MATHEMATICS - 2019

X-923
Maximum Marks : 100 **Time : 3 hours**

1. Choose the correct option and write it in your answer book.

(i) The H.C.F. of 96 and 404 is :
(A) 120 (B) 4 (C) 10 (D) 3

(ii) If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then the value of $\alpha \times \beta$ is
(A) $\frac{c}{a}$ (B) $\frac{a}{c}$ (C) $\frac{-c}{a}$ (D) $\frac{-a}{c}$

(iii) The zeroes of the polynomial $x^2 - 3$ will be
(A) $\pm\sqrt{3}$ (B) ± 3 (C) 3 (D) 9

(iv) When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the system of equation $a_1x + b_1y + c_1 = 0$. and $a_2x + b_2y + c_2 = 0$.
(A) has two solutions (B) has no solutions
(C) has infinitely many solutions (D) has unique solution

(v) Lines $x - 2y = 0$ and $3x + 4y - 20 = 0$ are :
(A) Intersect (B) Coincide (C) Parallel (D) None

Sol.
(i) **(B)**
HCF of 96 and 404 is using price factorization method.
Factors of $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3^1$.
Factors of $404 = 2 \times 2 \times 101 = 2^2 \times 101^1$
HCF = Product of smallest power of each common prime factor
$= 2^2 = 4$.

(ii) **(A)**
Given quadratic polynomial $ax^2 + bx + c$
zeroes = α and β.
$\therefore$ product of roots = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha . \beta = \frac{c}{a}$.

(iii) **(A)**
Given polynomial $x^2 - 3$
zeroes of $x^2 - 3$
$x^2 - 3 = 0$
$x^2 = 3$
Taking square root both sides.
$x = \pm \sqrt{3}$.

(iv) **(B)**
Given equations
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ shows equation has no solution.

(v) **(A)**

Given lines x – 2y = 0 and 3x + 4y – 20 = 0

Here $\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{-2}{4} = \frac{-1}{2}$

$\frac{c_1}{c_2} = \frac{0}{-20}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ Lines intersect at (4, 2).

2. Fill in the blanks

(i) A quadratic equation $ax^2 + bx + c = 0$ has no real root if ____.

(ii) The discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ is ____.

(iii) In the A.P. $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ ... the common difference d is ___.

(iv) The sum of the probabilities of all elementary events of an experiment is___.

(v) Formula of area of the sector of an angle θ is_____.

Sol.

(i) D < 0, where D is discriminant

$b^2 - 4ac < 0$.

(ii) Given equation $3x^2 - 2x + \frac{1}{3} = 0$

Discriminant $D = b^2 - 4ac$

$= (-2)^2 - 4(3)\left(\frac{1}{3}\right)$

$= 4 - 4$

$D = 0$.

(iii) Given AP = $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$,......

Common difference $d = a_2 - a_1 = a_3 - a_2$

$= \frac{1}{2} - \frac{3}{2}$

$= \frac{-2}{2} = -1$.

(iv) Sum of probabilities of an experiment = 1.

(v) Area of sector of angle θ = $\frac{\theta}{360^\circ} \times \pi r^2$.

3. Write true/false in the following :

(i) The perpendicular drawn from the center of a circle to a chord bisect the chord.

(ii) All squares are similar.

(iii) Area of right triangle = $\frac{1}{2}$ × base × altitude.

(iv) A line intersecting a circle in two points is called a secant.

(v) The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal, when we lower our head to look at the object.

Sol. **(i)** True
(ii) True
(iii) True
(iv) True
(v) False

4. Write the answers in one word/sentence.
(i) What will be the Arithmetic mean of 1, 2 , 3 , 4 , 5 ?
(ii) Write the formula of the median.
(iii) Find the value of probability of Event E + Probability of the EVENT " NOT E".
(iv) Write the formula of volume of a frustum of a cone.
(v) How many parallel tangents of a circle ?

Sol.
(i) Arithmetic mean of 1, 2, 3, 4, 5

Arithmetic mean = $\frac{\text{Sum of numbers}}{\text{Total numbers}} = \frac{1+2+3+4+5}{5} = \frac{15}{5}$ = 3.

(ii) Median is the value separating the higher half from lower half of a data sample.

Median formula is $\left(\frac{n+1}{2}\right)^{th}$ term

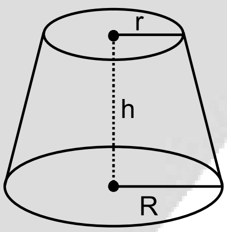
where n is number of items

If n is odd, median = $\left(\frac{n+1}{2}\right)^{th}$ term

If n is even, median = $\dfrac{\left(\frac{n}{2}\right)^{th}\text{term} + \left(\frac{n+1}{2}\right)^{th}}{2}$

(iii) P(E) + P$\left(\bar{E}\right)$ = 1

(iv) Volume of frustum of cone = $\frac{1}{3}\pi h$ (R^2 + Rr + r^2)



(v) There are always two parallel tangents about a diameter.

5. Match the correct column.

(Column "A")	(Column "B")
(i) $1+\cot^2\theta$	(a) $\sin\theta$
(ii) $\sec\theta$	(b) 0
(iii) $\sin^2\theta+\cos^2\theta$	(c) $\sqrt{3}$
(iv) $\tan 60°$	(d) 1
(v) $\cos(90-\theta)$	(e) $\operatorname{cosec}^2\theta$
	(f) $\frac{1}{\cos\theta}$
	(g) $\frac{1}{\sqrt{3}}$

Sol.

(i)	$1 + \cot^2\theta$	(e)	$\operatorname{cosec}^2\theta$
(ii)	$\sec\theta$	(f)	$\frac{1}{\cos\theta}$
(iii)	$\sin^2\theta + \cos^2\theta$	(d)	1
(iv)	$\tan 60°$	(c)	$\sqrt{3}$
(v)	$\cos(90° - \theta)$	(a)	$\sin\theta$

6. Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Sol. LCM and HCF of 6 and 20 by the prime factorisation method.

Factors of 6 and 20

$$\begin{array}{c|c} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$6 = 2 \times 3 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5 = = 2^2 \times 5^1$.

HCF = Product of smallest power of each common prime factor = 2^1.

Now we know

HCF × LCM = Product of two given numbers

$2 \times \text{LCM} = 6 \times 20$

$\text{LCM} = \frac{6 \times 20}{2} = 60$

LCM = 60

HCF = 2.

OR

Find the H.C.F. of 6 , 72 and 120 using the prime factorisation method.

Sol. HCF of 6, 72 and 120 using prime factorisation method.

Factors of 6, 72 and 120.

$$\begin{array}{c|c} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$6 = 2^1 \times 3^1$

$72 = 2^3 \times 3^2$

$120 = 2^3 \times 3^1 \times 5^1$

HCF = product of smallest power of each common prime factor

$2^1 \times 3^1 = 6$.

7. Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2.

Sol. Given sum of zeroes = – 3

product of zeroes = 3

We know quadratic polynomial

$= x^2 - (\text{sum of zeroes})\, x + (\text{product of zeroes})$

$= x^2 - (-3)x + (2)$

$= x^2 + 3x + 2$

OR

Divide $2x^2 + 3x + 1$ by $x + 2$.

Sol. Divide $2x^2 + 3x + 1$ by $x + 2$

Using long division method

$$\begin{array}{r|l} & 2x-1 \\ \hline x+2 & 2x^2+3x+1 \\ & 2x^2+4x \\ & -\quad\; - \\ \hline & \quad -x+1 \\ & \quad -x-2 \\ & \quad +\quad + \\ \hline & \qquad 3 \end{array}$$

Quotient = 2x – 1
Remainder = 3.

8. Find the distance between points (2 , 3) and (4 , 1).

Sol. Given points (2, 3) and (4, 1)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ Distance between given two points.

= $\sqrt{(4-2)^2 + (1-3)^2}$

= $\sqrt{(2)^2 + (-2)^2}$ = $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$.

OR

Find the area of a triangle whose vertices (1 , – 1) , (– 4 , 6) and (– 3 , – 5).

Sol. Given points (1, – 1), (–4, 6) and (–3, – 5)

Using area of triangle = $\left|\dfrac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{2}\right|$

= $\left|\dfrac{1\ [6-(-5)] + (-4)\ [-5-(-1)]\ \ + (-3)\,[(-1)-6]}{2}\right|$

= $\left|\dfrac{11 + (-4)\ (-4) +\ (-3)\ (-7)}{2}\right|$

= $\left|\dfrac{11 +\ 16 + 21}{2}\right|$ = $\left|\dfrac{48}{2}\right|$ = 24 sq. units.

9. Two players Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match.

Sol. Probability of Sangeeta winning the match P(S) = 0.62.

$\therefore$ Probability of Reshma winning the match

P(R) = 1 – P(S) = 1 – 0.62 = 0.38

OR

A box contains 3 blue , 2 white and 4 red marbles. If a marble is drawn at random from the box, What is the probability that it will be a (i) white ? , (ii) blue ? , (iii) red ?

Sol. Blue marbles in box = 3
White marbles in box = 2
Red marbles in box = 4
Total number of marbles = 9
$\therefore$ Probability of white marble = P(W)

P (W) = $\dfrac{\text{Favourable outcome}}{\text{Total number of outcomes}}$ = $\dfrac{2}{9}$

Probability of red marble = P(R)

P(R) = $\dfrac{4}{9}$.

Probability of blue marble = P(B)

P(B) = $\dfrac{3}{9}$ = $\dfrac{1}{3}$.

10. If P(E) = 0.05, what is the probability of (" not E") i.e., $P(\bar{E})$?

Sol. Given P(E) = 0.05

We know P(e) + $P(\bar{E})$ = 1

$\therefore$ $P(\bar{E})$ = 1 –P(E)

= 1 – 0.05

= 0.95

OR

One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will

(i) be an ace. (ii) not be an ace.

Sol. Total number of cards= 52.

Number of Ace = 4

$\therefore$ Probability of card drawn will be ace

$= \frac{\text{Favourable outcome}}{\text{Total number of outcome}}$

$= \frac{4}{52} = \frac{1}{13}$.

P(not be an ace) = 1 – P(ace)

$= 1 - \frac{1}{13} = \frac{12}{13}$.

11. Prove that : $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A

Sol. Given $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A

Solving LHS $\sqrt{\frac{1+\sin A}{1-\sin A}}$

Rationalizing the denominator

$\sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{(1)^2-(\sin A)^2}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

= secA + tanA = RHS

Hence proved.

OR

Evaluate the following :

sin60° × cos30° + sin30° × cos60°

Sol. Given

sin60° × cos30° + sin30° × cos60°

We know $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\cos 60° = \frac{1}{2}$

Substituting the values in given equation

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$

$= \frac{3}{4}+\frac{1}{4} = \frac{4}{4} = 1.$

Method - 2

We know

sin (A + B) = sinA cos B + cos A sin B

$\therefore$ sin (60° + 30°) = sin 60° cos 30° + sin 30° cos 60°

= sin (90) = 1.

12. Find the value of K, if the points A (2 , 3) , B (4 , K) and C (6 , – 3) are collinear.

Sol. Given points A(2, 3), B(4, k) and C(6, – 3) are collinear.

$\therefore$ Area of triangle is equal to zero.

$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

Substituting the values

2 [k – (–3)] + 4(– 3 – 3) + 6(3 – k) = 0

2k + 6 + (– 12) + (– 12) + 18 – 6k = 0

4k = 24 – 24 = 0

k = 0.

OR

Find the ratio in which the y-axis divides the line segment joining the points (5 , – 6) and (– 1 , – 4) also find the point of intersection.

Sol. Given points (5, – 6) and (–1, – 4)

Let y-axis divides the line segment in ratio m : n

Using section formula

$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + my_1}{m+n}\right)$

Now, $x = 0$

$\therefore \quad \frac{mx_2 + nx_1}{m+n} = 0$

$\Rightarrow \quad mx_2 + nx_1 = 0$

$m(-1) + n(5) = 0$

$m = 5n$

$\Rightarrow \quad \frac{m}{n} = \frac{5}{1}$

Now point of intersection

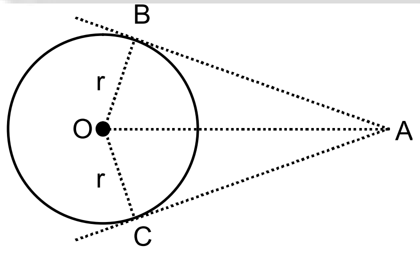
$y = \frac{my_2 + ny_1}{m+n} = \frac{5(-4)+1(-6)}{6}$

$= \frac{-20-6}{6} = \frac{-26}{6} = \frac{-13}{3} \quad \Rightarrow \quad \left(0, \frac{-13}{3}\right).$

13. The length of tangents drawn from an external point to a circle are equal.

Sol. We need to prove that the length of tangents drawn from an internal point to a circle are equal

Given circle (0, r)



AB and AC are two tangent on circle

To prove AB = AC

Proof : In triangle(s) AOB and AOC

$\angle$OBA = $\angle$OCA = 90° (Tangent is perpendicular to centre of circle)

OA = OA = {common side}

OB = OC = r [equal radius]

Using RHS congruence criterion rule

ΔAOB $\cong$ ΔAOC

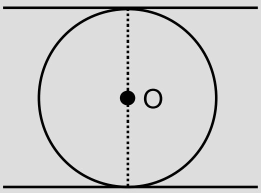
$\therefore$ AB = AC

Hence proved.

OR

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. We need to prove that the tangents drawn at the ends of a diameter of circle are parallel.



Given circle C(0, r)

AB is diameter. Two tangents PQ and RS drawn at points A and B respectively.

To prove PQ || RS

Proof : Radius will be perpendicular to these tangents

Thus OA $\perp$ PQ and OB $\perp$ RS

$\angle$OAQ = $\angle$OAP = $\angle$OBS = $\angle$OBR = 90°

Therefore

$\angle$OAQ = $\angle$OBR (Alternative interior angles)

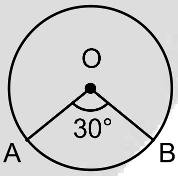
$\angle$OAP = $\angle$OBS (Alternative interior angles)

Since alternate interior angles are equal

$\therefore$ lines PQ and RS will be parallel.

14. Find the area of the sector of a circle with radius 4 cm and angle 30°. Also find the area of the corresponding major sector. (Use π = 3.14)

Sol. Given radius of circle = 4cm



Angle of sector θ = 30°

To find area of sector (minor and major)

Solution : Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{30}{360^\circ} \times (3.14)\,(4)^2$

$= \frac{1}{12} \times (3.14)\,(16)$

$= \frac{50.24}{12} = 4.187 \text{ cm}^2.$

Area of corresponding major sector

$= \left(\frac{360 - \theta}{360}\right) \times \pi r^2$

$= \frac{330}{360} \times (3.14)\,(4)^2$

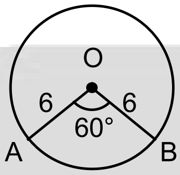
$= \frac{33}{36} \times 3.14 \times 16$

$= \frac{1657.92}{36} = 46.053\ cm^2.$

OR

Find the area of sector of a circle with radius 6 cm whose angle of sector is 60°.

Sol. Given radius of circle = 6 cm



Angle of sector $\theta = 60°$

To find area of sector of a circle

Solution : Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times pr^2$

$= \frac{1}{6} \times \frac{22}{7} \times (6)^2$

$= \frac{22 \times 6}{7} = \frac{132}{7} = 18.85\ cm^2.$

15. Prove that $5 - \sqrt{3}$ is irrational number.

Sol. Let $5 - \sqrt{3}$ is irrational number.

Hence $5 - \sqrt{3}$ can be written in form $\frac{a}{b}$

where a and b are coprime ($b \neq 0$).

Hence $5 - \sqrt{3} = \frac{a}{b}$

$- \sqrt{3} = \frac{a}{b} - 5$

$-\sqrt{3} = \frac{a-5b}{b}$

$\underbrace{\sqrt{3}}_{\text{Irrational}} = -\underbrace{\left(\frac{a-5b}{b}\right)}_{\text{Rational}}$

Here $\frac{-a+5b}{b}$ is a rational number.

But $\sqrt{3}$ is irrational

Since rational $\neq$ irrational

This is a contradiction

$\therefore$ Our assumption is incorrect.

Hence $5 - \sqrt{3}$ is irrational. Hence proved.

OR

Show that any positive odd integer is of the form 4q + 1 or 4q + 3 where q is an integer.

Sol. As per Euclid's division lenima

If a and b are two positive integers, then a = bq + r where $0 \leq r < b$

Let positive integer be a and b = 4.

Hence a = 4q + r

Where ($0 \leq r < 4$)

r is an integer greater than or equal to 0 and less than 4.

Hence r can be either 0, 1, 2, or 3.
If r = 1
a = 4q + 1
This will always be an odd integer.
If r = 3
a = 4q + 3
This will always be an odd integer.
Therefore any odd integer is of the form 4q + 1 or 4q + 3
Hence proved.

16. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

Sol. Given quadratic polynomial $x^2 + 7x + 10$
Using factorisation (splitting the middle term)
$x^2 + 7x + 10 = 0$
$x^2 + (5x + 2x) + 10 = 0$
$x^2 + 5x + 2x + 10 = 0$
$x(x + 5) + 2(x + 5) = 0$
$(x + 5)(x + 2) = 0$
Hence zeroes of given quadratic polynomial are – 5 and – 2.
Now we know

Sum of roots = $-\dfrac{(\text{coefficient of } x)}{\text{Coefficient of } x^2} = \dfrac{-b}{a}$

Product of roots = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a}$

Sum of zoroes = $(-5) + (-2) = -7$ ....(i)
Product of zeroes = $(-5) \times (-2) = 10$ ....(ii)
Given polynomial $x^2 + 7x + 10$

Here $\dfrac{-b}{a} = \dfrac{-7}{1}$ ....(iii)

and $\dfrac{c}{a} = 10$ ....(iv)

Hence from equation (i) and (iii) and equation (ii) and (iv)

Sum of zeroes = $\dfrac{-b}{a} = -7$

Product of zeroes = $\dfrac{c}{a} = 10$

Hence verified.

OR

Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

Sol. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$
Using long Division Method

$$
\begin{array}{r|l}
 & 3x - 5 \\
x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\
 & 3x^3 + 6x^2 + 3x \\
 & - \quad - \quad - \\
\hline
 & -5x^2 - x + 5 \\
 & -5x^2 - 10x - 5 \\
 & + \quad + \quad + \\
\hline
 & 9x + 10 \\
\hline
\end{array}
$$

Quotient = $3x - 5$
Remainder = $9x + 10$.

17. If the sum of the first 14 terms of an A. P. is 1050 and its first term is 10, find the 20^{th} term.

Sol. Given sum of first 14 terms $S_{14} = 1050$

$a = 10$

To find a_{20} (20^{th} term)

Solution $S_{14} = 1050$

$$\frac{14}{2}[2a + (14 - 1)d] = 1050$$

$7 [2(10) + 13d] = 1050$

$7 [20 + 13d] = 1050$

$$20 + 13d = \frac{1050}{7} = 150.$$

$13d = 150 - 20 = 130$

$$d = \frac{130}{13} = 10$$

Now $a_{20} = a + (20 - 1) d$

$= 10 + 19 (10)$

$= 10 + 190$

$= 200$

OR

Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and 16^{th} term is 73.

Sol. Given 11^{th} term $a_{11} = 38$

16^{th} term $a_{16} = 73$

To find a_{31}

Solution :

$a_{11} = 38$

$a + (11 - 1) d = 38$

$a + 10d = 38$ ....(i)

and $a_{16} = 73$

$a + (16 - 1) d = 73$

$a + 15d = 73$ ....(ii)

Solving equation (i) and (ii)

$a + 10d = 38$

$a + 15d = 73$

$-\quad -\quad -$

$-5d = -35$

$$d = \frac{-35}{-5} = \frac{35}{5} = 7$$

Now

$a + 10 d = 38$

$a = 38 - 10d$

$= 38 - 10 (7)$

$= 38 - 70 = -32$

Now

$a_{31} = a + (31 - 1) d$

$= a + 30d$

$= -32 + 210 = 178.$

18. From a point P on the ground the angle of elevation of the top of a 10 meter tall building is 30°. A flag is hosted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (You may take $\sqrt{3} = 1.732$)

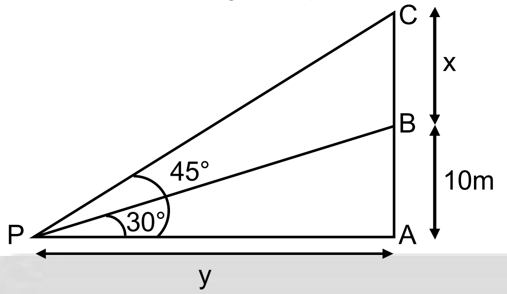
Sol. Given : Length of building = 10

Angle of elevation of top of building from point P on ground = 30°

Angle of elevation of top of flagstaff from point P on ground = 45°

To find : Length of flagstaff

Distance of building from point P.



Solution :

In $\triangle$APB

$\tan 30° = \frac{AB}{AP}$

$\tan 30° = \frac{10}{y}$

$y = \frac{10}{\tan 30°} = \frac{10}{\frac{1}{\sqrt{3}}} = 10\sqrt{3} = 10 \times 1.732 = 17.32$ m.

In $\triangle$APC

$\tan P = \frac{AC}{AP}$

$\tan 45° = \frac{AC}{AP}$

$1 = \frac{AC}{AP}$

AP = AC

y = 10 + x

17.32 = 10 + x

x = 7.32 m

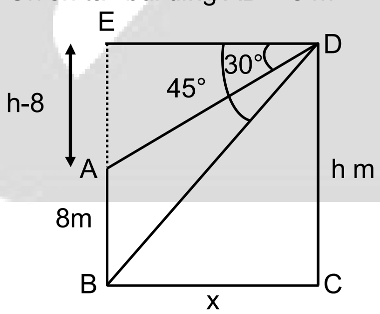
Length of flagstaff = 7.32 m

Distance of building from point P = 17.32 m.

OR

The angle of depression of the top and the bottom of an 8m. tall building from the top of a multi-stoyered building are 30° and 45° respectively. Find the height of the multi-stoyered building and distance between the two buildings.

Sol. Given tall building AB = 8 m



$\angle$EDA = 30°

$\angle$EDB = 45°

To find length of multi strong building = hm

distance between two building xm.

Solution : In ΔEDA tan 30° = $\frac{EA}{ED}$

tan 30° = $\frac{h-8}{x}$

$\frac{1}{\sqrt{3}} = \frac{h-8}{x}$

$x = \sqrt{3}(h-8)$ ....(i)

In ΔEBD tan 45° = $\frac{EB}{ED}$

$1 = \frac{h}{x}$

h = x ....(ii)

Solving (i) and (ii)

$x = \sqrt{3}(x-8)$

$x = x\sqrt{3} - 8\sqrt{3}$

$8\sqrt{3} = x(\sqrt{3}-1)$

$x = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{2} = 4\sqrt{3}(\sqrt{3}+1)$ m

$h = 4\sqrt{3}(\sqrt{3}+1)$ m.

19. Use Elimination method to find all possible solutions of the following pair of linear equations :

2x + 3y = 8

4x + 6y = 7

Sol. Given equations 2x + 3y = 8 ....(i)

4x + 6y = 7 ....(ii)

Using elimination method

Multiplying the (i) equation by 2

(2x + 3y = 8) × 2

(4x + 6y = 7) × 1

$\therefore$ 4x + 6y = 16

and 4x + 6y = 7

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ the given system of equations have no solution.

OR

The cost of 5 oranges and 3 apples is Rs. 35 and the cost of 2 oranges and 4 apples is Rs. 28. Let us find the cost of an orange and an apple.

Sol. **Given :** Cost of 5 orange and 3 apple = Rs. 35

Cost of 2 orange and 4 apple = Rs. 28

To find cost of an orange and an apple

Solution :

Let cost of an orange be Rs. x

and cost of an apple be Rs. y

$\therefore$ A/C to problem

[5x + 3y = 35] ....(i)

[2x + 4y = 28] ....(ii)

Using elimination Method

Multiplying equation (i) by 2 and equation (ii) by 5.

2 × [5x + 3y = 35]

10x + 6y = 70 ....(iii)

$5 \times [2x + 4y = 28]$
$10x + 20y = 140$ ....(iv)
Solving (iii) and (iv)

$$\begin{array}{r} 10x + 6y = 70 \\ 10x + 20y = 140 \\ - \quad - \quad - \\ \hline -14y = -70 \\ \hline \end{array}$$

$y = \frac{70}{14} = 5$

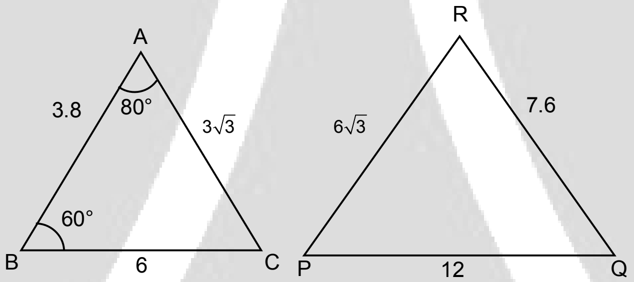
substitute in equation (i)

$x = \frac{35-3y}{5} = \frac{35-15}{5}$

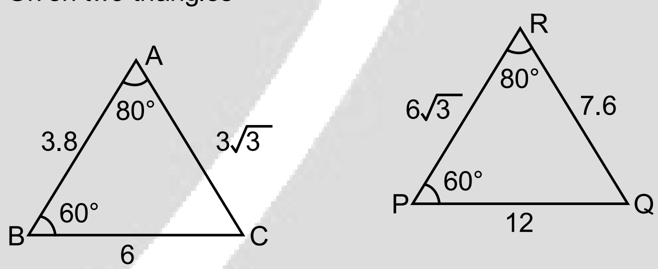
$= \frac{20}{5} = 4.$

Cost of an orange = Rs. 4
Cost of an apple = Rs. 5.

20. Observe in figure , find $\angle$ P.



20. Given two triangles



To find $\angle$P

Solution :

In ΔABC and ΔPQR

$\frac{AB}{QR} = \frac{AC}{RP} = \frac{BC}{PQ} = \frac{1}{2}$ (each).

If in two triangles, ratio of sides are equal then triangles are similar.

$\therefore$ $\Delta ABC \sim \Delta RQP$

$\Rightarrow$ $\angle A = \angle R = 80°$

$\angle B = \angle Q = 60°$

$\angle C = \angle P = 180° - (80° + 60°)$

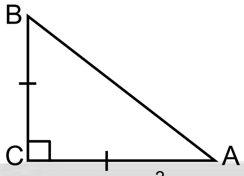
$= 180° - 140° = 40°.$

OR

ABC is an Isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Sol. Given ABC is an isosceles triangle right angled at C.

AC = BC



To prove $AB^2 = 2AC^2$

Proof :

In $\triangle ABC$

Using Pythagoras theorem

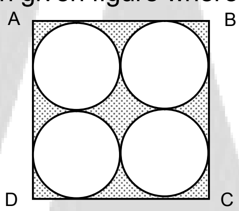
$(AB)^2 = (AC)^2 + (BC)^2$

$(AB)^2 = (AC)^2 + (AC)^2$ [$\because$ AC = BC]

$(AB)^2 = 2(AC)^2$

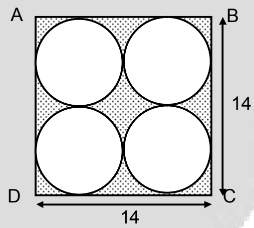
Hence proved.

21. Find the area of the shaded region in given figure where ABCD is a square of side 14 cm.



Sol. **Given :** ABCD is a square AB = 14 cm.

To find : Area of shaded region



Solution : In order to find area of shaded region = Area of square – (area of 4 circles)

Now area of square = 14 × 14 = 196 cm^2

Area of 4 circles = 4 × area of 1 circle

$$= 4 \times \pi \times \left(\frac{7}{2}\right)^2 \quad [\text{as } d = 7]$$

$$= 4 \times \frac{22}{7} \times \frac{49}{4}$$

$= 154\ cm^2$

$\therefore$ Area of shaded region = 196 – 154 cm^2 = 42 cm^2.

OR

In a circle of radius 21 cm an arc subtends an angle of 60° at the centre find :

(i) the length of the arc. **(ii)** area of the sector.

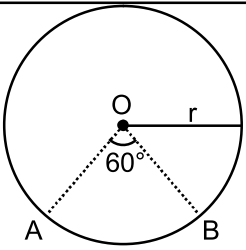
Sol. **Given :** C(0, r), r = 21 cm

$\overset{\frown}{AB}$ is arc

$\angle AOB = 60°$

To find : length of arc AB and area of sector AB

area of sector AB



Solution : Length of arc $\widehat{AB} = \frac{\theta}{360} \times 2\pi r$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 = 22 \text{ cm.}$$

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = 231 \text{ cm}^2.$$

22. Find the roots of the following equation :

$x + \frac{1}{x} = 3 , x \neq 0$

Sol. Given equation $x + \frac{1}{x} = 3 \qquad x \neq 0$

$$\frac{x^2+1}{x} = 3$$

$x^2 + 1 = 3x$

$x^2 - 3x + 1 = 0$

$D = b^2 - 4ac$

$= 9 - 4(1)(1)$

$D = 5$

$D > 0$

$\therefore$ Real and distinct roots

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-3) \pm \sqrt{5}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$

Hence $x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$

OR

Find two consecutive odd positive integers, sum of whose squares is 290.

Sol. Let two consecutive odd positive integers be x and x + 2.

Now, A/c to problem

$(x)^2 + (x + 2)^2 = 290$

$x^2 + x^2 + 4 + 4x = 290$

$2x^2 + 4x + 4 - 290 = 0$

Divide the equation by 2

$\frac{2x^2}{2} + \frac{4x}{2} - \frac{286}{2} = 0$

$x^2 + 2x - 143 = 0$

Using quadratic formula

$D = b^2 - 4ac$

$= (2)^2 - 4(1)(-143)$

$= 4 + (4 \times 143)$

$D = 576$

$\therefore \qquad D = \frac{-b \pm \sqrt{D}}{2a}$

$= \dfrac{-2 \pm \sqrt{576}}{2(1)}$

$= \dfrac{-2 \pm 24}{2}$

$\therefore \quad x = \dfrac{22}{2} = 11 \quad$ or $x = \dfrac{-26}{2} = -13$

x is a positive integer

$\therefore \quad x = 11$, 11 and 13.

23. If $\sin A = \dfrac{3}{4}$, calculate cos A and tan A.

Sol. **Given :** $\sin A = \dfrac{3}{4}$

M – 1 : $\therefore \sin^2 A = \dfrac{9}{16}$

Using algebraic identities

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

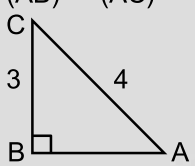
$= 1 - \dfrac{9}{16} = \dfrac{7}{16}$

$\therefore \quad \cos A = \dfrac{\sqrt{7}}{4}$

Now $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \dfrac{3}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{3\sqrt{7}}{7}$.

M – 2 :

$(AB)^2 = (AC)^2 - (BC)^2 = 16 - 9$



$(AB)^2 = 7$

$AB = \sqrt{7}$

Now $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{7}}{4}$

$\tan A = \dfrac{BC}{AB} = \dfrac{3}{\sqrt{7}}$

OR

Evaluate the following :

$2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Sol. $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

We know

$\tan 45° = 1$

$\cos 30° = \dfrac{\sqrt{3}}{2}$

$\sin 60° = \dfrac{\sqrt{3}}{2}$

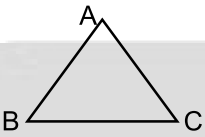
Substitute in the given equation

$2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2 = 2.$

24. Construct a triangle similar to a given triangle ABC with its side equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC.

Sol. Given ΔABC

Scale factor = $\frac{5}{3} > 1$



Steps of construction :

1. Draw any ray BX making an acute angle with BC, on the side opposite to the vertex A.

2. Mark 5 (the greater of 5 and 3 in $\frac{5}{3}$) points B_1, B_2, B_3, B_4, B_5 on BX so that $BB_1 = B_1B_2 = B_3B_4 = B_4B_5$.

3. Join B_3C and draw a line through B_5 parallel to B_3C, to intersect BC extended at C′.

4. Draw a line through C′ parallel to the line CA to intersect BA extended at A′.

Thus ΔA′B′C′ is the required triangle.

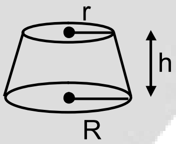
OR

Draw a circle with help of bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

Sol.
1. Draw a circle with the help of a bangle.
2. Draw a secant ARS from an external point A, produce RA to C such that AR = AC.
3. With CS as diameter, draw a semi-circle.
4. At the point A, draw AB $\perp$ AS, cutting the semi circle at B.
5. With A as centre and AB as radius, draw an arc to intersect the given circle, in T and T′.
6. Join AT and AT′
AT and AT′ are the required tangent lines.

25. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm respectively, find the volume.

Sol. Radii of ends of frustum of cone are R = 28 cm , r = 7 cm, h = 45 cm



To find : Volume

Solution :

Volume of frustum = $\frac{1}{3}\pi h\,(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 45\,[(28)^2 + (7)^2 + (28)\,7]$

$= \frac{22 \times 15}{7}\,[784 + 49 + 196]$

$= \frac{22 \times 15 \times 1029}{7}$

$= 22 \times 15 \times 147$

$= 48{,}510\ \text{cm}^2$.

OR

A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is 3 m in diameter ? (Take $\pi = \frac{22}{7}$)

Sol. Given hemispherical tank $r = \frac{3}{2}$ m

Rate = $3\frac{4}{7}$ litres

To find : Time taken to empty the tank.

Solution : Tank is in form of hemisphere with D = 3 m

$r = \frac{3}{2}$ m

Volume of tank = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27}{8} = \frac{99}{14}\ m^3$$

$$= \frac{99}{14} \times 1000 \text{ litres}$$

$$= \frac{99000}{14} \qquad (1m^3 = 1000 \text{ litre})$$

Volume of water to be emptied = $\frac{1}{2}$ × Volume of tank

$$= \frac{1}{2} \times \frac{99000}{14} \text{ litres}$$

$$= \frac{99000}{28} \text{ litres}$$

Now it is given that tank is emptied at $3\frac{4}{7}$ litre per second = $\frac{25}{7}$ litres per second

Time taken to empty $\frac{25}{7}$ litre = 1 second.

Time taken to empty 1 litre = 1 × $\frac{7}{25}$ second.

Time taken to empty $\frac{99000}{28}$ litre

$$= \frac{7}{25} \times \frac{99000}{28}$$

$$= \frac{693000}{700} = 990 \text{ second} = 16.5 \text{ minutes.}$$

26. A survey conduct on 20 households in a locality by a group of students resulted in the following frequency table for the number of family member in a household.

Family Size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Number of Families	7	8	2	2	1

Find the Mode of this data

Sol. Given

Family size	**Number of families**	
1 – 3	7	f_0
3 – 5	8	f_1
5 – 7	2	f_2
7 – 9	2	
9 – 11	1	

We know

Mode = $\ell = \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Modal class = 3 – 5

ℓ = lower limit of modal class = 3

h = class interval = 3 – 1 = 2

$f_1 = 8$

$f_0 = 7$

$f_2 = 2$

Mode = $3 + \frac{8-7}{2(8)-7-2} \times 2$

$= 3 + \frac{1}{16-9} \times 2$

$= 3 + \frac{1}{7} \times 2$

$= 3 + \frac{2}{7}$

= 3.286.

OR

In the given data

Class Interval	No. of Students
10 – 25	2
25 – 40	3
40 – 55	7
55 – 70	6
70 – 85	6
85 – 100	6

Find the Arithmetic Mean of this data

Sol.

Class Interval	Mid value x_i	No. of students f_i	$f_i x_i$
10 – 25	17.5	2	35
25 – 40	32.5	3	97.5
40 – 55	47.5	7	332.5
55 – 70	62.5	6	375
70 – 85	77.5	6	465
85 – 100	92.5	6	555
		$\Sigma f_i = 30$	$\Sigma f_i x_i = 1860$

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1860}{30}$

$\bar{x} = 62$