ASSAM BOARD
SUBJECT : MATHEMATICS
SUBJECT CODE – C2

B19 – GM (ENGLISH) **CLASS : X**

HINTS & SOLUTIONS

Section - A

1. (a) = $\frac{3}{8} = \frac{3}{2^3}$.

(b) = $\frac{7}{8} = \frac{1}{2^4 \times 5}$

(c) = $\frac{64}{455} = \frac{1}{5 \times 7 \times 13}$

(d) = $\frac{124}{625} = \frac{124}{5^4}$.

Since in option (c) denominator is not of the form $2^m \times 5^n$. So it is non-terminating repeating decimal.

2. $x^2 - 15$

$= \left(x - \sqrt{15}\right)\left(x + \sqrt{15}\right)$

$\therefore$ zeros are $\sqrt{15}$, $-\sqrt{15}$

Product of the zeros = $\sqrt{15} \times \left(-\sqrt{15}\right) = -15$

3.

(i) $3x + 2y = 5$, $2x + 3y = 5$

$\frac{3}{2} \neq \frac{2}{3} \neq \frac{-5}{-5} \rightarrow$ unique solution

$\therefore$ consistent.

(ii) $2x - 3y = 7$, $2x - 3y = 8$

$\frac{2}{2} = \frac{-3}{-3} \neq \frac{-7}{8} \rightarrow$ No solution.

$\therefore$ incosistent.

4. 10, 7, 4,..........

$a = 10$, $d = 7 - 10 = -3$

$a_{30} = 10 + (30 - 1)(-3)$

$10 - 87 = -77$.

option (b).

5. AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm

$(AB)^2 + (BC)^2 = \left(6\sqrt{3}\right)^2 + (6)^2$

$108 + 36 = 144$

$(AC)^2 = (12)^2 = 144$

$\because (AB)^2 + (BC)^2 = (AC)^2$

$\therefore \angle B = 90°$

6. Distance = $\sqrt{(-8-0)^2+(6-0)^2}$

$\sqrt{64+36}$ = $\sqrt{100}$ = 10.

7.

(a) This statement is wrong

$\because$ tan 45° = 1 and tan 60° = $\sqrt{3}$ = 1.732

(b) No, $\because$ cot 60° = $\frac{1}{\sqrt{3}}$ = 0.577 < 1.

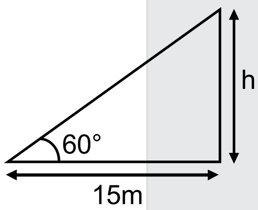
(c) No, since value of sinθ lies between 1 to – 1.

(d) Yes, secθ = $\frac{\text{Hypotenuse}}{\text{Base}}$.

Hypotenuse > Base

$\therefore$ secA = $\frac{12}{5}$ can be possible.

8. tan 60° = $\frac{h}{15}$



$\sqrt{3}$ = $\frac{h}{15}$

h = $15\sqrt{3}$ m

Option (b).

9. 2

Option (b) is correct.

10. θ = 1

Area of sector = $\frac{\theta}{360} \times \pi r^2$

$\frac{1}{360} \times \pi r^2 = \frac{\pi r^2}{360}$

Option (d).

11. Volume of sphere = 288π cm^3

$\frac{4}{3} \times \pi r^3 = 288\pi$

$r^3 = \frac{288 \times 3}{4}$

$r^3 = 216 \quad \Rightarrow \quad r = 6$ cm.

Option (c) is correct.

12. Probability of impossible event is O.
Option (a).

Section - B

13. $96 = 2^5 \times 3$
$404 = 2^2 \times 101$
HCF = $2^2 = 4$
LCM = $2^5 \times 3 \times 101 = 9696$.

14. Let width be x
length = x + 4
$\frac{1}{2} \times 2(\ell + b) = 36$
2x = 32
x = 16
Length = 20 m, Breadth = 16 m.

15. $a_{11} = 38$
$a + 10d = 38$ ....(i)
$a_{16} = 73$
$a + 15d = 73$ ....(ii)
Solving equation (i) and (ii) we get
d = 7.

16. Let the point be P (x, 0)
A(2, – 5), B(– 2, 9)
ATP
AP = BP
$AP^2 = BP^2$
$(x - 2)^2 + (0 + 5)^2 = (x + 2)^2 + (0 - 9)^2$
$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$
8x = – 56
x = – 7
$\therefore$ P(– 7, 0).

17. $\sin A = \frac{3}{4} = \frac{P}{H}$
$\therefore$ $B = \sqrt{(4)^2 - (3)^2} = \sqrt{16 - 9} = \sqrt{7}$
$\cos A = \frac{\sqrt{7}}{4}$, $\cot A = \frac{\sqrt{7}}{3}$.

18. $\frac{\sin 30° + \tan 45° - \text{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$
$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$
$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4} = \frac{43 - 24\sqrt{3}}{11}$.

19. tan 2A = cot (A – 18°)

cot (90 – 2A) = cot (A – 18°)

90 – 2A = A – 18

3A = 108°

A= 36°.

20. LHS = $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$\frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\ \cos A}$

$\frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\ \cos A}$

$\frac{2+2\sin A}{(1+\sin A)\ \cos A}$ $(\because \sin^2 A + \cos^2 A = 1)$

$\frac{2(1+\sin A)}{(1+\sin A)\ \cos A} = \frac{2}{\cos A}$ = 2 secA.

21. Red balls = 3

Black balls = 5

Total balls = 8.

(a) P(red) = $\frac{3}{8}$

(b) P(not red) = $\frac{5}{8}$

Section - C

22. Let us assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that $\sqrt{2} = \frac{a}{b}$ where, a and b are coprimes i.e. their HCF is1.

$\Rightarrow \quad (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \quad \Rightarrow \quad 2 = \frac{a^2}{b^2}$

$\Rightarrow \quad a^2 = 2b^2 \quad \Rightarrow \quad a^2$ is a multiple of 2

$\Rightarrow$ a is a multiple of 2 .....(i)

a = 2c for some integer c.

$\Rightarrow \quad a^2 = 4c^2 \quad \Rightarrow \quad 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2 \quad \Rightarrow \quad b^2$ is a multiple of 2

$\Rightarrow$ b is a multiple of 2 ....(ii)

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

23. $p(x) = x^3 - 3x^2 + 5x - 3$, $q(x) = x^2 - 2$

$$
\begin{array}{r|l}
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \quad (x - 3 \\
 & \underline{\overset{-}{x^3} \qquad\quad \overset{+}{-\,2x}} \\
 & -3x^2 + 7x - 3 \\
 & \underline{\overset{+}{-3x^2} \qquad\quad \overset{-}{+\,6}} \\
 & 7x - 9
\end{array}
$$

Quotient = x – 3

Remainder = 7x – 9.

24. Let the fraction be $\frac{x}{y}$

ATP

$\frac{x+1}{y-1}=1$

x + 1 = y – 1

x – y = – 2 ....(i)

$\frac{x}{y-1}=\frac{1}{2}$

2x = y + 1

2x – y = 1 ....(ii)

Solving equation (i) and (ii) we get

x = 3, y = 5

$\therefore$ fraction = $\frac{3}{5}$.

25. $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\sqrt{2}x\left(x+\sqrt{2}\right)+5\left(x+\sqrt{2}\right)=0$

$\left(x+\sqrt{2}\right)\left(\sqrt{2}x+5\right)=0$

x = $-\sqrt{2}$, x = $\frac{-5}{\sqrt{2}}$.

26. Let two natural numbers be x and y

ATP

$x^2 - y^2 = 180$ ....(i)

$y^2 = 8x$ ....(ii)

From equation (i) and (ii), we get

$x^2 - 8x = 180$

$x^2 - 8x - 180 = 0$

$x^2 - 18x + 10x - 180 = 0$

x(x – 18) + 10(x – 18) = 0

(x – 18) (x + 10)

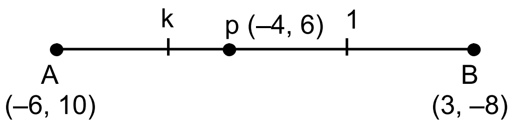
x = 18, x = – 10 (Reject)

$y^2 = 8 \times 18$

y = 12

$\therefore$ Two natural numbers are 18 and 12.

27. Let the ratio be K : 1



$\frac{3k-6}{k+1}$ = – 4

3k – 6 = – 4k – 4

7k = 2

$k = \frac{2}{7}$

$\therefore$ k : 1 = $\frac{2}{7}$: 1 = 2 : 7.

28. d = 7

$a_{22} = 149$

a + 21d = 149

a + 21(7) = 149

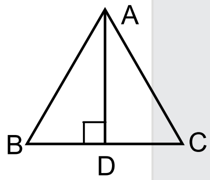
a = 2

$S_{22} = \frac{22}{2}$ [2 × 2 + (22 – 1) 7]

= 11 [4 + 147] = 11 × 151

= 1661.

29. Let the side of an equilateral triangle be (2x)



AB = AC = BC = 2x

BD = x

$AD = \sqrt{4x^2 - x^2} = \sqrt{3x^2} = \sqrt{3}x$

To prove $3AB^2 = 4AD^2$

LHS = $3(2x)^2 = 12x^2$

RHS = $4\left(\sqrt{3}x\right)^2 = 12x^2$

LHS = RHS

Hence proved.

30. A(8, 1) B(K, – 4) C(2, – 5)

Points are collinear

$\therefore$ area(ΔABC) = 0

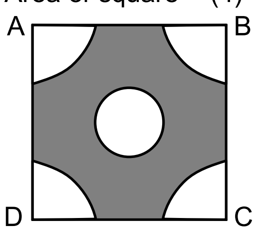
$\frac{1}{2}$ | [8(– 4 – 1) + K (– 5 + 4) + 2(1 + 4)] | = 0

$\frac{1}{2}$ | – 40 – k + 10 | = 0

– 30 – k = 0

k = – 30

31. Area of square = $(4)^2 = 16\ cm^2$



Area of 4 quadrant = $4 \times \frac{1}{4} \times \pi (1)^2 = \frac{22}{7}\ cm^2$

Area of circle = $\frac{22}{7} \times (1)^2 = \frac{22}{7}$

Area of remaining portion of the square = $16 - \frac{22}{7} - \frac{22}{7} = \frac{68}{7}$ cm^2.

Section - D

32. Let $\frac{1}{x-1} = u$, $\frac{1}{y-2} = u = v$

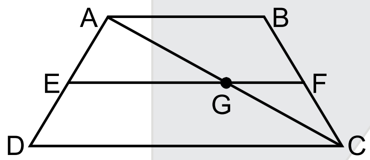
$5u + v = 2$ ....(1)
$6u - 3v = 1$ ....(2)
Multiplying equation (1) by 3
$15u + 3v = 6$ ....(3)
$6u - 3v = 1$ ....(4)
Solving equation (3) and (4) we get
$21u = 7$

$u = \frac{1}{3}$ and $v = \frac{1}{3}$.

$\frac{1}{x-1} = \frac{1}{3}$, $\frac{1}{y-2} = \frac{1}{3}$

$x - 1 = 3$, $y - 2 = 3$
$x = 4$, $y = 5$.

33. Join AC, let it intersect EF at G.



In ΔADC
EG || DC

$\frac{AE}{ED} = \frac{AG}{GC}$ ....(i)

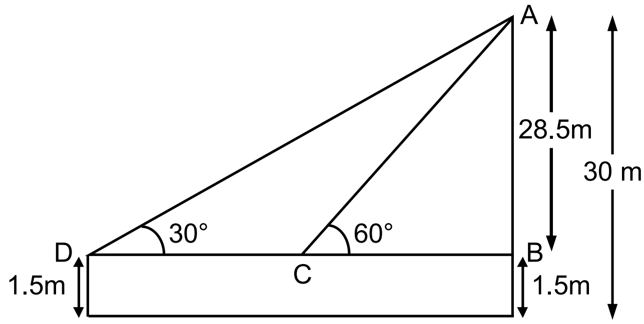
In ΔABC
GF || AB

$\frac{CF}{FB} = \frac{CG}{AG}$ OR $\frac{BF}{FC} = \frac{AG}{CG}$ ....(ii)

From equation (i) and (ii)

$\frac{AE}{ED} = \frac{BF}{FC}$.

34. In ΔABC



$\tan 60° = \frac{28.5}{BC}$

$\sqrt{3} = \frac{28.5}{BC}$

$BC = \frac{28.5}{\sqrt{3}}$

In ΔABD

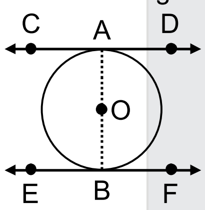
$\tan 30° = \frac{28.5}{DB}$

$\frac{1}{\sqrt{3}} = \frac{28.5}{\frac{28.5}{\sqrt{3}} + DC}$

$\frac{28.5}{\sqrt{3}} + DC = 28.5\sqrt{3}$

$DC = 28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3} = 19\sqrt{3}$ m.

35. CD is the tangent of the circle at the point A and EF is the tangent of the circle at the point B.



$CD \perp DA$

$\angle OAD = 90° \Rightarrow \angle BAD = 90°$ ....(i)

EF is the tangent to the circle at the point B.

$EA \perp OB$

$\angle OBE = 90°$, $\angle ABE = 90°$ ....(ii)

From (i) and (ii)

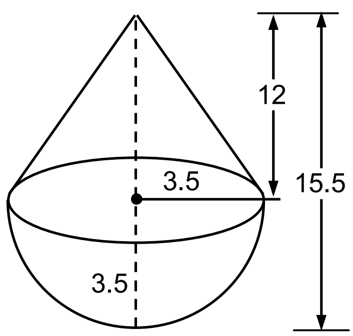
$\angle BAD = \angle ABE = 90°$.

$\because$ alternate angles are equal.

$\therefore$ CD || EF

Section - E

37.



$\ell = \sqrt{(12)^2 + (3.5)^2}$

$\sqrt{144+12.25}$ = $\sqrt{156.25}$ = 12.5 cm.

Total surface area of the toy = $2\pi r^2 + \pi r\ell$

$2\pi(3.5)^2 + \pi(3.5)(12.5)$

= 214.5 cm^2.

OR

Volume of new sphere = $\frac{4\pi}{3}(6)^3 + \frac{4\pi}{3}(8)^3 + \frac{4\pi}{3}(10)^3$

= $\frac{4}{3}\times\frac{22}{7}$ (216 + 512 + 1000)

$\frac{4}{3}\times\frac{22}{7}\times R^3 = \frac{4}{3}\times\frac{22}{7}\times 1728$

$R^2 = 1728$

R = 12 cm

Surface area of the new sphere = $4\pi R^2$

$4 \times \frac{22}{7} \times 12 \times 12$

= 1810.28 cm^2.

38. Modal class = Interval with highest frequency

= 40 – 55

Mode = $\ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

= $40 + \frac{7-3}{14-3-6} \times 15$

= $40 + \frac{4}{5} \times 15$

= 40 + 12 = 52.