MATHEMATICS
MARCH - 2019

SET NO. – 01 **12 (E)**
PART – A : TIME 1 HOUR/MARKS – 50
PART – B : TIME 2 HOURS/MARKS : 50

PART - A

1. Product of four consecutive positive integers is divisible by ___________
(A) 32 (B) 24 (C) 48 (D) 16

Sol. **(B)**
Quick theorem : Product of n consecutive integer is divisible by n! (Therefore product of 4 consecutive integer is divisible by 4!)
i.e., 4! = 4 × 3 × 2 × 1 $\Rightarrow$ 24. **Ans.**

2. The decimal expansion of $\frac{2517}{6250}$ will terminate after __________ digits
(A) 3 (B) 5 (C) 4 (D) 6

Sol. **(B)**
$\frac{2517}{6250} = \frac{2517}{2\times5^5} = \frac{2517}{2\times5\times5^5} = \frac{2517\times2^4}{2^5\times5^5} = \frac{2517\times16}{10^5} = \frac{40272}{10^5}$ = 0.40272
$\therefore$ it will terminate after 5 digit.

3. The zeros of a quadratic polynomial _____________ are 4 and 3.
(A) $x^2 + 7x – 12$ (B) $x^2 – 7x + 12$ (C) $x^2 + 7x + 12$ (D) $x^2 – 7x – 12$

Sol. **(B)**
Zeros are 4 and 3 i.e., α and β respectively.
Quadratic polynomial when zeros are known.
$x^2 – (\alpha + \beta) x + \alpha\beta$
$x^2 – (4 + 3) x + 4 \times 3$
$x^2 – 7x + 12$.

4. When $p(x) = 40x^2 + 11x – 63$ is divided by x + 2, then _______ is obtained as remainder
(A) 245 (B) 85 (C) 75 (D) – 75

Sol. **(C)**
$P(x) = 40x^2 + 11x – 63$ is divided by x + 2 then remainder will be
$x + 2 = 0 \Rightarrow x = –2$
By remainder theorem
Remainder = P(–2) = Value of polynomial P(x) at x = – 2.
$\therefore$ $P(– 2) = 40 (– 2)^2 + 11(– 2) – 63$
$\Rightarrow$ 160 – 22 – 63 = 75. **Ans.**

5. If α, β and γ are the zeros of a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ = __________.
(A) $-\frac{b}{d}$ (B) $\frac{c}{d}$ (C) $-\frac{c}{d}$ (D) $-\frac{c}{a}$

Sol. **(C)**
α, β, γ are zeros of cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$.
$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ = ?
$\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ = ? ....(i)

We know that for
$P(x) = ax^3 + bx^2 + cx + d$

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ ....(ii)

$\alpha\beta\gamma = -\frac{d}{a}$ ....(iii)

$\therefore$ Putting the values from (ii) and (iii) in (i)

$\Rightarrow \frac{\frac{c}{a}}{\frac{-d}{a}} = \frac{-c}{d}$. **Ans.**

6. If 3x + 2y = 7 and 2x + 3y = 3, then x – y = ______________
(A) 4 (B) – 4 (C) 2 (D) – 2

Sol. **(A)**
3x + 2y = 7 ....(i)
2x + 3y = 3 ....(ii)
then x – y = ?
Sub (ii) from (i)
$\therefore$

$$\begin{array}{r} 3x + 2y = 7 \\ 2x + 3y = 3 \\ - \quad - \quad - \\ \hline x - y = 4 \\ \hline \end{array}$$

$\therefore$ $\boxed{x - y = 4}$.

7. If, in a two digit number, the digit at unit place is x and the digit at tens place is 4, then the two digit number is ___________
(A) 40 + x (B) 4x (C) 40x + 4 (D) 10x + 4

Sol. **(A)**
Expanded form of two digit number = 10y + x ....(i)
where, digit on unit place = x
digit on ten's place = y = 4
$\therefore$ Putting y = 4 in equation (i)
Number = 10 × 4 + x = 40 + x. **Ans.**

8. _____________ is a solution of the linear equation of two variable 2x – y = 5.
(A) (3, 1) (B) (–3, – 1) (C) (–3, 1) (D) (3, – 1)

Sol. **(A)**
Solution of 2x – y = 5 ....(i)
putting points (3, 1) in equation (1) R.H.S
2(3) – (1) $\Rightarrow$ 6 – 1 = 5
$\therefore$ L.H.S. = R.H.S.
So solution is (3, 1). **Ans.**

9. The graph of linear polynomial p(x) = 5x + 3, x $\in$ R is __________
(A) Ray (B) Line
(C) Parabola open downward (D) Parabola open upward

Sol. **(B)**
P(x) = 5x + 3
It is a linear polynomial
$\therefore$ graph will be a line. **Ans.**

10. The solution set of the given pair of linear equation 5x – 5y = – 5 and $\frac{3x}{2}-\frac{3y}{2}+\frac{3}{2}$ = 0 is

(A) $\left(\frac{5}{2}, 0\right)$ (B) empty set (C) infinite set (D) $\left(0, -\frac{3}{2}\right)$

Sol. **(C)**

$5x - 5y + 5 = 0$ ....(i)

$a_1x + b_1y + c_1 = 0$

$\frac{3x}{2}-\frac{3y}{2}+\frac{3}{2}=0$ ....(ii)

$a_2x + b_1y + c_1 = 0$

from equation (i) and (ii)

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

it satisfies the condition of coincident lines.

i.e., $\frac{5}{\frac{3}{2}}=\frac{-5}{\frac{-3}{2}}=\frac{5}{\frac{3}{2}} \Rightarrow \frac{10}{3}=\frac{10}{3}=\frac{10}{3}$

$\therefore$ They are coincident lines

$\therefore$ Solution set = infinite set. **Ans.**

11. If one root of quadratic equation $Kx^2 - 4\sqrt{5}x + 5 = 0$ is $\sqrt{5}$, then K = ___________

(A) 3 (B) – 3 (C) $-\sqrt{5}$ (D) 5

Sol. **(A)**

$Kx^2 - 4\sqrt{5}x + 5 = 0$ ....(i)

one root $\sqrt{5}$ (put $x = \sqrt{5}$ in equation (i))

$K\left(\sqrt{5}\right)^2 - 4\sqrt{5}\left(\sqrt{5}\right) + 5 = 0$

5K – 20 + 5 = 0

5K = 15

K = 3. **Ans.**

12. If ____________, then the quadratic equation has no real roots

(A) D = 0 (B) D > 0 (C) D < 0 (D) D = 1

Sol. **(C)**

Condition for no real roots of quadratic equation is $b^2 - 4ac < 0$ i.e., D < 0. **Ans.**

13. Discriminant D = ___________, for the quadratic equation $25x^2 - 10x + 1 = 0$

(A) 0 (B) 1 (C) – 10 (D) 25

Sol. **(A)**

$25x^2 - 10x + 1 = 0$

$ax^2 + bx + c = 0$

$\therefore$ a = 25, b = – 10, c = 1.

$\therefore$ Discriminant D = $b^2 - 4ac$

D = $(-10)^2 - 4(25)(1)$

$\Rightarrow$ 100 – 100 = 0. **Ans.**

14. The formula to find the root of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, by method of completing square was given by mathematician _________

(A) Pythagoras (B) Sridhar Acharya (C) Hilbert (D) Uclid

Sol. **(B)**

Sridhar Acharya

15. For an A.P. if $T_3 = 8$ and $T_7 = 24$, then T_{10} = ______________
(A) – 4 (B) 28 (C) 32 (D) 36

Sol. **(D)**
$T_3 = 8$ $T_n = a + (n - 1)d$, $T_{10} = ?$
$T_3 = a + 2d = 8$ ....(i)
$T_7 = a + 6d = 24$...........(ii) Sub (i) from (ii)

$$\begin{array}{l} a + 6d = 24 \\ a + 2d = 8 \\ \underline{- \quad - \quad -} \\ 4d = 16 \end{array}$$

$\boxed{d = 4}$
$\therefore a + 6 \times 4 = 24.$
$\boxed{a = 0}$.
$T_{10} = a + 9d$ $\Rightarrow 0 + 9 \times 4 = 36$. **Ans.**

16. If x + 2, 3x – 1, 4x + 1 are the three consecutive terms of an A.P., then x = _____________.
(A) 1 (B) $\frac{1}{5}$ (C) 5 (D) – 1

Sol. **(C)**
(a) (b) (c)
x + 2, 3x –1 4x + 1 are in AP
by using
$\therefore 2b = a + c$
$2(3x - 1) = (x + 2) (4x + 1)$
$6x - 2 = 5x + 3$
$\boxed{x = 5}$. **Ans.**

17. If 5 + 7 + 9 + ... n = 437, then n = _____________
(A) 19 (B) 20 (C) 21 (D) 22

Sol. **(A)**
5 + 7 + 9 + ... upto n terms = 437 find = n.
$a = 5$, $d = 7 - 5 = 2$, $S_n = 437$, $n = ?$
$S_n = \frac{n}{2}(2a + (n - 1)d)$
$437 = \frac{n}{2}[2(5) + (n - 1)(2)]$
$437 = n [5 + n - 1]$
$437 = 4n + n^2$
$n^2 + 23n - 19n - 437 = 0$
$n(n + 23) - 19(n + 23) = 0$
$n - 19 = 0$ $n + 23 = 0$
$n = 19$ $n = -23$
n = 19. **Ans.**

18. – 4 and 3 are the roots of variable x of a quadratic equation ___________
(A) $x^2 - x - 12 = 0$ (B) $x^2 + x - 12 = 0$
(C) $x^2 - 7x - 12 = 0$ (D) $x^2 + 7x - 12 = 0$

Sol. **(B)**
$\alpha = -4, \beta = 3$
$\therefore$ Quadratic equation $= x^2 - (\alpha + \beta) x + \alpha\beta = 0$
$= x^2 - (-4 + 3)x + (-4) (3) = 0$
$\Rightarrow$ $x^2 + x - 12 = 0$. **Ans.**

19. In ΔABC and ΔDEF, $ABC \leftrightarrow DEF$ is a similarity. If AB + BC = 10 and DE + EF = 12, AC = 6, then DF = ____________

(A) 6 (B) 5 (C) 16 (D) 7.2

Sol. **(D)**

$\Delta ABC \sim \Delta DEF$

$\therefore \frac{AC}{DF} = \frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta DEF}$

$\frac{6}{DF} = \frac{AB+BC+AC}{DE+EF+DF}$

$\frac{6}{DF} = \frac{10+6}{12+DF}$

72 + 6DF = 16DF

72 = 10DF

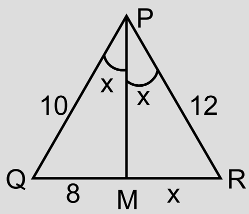
7.2 = DF. **Ans.**

20. In ΔPQR, the bisector of $\angle P$ intersects $\overline{QR}$ in M. If PQ = 10, PR = 12, QM = 8, then QR = _________.

(A) 9.6 (B) 17.6 (C) 10 (D) 18

Sol. **(B)**

QR = ?



Using angle bisector theorem

$\frac{PQ}{PR} = \frac{QM}{MR} \Rightarrow \frac{10}{12} = \frac{8}{x}$

$\Rightarrow$ 10x = 96

x = 9.6

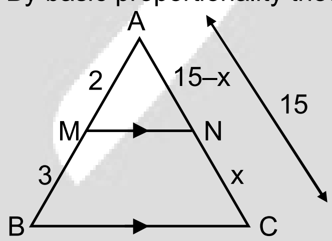
$\therefore$ QR = QM + MR = 8 + 9.6 $\Rightarrow$ 17.6. **Ans.**

21. In ΔABC, A – M – B, A – N – C, $\overline{MN} \parallel \overline{BC}$. If AM : AB = 2 : 3 and AC = 15, then NC = __________

(A) 3 (B) 6 (C) 9 (D) 5

Sol. **(C)**

By basic proportionality theorem



$\frac{AM}{MB} = \frac{AN}{NC}$

$\frac{2}{3} = \frac{15-x}{x}$

2x = 45 – 3x

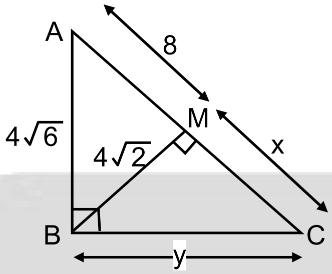
5x = 45

x = 9. **Ans.**

22. In $\triangle ABC$, $m\angle B = 90°$ and $\overline{BM}$ is altitude. If AB = $4\sqrt{6}$, AM = 8, then AC = ____________.

(A) 4 (B) 12 (C) 3 (D) 11

Sol. **(B)**



By using Pythagoras theorem

In $\triangle AMB$

$AB^2 = AM^2 + BM^2$

$\left(4\sqrt{6}\right)^2 = (8)^2 + BM^2$

$96 = 64 + BM^2$

$4\sqrt{2} = BM$

In $\triangle BMC$

$BM^2 + MC^2 = BC^2$

$\left(4\sqrt{2}\right)^2 + x^2 = y^2$

$32 = y^2 - x^2$ ....(i)

In $\triangle ABC$

$AB^2 + BC^2 = AC^2$

$96 + y^2 = (8 + x)^2$

$96 + y^2 = 64 + x^2 + 16x$

$96 - 64 + y^2 - x^2 = 16x$

$32 + 32 = 16x$

$\frac{64}{16} = x$

$4 = x$

$\therefore$ AC = AM + MC

AC = 8 + 4 = 12. **Ans.**

23. $\overline{XM}$ is a median in $\triangle XYZ$, $XY^2 + XZ^2 = 328$ and XM = 8, then YZ = ____________

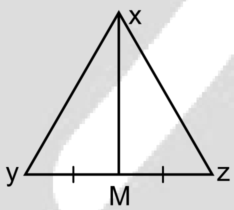
(A) 10 (B) 22 (C) 20 (D) 5

Sol. **(C)**

$XY^2 + XZ^2 = 328$

XM = 8

then YZ = ?



Using Apollonius theorem

$XY^2 + XZ^2 = 2\left(\frac{YZ}{2}\right)^2 + 2XM^2$

$328 = \frac{YZ^2}{2} + 2(8)^2$

$328 - 128 = \frac{YZ^2}{2}$

$400 = YZ^2$

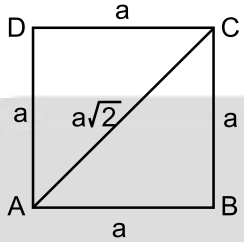
$\therefore$ YZ = 20. **Ans.**

24. The perimeter of a square ABCD is 32. Then the measure of its diagonal $\overline{AC}$ = ____________.

(A) $8\sqrt{2}$ (B) $2\sqrt{8}$ (C) $\sqrt{8}$ (D) $\frac{\sqrt{8}}{2}$

Sol. **(A)**

Perimeter of square (ABCD) = 32



4a = 32

a = 8.

Diagonal of square = $a\sqrt{2} \Rightarrow 8\sqrt{2}$. **Ans.**

25. If A(3, 5) and B(8, 9) are given points, then _________ is the mid point of $\overline{AB}$

(A) (4, 7) (B) (3, 9) (C) (11, 14) (D) $\left(\frac{11}{2}, 7\right)$

Sol. **(D)**

A(3, 5), B(8, 9)

$(x_1\ y_1)\ (x_2\ y_2)$

Mid point = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \Rightarrow \left(\frac{3+8}{2}, \frac{5+9}{2}\right) \Rightarrow \left(\frac{11}{2}, 7\right)$.

26. If the distance between the points (2, 3) and (a, 0) is 3 then a = ____________

(A) 2 (B) 3 (C) 5 (D) 1

Sol. **(A)**

(2, 3) and (a, 0) distance = 3

Distance = $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$3^2 = (a - 2)^2 + (0 - 3)^2$

$0 = (a - 2)^2$

$\therefore a - 2 = 0 \quad \Rightarrow a = 2$. **Ans.**

27. A(0, 0), B(2, 0), C(0, –2) are the vertices of a ____________ triangle

(A) Equilateral (B) Obtuse angled

(C) Right angled isosceles (D) Acute angled

Sol. **(C)**

A(0, 0), B(2, 0), C(0, – 2)

$(x_1\ y_1)\quad (x_2\ y_2)\quad (x_3, y_3)$

By using distance formula

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$AB^2 = (2 - 0)^2 + (0 - 0)^2$

$AB^2 = 4$ ....(i)

$BC^2 = (2 - 0)^2 + (0 + 2)^2$

$BC^2 = 4 + 4 = 8$

$BC^2 = 8$ ....(ii)

$AC^2 = (0 - 0)^2 + (0 + 2)^2$

$AC^2 = 4$ ....(iii)

from equation (i), (ii), (iii)

$AB^2 + AC^2 = BC^2$

i.e., 4 + 4 = 8

$\therefore$ Two sides are equal it is satisfying Pythagoras theorem

$\therefore$ It is a right angled isosceles.

28. A(2, 4), B(3, 5), C(4, 3) are the vertices of ΔABC. Hence the coordinate of centroid of the triangle is ______________

(A) (4, 3) (B) (3, 4) (C) (9, 12) (D) (4.5, 6)

Sol. **(B)**

A(2, 4), B(3, 5), C(4, 3)

x_1, y_2 x_2, y_2 x_3, y_3

Centroid $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\left(\frac{2+3+4}{3}, \frac{4+5+3}{3}\right)$ $\Rightarrow$ (3, 4). **Ans.**

29. If 5 sin θ = 4 cos θ then tan θ = ______________

(A) $\frac{5}{4}$ (B) 5 (C) 4 (D) $\frac{4}{5}$

Sol. **(D)**

5 sinθ = 4cosθ

$\frac{\sin\theta}{\cos\theta} = \frac{4}{5}$

$\therefore$ tanθ = $\frac{4}{5}$. **Ans.**

30. $(1 + \tan^2\theta)(1 - \sin^2\theta)$ = ______________.

(A) 1 (B) 0 (C) – 1 (D) 2

Sol. **(A)**

$(1 + \tan^2\theta)(1 - \sin^2\theta)$

By using

$1 + \tan^2\theta = \sec^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$(\sec^2\theta)(\cos^2\theta)$

$\frac{1}{\cos^2\theta} \times \cos^2\theta = 1$. **Ans.**

31. For ΔABC, sin $\left(\frac{B+C}{2}\right)$ = ______________

(A) cos A (B) sin A (C) cos $\frac{A}{2}$ (D) sin $\frac{A}{2}$

Sol. **(C)**

In ΔABC, sin $\left(\frac{B+C}{2}\right)$ = ? ....(i)

We know that A + B + C = 180°

$\frac{B+C}{2} = \frac{180+A}{2}$

$\frac{B+C}{2} = 90 - \frac{A}{2}$

Put in equation (i)

$\text{Sin}\left(90 - \frac{A}{2}\right) \Rightarrow \cos\frac{A}{2}$.

32. If $\tan 7\theta \cdot \tan 3\theta = 1$ then θ = ___________

(A) 0 (B) 9 (C) 10 (D) 18

Sol. **(B)**

$\tan 7\theta \cdot \tan 3\theta = 1$

$\tan 3\theta = \dfrac{1}{\tan 7\theta}$

$\tan 3\theta = \cot 7\theta$

$\tan 3\theta = \tan(90 - 7\theta)$

$3\theta = 90 - 7\theta$

$10\theta = 9\theta$

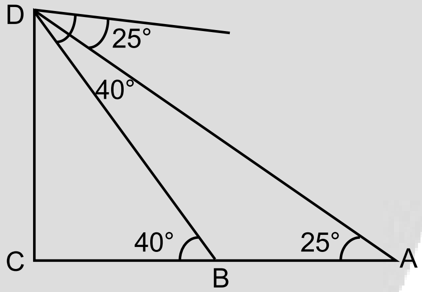
$\theta = 9$. **Ans.**

33. As observed from the top of the lighthouse, the angle of depression of the two ships A and B have measures 25 and 40 respectively. Then from the lighthouse __________

(A) A and B are at equal distance (B) The distance of B is more than A

(C) The distance of A is more than B (D) The distance of B is twice the distance of A

Sol. **(C)**

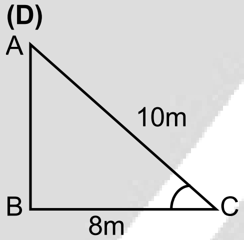


The distance of A is more than B.

34. 10 m long ladder $\overline{AC}$ leans on the wall such that its lower end (C) remains 8 m away from the base of the wall, then sin C = __________

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{3}{5}$

Sol. **(D)**



Using Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + 8^2 = 10^2$

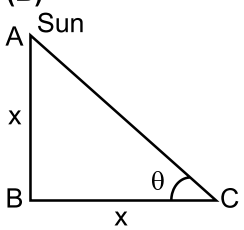
$AB^2 = 36 \quad \Rightarrow \quad AB = 6$ m

$\sin C = \dfrac{P}{H} = \dfrac{AB}{AC} = \dfrac{6}{10} = \dfrac{3}{5} = \sin C$. **Ans.**

35. When the length of the shadow of a tree becomes equal to the height of the tree, then the angle of elevation of the sun becomes ______________

(A) 90° (B) 45° (C) 30° (D) 60°

Sol. **(B)**



$\tan\theta = \frac{P}{B} = \frac{AB}{BC} = \frac{x}{x} = 1.$

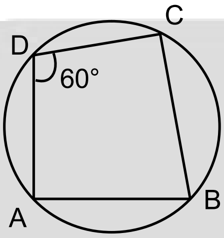
$\tan\theta = 1$

$\theta = 45°$. **Ans.**

36. If all the four vertices of a quadrilateral ABCD lie on the circle and $m\angle D = 60°$, then $m\angle B$ = ______

(A) 30° (B) 90° (C) 120° (D) 100°

Sol. **(C)**



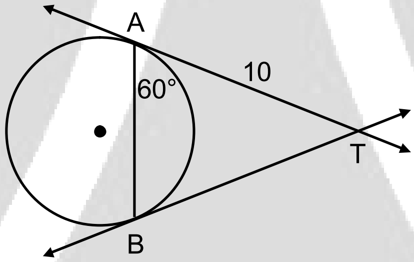
ABCD is a cyclic quadrilateral

$\therefore \quad \angle B + \angle D = 180°$ (Sum of opposite angle = 180°)

$60 + \angle B = 180°$

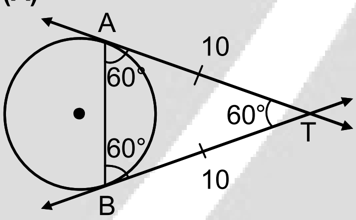
$\angle B = 120°$. **Ans.**

37. In the following figure, $\overleftrightarrow{TA}$ and $\overleftrightarrow{TB}$ are the tangents drawn from the exterior point T to the circle. If TA = 10 and $m\angle TAB = 60°$, then the length of the chord $\overline{AB}$ is __________



(A) 10 (B) 20 (C) 5 (D) 8

Sol. **(A)**



Since length of tangents drawn to a circle from an external point are equal.

$\therefore$ AT = TB = 10

Angle opposite to equal sides are equal

$\therefore \angle A = \angle B = 60°$ and $\angle C = 60°$

Using angle sum property in ΔABT

$\therefore \Delta ABT$ is an equilateral Δ and hence AB = 10 Ans.

38. In a $\odot(O,r)$ if the angle subtended by the minor arc at the centre of the circle is θ, then the length of the minor are (ℓ) is __________

(A) $\frac{\pi^2\theta}{90}$ (B) $\frac{\pi r\theta}{360}$ (C) $\frac{\pi r^2\theta}{360}$ (D) $\frac{\pi r\theta}{180}$

Sol. **(D)**

Length of minor arc = $\frac{\theta}{360°} \times 2\pi r = \frac{\theta\pi r}{180°}$.

39. The length of an arc of a circle having radius 15 cm is 20 cm. Hence its area of minor sector is _____ cm^2.
(A) 150 (B) 300 (C) 200 (D) 125

Sol. **(A)**
Length of arc = 20 cm, r = 15 cm, Area of minor sector = ?

Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

$20 = \frac{\theta}{360^\circ} \times 2 \times \pi \times 15$

$\frac{10}{15} = \frac{\pi\theta}{360^\circ}$ ....(i)

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$\frac{10}{15} \times 15 \times 15 \Rightarrow$ 150 cm^2. **Ans.**

40. If the radius of the circle is increased by 20% then the corresponding increase in the area of the circle is _________ (π = 3.14)
(A) 20% (B) 44% (C) 40% (D) 21%

Sol. **(B)**

Let radius = r

Area = πr^2

radius is increased by = 20%

$r' = r + \frac{20r}{100}$

$r' = \frac{6}{5}r$

Area = $\pi\left(\frac{6}{5}r\right)^2 = \frac{\pi 36r^2}{25}$.

Increase % in area = $\frac{\text{increase}}{\text{original}} \times 100$

$= \frac{\frac{11r^2\pi}{25}}{\pi r^2} \times 100 = 44\%$. **Ans.**

41. The ratio of the area of two circles is 9 : 16, then the ratio of their circumference ___________.
(A) 9 : 16 (B) 4 : 3 (C) 3 : 4 (D) 16 : 9

Sol. **(C)**

$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{16}$

$\frac{r_1^2}{r_2^2} = \frac{9}{16}$

$\therefore \frac{r_1}{r_2} = \frac{3}{4}$

$\therefore \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$.

3 : 4. **Ans.**

42. The formula to find volume of a cone is __________

(A) $\frac{4}{3}\pi r^3$ (B) $\pi r^2 h$ (C) $\frac{2}{3}\pi r^3$ (D) $\frac{1}{3}\pi r^2 h$

Sol. **(D)**

Volume of cone = $\frac{1}{3}\pi r^2 h$.

43. The formula to find curved surface area of a one rupee coin is __________

(A) $2\pi rh$ (B) $\pi r^2 h$ (C) $\pi r(h + r)$ (D) $2\pi r\,(h + r)$

Sol. **(A)**

C.S.A. of one rupee coin (cylinder) = $2\pi rh$.

44. The volume of a cylinder is 1408 cm^3 and its height is 7 cm. Hence its radius is _________ cm.

(A) 5 (B) 8 (C) 12 (D) 10

Sol. **(B)**

Volume of cylinder = 1408, h = 7 cm, r = ?

Volume = $\pi r^2 h$

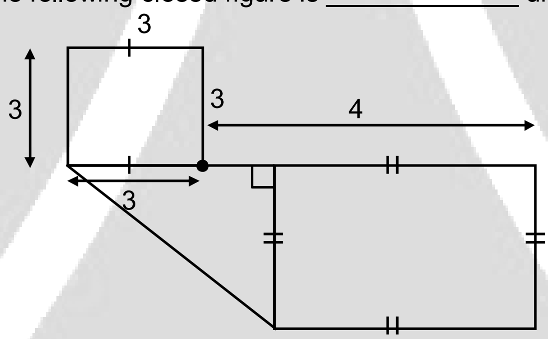
$1408 = \frac{22}{7} \times r^2 \times 7$

$\frac{1408}{22} = r^2$

$64 = r^2$

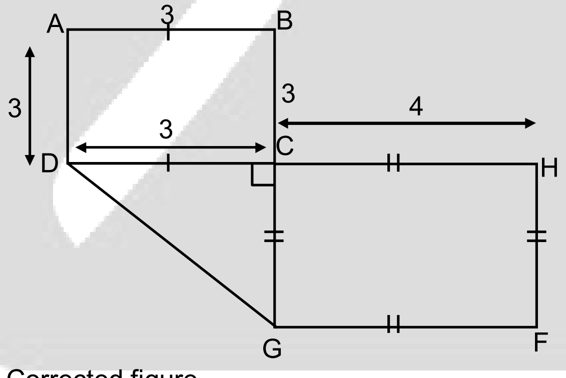
$\therefore$ 8 cm = r. **Ans.**

45. Total surface area of the following closed figure is _____________ $units^2$.



(A) 25 (B) 45 (C*) 31 (D) 40

Sol.



Corrected figure

Area of square ABCD = 3 × 3 = 9 $unit^2$

Area of square CGHF = 4 × 4 = 16 $unit^2$

Area of ΔCDG = $\frac{1}{2}$ × 4 × 3 = 6 $unit^2$

Total surface area = 9 + 16 + 6 = 31 $unit^2$

46. For some data, Z = 20 and M = 30, then $\bar{X}$ = ____________

(A) 25 (B) 35 (C) 37.5 (D) 32.5

Sol. **(B)**

Z = 20 and M = 30, then $\bar{X}$ = ?

Mode = 3 median – 2 mean

20 = 3 × 30 – 2 mean

2 mean = 70

mean = 35. **Ans.**

47. For $M+\bar{X}=22$ and $M-\bar{X}=2$, we have Z = ___________.

(A) 16 (B) 14 (C) 10 (D) 12

Sol. **(A)**

$M+\bar{X}$ = 22

$M-\bar{X}$ = 2

2M = 24

M = 12 , $\bar{X}$ = 12

Mode = 3 median – 2 mean

Z = 3(12) – 2(10)

Z = 36 – 20

Z = 16.

48. The modal class of the frequency distribution given below is __________.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	7	15	13	17	10

(A) 10 – 20 (B) 20 – 30 (C) 30 – 40 (D) 40 – 50

Sol. **(C)**

Modal class = highest frequency

$\therefore$ 30 – 40. **Ans.**

49. On tossing a balanced dice once, the probability of a number obtained as multiple of 3 is _______

(A) $\frac{1}{6}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$

Sol. **(C)**

D = [1, 2, 3, 4, 5, 6]

Multiple of 3 = {3, 6}

P(multiple of 3) = $\frac{2}{6} = \frac{1}{3}$. **Ans.**

50. If P(C) = $\frac{3}{5}$, then $P(\bar{C})$ = _________.

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{5}$ (D) 1

Sol. **(A)**

P(C) = $\frac{3}{5}$

$P(\bar{C})$ = 1 – P(C)

$1 - \frac{3}{5} = \frac{2}{5} = P(\bar{C})$. **Ans.**

PART – B

SECTION - A

Answer the following questions number 1 to 8 with calculations in brief.
[Each question carries 2 marks]

1. Find the square root : $14+6\sqrt{5}$

Sol. Square root $14+6\sqrt{5}$

$(a + b)^2 = a^2 + b^2 + 2ab$

$14+6\sqrt{5} = a^2 + b^2 + 2ab$

$2ab = 6\sqrt{5}$

$2 \times a \times b = 2 \times 3 \times \sqrt{5}$

$a = 3,\ b = \sqrt{5}$

$14+6\sqrt{5} = \left(3+\sqrt{5}\right)^2$

Square root of $\sqrt{\left(3+\sqrt{5}\right)^2}$ $\Rightarrow$ $3+\sqrt{5}$. **Ans.**

2. Find zeros of $p(x) = x^2 + 9x + 14$. Also find the sum and product of the zeros.

Sol. $x^2 + 9x + 14$

Let α and β are zeros

$x^2 + 7x + 2x + 14 = 0$

$x(x + 7) + 2(x + 7) = 0$

$(x + 2)(x + 7) = 0$

$\alpha = -2,\ \beta = -7$

Sum of zeros

$\Rightarrow$ $\alpha + \beta = -2 - 7 = -9$

product of zeros

$\Rightarrow$ $\alpha\beta = -2 \times -7$ $\Rightarrow$ 14. **Ans.**

3. Find the solution set for the given pair of equations $x + y = 7$, $3x - 2y = 11$.

Sol. $a_1\ b_1\ c_1$ $\qquad$ $a_2\ b_2\ c_2$

$x + y = 7$; $\qquad 3x - 2y = 11$

$\frac{a_1}{a_2} = \frac{1}{3} \neq \frac{1}{-2} \neq \frac{7}{11}$.

$3x + 3y = 21$

$3x - 2y = 11$

$-\quad +\quad -$

$5y = 10$.

$\therefore$ $y = 2,\ x = 5$.

Hence $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ it will have unique solution.

4. For an A.P. 1, 1.5., 2, 2.5,.............. find the sum of the first 16 terms.

Sol. AP 1, 1.5, 2., 2.5 $\quad S_{16} = ?$

$S_n = \frac{n}{2}\left(20+(n-1)\ d\right)$

$a = 1,\ d = a_2 - a_1 = 1.5 - 1 = 0.5,\ n = 16$

$S_{16} = \frac{16}{2}\left[2+(16-1)\ 0.5\right]$ $\Rightarrow$ $8[2 + 7.5] = 8 \times 9.5 = 76.0$. **Ans.**

OR

Find 10^{th} term of an A.P. 115, 100, 85, 70,........... .

Sol. AP $\rightarrow$ 115, 100, 85, 70,..........., $a_{10} = ?$

$a = 115,\ d = a_2 - a_1 = 100 - 115 = -15$

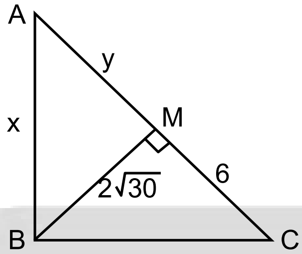
$a_n = a + (n - 1)d$

$a_{10} = a + 9d$ $\Rightarrow$ $115 + 9 \times (-15)$ $\Rightarrow$ $115 - 135 = -20$.

$a_{10} = -20$. **Ans.**

5. In ΔABC, $m\angle B = 90°$ and $\overline{BM}$ is altitude of $\overline{AC}$. If $BM = 2\sqrt{30}$. MC = 6, then find AC.

Sol.



Using Pythagoras theorem

In ΔMBC

$P^2 + B = H^2$

$MB^2 + MC^2 = BC^2$

$\left(2\sqrt{30}\right)^2 + (6)^2 = BC^2$

$120 + 36 = BC^2$

$156 = BC^2$

In ΔABC

$AB^2 + BC^2 = AC^2$

$x^2 + 156 = (6 + y)^2$

$x^2 + 156 = 36 + y^2 + 12y$

$x^2 - y^2 + 120 = 12y$ ....(i)

In ΔABM

$\left(2\sqrt{30}\right)^2 + y^2 = x^2$

$120 = x^2 - y^2$ ....(ii)

Put (ii) in (i)

$120 + 120 = 12y$

$240 = 12y$

$20 = y = AM$

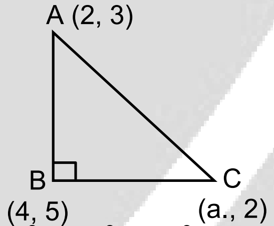
$AC = AM + MC$

$AC = 20 + 6 = 26$

AC = 26. **Ans.**

6. In ΔABC, $m\angle B = 90°$, A(2, 3), B(4, 5) and C(a, 2). Then find a.

Sol. ΔABC is a right angled Δ



$AB^2 + BC^2 = AC^2$ ....(i)

$AB^2 = (4-2)^2 + (5-3)^2 \Rightarrow 4 + 4 = 8$

$BC^2 = (a-4)^2 + (2-5)^2 = a^2 + 16 - 8a + 9$

$AC^2 = (a-2)^2 + (2-3)^2 = a^2 + 4 - 4a + 1$

Putting values in equation (i)

$8 + a^2 + 16 - 8a + 9 = a^2 + 4 - 4a + 1$

$28 = 4a$

$a = 7$.

7. Prove that $\frac{\sin 70}{\cos 20} + \frac{\text{cosec} 20}{\sec 70} - 2\cos 70 - \text{cosec } 20 = 0$.

Sol. $\frac{\sin 70}{\cos 20} + \frac{\text{cosec} 20}{\sec 70} - 2\cos 70 - \text{cosec } 20 = 0$

L.H.S

$\frac{\sin 70}{\cos 20} + \frac{\text{cosec} 20}{\sec 70} - 2\cos 70 \times \text{cosec} 20$

$\frac{\sin 70}{\sin(90-20)} + \frac{\text{cosec}20}{\text{cosec}\ (90-70)}$ – 2 cos 70 × cosec20

$\frac{\sin 70}{\sin 70} + \frac{\text{cosec}20}{\text{cosec}20}$ – 2 cos70 × cosec 20

1 + 1 – 2 cos 70 $\frac{1}{\sin 20}$

1 + 1 – 2 cos 70 × $\frac{1}{\cos(90-20)}$

1 + 1 – 2 cos 70 × $\frac{1}{\cos 70}$ $\Rightarrow$ 2 – 2 = 0.

OR

Prove that $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Sol. $\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta + \text{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$.
$1 + \text{cosec}^2\theta + 2 + 2 + \sec^2\theta$.
$1 + 1 + \cot^2\theta + 2 + 2 + 1 + \tan^2\theta$
$7 + \tan^2\theta + \cot^2\theta$.

8. For certain data, if $\overline{X} = 35.8$, C = 10, $\sum f_i u_i = 4$, $\sum f_i = 50$, then find assumed mean A.

Sol. $\overline{X} = 35.8$, C or h = 10, $\sum f_i u_i = 4$, $\sum f_i = 50$

Mean $\Rightarrow$ a + $\frac{\sum f_i u_i}{\sum f_i} \times h$

35.8 = a + $\frac{4}{50} \times 10$
35.8 = a + 0.8
35 = a. **Ans.**

SECTION – B

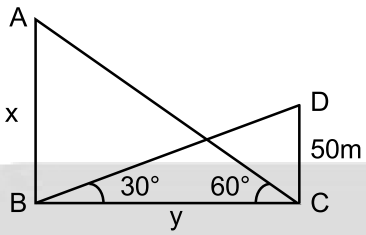
Answer the following questions number 9 to 12 with calculations.
[Each question carries 3 marks]

9. The sum and product of two numbers are 27 and 182 respectively. Find the two numbers.

Sol. a + b = 27 ....(i)
a × b = 182
$(a = b)^2 = (27)^2$
$a^2 + b^2 + 2ab = 729$
$a^2 + b^2 + 2 \times 182 = 725$
$a^2 + b^2 = 729 - 364$
$a^2 = b^2 = 365$
$(a - b)^2 = a^2 + b^2 - 2ab$
$(a - b)^2 = 365 - 2 \times 182$
$(a - b)^2 = 365 - 364$
a – b = 1 ....(ii)
Solving (i) and (ii)
a + b = 27
a – b = 1
——————
2a = 28
——————
a = 14
b = 13. **Ans.**

10. The angle of elevation of the top of a tower as observed from the foot of a temple has measure 60. The angle of elevation of the top of the temple as observed from the foot of the tower has measure 30. If the temple is 50 m high, find the height of the tower.

Sol.



In $\triangle ABC$

$\frac{AB}{BC}$ = tan 60°

$\frac{x}{y} = \sqrt{3}$ ....(i)

In $\triangle DBC$

$\frac{50}{y}$ = tan 30°

$\frac{50}{y} = \frac{1}{\sqrt{3}}$

y = $50\sqrt{3}$ ....(ii)

Put (ii) in (i)

$\frac{x}{50\sqrt{3}} = \sqrt{3}$

x = 50 × 3 = 150 m.

Height of tower = 150 m. **Ans.**

11. Find the median of the following data :

Class	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28
Frequency	9	16	12	7	15	1

Sol.

Class	Frequency	cf
4 – 8	9	9
8 – 12	16	25
12 – 16	12	37
16 – 20	7	44
20 – 24	15	59
24 – 28	1	60
	$\Sigma f_i = 60$	

Median = $\frac{n}{2}$ term, $\frac{n}{2}$ + 1 term

$\Rightarrow$ $\frac{60}{2}$, $\frac{60}{2}$ + 1 $\Rightarrow$ 30, 31th term

Median = $\ell + \left(\frac{\frac{N}{2} - cf}{F}\right) \times h$

l = 12, N = 60, cf = 25 (upper class) F = 12, h = 4.

Median = $12 + \left(\frac{\frac{60}{2} - 25}{12}\right) \times 4$ $\Rightarrow$ $12 + \left(\frac{5}{12}\right) \times 4$ $\Rightarrow$ 12 + $\frac{5}{3}$

$\Rightarrow$ 12 + 0.6 $\Rightarrow$ 12.6. **Ans.**

OR

Find the mode of the following data

Class	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	12	17	28	23	7	8	5

Sol.

Class	f	
30 – 40	12	
40 – 50	17	f_0
50 – 60	28	f_1
60 – 70	23	f_2
70 – 80	7	
80 – 90	8	
90 – 100	5	

$\ell = 50$, h = 10

Mode = $\ell + \dfrac{F_1 - F_0}{2F_1 - F_0 - F_2} \times h$

Mode = $50 + \dfrac{11}{56 - 40} \times 10$

$\Rightarrow \quad 50 + \dfrac{11}{16} \times 10$

$\Rightarrow \quad 50 + \dfrac{110}{16}$

= 50 + 6.875 = 56.875. **Ans.**

12. A box contains 8 black, 7 white and 6 yellow balls in it. One ball is taken out from the box at random. What is the probability that the ball taken out is :

(i) Yellow ? (ii) Not a black ? (iii) White ?

Sol. 8 black, 7 white, 6 yellow

Total = 21 balls

(i) P(yellow) = $\dfrac{6}{21}$

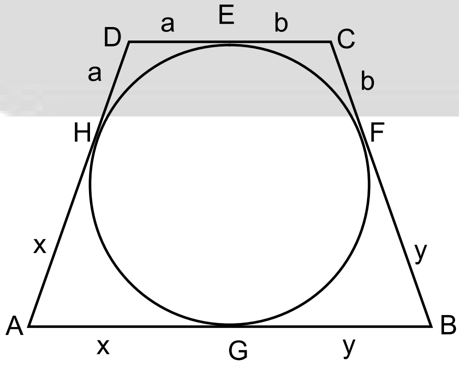
(ii) P(not black) = $\dfrac{13}{21}$

(iii) P(white) = $\dfrac{7}{21}$.

SECTION – C

13. A circle touches all the sides of □ABCD. If $\overline{AB}$ is the largest side then prove that $\overline{CD}$ is the smallest side.

Sol. AB is the longest side

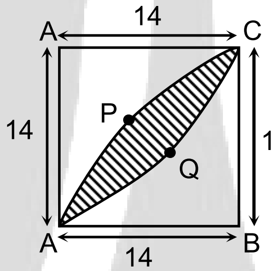


$$\left.\begin{array}{l} AH = AG = x \\ BG = BF = y \\ FC = CE = b \\ HD = DE = a \end{array}\right\} \because \text{ tangent drawn from an external point to a circle are equal.}$$

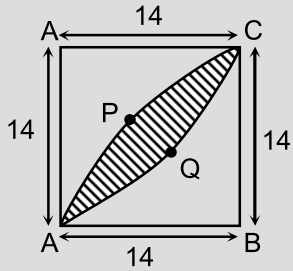
AB = x + y	AB > BC
BC = y + b	x + y > y + b
CD = a + b	x > b , AB > AD ....(i)
AD = x + a	x + y > x + a
	y > a ....(ii)

a + b < x + a
a + b < y + b
$\therefore$ CD is the smallest side.

14. What will be the cost of making design in the coloured region at the rate of Rs. 25 per cm^2 ?



Sol.



Area of segment AQC = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta$

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{-1}{2} \times 14 \times 14 \times \sin 90°$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

$= \frac{1}{2} \times 14 \times 14 \left(\frac{11}{7} - 1\right)$

$= 98 \left(\frac{4}{7}\right)$

= 14 × 4 = 56.
Similarly
Area of segment APC = 56
$\therefore$ Shaded region = 56 + 56 = 128. **Ans.**

15. A metallic sphere of radius 5.6 cm is melted to make a cylinder having radius 6 cm. Find the height of the cylinder.

Sol.

Sphere	Cylinder
r = 5.6 cm	R = 6 cm, h = ?

$\frac{4}{3}\pi r^3 = \pi R^2 h$

$\frac{4}{3} \times \frac{56}{10} \times \frac{56}{10} \times \frac{56}{10} = 6 \times 6 \times h$

$\frac{56 \times 56 \times 56}{30 \times 30 \times 30} = h$

$\frac{175616}{27000} = h$

6.50 = h.

OR

A cylindrical tank with hemispherical ends having radius 0.42 m and total height 3.84m. Find total surface area of the closed tank.

Sol.

T.S.A = $2\pi r^2 + 2\pi r^2 + 2\pi rh$

$\Rightarrow \quad 4\pi r^2 + 2\pi rh$

$\Rightarrow \quad 2 \times \frac{22}{7} \times \frac{42}{100}\left(2 \times \frac{42}{100} + \frac{384}{100}\right)$

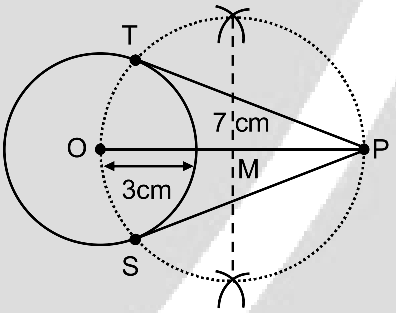
$\Rightarrow \quad \frac{264}{100}\left(\frac{468}{100}\right) \quad \Rightarrow \quad$ 12.3552. **Ans.**

SECTION – D

Answer the following questions no. 16 to 17 [Each question carries 5 marks]

16. Draw $\odot(O, 3\text{ cm})$. Construct a pair of tangents from point P at a distance of 7 cm from the centre O. Also write points of constructions.

Sol.

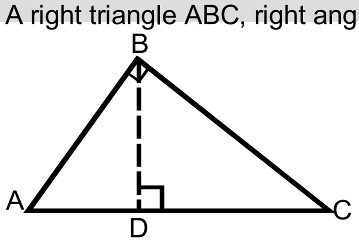


Step-1 : Draw a circle of radius 3 cm with centre O.
Step-2 : Mark a point P outside the circle at a distance 7 cm from the centre O.
Step-3 : Find a mid point M of line segment OP by drawing perpendicular bisector of it.
Step-4 : With centre M and radius OM draw a circle.
Step-5 : Mark the intersection point of two circle as T and S and join PT and PS.
$\therefore$ PT and PS are our required tangents..

17. State and prove Pythagoras theorem.

Sol. **Statement :** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : A right triangle ABC, right angled at B.



To prove : $AC^2 = AB^2 + BC^2$

Construction : $BD \perp AC$

Proof : Δ ADB & Δ ABC

$\angle DAB = \angle CAB$ [Common]

$\angle BDA = \angle CBA$ [90º each]

So, $\Delta ADB \sim \Delta ABC$ [By AA similarity]

$\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

or, $AD . AC = AB^2$ ... (i)

Similarily $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or $CD . AC = BC^2$ ... (ii)

Adding (i) and (ii),

$AD . AC + CD . AC = AB^2 + BC^2$

or, $AC (AD + CD) = AB^2 + BC^2$

or, $AC. AC = AB^2 + BC^2$

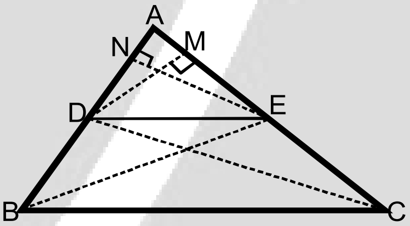
or, $\mathbf{AC^2 = AB^2 + BC^2}$ **Hence Proved.**

OR

If a line parallel to one of the sides of a triangle intersects the other two sides in distinct points, then the segments of the other two sides in one half plane are proportional to the segments in the other half plane.

Sol. **Given :** A ΔABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$.



Construction : Join BE and CD and draw DM $\perp$ AC and EN $\perp$ AB.

Proof : Area of Δ ADE $= \frac{1}{2}$ (base × height) $= \frac{1}{2}$ AD × EN.

Area of Δ ADE is denoted as ar(ADE).

So, ar(ADE) $= \frac{1}{2}$ AD × EN and ar(BDE) $= \frac{1}{2}$ DB × EN.

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ... (i)

Similarly, ar(ADE) $= \frac{1}{2}$ AE × DM and ar(DEC) $= \frac{1}{2}$ EC × DM.

And $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ... (ii)

Note that Δ BDE and Δ DEC are on the same base DE and between the two parallel lines BC and DE.

So, ar(BDE) = ar(DEC) ... (iii)

Therefore, from (i), (ii) and (iii), we have :

$\frac{AD}{DB} = \frac{AE}{EC}$ **Hence Proved.**