occupied
If we start placing rooks 12 × 6 + 1 = 73
If we place 73 rooks
Then are will always final 7 rooks in non-attacking position