SET-1

Series JBB/5 **Code No. 30/5/1**

Roll No. | | | | | | | |

Note :

(I) Please check that this question paper contains **20** printed pages.

(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **40** questions.

(IV) Please write down the Serial Number of the questions in the answer book before attempting it.

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS (STANDARD)
HINTS & SOLUTIONS

Time allowed : 3 hours **Maximum Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them.

(i) This question paper comprises **Four** Sections — **A, B, C and D** There are 40 questions in the question paper. All questions are compulsory.

(ii) **Section A —** Questions no. **1** to **20** comprises of **20** questions of **one** mark each.

(iii) **Section B —** Questions no. **21** to **26** comprises of **6** questions of **two** mark each.

(iv) **Section C —** Questions no**. 27** to **34** comprises of **8** questions of **three** mark each.

(v) **Section D —** Questions no**. 35** to **40** comprises of **6** questions of **four** mark each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark, 2 questions of two marks, 3 questions of three marks and 3 questions of four marks. You have to attempt only one of the choices in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

SECTION-A

Question numbers 1 to 20 carry 1 mark each.
Question numbers 1 to 10 are multiple choice questions.
Choose the correct option.

1. On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is

(A) $3x^2 + x - 12$ (B) $x^3 - 4x + 3$ (C) $x^2 + 3x - 4$ (D) $x^3 - 4x - 3$

Sol. Let p(x) in divided by $g(x) = x^2 - 4$ and quotient is q (x) = x and remainder is r (x) = 3
by division Algorithm
Divided = divisor × quotient + Reminder
$p(x) = g(x) \times q(x) + r(x)$
$p(x) = (x^2 - 4) \times x + 3$
$p(x) = x^3 - 4x + 3$
Option (B)

2. In figure - 1, ABC is an isosceles triangle, right angled at C. Therefore

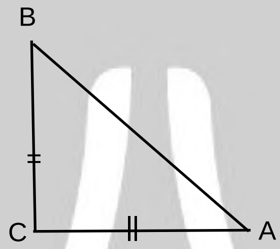


figure - 1

(A) $AB^2 = 2AC^2$ (B) $BC^2 = 2AB^2$ (C) $AC^2 = 2AB^2$ (D) $AB^2 = 4AC^2$

Sol. Given that Δ ABC is an isosceles right angled triangle with AC = BC
$\therefore$ $AB^2 = AC^2 + BC^2$ {by Pythagoras theorem}
$AB^2 = AC^2 + AC^2$ {given AC = BC}
$AB^2 = 2AC^2$
Option (A)

3. The point on the x-axis which is equidistant from (– 4, 0) and (10, 0) is

(A) (7, 0) (B) (5, 0) (C) (0, 0) (D) (3, 0)

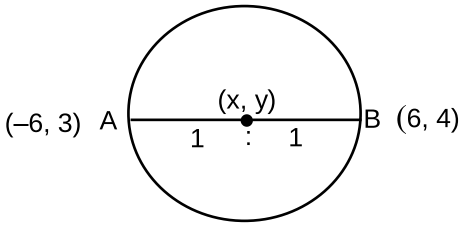
Sol. Let the point be P(x, 0) which in equidistant from point A(–4, 0) & point B (10,0)
$\therefore$ PA = PB by distance formula
$\sqrt{(x+4)^2 + (0-0)^2} = \sqrt{(x-10)^2 + (0-0)^2}$
Squaring on the both sides
$(x + 4)^2 = (x - 10)^2$
$\Rightarrow x^2 + 2x + 16 = x^2 - 20x + 100$
$22x = 84$
$x = 3$
required point is (3, 0)
Option (D)

OR

The centre of a circle whose end points of a diameter are (– 6, 3) and (6, 4) is

(A) (8, - 1) (B) (4, 7) (C) $\left(0, \frac{7}{2}\right)$ (D) $\left(4, \frac{7}{2}\right)$

Sol. Let AB in diameter and O is the centre
We know that
Centre O is the mid point of diameter so



By section formula

$x = \frac{6-6}{1+1}$ $y = \frac{4+3}{1+1}$ $\left\{x = \frac{mx_2 + nx_1}{m+n}\right\}$ $y = \left\{y = \frac{my_2 + ny_1}{m+n}\right\}$

x = 0 $y = \frac{7}{2}$

O $(0, \frac{7}{2})$

Option (C)

4. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(A) 4 (B) ± 4 (C) – 4 (D) 0

Sol. given $2x^2 + Kx + 2 = 0$ has equal roots

So D = 0 a = 2, b = k. C = 2

$b^2 - 4\,ac = 0$

$(k^2) - 4\,(2)\,(2) = 0$

$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm 4$

Option (B)

5. Which of the following is not an A.P. ?

(A) – 1.2, 0.8, 2.8,....... (B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, ...$ (D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, ...$

Sol. By checking options, option (C) is not in A.P

$\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, ...$

$d = a_2 - a_1 = \frac{7}{3} - \frac{4}{3} = \frac{3}{3} = 1$

$d = a_3 - a_2 = \frac{9}{3} - \frac{7}{3} = \frac{2}{3}$

Difference is not same so this is not an A.P.

6. The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is

(A) consistent (B) inconsistent

(C) consistent with one solution (D) consistent with many solutions.

Sol. $\frac{3x}{2} + \frac{5y}{3} - 7 = 0$ … (1)

$9x + 10y - 14 = 0$ … (2)

$a_1 = \frac{3}{2}$ $b_1 = \frac{5}{3}$ $c_1 = -7$

$a_2 = 9$ $b_2 = 10$ $c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}$ … (A)

$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{10} = \frac{1}{6}$ … (B)

$\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$ … (C)

Equation (A), (B) & (C)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So the system is inconsistent Option (B)

7. In figure - 2, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

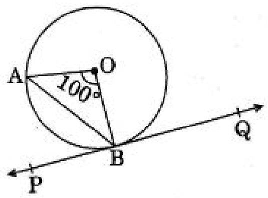


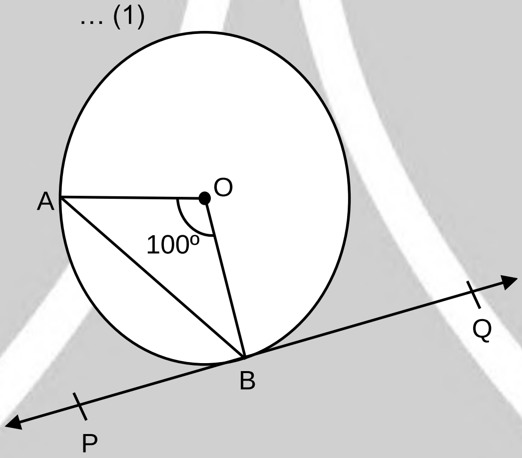
Figure-2

(A) 50° (B) 40° (C) 60° (D) 80°

Sol. In ΔAOB

OA = OB = radius

So $\angle OAB = \angle OBA$ … (1)



Now

$\angle AOB + \angle OAB + \angle OBA = 180°$ [By ASP]

$100 + 2\angle OBA = 180°$ [By equation (1)]

$2\angle OBA = 80°$

$\angle OBA = 40$ … (2)

$\Rightarrow$ $\angle OBP = 90°$ [∵A tangent to a circle is perpendicular to the radius through the point of contact]

$\angle OBA + \angle ABP = 90°$

$40 + \angle ABP = 90°$

$\angle ABP = 90 - 40$

$\angle ABP = 50°$ Option (A)

8. The radius of a sphere (in cm) whose volume is 12π cm^3, is

(A) 3 (B) $3\sqrt{3}$ (C) $3^{\frac{2}{3}}$ (D) $3^{\frac{1}{3}}$

Sol. Let the radius of sphere be r cm given that

Volume of sphere = 12π cm^3

$\Rightarrow \frac{4}{3}\pi r^3 = 12\pi$

$r^3 = 9$
$r^3 = 3^2$
$r = 3^{\frac{2}{3}}$
Option (C)

9. The distance between the points (m, – n) and (– m, n) is

(A) $\sqrt{m^2+n^2}$ (B) m + n (C) $2\sqrt{m^2+n^2}$ (D) $\sqrt{2m^2+2n^2}$

Sol. Let point P(m_1 – n) and Q(–m_1 n)

So $PQ = \sqrt{(-m-m)^2+(n+n)^2}$

$PQ = \sqrt{(-2m)^2+(2n)^2}$

$= \sqrt{4m^2+4n^2}$

$PQ = 2\sqrt{m^2+n^2}$ Option (C)

10. In figure - 3, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle$QPR = 90°, then length of PQ is

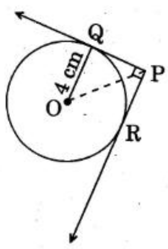


Figure-3

(A) 3 cm (B) 4 cm (C) 2 cm (D) $2\sqrt{2}$ cm

Sol. In the given figure QP and PR are the tangent so $\angle$OQP and $\angle$ORP = 90

In quadrilateral

$\angle$P = $\angle$Q = $\angle$R = 90º

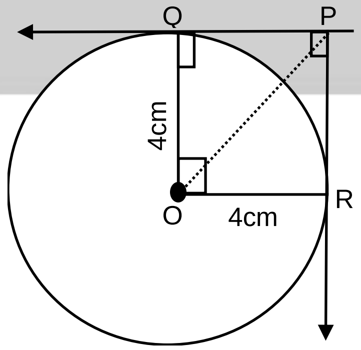
By ASP

$\angle$P + $\angle$Q + $\angle$R + $\angle$Q = 360º

90 + 90 + 90 +$\angle$O = 360º

$\angle$O = 90º

All angles of quadrilateral PQR are of 90º so it is rectangle and rectangle OPQR having adjacent sides equal to it is a square



Fill in the blanks in questions numbers 11 to 15.

11. The probability of an event that is sure to happen, is _______________

Sol. 1

12. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is ____________

Sol. $\frac{1+\tan^2 A}{1+\cot^2 A}$ $\quad\because\quad \sec^2\theta = 1 + \tan^2\theta$

$\because\quad \text{cosec}^2\theta = 1 + \cot^2\theta$

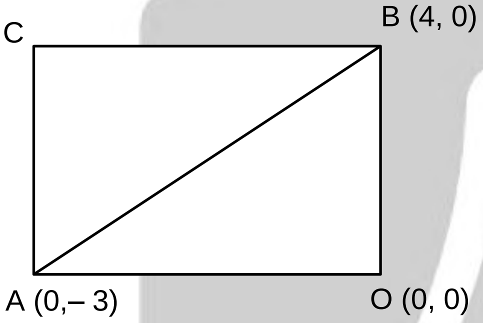
$\Rightarrow \frac{\sec^2 A}{\text{cosec}^2 A}$ $\quad\because\quad \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \frac{\sin^2 A}{\cos^2 A}$ $\quad\because\quad \text{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \tan^2 A$ $\quad\because\quad \frac{\sin\theta}{\cos\theta} = \tan\theta$

13. AOBC is a rectangle whose three vertices are A(0, – 3), O(0, 0) and B(4, 0). The length of its diagonals is ________________.

Sol.



AB = $\sqrt{(4-0)^3 + (0+3)^2}$

$\sqrt{16+9} = \sqrt{25}$

AB = 5 units

14. In the formula $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h_i, u_i$ = ___________.

Sol. $U_i = \frac{x_i - a}{h}$

15. All concentric circles are ________________ to each other.

Sol. Similar

Answer the following question numbers 16 to 20

16. Find the sum of the first 100 natural numbers.

Sol. First 100 natural numbers one

1, 2, 3, 4………………99, 100

So this should be can AP

a = 1 d = 1 ℓ = 100 n = 100

$S_{100} = \frac{m}{2}(a+\ell)$

$S_{100} = \frac{100}{2}(1+100)$

$S_{100} = 5050$

17. In figure - 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

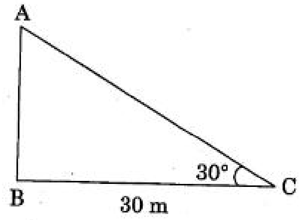


Figure-4

Sol. $\tan 30^o = \frac{AB}{30}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30} \Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

Length of lower = $10\sqrt{3}$ m

18. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

Sol. Let the other number be a.

LCM (a, b) × HCF(a, b) = a × b

182 × 13 = a × 26

$a = \frac{182 \times 13}{26}$

a = 91

19. Form a quadratic polynomial, the sum and product of whose zeroes are (– 3) and 2 respectively.

Sol. Given that sum of zeros (–3) and product of zeros is 2.

Quadratic polynomial

$P(x) = k [x^2 - (\text{Sum of zeros}) x + \text{Product of zeros}]$

$P(x) = k [x^2 - (-3) x + 2]$

$P(x) = k [x^2 + 3x + 2]$

OR

Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$?

Sol.

$$\begin{array}{l} x^2 + 3 \overline{\smash{)}\, x^4 - 3x^2 + 5x - 9 \,} \left(x^2 - 6 \right. \\ \quad\quad\;\; \underline{\underset{-}{x^4} \underset{-}{+ 3x^2}} \\ \quad\quad\quad\quad -6x^2 + 5x - 9 \\ \quad\quad\quad\quad \underline{\underset{+}{-6x^2} \quad \underset{+}{-18}} \\ \quad\quad\quad\quad\quad\quad 5x + 9 \end{array}$$

No, $x^2 - 1$ in not the remainder when divided by $x^2 + 3$.

20. Evaluate : $\frac{2\tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ}$

Sol. $\frac{2\tan 45^o \times \cos 60^o}{\sin 30^o}$

$\frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}}$ **= 2**

$\left\{ \begin{array}{l} \because \tan 45^o = 1 \\ \cos 60^o = \frac{1}{2} \\ \sin 30^o = \frac{1}{2} \end{array} \right\}$

SECTION-B

Question numbers 20 to 26 carry 2 marks each.

21. In the figure - 5, DE || AC and DF || AE.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$

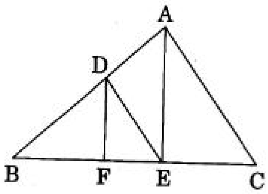
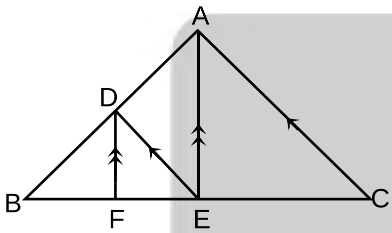


Figure-5

Sol.



In ΔBEA

DF||AE

By BPT $\frac{BF}{FE} = \frac{BD}{AD}$ ..(1)

In ΔABC

DE || AC

By BPT $\frac{BE}{EC} = \frac{BD}{AD}$ ....(2)

From (1) and (2)

$\frac{BF}{FE} = \frac{BE}{EC}$ Hence proved.

22. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

Sol. Let $5+2\sqrt{7}$ is a rational number

$\therefore\ 5+2\sqrt{7} = \frac{P}{q}$ Where P, q are

integer ,$q \neq 0$

$2\sqrt{7} = \frac{P}{q} - 5$

$\sqrt{7} = \frac{1}{2}\left[\frac{P}{q} - 5\right]$

IN LHS we have $\sqrt{7}$ which is an irrational number and in RHS we have rational number. And we know a rational number is not equal to irrational number.

$\therefore$ LHS $\neq$ RHS

So our assumption is not correct

$\therefore\ 5+2\sqrt{7}$ is irrational number

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Sol. $12^n = (2^2 \times 3)^n$

$= 2^{2n} \times 3^n$

= (2 × 2 × 2 × ---------- up to 2 n times) (3 × 3 × ----- up to n times)

to get zero at unit place we required a pair of 2 & 5. but here we not get a pair of 2 × 5

So it never ends with digit 0.

23. If A, B and C are interior angles of a ΔABC, then show that $\cos\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right)$.

Sol. m.d LHS : $\cos\left(\frac{B+C}{2}\right)$

$= \cos\left(\frac{180-A}{2}\right)$

$= \left\{\begin{matrix} \text{In}\Delta ABC \\ \angle A + \angle B + \angle C = 180 \\ \angle B + \angle C = 180 - \angle A \end{matrix}\right\}$

$= \cos\left(90 - \frac{A}{2}\right)$

$= \sin\left(\frac{A}{2}\right)$ $\{\cos(90 - \theta) = \sin\theta\}$

24. In figure - 6, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

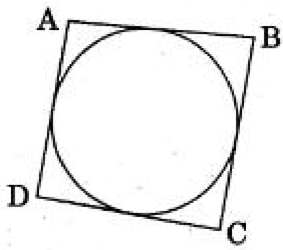
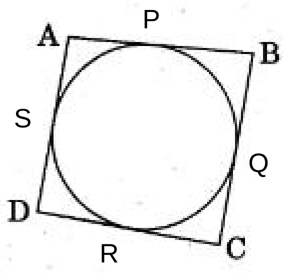


Figure-6

Sol. Sides AB, BC, CD and DA of a quadrilateral ABCD touch a circle at P, Q, R and S respectively.

To prove : AB + CD = AD + BC.



Proof :

AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS....(iv)

[Tangents drawn from an external point to a circle are equal]

Adding (1), (2), (3) and (4), we get

$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC.

OR

In figure - 7, find the perimeter of ΔABC, if AP = 12 cm.

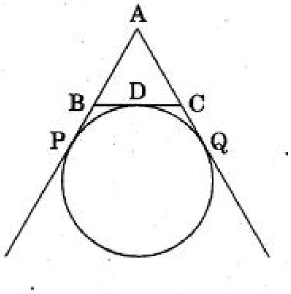


Figure-7

Sol. AP = AQ

[$\because$ Length of tangent drawn from external point are equal]

AB + BP = AC + CQ

AB + BD = AC + CD [$\because$ BP = BD, CQ = CD]

So, perimeter of ΔABC = AB + BD + DC + CA = 2AP = 2 (12) = 24 cm

25. Find the mode of the following distribution.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	4	6	7	12	5	6

Sol. Model Class = 30 – 40

$$\text{Mode} = \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{12 - 7}{2(12) - 7 - 5} \times 10$$

$$= 30 + \frac{5}{12} \times 10$$

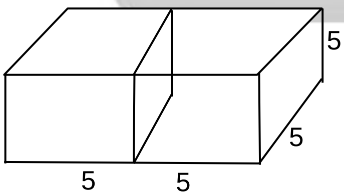
= 30 + 4.17 = 34.17

26. 2 cubes, each of volume 125 cm^3, are joined end to end. Find the surface area of the resulting cuboid.

Sol. Volume of cube = 125 cm^2

$(\text{edge})^3 = 125$

edge = 5 cm



$\therefore$ Length of cuboid = 10 cm

Breadth of cuboid = 5 cm

Height of cuboid = 5 cm

TSA of cuboid = 2 (10 × 5 + 5 × 5 + 5 × 10)

2 (50 + 25 + 50)

2 × 125

= 250 cm^2

SECTION-C

Question numbera 27 to 34 carry 3 marks each

27. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the fraction be $\frac{x}{y}$

Case-I $\frac{x-1}{y} = \frac{1}{3}$

$\Rightarrow$ $3x - 3 = y$

$\Rightarrow$ $3x - y = 3$ .... (i)

Case-II $\frac{x}{y+8} = \frac{1}{4}$

$\Rightarrow$ $4x = y + 8$

$\Rightarrow$ $4x - y = 8$ .... (ii)

By solving eq^n (i) & (ii)

$3x - y = 3$
$4x - y = 8$
$-\quad + \quad -$

$-x = -5$

$x = 5$ Put x = 5 in eq^n (i) then
$y = 12$

$\therefore$ Fraction = $\frac{x}{y} = \frac{5}{12}$

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Sol. Let Father's age = x
Son's age = y

Case-I $x = 3y + 3$

$\Rightarrow$ $x - 3y = 3$ .... (i)

Case-II $(x + 3) = 2(y + 3) + 10$

$\Rightarrow$ $x + 3 = 2y + 6 + 10$

$\Rightarrow$ $x - 2y = 13$ .... (ii)

By solving eq^n (i) & (ii)

$x - 3y = 3$
$x - 2y = 13$
$-\quad + \quad -$

$-y = -10$

$y = 10$ years Put y = 10 in eq^n (i) then

$x = 33$ years

Father's present age = x = 33 years
Son's present age = y = 10 years.

28. Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.

Sol. Let a &" b be any two positive integers and b = 3, so by applying EDL –

$a = bq' + r$; $\quad 0 \le b < r$

$a = 3q' + r$; $\quad 0 \le 3 < r$

Possible value of r = 0, 1, 2

r = 0	r = 1	r = 2
$a = 3q'$	$a = 3q' + 1$	$a = 3q' + 2$
$a^2 = (3q')^2$	$a^2 = (3q' + 1)^2$	$a^2 = (3q' + 2)^2$
$a^2 = 9q'^2$	$a^2 = 9q'^2 + 6q' + 1$	$a^2 = 9q'^2 + 12q' + 4$
$a^2 = 3(3q'^2)$	$a^2 = 3(3q'^2 + 2q') + 1$	$a^2 = 9q'^2 + 12q' + 3 + 1$
$a^2 = 3q$	$a^2 = 3q + 1$	$a^2 = 3(3q'^2 + 4q' + 1) + 1$
		$a^2 = 3q + 1$

29. Find the ratio in which the y-axis divides the line segment joining the points (6, – 4) and (–2, – 7). Also find the point of intersection.

Sol. Let the ratio be k : 1 and the point of intersection R(0, y)

3 : 1

k : 1

P ———————— Q

(6, –4) R (0, y) (–2, –7)

By section formula

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{k(-2)+(1)(6)}{k+1} = 0$$

$0 = -2k + 6$

$2k = 6$

$k = 3$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{k(-7)+(1)(-4)}{k+1} = \frac{-7k-4}{k+1}$$

Put k = 3

$$y = \frac{-21-4}{4} = \frac{-25}{4}$$

$$R\,(x, y) = \left(0, \frac{-25}{4}\right)$$

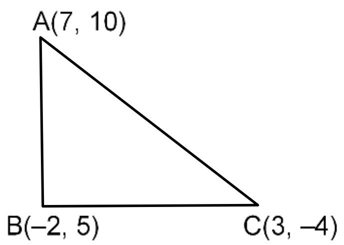
Ratio is k : 1 or 3 : 1

and point of intersection $R\left(0, \frac{-25}{4}\right)$

OR

Show that the points (7, 10), (–2, 5) and (3, – 4) are vertices of an isoceles right triangle.

Sol. In isosceles right triangle sum of square of two sides is equal to sphere of third side and two side are equal.



By distance formula

Now, $AB = \sqrt{(7-(-2))^2 + (10-5)^2}$

$= \sqrt{9^2 + 5^2}$

$= \sqrt{81+25} = \sqrt{106}$

$BC = \sqrt{(-2-3)^2 + [5-(-4)]^2}$

$= \sqrt{5^2 + 9^2}$

$= \sqrt{25+81} = \sqrt{106}$

AC = $\sqrt{(7-3)^2 + (-10-(-4))^2}$

$= \sqrt{4^2 + 14^2}$

$= \sqrt{16+196} = \sqrt{212}$

Hence, AB = BC = $\sqrt{106}$

and $AB^2 + BC^2 = (\sqrt{106})^2 + (\sqrt{106})^2$

$= 106 + 106 = 212 = AC^2$

If the sum of the squares of two sides is equal to the square of the third side then by triangle is right angled triangle.

So (7, 10), (–2, 5), (3, 4) are coordinates of isosceles right triangle.

30. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Sol. $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A

L.H.S.

$\Rightarrow \quad \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

$\Rightarrow \quad \sqrt{\frac{(1+\sin A)^2}{(1)^2 - \sin^2 A}} \quad \Rightarrow \quad \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$\Rightarrow \quad \frac{1+\sin A}{\cos A} \quad \Rightarrow \quad \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$\Rightarrow$ sec A + tan A R.H.S.

31. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3 and the nth term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P.

Sol. First term (a) = 5

common difference (d) = 3

nth term (a_n) = 50

$a_n = a + (n-1)d$

$\Rightarrow \quad 50 = 5 + (n-1)\,3$

$\Rightarrow \quad 45 = (n-1)\,3$

$\Rightarrow \quad (n-1) = 15$

$\Rightarrow \quad n = 16$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{16}{2}[2 \times 5 + (15)\,3]$

$= 8\,[10 + 45]$

$= 8 \times 55$

$S_n = 440$

32. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle$ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

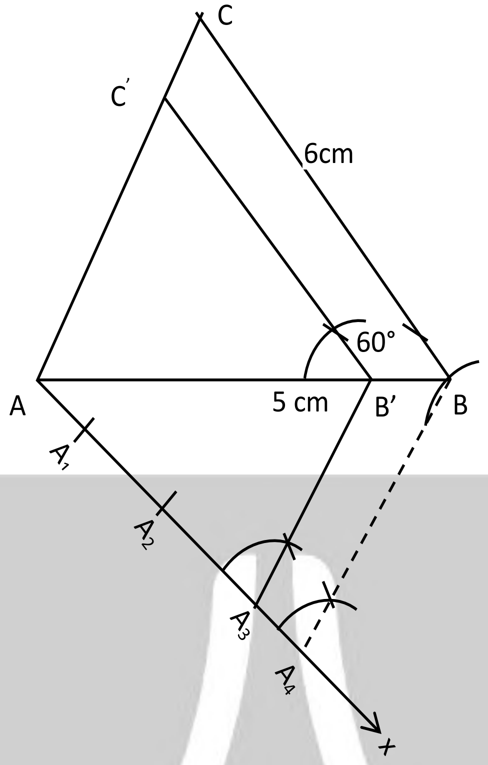
Sol. Step1. Construct a Δ ABC with given data

Step.2 draw a angle $\angle$BAX.

Step.3 put on 4 equal parts on AX such as .

$A A_1 = A_1 A_2 = A_2 A_3 = A_3 A_4$

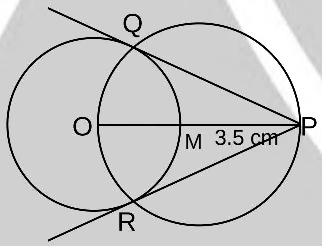
Step.4 join B to A_4, and draw line segment from A_3 such as $A_3B' \parallel A_4B$.
Step.5 draw a line segment B'C || BC thus AB'C' is required triangle



OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Sol.



Steps of contruction

(1) Take any point O of given plane as centre draw a circle of 3.5 cm radius, locate a point P 7 cm away from O join OP.
(2) Bisect OP, let M be the mid point of OP
(3) Taking M as centre and MO as raidus draw a circle
(4) Let this circle intersect the previous circle at Q and R.
(5) Join PQ and PR . PQ and PR are required tengents,

33. Read the following passage and answer the questions given at the end :

Diwali Fair

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bage are respresented in Figure - 8.

Prizes are given, when a black marbles is picked. Shweta plays the same once.

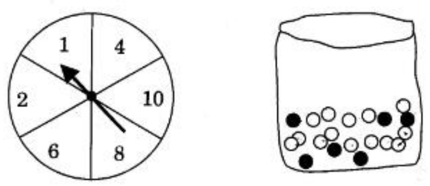


Figure-8

(i) What is the probability that she will be allowed to pick a marble from the bag ?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black ?

Sol. Total numbers = 6

(i) Favourable case = 5, {4,10,8,6,2}, P(topick marble from bag) = $\frac{5}{6}$

(ii) Favourable case = 6, Total case = 20, P(of getting prize) = $\frac{6}{20} = \frac{3}{10}$

34. In figure-9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region.

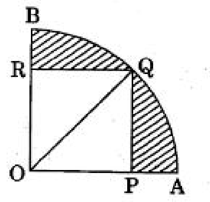
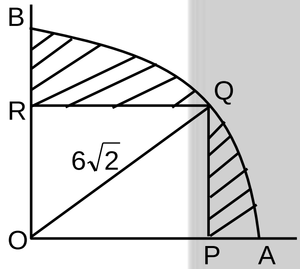


Figure-9

Sol. So area of OAQB = $\frac{1}{4}\pi r^2$



$= \frac{1}{4} \times \frac{22}{7} \times \left(6\sqrt{2}\right)^2 \qquad \left(r = 6\sqrt{2}\right)$

$= \frac{1}{4} \times \frac{22}{7} \times 36 \times 2 = 56.57 \text{ cm}^2 \qquad \ldots(i)$

OR = OP = PQ = RQ (square)

So $(OP)^2 + (PQ)^2 = \left(6\sqrt{2}\right)^2 = 36 \times 2$

$a^2 + a^2 = 72$

$2a^2 = 72$

a = 6

area of (OPQR) = $a^2 = (6)^2 = 36 \text{ cm}^2$ …(ii)

area of shaded region = area of quadrant – area of square [by equation (i) & (ii)]

= 56.57 – 36

= 20.57 cm^2

SECTION-D

Question number 35 to 40 carry 4 marks each.

35. Obtain other zeroes of the polynomial $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Sol. P(x) = 2x4 – x3 – 11x2 + 5x + 5

As $x = \sqrt{5}$ is zero, $\left(x - \sqrt{5}\right)$ will be factor, of P(x)

As $x = -\sqrt{5}$ is zero $\left(x + \sqrt{5}\right)$ will be factor , of P(x)

Therefore x2 – 5 will be factor of P(x)

$$\begin{array}{l} \quad\quad 2x^2 - x - 1 \\ x^2 - x\overline{)2x^4 - x^3 - 11x^2 + 5x + 5} \\ \quad\quad 2x^4 \quad\quad - 10x^2 \\ \quad\quad - \quad\quad\quad + \\ \hline \quad\quad -x^3 - x^2 + 5x + 5 \\ \quad\quad -x^3 \quad\quad + 5x \\ \quad\quad + \quad\quad\quad - \\ \hline \quad\quad\quad -x^2 + 5 \\ \quad\quad\quad -x^2 + 5 \\ \quad\quad\quad + \quad - \\ \hline \quad\quad\quad\quad 0 \end{array}$$

$P(x) = (x^2 - 5)(2x^2 - x - 1) + 0$
$= (x^2 - 5)(2x^2 - 2x + x - 1)$
$= (x^2 - 5)[2x(x -) + 1(x -)]$
$= (x^2 - 5)(x - 1)(2x + 1)$

Therefore other zeros will be $x = 1, x = -\frac{1}{2}$

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?

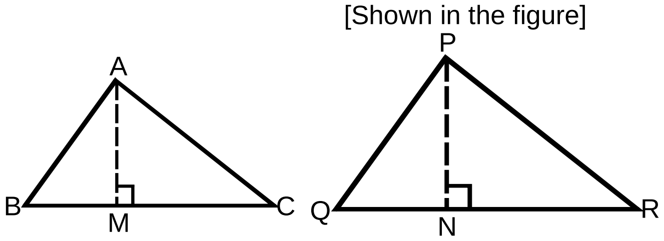
Sol.

$$\begin{array}{l} x^2 - 4x + 8 \overline{)\,2x^3 - 3x^2 + 6x + 7\,}(2x + 5 \\ \quad\quad\quad\quad 2x^3 - 8x^2 + 16x \\ \quad\quad\quad - \quad\quad + \quad - \\ \hline \quad\quad\quad\quad\quad 5x^2 - 10x + 7 \\ \quad\quad\quad\quad\quad 5x^2 - 20x + 40 \\ \quad\quad\quad\quad\quad - \quad\quad + \quad\quad - \\ \hline \quad\quad\quad\quad\quad\quad\quad 10x - 33 \end{array}$$

So $-10x + 33$ should be added to polynomial so that resulting polynomial will be divisible by $x^2 - 4x + 8$.

36. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their correspoinding sides.

Sol. Given : Two triangles ABC and PQR such that
$\Delta ABC \sim \Delta PQR$ [Shown in the figure]



To prove : $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$.

Construction : Draw altitudes AM and PN of the triangle ABC and PQR.

Proof : $ar(ABC) = \frac{1}{2}\ BC \times AM$ and $ar(PQR) = \frac{1}{2}\ QR \times PN$

So, $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} BC \times AM}{\frac{1}{2} QR \times PN} = \frac{BC \times AM}{QR \times PN}$ ... (i)

Now, in Δ ABM and Δ PQN,

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle M = \angle N$ [90º each]

So, $\Delta ABM \sim \Delta PQN$ [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ... (ii)

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ... (iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ}$ [From (i) and (ii)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (iii), we get

$$\frac{\mathbf{ar(\Delta ABC)}}{\mathbf{ar(\Delta PQR)}} = \left(\frac{\mathbf{AB}}{\mathbf{PQ}}\right)^2 = \left(\frac{\mathbf{BC}}{\mathbf{QR}}\right)^2 = \left(\frac{\mathbf{CA}}{\mathbf{RP}}\right)^2.$$

37. Sum of the areas of two squares is 544 m^2. If the diffeence of their perimeter is 32 m, find the sides of the two squares.

Sol. Let a, b are the sides of two square. Area's will be a^2 & b^2, Perimeter will be 4a, 4b

Given $a^2 + b^2 = 544$ …(i)

and $4a - 4b = 32$

$a - b = 8$ …(ii)

Put a from (ii) in equation (i)

$(8 + b)^2 + b^2 = 544$

$64 + b^2 + 16b + b^2 = 544$

$2b^2 + 16b - 480 = 0$

$b^2 + 8b - 240 = 0$

$b^2 + 20b - 12b - 240 = 0$

$b(b + 20) - 12(b + 20) = 0$

$(b + 20)(b - 12) = 0$

$b = 12$ or $b \neq -20$ as it is side

from equation (ii)

$a - 12 = 8$

$a = 20$

Sides will be 20m & 12m

OR

A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Speed of boat in still water (x) = 18 km/hr

Speed of strean = y km/hr

$$\frac{24}{18-y} - \frac{24}{18+y} = 1$$

$$\frac{1}{18-y} - \frac{1}{18+y} = \frac{1}{24}$$

$$\frac{18+y-18+y}{324-y^2} = \frac{1}{24}$$

$$\frac{2y}{24-y^2} = \frac{1}{24}$$

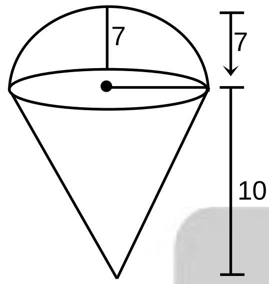
$48y = 324 - y^2$

$y^2 + 48y - 324 = 0$

$y^2 + 54y - 6y - 324 = 0$
$y(y + 54) - 6(y + 54) = 0$
$(y + 54)(y - 6) = 0$
$y = 6$ km/hr $\quad y = -54$ km/hr (Not possible)

38. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy. $\left(\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{149} = 12.2\right)$

Sol. r = 7 cm, h = 10 cm



Volume to Toy = Volume of hemisphere + Volume of cone.

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 + \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 10$

$= \frac{2156}{3} + \frac{1540}{3} = \frac{3696}{3}$

$= 1232 \text{ cm}^3$

$=$ slant height $\ell = \sqrt{10^2 + 7^2} = \sqrt{149}$

Area of colour sheet required = C.S.A of cone + C.S.A of hemisphere

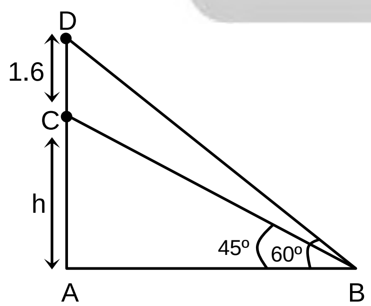
$= \pi r\ell + 2\pi r^2$

$= \frac{22}{7} \times 7 \times \sqrt{149} + 2 \times \frac{22}{7} \times 7 \times 7$

$= 22 \times 12.2 + 308 = 576.4 \text{ cm}^2$

39. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. $\left(\text{Use } \sqrt{3} = 1.73\right)$.

Sol.



Height of pedestal
AC = h + 1.6
In ΔACB

$\tan 45^\circ = \frac{AC}{AB} = 1$

$\frac{h}{AB} = 1$

h = AB … (i)

Now in ΔADB

$\frac{AD}{AB} = \tan 60° \Rightarrow \sqrt{3} = \frac{h+1.6}{h} = \sqrt{3}$ [By eqn (i)]

$\Rightarrow \sqrt{3}h - h = 1.6$

$h\left[\sqrt{3}-1\right] = 1.6$

$h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6\left[\sqrt{3}+1\right]}{2}$

$\Rightarrow 0.8\left[\sqrt{3}+1\right]$m.

$\Rightarrow$ 0.8 (1.73 +1)

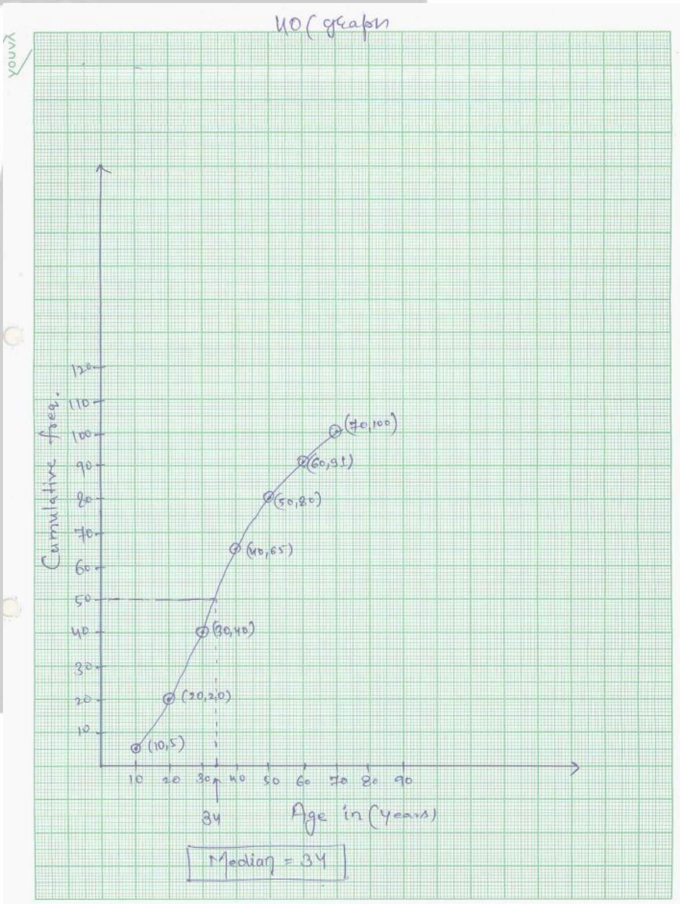
$\Rightarrow$ 0.8 (2.73) = 2.184 m Ans.

40. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (in years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of persons	5	15	20	25	15	11	9

Sol.

Age (In Years)	Less than C.F.
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	65
Less than 50	80
Less than 60	91
<u>Less than 70</u>	<u>100</u>



OR

The distribution given below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Sol.

No.of Wickets	No.of bowlers(f_i)	x_i	$f_i x_i$
20 – 60	7	40	280
60 – 100	5	80	400
100 – 140	16	120	1920
140 – 180	12	160	1920
180 – 220	2	200	400
220 – 260	3	240	720
	$\sum f_i = 45$	$\sum x_i = 840$	$\sum f_i x_i = 5640$

Mean = $(\overline{X}) = \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \dfrac{5640}{45}$ = 125.33

No.of wi	f_i	c.f.
20 – 60	7	7
60 – 100	5	12
100 – 140	16	28
140 – 180	12	40
180 – 220	2	42
220 – 260	3	45

Median $\Rightarrow$ $m = \ell + \left(\dfrac{\frac{N}{2} - c.f}{2}\right) \times h$

h = 40, $\dfrac{N}{2} = \dfrac{\sum f_i}{2} = \dfrac{45}{2}$ = 22.5

c.f. = 12
f = 16
ℓ = 100

m = 100 + $\left(\dfrac{22.5 - 12}{16}\right) \times 40$

$\Rightarrow$ 100 + $\left(\dfrac{10.5}{16}\right) \times 40$

$\Rightarrow$ 100 + 26.25
m $\Rightarrow$ 126. 25