ICSE EXAMINATION-2019
SUBJECT : MATHEMATICS

CODE NO. T19/511 **CLASS : X**

HINTS & SOLUTIONS

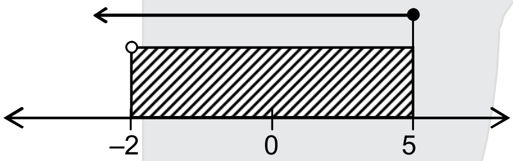
SECTION – A (40 MARKS)

1.

(a) $11x - 4 < 15x + 4$ $\qquad$ $15x + 4 \le 13x + 14$

$4x > -8$ $\qquad$ $2x \le 10$

$x > -2$ $\qquad$ $x \le 5$



$x \in (2, 5]$ and $x \in \omega$ $\Rightarrow$ $x = -1, 0, 1, 2, 3, 4, 5$

(b)

(i) Cost of 1 share = $100 \times \frac{90}{100}$ = Rs. 90

Number of share he purchase = $\frac{4500}{90}$ = 500 shares.

(ii) Profit on 1 share = $100 \times \frac{7.5}{100}$ = Rs. 7.5

Total income = 50 × 7.5 = Rs. 375.

(c)

(i)

x_i	f_i	c.f.
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40

N = 40

$\frac{N}{2} = 20$

Median = 6.

(ii) Highest frequency is 10 so mode will be 6.

2. **By factor theorem**

(a) At x = 2, f(2) = 0

$f(x) = x^3 + x^2 - 4x - 4$

$f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 0$

$f(x) = (x - 2)(x^2 + 3x + 2)$

$f(x) = (x - 2)(x^2 + 2x + x + 2)$

$f(x) = (x - 2)(x + 2)(x + 1)$.

(b) LHS : $(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\cot\theta} - \cot\theta\right)$$

$$\frac{(1-\sin^2\theta)}{\sin\theta} \cdot \frac{(1-\cos^2\theta)}{\cos\theta} \cdot \frac{(1-\cot^2\theta)}{\cot\theta}$$

$$\frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\text{cosec}^2\theta}{\cot\theta}$$

$$\sin\theta \cdot \cos\theta \cdot \frac{1}{\sin^2\theta} \cdot \sin\theta = 1 = \text{R.H.S. H.P.}$$

(c) a + 3d = 8, a → first term

a + 5d = 14 d → common difference

2d = 6 ⇒ d = 3.

a = 8 – 3 × 3 = 8 – 9 = – 1.

a = – 1

$$s_{20} = \frac{n}{2}\left[20 + (n-1)\ d\right] = \frac{20}{2}\left[2(-1) + (20-1)\ 3\right]$$

$S_{20} = 10\,[-2 + 57]$

$S_{20} = 10(55) = 550$.

3.

(a) $$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix} + \cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

⇒ $\sin A\,[\sin^2 A + \cos^2 A] + \cos A\,[\cos^2 A + \sin^2 A]$

⇒ $\sin A\,(1) + \cos A\,(1) = \sin A + \cos A$.

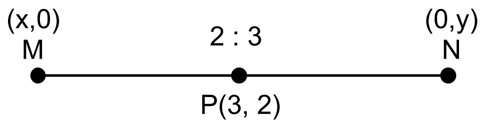
(b) **(i)** Let M(x, 0) and N(0, y)

By section formula

$$\therefore \quad x = \frac{mx_2 + nx_1}{m+n}.$$

$$3 = \frac{3(x) + 2(0)}{3+2}$$

$$\frac{3x}{5} = 3 \quad \Rightarrow \quad x = 5.$$



$$y = \frac{my_2 + ny_1}{m+n}$$

$$2 = \frac{3(0) + 2(y)}{3+2}$$

2y = 10 ⇒ y = 5.

So, point M(5, 0) and N(0, 5)

(ii) Slope of line MN = $\frac{5-0}{0-5}$ = – 1

(c) (Volume of remaining solid) = (Total volume of solid) – (volume of cone) – (Volume of hemisphere)

Total volume of solid = $\pi r^2 h = \frac{22}{7} \times 3 \times 3 \times 7 = 198$ cm^3.

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{198}{7}$ cm^3.

Volume of hemisphere = $\frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{396}{7}$ cm^3.

(Remaining Volume) = $198 - \left(\frac{198}{7} + \frac{396}{7}\right)$.

$\Rightarrow \quad 198 - \frac{594}{7} \quad \Rightarrow \quad \frac{1386 - 594}{7} = \frac{792}{7} = 113.1428$ cm^3 $\simeq$ 113 cm^3.

4.

(a) Product of extremes = Product of means

$(k + 3)(2k - 3) = (k - 12)(3k - 7)$

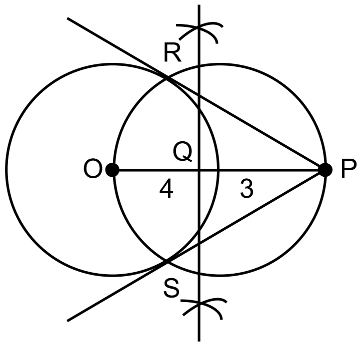
$k^2 - 46k + 93 = 0$

(b) $x^2 - 4x - 8 = 0$

$x = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

(+)	$x = 2 + 2\sqrt{3}$	(–)	$x = 2 - 2\sqrt{3}$
	$x = 2 + 2 \times 1.732$		$x = 2 - 2 \times 1.732$
	$x = 2 + 3.464$		$x = 2 - 3.464$
	$x = 5.464$		$x = -1.464$.

(c)



Steps :

(1) Draw a circle of 4 cm radius.
(2) Draw a point outside the circle at a distance of 7 cm from the centre.
(3) Join O to P.
(4) Make a perpendicular bisector of OP which cuts OP at Q.
(5) Taking Q as centre and QR as a radius draw a circle.
(6) It cuts the circle at two points R and S respectively.
(7) Join R to P and S to P.
(8) So PR and PS are the required tangnets.

5.

(a) **(i)** P(disc is odd) = $\frac{13}{25}$

(ii) P(divisible by both 2 and 3) = $\frac{4}{25}$

(iii) P(lies than 15) = $\frac{15}{25} = \frac{3}{5}$.

(b) P for 1 month = x $\frac{(n(n+1))}{2}$

$= x\left(\frac{20\times21}{2}\right) = 210\,x.$

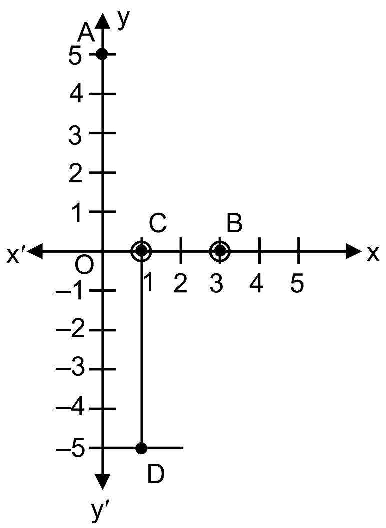
$I = \frac{210x\times1\times9}{12\times100} = \frac{63x}{4}$

$\frac{63x}{4} = 441 \Rightarrow x = \text{Rs. } 28$

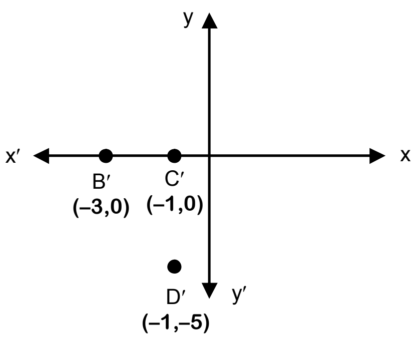
$\Rightarrow$ Monthly installment = Rs. 28.

(c)

(i)

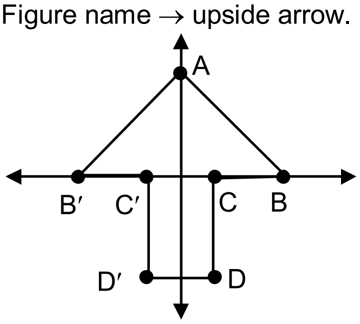


(ii)

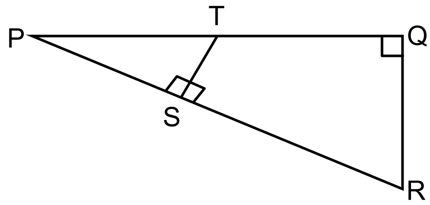


(iii) B′ ≡ (–3, 0)
C′ ≡ (–1, 0)
D′ ≡ (–1, –5)

(iv) Figure name → upside arrow.



6.



(i) Given ∠PQR = ∠PST = 90°

⇒ PQ = 5 cm, PS = 2 cm

Prove that (1) ΔPQR ~ ΔPST

(2) Area of traingle ΔPQR : Area of quadrilateral SRQT

⇒ Proof : In ΔPQR and ΔPST

∠PQR = ∠PST

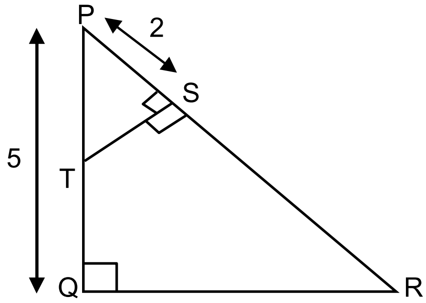
We know ∠PST = ∠PQR = 90°

⇒ ∠SPT = ∠QPR (common angle)

By angle angle similarity

ΔPQR ~ ΔPST

(ii) PQ = 5 cm, PS = 2 cm (given)



⇒ $\dfrac{\text{Area of } \Delta PST}{\text{Area of } \Delta PQR} = \left(\dfrac{2}{5}\right)^2 = \dfrac{4}{25}$

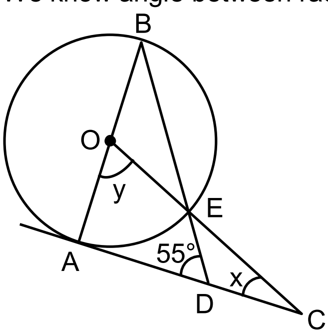
⇒ $\dfrac{\text{Area of } \Delta PQR}{\text{Area of quad. SRQT}} = \dfrac{25k}{25k - 4k}$ [k = constant].

$= \dfrac{25k}{21\ k} = \dfrac{25}{21}$

$\dfrac{\text{Area of } \Delta PQR}{\text{Area of quad. SRQT}} = \dfrac{25}{21}$.

7.

(a) We know angle between radius and tangent is 90°. Hence ∠OAC = 90°



in ΔABD = 90°

∠ABD = 180° – (90° + 55°) = 35°

and ∠AOE = 2 × ∠ABD

y° = 70°.

{Angle subtended at the centre is doubled the angle subtended at the circumference by same arc.}
Now in ΔAOC
$\angle OCA = x° = 180° – (90 + y) = 180 – (90 + 70°)$
$x = 20°$.

(b) **(i)** Scale factor 1 : 30

$$\frac{\text{actual height}}{\text{Model height}} = \frac{30}{1}$$

$$\text{actualy height} = \frac{30}{1} \times 80 = 2400 \text{ cm.}$$

(ii)
$$\frac{\text{Volume of building}}{\text{Volume of model}} = \left(\frac{30}{1}\right)^3$$

$$\text{Volume of model} = \frac{27 \times (100)^3}{(30)^3} \text{ cm}^3$$

$$\Rightarrow \frac{27 \times 1000}{27} = 1000 \text{ cm}^3.$$

(c) **(i)** order = 2 × 2

(ii) Let $M \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6.\, I$$

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 4a+4c & 2b+2d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

By comparing bot maxtrix
$4a + 4c = 6$, $2b + 2d = 0$
$–a + c = 0$, $–b + d = 6$
By solving

$$a = c = \frac{3}{4}, \qquad b = -3 \text{ and } d = 3.$$

$$M = \begin{bmatrix} \frac{3}{4} & -3 \\ \frac{3}{4} & 3 \end{bmatrix}.$$

8.
(a) $S_3 = 42$

$\frac{3}{2}$ [2a + (3 – 1)d] = 42
2a + 2d = 8
a + d = 14 ....(i)
and
$a \times T_3 = 52$
a [a + 2d] = 52
from equation (i)
a [a + 2(14 – a)] = 52
a [a + 28 – 2a] = 52
a [28 – a] = 52
$a^2 – 28a + 52 = 0$
(a – 26) (a – 2) = 0
a = 2 or 26
Ifa = 2 then d = 14 – 2 = 12 and if a = 26 then d = 14 – 26 = – 12.

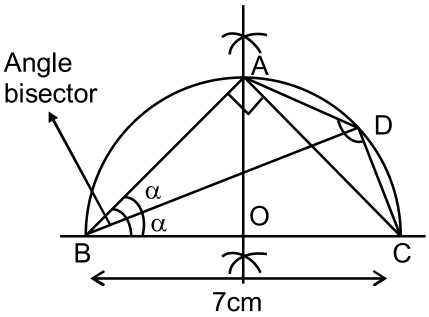
(b) Slope of BC = $\frac{-6-2}{6-(-1)} = \frac{-8}{7}$.

slope of perpendicular to line BC = $\frac{1}{\left(\frac{-8}{7}\right)} = \frac{7}{8}$

Hence equation whose passing through A is y – 8 = $\frac{7}{8}$ (x – 3)

8y – 64 = 7x – 21
7x – 8y + 43 = 0.

(c) Construction.



9.
(a) Here a = 45, h = 10

Number of Patients	Number of Days(f)	x_i	$d_i = x_1 - a$	$u_i = \frac{x_i - a}{h}$	f_iu_i
10 – 20	5	15	–30	–3	–15
20 – 30	2	25	–20	–2	–4
30 – 40	7	35	–10	–1	–7
40 – 50	9	45	0	0	0
50 – 60	2	55	10	1	2
60 – 70	5	65	20	2	10
	30				–14

$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\bar{x} = 45 + \frac{(-14)}{(30)} \times 10 = 45 - 14$

$\bar{x} = \frac{121}{3} = 40.33.$

(b) $\frac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = \frac{4}{1}$

By C and D

$\frac{2\sqrt{5}\ \ x}{2\sqrt{2x-6}} = \frac{4+1}{4-1}$

$\frac{\sqrt{5x}}{\sqrt{2x-6}} = \frac{5}{3}$

By squaring

$$\frac{5x}{2x-6} = \frac{25}{9}$$

9x = 10x – 30

x = 30

(c) **(i)** Share of first kind

FV = 100, Divided 90 = 10, MV = 170

No share purchased = $\frac{\text{Amount invested}}{\text{MV}} = \frac{8500}{170} = 50$

Income from first kind of share = $\frac{10}{100} \times 100 \times 50$

Amount recieved = selling price × number of shares

= 200 × 50 = 10000

∴ Sale proceeds = Rs. 10,000

(ii) Market value of second kind of share = Rs. 125

Number of shares bought of second kind = Rs. $\frac{10,000}{125}$ = Rs. 80

Income from second kind of share = $\frac{12}{100} \times 100 \times 80$ = Rs. 960

(iii) Change in annual income = 960 – 500 = Rs. 460 .

10.
(a)

(i)

Marks	Number of Students	c.f.
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113
80 – 90	4	117
90 – 100	3	120

$\ell = 40$ $\frac{N}{2} = \frac{120}{2} = 60$

c.f. = 52

f = 26

h = 10

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - (f)}{f}\right) \times h$$

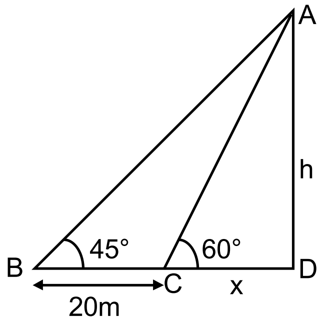
$$\Rightarrow 40 + \left(\frac{60-52}{26}\right) \times 10 \quad \Rightarrow 40 + \frac{8}{26} \times 10$$

$$\Rightarrow 40 + \frac{40}{13} = 43.076$$

(ii) Number of students who did not pas the test = 5 + 19 + 16 + 28 + 26 = 78.

(iii) $Q_3 = \frac{3n}{4} = \frac{3}{4} \times 120 = 90^{th}$.

(b) Let height of tanθ = hm, Let CD = x m



$\tan 45° = \frac{h}{20+x}$

h = 20 + x ....(i)

$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ ....(ii)

from (i) and (ii)

$h = 20 + \frac{h}{\sqrt{3}}$

$h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 20$

$h = \left(\frac{20\sqrt{3}}{\sqrt{3}-1}\right) = \frac{34.64}{0.732} = 47.32$ m.

11.

(a) x + 1 = 0

x = – 1

By remainder theorem f(– 1) = remainder

$f(x) = x^3 + (kx + r)x + k$

$f(-1) = (-1)^3 + (k(-1) + 8)(-1) + k$

$f(-1) = -1 + k - 8 + k = 2k - 9 = R_1$ (say)

$x - 2 = 0 \Rightarrow x = 2.$

By remainder theorem f(2) = remainder

$f(2) = (2)^3 + (2k + 8)\,2 = k$

$= 8 + 4k + 16 + k = 5k + 24 = R_2$ (say)

Given $R_1 + R_2 = 1$

$2k - 9 + 5k + 24 = 1 \Rightarrow 7k = -14 \Rightarrow k = -2$

(b) Let number are 3N (3N + 3)

3N(3N + 3) = 810

N(N + 1) = 90

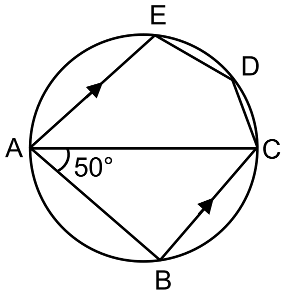
$N^2 + N - 90 = 0$

$N^2 + 10N - 9N - 90 = 0$

$(N + 10)(N - 9) = 0 \Rightarrow N = 9$ (N → Natural number so N = – 10 will be rejected)

Number are → 27, 30.

(c) **(i)** $\angle ABC = 90°$ (angle made by diameter on arc = 90°)



(ii) In $\Delta ABC \rightarrow \angle ABC + \angle BCA + \angle CAB = 180°$

$90 + \angle BCA + 50° = 180°$

$\angle BCA = 180°\ 2\ (90 + 50°) = 40°$.

BC || AE : $\angle EAC = \angle ACB = 40$ (Alternate angles)

$\angle EDC = 180° – (\angle EAC)$ [cylic quadrilateral]

$= 180° – 40° = 140°$.

(iii) If $\angle BAC = 50° + 40° = 90°$, than we can say BE will also a diameter.

$\angle BAC = \angle BEC = 50°$ (Angle made in same segment are equal).