

HINTS & SOLUTIONS

SECTION – A

OBJECTIVE TYPE QUESTIONS

1. First 5 odd numbers are 1, 3, 5, 7, 9

$$\text{Mean} = \frac{1+3+5+7+9}{5} = \frac{25}{5} = 5.$$

2. $x + 2y = 3$ and $5x + ky = 15$
for infinite solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{5} = \frac{2}{k} = \frac{3}{15}$$

$$\text{so } k = 10.$$

3. $\sqrt{\frac{64}{81}} = \frac{8}{9}$ is not irrational.

4. $x^2 = ax - b$

Zeroes of polynomial are equal but of opposite sign, let zeroes are α and $-\alpha$

$$\text{so } \alpha + (-\alpha) = -\frac{a}{1}$$

$$-a = 0 \Rightarrow a = 0.$$

5. $\frac{6}{15} = \frac{2}{5} = 0.4$

so $\frac{6}{15}$ is terminating decimal expansion.

6. $x^2 - 9x + a$ product of zeroes = 9

$$\frac{a}{1} = 9 \Rightarrow a = 9.$$

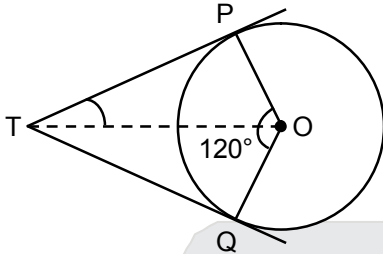
7. $d_1 = 30$ cm ; $d_2 = 40$ cm

$$\text{side of rhombus} = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{(15)^2 + (20)^2} = \sqrt{225 + 400} = \sqrt{625} = 25 \text{ cm.}$$

8. $\operatorname{cosec}(90 - \theta) \sin(90 - \theta)$
 $= \sec\theta \cdot \cos\theta = 1.$

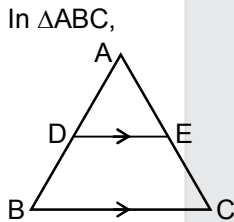
9. $x^2 + 3x + 2$
 for zeroes : $x^2 + 2x + x + 2 = 0$
 $x(x + 2) + 1(x + 2) = 0$
 $(x + 1)(x + 2) = 0$
 so zeroes are -1 and -2 .

10.



$\therefore \angle POQ = 120^\circ$
 $\therefore \angle PTQ = 180^\circ - 120^\circ = 60^\circ$
 and $\angle OTP = \frac{1}{2} \angle PTQ = \frac{60^\circ}{2} = 30^\circ$.

11.



In $\triangle ABC$,
 $DE \parallel BC$
 $\frac{AD}{DB} = \frac{3}{5}$ and $AE = 4.8$ cm
 by BPT,
 $\frac{AD}{DB} = \frac{AE}{EC}$
 $\frac{3}{5} = \frac{4.8}{EC} \Rightarrow EC = 8$ cm.

12.

On throwing a dice
 prime numbers are 2, 3, 5 probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$.

13.

Circumference = 462 cm
 $2\pi r = 462$
 $r = \frac{462 \times 7}{44} \Rightarrow r = 73.5$ cm.

14.

If two linear equations in two variables have infinite solutions, then their graphs will be two coincident lines.

15.

$kx^2 + 4x + 1 = 0$ for real and distinct roots
 $D > 0$
 $b^2 - 4ac > 0$
 $16 - 4k > 0$
 $k < 4$.

$$16. \quad \frac{1 + \cot^2 A}{1 + \tan^2 A} = \frac{\operatorname{cosec}^2 A}{\sec^2 A} = \frac{\left(\frac{1}{\sin^2 A}\right)}{\left(\frac{1}{\cos^2 A}\right)} = \frac{\cos^2 A}{\sin^2 A} = \cot^2 A.$$

17. If a line touches a circle at only one point, then it is known as tangent.

18. Let $x = 3.\overline{27}$
 $100x = 327.\overline{27}$
 $99x = 324$
 $x = \frac{324}{99} = \frac{36}{11}$

So $3.\overline{27}$ is a rational number.

19. $\cos A = \frac{1}{2}$

$$1 - 2\cos^2 A = 1 - 2\left(\frac{1}{4}\right) = 1 - \frac{1}{2} = \frac{1}{2}.$$

20. If all sides of a parallelogram touch a circle then the parallelogram will be a rhombus.

21. Ratio of side of two similar triangles = 3 : 5. ratio of their areas will be $(3)^2 : (5)^2 = 9 : 25$.

22. Volume of right circular cylinder = $\pi r^2 h$.

23. C.S.A of sphere = $144\pi \text{ cm}^2$
 $4\pi r^2 = 144\pi$
 $r = 6 \text{ cm}.$

24. Cumulative frequency curve is known as ogive.

25. In $\triangle ABC$, $\angle A = 90^\circ$, $BC = 13 \text{ cm}$, $AB = 12 \text{ cm}$ by pythagoras theorem,
 $(BC)^2 = (AB)^2 + (AC)^2$
 $AC = \sqrt{BC^2 - AB^2} = \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} = \sqrt{25} = 5.$

26. $a_6 = 13 \quad \Rightarrow \quad a + 5d = 13 \quad \dots\dots(i)$

$a_{12} = 25 \quad \Rightarrow \quad a + 11d = 25 \quad \dots\dots(ii)$

Equation (ii) – equation (i)

$\Rightarrow \quad 6d = 12 \quad \Rightarrow \quad d = 2$

then $a = 13 - 5(2) = 13 - 10$

$a = 3$ or first term is 3.

27. $\operatorname{cosec} 45^\circ = \sqrt{2}.$

28. $15 \cot A = 8$

$$\cot A = \frac{8}{15}$$

$$\operatorname{cosec} A = \sqrt{1 + \cot^2 A} = \sqrt{1 + \frac{64}{225}} = \sqrt{\frac{289}{225}} = \frac{17}{15}.$$

29. $\cos\theta = \sqrt{1 - \sin^2\theta}$

30. $P(x) = x^2 - 2x - 6$

if α, β are the zeroes then $\alpha\beta = \frac{c}{a} = \frac{-6}{1} = -6$.

31. Maximum value of probability of an event is 1.

32. $P + 1, 2P + 1, 4P - 1$ are in A.P. $\{a, b, c$ in A.P. so $2b = a + b\}$
 $2(2P + 1) = (P + 1) + (4P - 1)$

33. Areas of two circles are in ratio 4 : 1

$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{1}$$

$$\frac{r_1}{r_2} = \frac{2}{1} \text{ or } r_1 : r_2 = 2 : 1.$$

34. Probability of an impossible event is 0.

35. $\frac{\tan 65^\circ}{\cot 25^\circ} = \frac{\tan(90^\circ - 25^\circ)}{\cot 25^\circ} = \frac{\cot 25^\circ}{\cot 25^\circ} = 1$.

36. $\tan^2\theta - \sec^2\theta = -1$.

37. $x + 3y - 4 = 0$
 $2x - 5y - 1 = 0$

here $\frac{a_1}{a_2} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-3}{5}$ and $\frac{c_1}{c_2} = \frac{4}{1}$.

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

so unique solution and the pair of linear equation is consistent.

38. Form of even integer is $2m$.

39. Volume of cone = 1570 cm^3 and base area = 314 cm^2

$$\frac{1}{3}\pi r^2 h = 1570 \text{ and } \pi r^2 = 314.$$

$$\frac{1}{3} \times 314 \times h = 1570$$

$$h = \frac{1570 \times 3}{314} = 15 \text{ cm.}$$

40. Distance of $P(2, 3)$ from origin is $\sqrt{(2)^2 + (3)^2} = \sqrt{13}$.

41. $P(3, -4)$; ordinate is -4 .

42. In $\triangle ABC$, $A(2, 3)$, $B(1, -3)$; centroid is $(3, 0)$.

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$

$$3 = \frac{2+1+x_3}{3} \text{ and } 0 = \frac{3-3+y_3}{3}$$

$$x_3 = 6 \text{ and } y_3 = 0$$

$$\text{So } c(x_3, y_3) = c(6, 0).$$

43. $A(-2, 3)$ and $B(4, 1)$

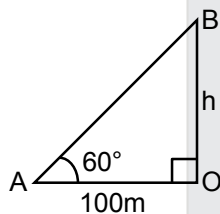
Let mid point of AB is $C(x, y)$

$$x = \frac{-2+4}{2} = 1 \text{ and } y = \frac{3+1}{2} = 2$$

So, $C(1, 2)$.

44. 29 is a prime number.

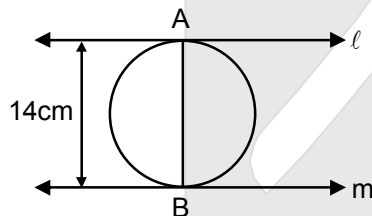
45. Angle of elevation = 60° and in $\triangle AOB$, $\tan 60^\circ = \frac{h}{100}$.



$$h = 100\sqrt{3} \text{ m.}$$

parallelogram $AB = 14 \text{ cm.}$

46.



$$r = 7 \text{ cm.}$$

47. Point $(4, 3)$ lies in 1st quadrant.

48. 3, 5, 4, 3, 2, 3, 1, 3

mode = 3.

49. Perimeter of semicircle = 72 cm

$$\pi r + 2r = 72$$

$$r \left(\frac{22}{7} + 2 \right) = 72$$

$$r = \frac{72 \times 7}{36} = 14 \quad \Rightarrow \quad r = 14 \text{ cm.}$$

50. $\cot(90^\circ - \theta) = \tan \theta$.



SECTION – B

NON - OBJECTIVE TYPE QUESTIONS SHORT ANSWER TYPE QUESTIONS

1. $13 \times 182 = 26 \times x$
 $\Rightarrow x = \frac{13 \times 182}{26} = 91.$

2. $3x - 5y = 4$ (i) $\times 3$
 $9x - 2y = 7$ (ii)
 \Rightarrow $9x - 15y = 12$
 $9x - 2y = 7$
 $\frac{-}{+} \frac{-}{-}$
 $\hline -13y = 5$

$y = -\frac{5}{13}$

then,

$9x - 2 \times \left(-\frac{5}{13}\right) = 7$

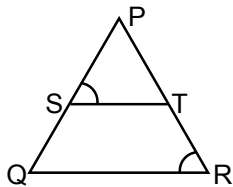
$9x = 7 + \frac{10}{13}$

$x = \frac{101}{117}.$

3. LHS
 $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$
 $\Rightarrow \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A}$
 $\Rightarrow \frac{\cos^2 A + \sin^2 A + 2 \sin A + 1}{(1 + \sin A) \cos A}$
 $\Rightarrow \frac{2 + 2 \sin A}{(1 + \sin A) \cos A} = \frac{2(1 + \sin A)}{(1 + \sin A) \cos A}$
 $\Rightarrow \frac{2}{\cos A} = 2 \sec A.$

4. $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$

So by converse of B.P.T



$ST \parallel QR$

$\therefore \angle PST = \angle PQR$ (i) [corresponding angle]

But $\angle PST = \angle PRQ$ (ii)

By (i) and (ii)

$\angle PQR = \angle PRQ$

$\therefore PR = PQ$

$\therefore \Delta PQR$ is isosceles Δ .

5. $963 = 657 \times 1 + 306$

$$657 = 306 \times 2 + 45$$

$$306 = 45 \times 6 + 36$$

$$45 = 36 \times 1 + 9$$

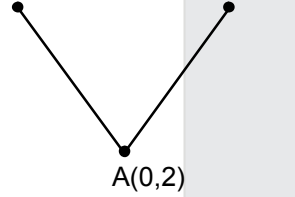
$$36 = 9 \times 4 + 0$$

So, HCF = 9.

6. $AB = AC$

$B(3, a)$

$C(a, 5)$



$A(0, 2)$

$$\Rightarrow \sqrt{(3-0)^2 + (a-2)^2} = \sqrt{(a-0)^2 + (5-2)^2}$$

squaring on both sides

$$\Rightarrow (3)^2 + (a-2)^2 = (a)^2 + (3)^2$$

$$\Rightarrow 9 + a^2 + 4 - 4a = a^2 + 9$$

$$\Rightarrow 4 - 4a = 0$$

$$\Rightarrow a = 1.$$

7.
$$\frac{5 \cos^2 60 + 4 \sec^2 30 - \tan^2 45}{\sin^2 30 + \cos^2 30}$$

$$\Rightarrow \frac{5 \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{12}{1}}{\frac{1}{1}} \Rightarrow \frac{67}{12}$$

8. $f(x) = x^2 - 5x + 6$

$$= x^2 - 3x - 2x + 6$$

$$= x(x-3) - 2(x-3)$$

$$= (x-3)(x-2)$$

so zeroes are 3 and 2.

9. Length of wire = circumference of circle.

$$\text{length of wire} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm.}$$

perimeter of square = length of wire

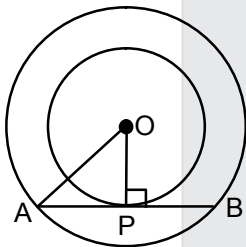
$$4 \times \text{side} = 264$$

$$\text{side} = \frac{264}{4} = 66.$$

10. For unit's digit to be 0. then 4^n should have 2 and 5 as its prime factor, but $4^n = (2^2)^n$ does not contain 5 as one of its factor.
Hence, 4^n cannot end with 0.

11. $\tan 2A = \cot (A - 18)$
 $\cot (90 - 2A) = \cot (A - 18)$
 $\therefore 90 - 2A = A - 18$
 $-3A = -108$
 $A = 36^\circ.$

12. Here $OP = 3$ cm is radius of smaller circle and $OA = 5$ cm is radius of larger circle.

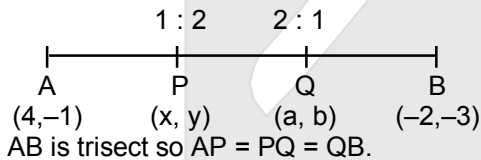


AB chord touches the smaller circle. So $OP \perp AB$ and bisect the AB.

$$\therefore AP = PB.$$

Now, $OA^2 = OP^2 + AP^2$
 $AP^2 = (5)^2 - (3)^2$
 $= 25 - 9 = 16.$
 $AP = \sqrt{16} = 4$ cm.
So, $AB = 2 \times 4 = 8$ cm.

- 13.



AB is trisect so $AP = PQ = QB$.

Now, P divides the segment AB in ratio of $\frac{AP}{PB} = \frac{1}{2}$.

So, coordinates of P(x, y)

$$x = \frac{1 \times (-2) + 2 \times 4}{1 + 2} = \frac{6}{3} = 2.$$

$$y = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2} = -\frac{5}{3}.$$

$$\text{Hence, } P(x, y) = P \left(2, -\frac{5}{3} \right)$$

Now, Q(a, b) divides the segment AB in ratio of $\frac{AQ}{QB} = \frac{2}{1}$.

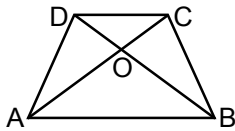
So, coordinates of Q(a, b)

$$a = \frac{2 \times (-2) + 1 \times 4}{2 + 1} = 0$$

$$b = \frac{2 \times (-3) + 1 \times (-1)}{2+3} = -\frac{7}{3}.$$

$$\text{Hence, } Q(a, b) = Q\left(0, -\frac{7}{3}\right).$$

14. ABCD is a trapezium where $AB \parallel CD$ and AC and BD intersect at O.



Now in $\triangle AOB$ and $\triangle COD$

$$\angle OAB = \angle OCD \quad [\text{A.I.A}]$$

$$\angle OBA = \angle ODC \quad [\text{A.I.A}]$$

\therefore By AA criterion

$$\triangle AOB \sim \triangle COD$$

\therefore ratio of area of $\triangle AOB$ and $\triangle COD$

$$\frac{\triangle AOB}{\triangle COD} = \frac{(AB)^2}{(CD)^2}, \text{ but } AB = 2CD$$

$$\therefore \frac{\triangle AOB}{\triangle COD} = \frac{(2 \text{ CD})^2}{(CD)^2} = \frac{4 \text{ CD}^2}{\text{CD}^2} = \frac{4}{1}$$

15. Let us assume that $3 + 2\sqrt{5}$ is rational number.

Hence, $3 + 2\sqrt{5} = \frac{a}{b}$ where a, b are co-prime and $b \neq 0$.

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a-3b}{b}$$

$$\sqrt{5} = \frac{a-3b}{2b}$$

Now, $\frac{a-3b}{2b}$ is a rational number but $\sqrt{5}$ is an irrational number. Since rational \neq irrational.

\therefore our assumption is incorrect. Hence $3 + 2\sqrt{5}$ is irrational.

16. $3x^2 - 2\sqrt{6}x + 2 = 0$

$$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

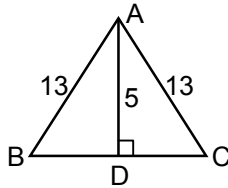
$$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\sqrt{3}x - \sqrt{2} = 0 \quad \Bigg| \quad \sqrt{3}x - \sqrt{2} = 0$$

$$x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$x = \frac{\sqrt{2}}{\sqrt{3}}$$

17. $\triangle ABC$ is an isosceles triangle



where $AB = AC = 13$ cm and altitude $AD = 5$ cm.

In isosceles \triangle altitudes drawn to the unequal side from opposite vertex bisect the side.

$$BD = DC.$$

$$\text{Now, } (BD)^2 = (AB)^2 - (AD)^2 \\ = (13)^2 - (5)^2$$

$$(BD)^2 = 169 - 25 = 144.$$

$$BD = 12$$

Hence, side $BC = 2 \times 12 = 24$ cm.

18. Height of cone = $8 : 4$ cm = h
 base radius of cone = 2.1 cm = r_2
 Now, volume of sphere = volume of cone

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi \times (2.1)^2 \times (8.4)$$

$$4r_1^3 = 2.1 \times 2.1 \times 8.4$$

$$r_1^3 = \frac{2.1 \times 2.1 \times 8.4}{4}$$

$$r_1 = \sqrt[3]{2.1 \times 2.1 \times 2.1}$$

$$r_1 = 2.1 \text{ cm.}$$

19. Dice is thrown once.
 \therefore Total events = $\{1, 2, 3, 4, 5, 6\}$
 Getting an even number.
 Favourable event = $\{2, 4, 6\}$
 Hence probability of even is $P(E) = \frac{3}{6} = \frac{1}{2}$.

20. Condition of collinearity is $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
 $\Rightarrow x(y - 1) + 0 + 1(0 - y) = 0$
 $\Rightarrow xy - x - y = 0$
 $\Rightarrow xy = x + y$
 dividing by xy , we get
 $\Rightarrow \frac{xy}{xy} = \frac{x}{xy} + \frac{y}{xy} \Rightarrow 1 = \frac{1}{y} + \frac{1}{x}$.

- 21.

x	y or f	fx
2	3	6
4	2	8
6	3	18
10	1	10
$P + 5$	2	$2P + 10$
$\Sigma f = 11$		$\Sigma fx = 2P + 52$

$$\begin{aligned} \text{mean} &= \frac{\Sigma fx}{\Sigma f} = \frac{2P + 52}{11} \\ \Rightarrow 6 &= \frac{2P + 52}{11} \\ \Rightarrow 66 &= 2P + 52 \\ \Rightarrow 66 - 52 &= 2P \\ \Rightarrow P &= \frac{14}{2} = 7. \end{aligned}$$

22. Volume of cuboid = $n \times$ volume of spheres

$$\begin{aligned} 9 \times 11 \times 12 &= n \times \frac{4}{3} \pi (r)^3 \\ 9 \times 11 \times 12 &= n \times \frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \\ 11 \times 12 &= n \times \frac{11}{7} \\ n &= 84. \end{aligned}$$

23. A.P. = 3, 10, 17, 24,.....
 where $a = 3$, $d = 10 - 3 = 7$.
 $\therefore a_{13} = a + (13 - 1)d = 3 + 12 \times 7 = 87$
 Now term which is 84 greater than 13th term is
 $a_n = a_{13} + 84 = 87 + 84 = 171$.
 $a + (n - 1)d = 171$.
 $\Rightarrow 3 + (n - 1)7 = 171$
 $\Rightarrow (n - 1)7 = 168$
 $\Rightarrow n - 1 = \frac{168}{7} = 24$.
 $\Rightarrow n = 24 + 1 = 25$.
 So, 25th term is 84 more than 13th term.

24. $T(E) = 52$ and event of having a club card is $F(E) = 13$.
 Hence, probability of event.

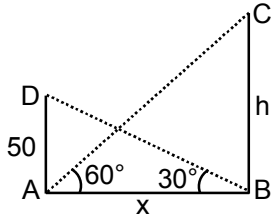
$$P(E) = \frac{13}{52} = \frac{1}{4}.$$

25.

Variable x	f	fx
2	3	6
4	4	16
3	2	6
4	4	28
8	6	48
	$\Sigma f = 19$	$\Sigma fx = 104$

$$\text{Hence, mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{104}{19} = 5.47.$$

26. Let the height of mountain is h m and height of tower is 50m. The distance between tower and mountain is x m.



Now, In $\triangle ADB$

$$\tan 30^\circ = \frac{50}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$\Rightarrow x = 50\sqrt{3} \text{ m and in } \triangle ABC$$

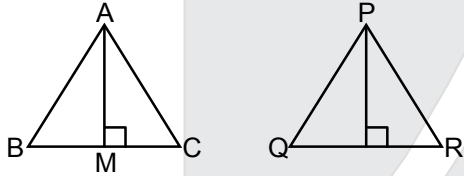
$$\tan 60 = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x \cdot 50\sqrt{3}}$$

$$\Rightarrow h = 50\sqrt{3} \times \sqrt{3} = 150 \text{ m.}$$

Hence, height of mountain is 150 m.

27. Given : two triangles ABC and PQR such that $\triangle ABC \sim \triangle PQR$.



$$\text{To prove : } \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$$

Construction : Draw altitudes AM and PN of $\triangle ABC$ and $\triangle PQR$

$$\text{Proff : area } (\triangle ABC) = \frac{1}{2} \times BC \times AM$$

$$\text{and area } (\triangle PQR) = \frac{1}{2} \times QR \times PN$$

$$\text{So, } \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} \quad \dots\dots\dots(i)$$

Now, In $\triangle ABM$ and $\triangle PQN$

$$\angle B = \angle Q \quad [\because \triangle ABC \sim \triangle PQR]$$

$$\angle M = \angle N \quad [90^\circ \text{ each}]$$

$$\therefore \triangle ABM \sim \triangle PQN \quad [\text{AA-criterion}]$$

$$\text{Therefore } \frac{AM}{PN} = \frac{AB}{PQ} \quad \dots\dots\dots(ii)$$

also $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR} \quad \dots\dots\dots(iii)$$

$$\text{Therefore, } \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ} \quad [\text{by (i) and (ii)}]$$

$$= \frac{AB}{PQ} \times \frac{AB}{PQ} \quad [\text{from (iii)}]$$

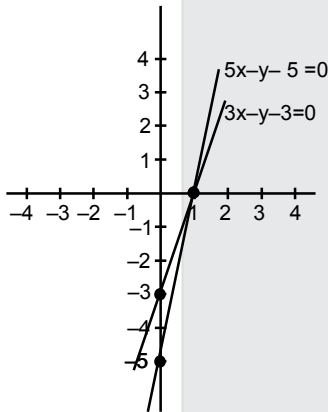
$$= \left(\frac{AB}{PQ}\right)^2$$

Now, by equation (iii)

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2.$$

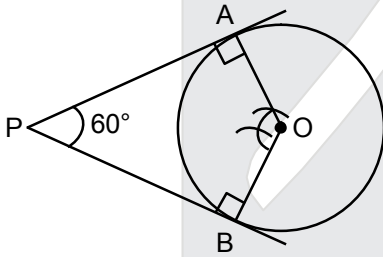
28. $5x - y - 5 = 0$ where $\begin{array}{c|c|c} x & 0 & 1 \\ \hline y & -5 & 0 \end{array}$

and $3x - y - 3 = 0$ where $\begin{array}{c|c|c} x & 0 & 1 \\ \hline y & -3 & 0 \end{array}$



So graphically solution of (x, y) is $(1, 0)$.

29.



30. LHS

$$\frac{\sin \theta}{1 - \cos \theta} + \frac{\tan \theta}{1 + \cos \theta}$$

$$\Rightarrow \frac{\sin \theta (1 - \cos \theta) + \tan \theta (1 - \cos \theta)}{(1 - \cos \theta) (1 + \cos \theta)}$$

$$\Rightarrow \frac{\sin \theta + \sin \theta \cos \theta + \tan \theta - \sin \theta}{1 - \cos \theta}$$

$$\Rightarrow \frac{\sin \theta \cos \theta + \tan \theta}{\sin^2 \theta}$$

$$\Rightarrow \frac{\sin \theta \cos \theta}{\sin^2 \theta} + \frac{\tan \theta}{\sin^2 \theta}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} + \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\Rightarrow \cot \theta + \sec \theta \cdot \operatorname{cosec} \theta = \text{RHS.}$$

31. Let the fraction is $\frac{x}{y}$

Now, from 1st condition

$$\frac{x-1}{y} = \frac{1}{3}$$

$$\Rightarrow 3x - 3 = y \quad \dots\dots\dots(i)$$

and from 2nd condition

$$\frac{x}{y+8} = \frac{1}{4} \quad \dots\dots\dots(ii)$$

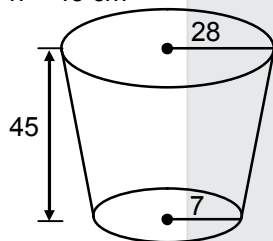
Putting value of y from equation (i)

$$\Rightarrow 4x = 3x - 3 + 8$$

$$\Rightarrow x = 5 \text{ and } y = 12.$$

$$\therefore \text{fraction } \frac{x}{y} = \frac{5}{12}$$

32. R = 28 cm
r = 7 cm
h = 45 cm



So, volume of glass

$$= \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 45 [(28)^2 + (7)^2 + 28 \times 7]$$

$$= \frac{22 \times 15}{7} [784 + 49 + 196]$$

$$= \frac{22 \times 15}{7} \times 1029$$

$$= 22 \times 15 \times 147$$

$$= 48510 \text{ cm}^3.$$

33. Diameter of hemispherical tank = 3 m.

$$\therefore \text{radius} = \frac{3}{2} \text{ m}$$

So capacity of tank

$$= \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$$

$$= \frac{99}{14} \text{ m}^3$$

$$\therefore 1\text{m}^3 = 1000 \text{ litre.}$$

$$\therefore \text{capacity} = \frac{99}{14} \times 1000 \text{ litre.}$$

$$\text{half of tank capacity} = \frac{\frac{99}{14} \times 1000}{2} = \frac{99}{14} \times 500 \text{ litre.}$$

Now,

$\frac{27}{7}$ litre emptied per second so time taken to emptied the half tank is

$$= \frac{\frac{99}{14} \times 500}{\frac{27}{7}} = 990 \text{ seconds.}$$

$$\text{or } \frac{990}{60} \text{ min} = \frac{33}{2} \text{ min.}$$

