WEST BENGAL BOARD

SUBJECT : MATHEMATICS **CLASS : X**

HINTS & SOLUTIONS

1.

(i) Let profit of friend 1 be $\frac{1}{2}x$

Profit of friend 2 be $\frac{1}{3}x$

We know that
Ratio of investment = Ratio of profit

$$\frac{\text{Investment of friend 1}}{\text{Investment of friend 2}} = \frac{\frac{1}{2}x}{\frac{1}{3}x} \Rightarrow \frac{3}{2}.$$

i.e., 3 : 2

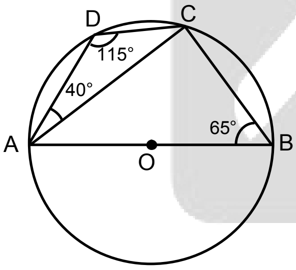
(ii) $p + q = \sqrt{13}$
squaring both sides
$(p + q)^2 = \left(\sqrt{13}\right)^2$
$p^2 + q^2 + 2pq = 13$ ....(i)

$p - q = \sqrt{5}$
Squaring both sides
$(p - q)^2 = \left(\sqrt{5}\right)^2$
$p^2 + q^2 - 2pq = 5$ ....(ii)

Subtract (ii) from (i) or solving it

$$\begin{array}{r} p^2 + q^2 + 2pq = 13 \\ p^2 + q^2 - 2pq = 5 \\ - \quad - \quad + \quad\quad - \\ \hline 4pq = 8 \\ pq = 2 \end{array}$$

(iii)



In cyclic quadrilateral
Sum of opposite angles = 180°
$\therefore \angle B + \angle D = 180°$
$65 + \angle D = 180°$
$\angle D = 180° - 65°$
$\angle D = 115°$
Angle in semi-circle = 90°
$\therefore \angle ACB = 90°$
In Δ ADC
$40 + 115 + \angle ACD = 180°$
$\angle ACD = 25°$
$\therefore \angle BCD = \angle BCA + \angle ACD = 90 + 25 \Rightarrow = 115°$

(iv) $\tan\alpha + \frac{1}{\tan\alpha} = 2$

$\tan^2\alpha + 1 = 2\tan\alpha$

$\tan^2\alpha - 2\tan\alpha + 1 = 0$

$\tan^2\alpha - \tan\alpha - \tan\alpha + 1 = 0$

$\tan\alpha(\tan\alpha - 1) - (\tan\alpha - 1) = 0$

$(\tan\alpha - 1)^2 = 0$

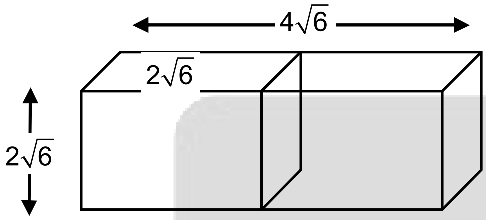
$\tan\alpha - 1 = 0$

$\tan\alpha = 1$ then $\cot\alpha = 1$

$\because \alpha = 45°$

$(1)^{13} + (1)^{13} = 2$

(v)



$\ell = 4\sqrt{6}$, $b = 2\sqrt{6}$, $h = 2\sqrt{6}$

Diagonal = $\sqrt{\ell^2 + b^2 + h^2} = \sqrt{96 + 24 + 24} = \sqrt{144} = 12$ cm

(vi) Mean = $\frac{\text{Sum}}{\text{Total}}$

$20 = \frac{x_1 + x_2 + x_3 + ... + x_{10}}{10}$

$200 = x_1 + x_2 + x_3 ... + x_{10}$

Mean = $\frac{x_1 + 4 + x_2 + 4 + x_3 + 4 ... + x_0 + 4}{10} = \frac{200 + 4 \times 10}{10} = 24$

2.

(i) $A = \left(1 + \frac{R}{100}\right)^n$

$121 = 100\left(1 + \frac{R}{100}\right)^2$

$\left(\frac{11}{10}\right)^2 = \left(1 + \frac{R}{100}\right)^2$

$\frac{11}{10} - 1 = \frac{R}{100}$

$\Rightarrow R = 10\%$

(ii) Conjucate surds

(iii) Equal

(iv) $\frac{\cos 53°}{\sin 37°} = \frac{\sin 37°}{\sin 37°} = 1$ (using $\cos\theta = \sin(90 - \theta)$)

(v) 3 surfaces

(vi) $\frac{x_{50} + x_{51}}{2}$

3.

(i) CI – SI = 0 [$\because$ for first year CI = SI]

$\therefore$ False

(ii) Compounded ratio = $\frac{ab}{c^2} \times \frac{bc}{a^2} \times \frac{ca}{b^2}$

$\Rightarrow \frac{a^2b^2c^2}{a^2b^2c^2} = \frac{1}{1}$ i.e., 1 : 1

$\therefore$ True

(iii) True

(iv) sin30° + sin60° > sin90°

$\frac{1}{2} + \frac{\sqrt{3}}{2} > 1$

$\frac{2.732}{2} > 1$

1.366 > 1 True

(v) Volume of cone : Volume of cylinder

$\frac{1}{3}\pi r^2 h : \pi r^2 h$

1 : 3 (True)

(vi) 2 , 3 , 3 , $\boxed{9}$ 9 , 9 , 10 Median = 9

False

4.

(i) SI = $\frac{P \times R \times T}{100}$

I = $\frac{P \times 5}{100} \times \frac{1}{12}$

Rs. 240

(ii) Capital of 1st men = 3x

Capital of 2nd men = 5x

Capital of 3rd men = 8x

$\because$ Ratio of capital = Ratio of profit

3x = 8x – 60

60 = 5x $\Rightarrow$ x = 12

Total profit = 3x + 5x + 8x = 16x = 16 × 12 $\Rightarrow$ Rs. 192

(iii) Let $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 3b + 4c}{p} = k$

a = 2k , b = 3k , c = 4k , 2a – 3b + 4c = pk

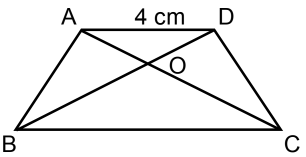
2(2k) – 3(3k) + 4(4k) = pk $\Rightarrow$ p = 11

(iv) $y^2 = kx$

$(2a)^2$ = ka on solving $\therefore y^2 = 4ax$

k = 4a

(v)



Given $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$

and $\angle$ AOD = $\angle$ BOC $\Rightarrow$ Vertically opposite angle

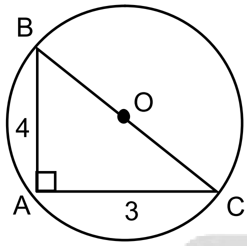
$\therefore$ Δ AOD ~ Δ COB by SAS

$\therefore \dfrac{AD}{BC} = \dfrac{1}{2}$

$\dfrac{4}{BC} = \dfrac{1}{2}$

BC = 8 cm

(vi)



$\because$ Angle formed in a semicircle is 90°

$\therefore$ BOC is diameter

In Δ ABC

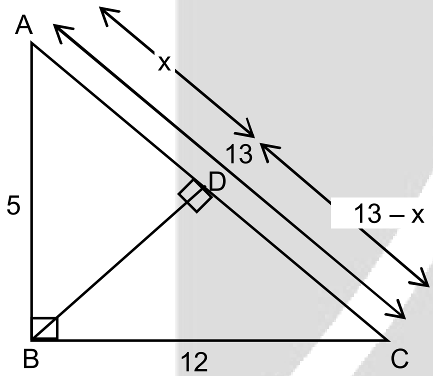
$AB^2 + AC^2 = BC^2$

$4^2 + 3^2 = BC^2$

BC = 5 cm

$\therefore$ radius = 2.5 cm

(vii)



In Δ ABC,

By pythagoras theorem

AC = 13 cm

In Δ ABD

$x^2 + y^2 = 5^2$ _____(1)

In Δ BDC

$y^2 + (13 - x)^2 = 12^2$ ____(2)

Solving (1) & (2)

$25 - x^2 + 169 + x^2 - 26x = 144$

$x = \dfrac{25}{13}$

$\left(\dfrac{25}{13}\right)^2 + y^2 = 5^2$

$y^2 = 25 - \dfrac{625}{169} = \dfrac{3600}{169}$

$y = \dfrac{60}{13}$ cm = BD

(viii) $2\sin\theta\cos\theta = \cos\theta$ $\quad 0° \le \theta \le 90°$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

(ix) $\sin 10\theta = \cos 8\theta$

$\sin 10\theta = \sin(90 - 8\theta)$

$10\theta = 90 - 8\theta$

$\theta = 5$

$\therefore \tan 9\theta = \tan 45° = 1$

(x) $a + b + c = 25$ ___(1)

$ab + bc + ca = 240.5$

longest root = $\sqrt{a^2 + b^2 + c^2}$

Now $(a + b + c)^2 = 25^2$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 625$

$a^2 + b^2 + c^2 + 2 \times 240.5 = 625$

$a^2 + b^2 + c^2 = 144$

$\therefore$ longest chord = $\sqrt{144}$ = 12 cm

(ix) C.S.A. of cone = $\sqrt{5}$ Area of base

$\pi r\ell = \sqrt{5} \times \pi r^2$

$\sqrt{r^2 + h^2} = \sqrt{5}\, r$

$r^2 + h^2 = 5r^2$

$\frac{h^2}{r^2} = \frac{4}{1}$

$\Rightarrow \frac{h}{r} = \frac{2}{1}$

h : r = 2 : 1

(xii) Median = $\frac{n+1}{2}$

$\frac{n+108}{3} = \frac{2n+2}{2}$

$n + 103 = 3n + 3$

$n = 50$

5.

(i) Compounded half-yearly

T = 2t

P = 8000 , $T = 2 \times \frac{3}{2}$ = 3 years , r = 10% = $\frac{r}{2} = \frac{10}{2}$ = 5%

$A = P\left(1 + \frac{R}{100}\right)^n$

$A = 8000\left(1 + \frac{5}{100}\right)^3$

A = Rs. 9261

C.I. = A – P = 9261 – 8000 = Rs. 1261

(ii) Investment by 1st friend = Rs. 40000 | Profit = 4x = 4/9

Investment by 2nd friend = Rs. 50000 | Profit = 5x = 5/9

$\therefore$ Profit is in the ratio = 4 : 5   Total profit = 9x

$$\frac{9x}{2}\left(\frac{4}{9}\right) = \frac{9x}{2}\left(\frac{5}{9}\right) - 800$$

$$\frac{4x}{2} - \frac{5x}{2} = -800$$

x = 1600

Total profit = 9 × 1600 = 14400

6.

(i) $x^2 + x + 1 = 0$

$\alpha + \beta = -1$

$\alpha\beta = 1$

$(\alpha + \beta)^2 = 1$

$\alpha^2 + \beta^2 + 2\alpha\beta = 1$

$\alpha^2 + \beta^2 = -1$, $\alpha^2\beta^2 = 1$

so $\therefore$ equations

$x^2 (\alpha^2 + \beta^2) x + \alpha^2 \beta^2 = 0$

$x^2 + x + 1 = 0$

(ii) Let the price of 12 pen be x

after reduction 12 pen cost = x – 6

cost of (12 + 3) 15 pen = 30 ____(1)

cost of 15 pen = $\frac{x-6}{12} \times 15$ ____(2)

from (i) & (ii)

$$30 = \frac{x-6}{12} \times 15$$

x = 30

7.

(i) $$\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-\sqrt{12}}$$

$$= \frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-3\sqrt{2}} - \frac{3\sqrt{2}}{3-2\sqrt{3}}$$

$$= \frac{4\sqrt{3}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} - \frac{30}{4\sqrt{3}-3\sqrt{2}} \times \frac{4\sqrt{3}+3\sqrt{2}}{4\sqrt{3}+2\sqrt{2}} - \frac{3\sqrt{2}}{3-2\sqrt{3}} \times \frac{3+2\sqrt{3}}{3+2\sqrt{3}}$$

$$= 4\sqrt{3} + 2\sqrt{6} - 4\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} + 2\sqrt{6}$$

$$= 4\sqrt{6}$$

(ii) $$\frac{y-x}{xy} \propto \frac{1}{x-y}$$

$$= \frac{-(x-y)^2}{xy} = k$$

$-x^2 - y^2 + 2xy = kxy$

$(2 - k)\, xy = x^2 + y^2$

$\therefore$ we can say that

$xy \propto x^2 + y^2$

8.

(i) $18x - 12y = 5x + 15y$

$13x = 27y$

$$x = \frac{27y}{13}$$

$$\frac{2\left(\frac{27y}{13}\right)+5y}{3\left(\frac{27y}{13}\right)+4y} \Rightarrow \frac{54y+65y}{81y+52y} \Rightarrow \frac{119}{133} = \frac{17}{19}$$

(ii) $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z} = k$

$b + c - a = k(y + z - x)$ ___(1)

$c + a - b = k(z + x - y)$ ___(2) Add

$a + b - c = k(x + y - z)$ ___(3)

$a + b + c = k(x + y + z)$ ___(4)

Sum from (4)

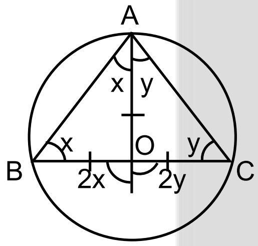
$2a = 2kx$

$\frac{a}{x} = k$ similarly $\frac{b}{y} = k$, $\frac{c}{z} = k$

$\therefore \frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

9.

(i) Semicircular angle is right angle



OA = OB = OC radius = r

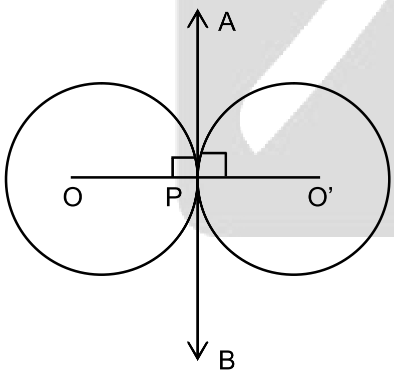
Angle opposite to equal sides are equal

$2x + 2y = 180°$

$x + y = 90°$

Hence proved

(ii)



Radius is $\perp$ to tangent

$\angle OPA = \angle O'PA = 90°$

$(\angle OPA + \angle O'PA = 180°)$

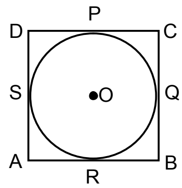
$\therefore \angle OPO' = 180°$

hence OPO' is a straight line

$\therefore$ O, P O' are collinear.

10.

(i)



SD = PD ....(i)

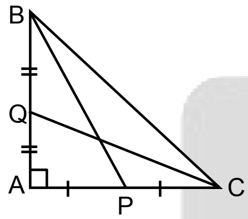
CQ = PC ....(ii)

OB = BR ....(iii)

AS = RS ....(iv)

Add AD + BC = AB + CD. Hence proved.

(ii)



$BC^2 = AB^2 + AC^2$ ....(i)

$BP^2 = AB^2 + AP^2$

$BP^2 = AB^2 + \frac{AC^2}{4}$ ....(ii)

$CQ^2 = AC^2 + AQ^2$

$CQ^2 = AC^2 + \frac{BA^2}{4}$ ....(iii)

R.H.S.

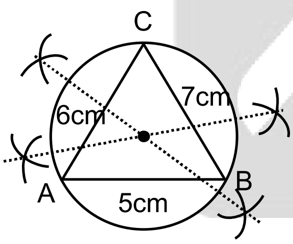
$4(BP^2 + CQ^2)$

$4\left(AB^2 + \frac{AC^2}{4} + AC^2 + \frac{BA^2}{4}\right)$

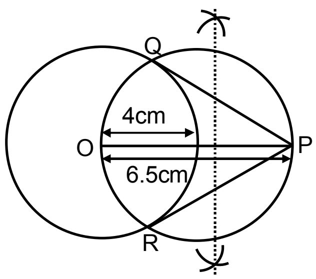
$5(AB^2 + AC^2) \Rightarrow 5BC^2$ Hence proved

11.

(i)



(ii)

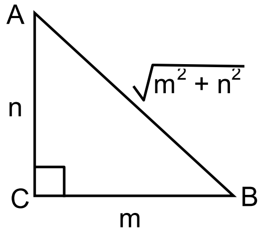


PQ and PR are required tangent.

NOTE : Steps of construction are in Q.No. 15 Part (ii).

12.

(i)



To prove

$m \sin A + n \sin B = \sqrt{m^2 + n^2}$

L.H.S.

$m\left(\frac{m}{\sqrt{m^2+n^2}}\right) + n\left(\frac{n}{\sqrt{m^2+n^2}}\right)$

$\frac{m^2+n^2}{\sqrt{m^2+n^2}} \quad \Rightarrow \quad \sqrt{m^2+n^2}$. Hence proved.

(ii) $\frac{4}{3}\cot^2 30^\circ + 3\sin^2 60^\circ - 2\operatorname{cosec}^2 60^\circ - \frac{3}{4}\tan^2 30^\circ$

$\frac{4}{3}\left(\sqrt{3}\right)^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$

$4 + \frac{9}{4} - \frac{8}{3} - \frac{1}{4} \quad \Rightarrow \quad 6 - \frac{8}{3}$

$\frac{10}{3}$.

(iii) $\angle P + \angle Q = 90^\circ$ $\quad 90 - Q = \angle P$ $\quad [\because \cos Q = \sin\ (90^\circ - Q)]$

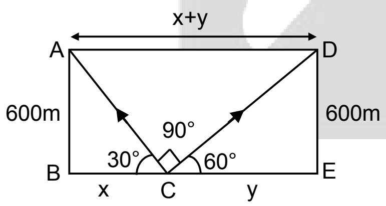
$\sqrt{\frac{\sin P}{\cos Q}} - \sin P \cos Q = \cos P$

L.H.S. $= \sqrt{\frac{\sin P}{\sin(90 - Q)}} - (\sin P)\ (\sin\ (90^\circ - Q))$

$1 - \sin^2 P = \cos^2 P$. **Ans.**

13.

(i)



$\angle ACB + \angle ACD + \angle DCE = 180^\circ$ (Lie on straight line)

$30^\circ + 90^\circ + \angle DCE = 180^\circ$

therefore $\angle DCE = 60^\circ$

In ΔABC

$\tan 30^\circ = \frac{600}{x}$

$x = 600\sqrt{3}$

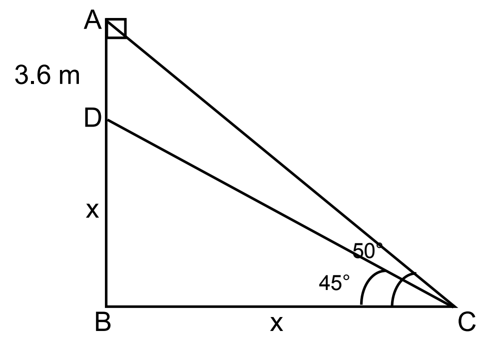
In ΔDEC

$\tan 60^\circ = \frac{600}{y}$

$y = \frac{600}{\sqrt{3}}$

$\therefore x + y = 600\sqrt{3} + \frac{600}{\sqrt{3}} = 800\sqrt{3}$ m. **Ans.**

(ii)



tan 50° = 1.2

In ΔDBC

$\frac{DB}{BC}$ = tan 45°

x = BC

In ΔABC

$\frac{3.6+x}{x}$ = tan 50

$\frac{3.6+x}{x}$ = 1.2

3.6 = 0.2x

18m = x

Height of building = xm.

14.

(i) $64 \times \pi r^2 h = \frac{2}{3} a^3$

Capacity = volume of bucket

$\pi r^2 h = \frac{2}{3} \times \frac{12}{10} \times \frac{12}{10} \times \frac{1}{64}$

$1m^3 = 1000l$

$= \frac{18}{1000}\ m^3$ or 18 l

(ii)

Let diameter be d

so $r = \frac{d}{2}$

$\text{Volume} = \pi\left(\frac{d}{2}\right)^2 \times h_1$

diameter $= \frac{d}{2}$

$r = \frac{d}{4}$

$\left(\frac{d}{2}\right)^2 \times h_1 = \pi\left(\frac{d}{4}\right)^2 \times h_2$

$4h_1 = h_2$

Increase% = $\frac{\text{Increase}}{\text{Original}} \times 100$

$= \frac{3h_1}{h_2} \times 100 = 300\%$

(iii) $\pi r \ell = 77$

$\frac{22}{7} \times r \times 7 = 77$

$r = \frac{7}{2}$

Area of the base = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} = 38.5\ m^2$

15.(I)

(i)

Class	Frequncy	x	$f_i\ x_i$
$0-20$	7	10	70
$20-40$	11	30	330
$40-60$	k	50	50k
$60-80$	9	70	630
$80-100$	13	90	1170
	$\Sigma f_i = 40+k$		$\Sigma f_i x_i = 2200+50k$

Mean = $\dfrac{\sum f_i x_i}{\sum f_i}$

$54 = \dfrac{2200+50k}{40+k}$

2160 + 54 k = 2200 + 50k

4k = 40

k = 10

(ii)

	Cf	f	
$0-10$	4	4	
$10-20$	16	12	
$20-30$	40	24	f_0
$30-40$	76	36	f_1
$40-50$	96	20	f_2
$50-60$	112	16	
$60-70$	120	8	
$70-80$	125	5	

Mode = $\ell + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\dfrac{12}{28}\right) \times 10$

$= 30 + \dfrac{30}{7} = \dfrac{240}{7} = 34.2$

(iii)

x_i Marks	f_i	$f_i x_i$
30	4	120
33	7	231
35	10	350
40	15	600
43	8	344
45	5	225
48	3	144
	$\Sigma f_i = 52$	$\Sigma f_i x_i = 2014$

Mean = $\dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{2014}{52} = 38.73$

By assumed mean method

x_i	$d_i = x_i - a$ $(a = 40)$	f_i	$f_i d_i$
30	–10	4	–40
33	–7	7	–49
35	–5	10	–50
40	0	15	0
43	3	8	24
45	5	5	25
48	8	3	24
		$\Sigma f_i = 52$	$\Sigma f_i d_i = -66$

Mean = $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ = 40 + $\left(\frac{-66}{52}\right)$ = $\frac{2014}{52}$ = 38 .73

15(II) Step of construction

(i)

(a) Construct a triangle ABC of the given sides

(b) Draw perpendicular of side AB and AC.

(c) Taking the intersection point of the perpendicular bisectors of the sides AB and AC as centre O and radius as OA draw a circle.
Therefore the circle with centre O is required circumcircle of the triangle.

(ii)

(a) Draw a circle of given radius with centre O.

(b) Take a point outside the circle of given distance from the centre and mark it as P.

(c) Draw perpendicular bisector of OP and taking it as a centre M and OM as radius draw another circle.

(d) Join P to the intersection points of the two circles Q and R and therefore PQ and QR will be the required tangent.

16.(a)

(i) Let p = 5x , q = 7x

5x + 7x = – 4

x = $\frac{-1}{3}$

$\therefore$ P = $\frac{-5}{3}$, q = $\frac{-7}{3}$

then 3p + 2q

3 × $\left(\frac{-5}{3}\right)$ + 2 × $\left(\frac{-7}{3}\right)$

$\Rightarrow \frac{-29}{3}$

(ii) P = P, SI = $\frac{3P}{5}$, R = 10%, T = ?

SI = $\frac{P \times R \times T}{100}$

$\frac{3P}{5} = \frac{P \times 10 \times T}{100}$ T = 6 years

(iii) $\frac{360}{12}$ = 30° = 1 hr rotation

(iv) y = $\frac{-1}{2+\sqrt{5}}$ $\Rightarrow 2 - \sqrt{5}$

x – y

2 + $\sqrt{5}$ – 2 + $\sqrt{5}$

2$\sqrt{5}$

(b)

(i) $x^2 + ax + 3 = 0$ root = 1

$(1)^2 + a(1) + 3 = 0$

$a = -4$

(ii) $abc = 64$ and $b^2 = ac$

$b^3 = 64$

$\therefore b = 4$

(iii) $x^2 - kx + 4 = 0$

roots are real equal

$\therefore b^2 - 4ac = 0$

$k^2 - 4 \times 4 \times 1 = 0$

$k^2 = 16 \Rightarrow k = \pm 4$

(iv) Median of 1 , 2 , 3 , 4 , $\boxed{5}$, 6 , 8 , 9 , 11

$\Rightarrow \boxed{5}$

(v) Investment by A = 600 × 9 = Rs. 5400

Investment by B = 700 × 5 = Rs. 3500

Ratio of profit : Ratio of investment

5400 : 3500 $\Rightarrow$ 54 : 35