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JEE MAIN 2015

ONLINE EXAMINATION

DATE : 10-04-2015

TEST PAPER
WITH SOLUTIONS & ASNWER KEY

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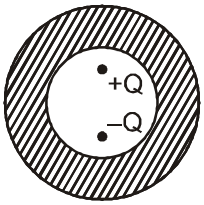
PART - A : PHYSICS

1. Shown in the figure are two point charges +Q and – Q inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then



(1) $\sigma_1 \neq 0, Q_1 = 0$ $\sigma_2 \neq 0, Q_2 = 0$ (2) $\sigma_1 = 0, Q_1 = 0$ $\sigma_2 = 0, Q_2 = 0$ (3) $\sigma_1 \neq 0, Q_1 \neq 0$ $\sigma_2 \neq 0, Q_2 \neq 0$ (4) $\sigma_1 \neq 0, Q_1 = 0$ $\sigma_2 = 0, Q_2 = 0$

Ans. **(2)**

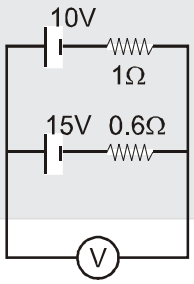
Sol. Net charge inside cavity is zero

$\therefore Q_1 = 0$ and $\sigma_1 = 0$

There is no effect of +Q, –Q and induced charge on inner surface on the outer surface hence

$Q_2 = 0$, $\sigma_2 = 0$

2. A 10V battery with internal resistance 1Ω and a 15V battery with internal resistance 0.6Ω are connected in parallel to a voltmeter (see figure). The reading in the voltmeter will be close to



(1) 11.9 V (2) 13.1 V (3) 12.5 V (4) 24.5 V

Ans. **(2)**

Sol. $E = \dfrac{\frac{10}{1} + \frac{15}{0.6}}{\frac{1}{1} + \frac{1}{0.6}} = \dfrac{10 + 25}{2.67} = 13.1V$

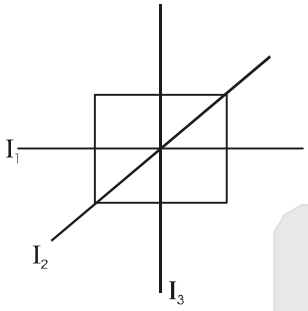
3. Consider a thin uniform square sheet made of a right material . If its side is 'a', mass m and moment of inertia I about one of its diagonals, then :

(1) $I > \frac{ma^2}{12}$ (2) $I = \frac{ma^2}{12}$ (3) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$ (4) $I = \frac{ma^2}{24}$

Ans. **(2)**

Sol. For uniform thin square sheet

$$I_1 = I_2 = I_3 = \frac{ma^2}{12}$$



4. If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water = 10^3 kg/m^3 and viscosity of water = 10^{-3}Pa.s) close to

(1) 11,000 (2) 550 (3) 1100 (4) 5500

Ans. **(4)**

Sol. $\frac{dm}{dt} = SAV$

$$\frac{15}{5\times 60} = 10^3 \times \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4} V$$

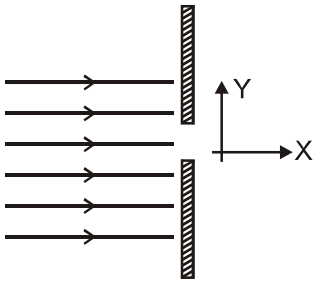
$V = 0.05$ m/s

$$R_e = \frac{SVD}{\mu}$$

$$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}}$$

$\cong 5500$ **Ans.**

5. A parallel beam of electrons travelling in x-direction falls on a slit of width d (see figure). If after passing the slit, an electron acquires momentum p_y in the y-direction then for a majority of electrons passing through the slit (h is Planck's constant) :



(1) $|p_y|\, d > h$ (2) $|p_y|\, d >> h$ (3) $|p_y|\, d < h$ (4) $|p_y|\, d \simeq h$

Ans. **(1)**

Sol. $d \sin\theta = \lambda$.

$$\sin\theta = \frac{\lambda}{d} < 1$$

$$\lambda < d \qquad \left[\lambda = \frac{h}{|p_y|}\right]$$

$$\frac{h}{|p_y|} < d$$

$h < |p_y|\, d$

Option (1)

6. A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force F = sin t N. If the oscillator is at rest in its equilibrium position at t = 0, its position at later times is proportional to :

(1) $\cos t - \frac{1}{2}\sin 2t$ (2) $\sin t + \frac{1}{2}\cot 2t$ (3) $\sin t + \frac{1}{2}\sin 2t$ (4) $\sin t - \frac{1}{2}\sin 2t$

Ans. **(4)**

Sol. $F = ma$

$a \propto \sin t$

$$\frac{dv}{dt} \propto \sin t$$

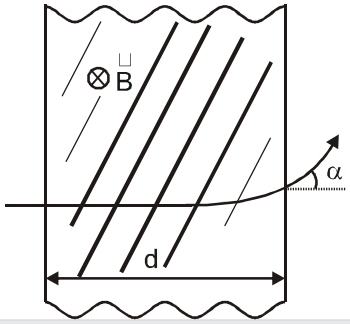
$$\int_0^0 dV \propto \int_0^t \sin t\, dt$$

$V \propto -\cos t + 1$

$$\int_0^x dx = \int_0^t (-\cos t + 1)dt$$

$x = -\sin t + t$

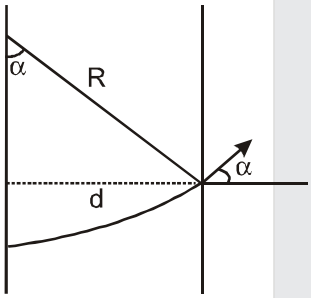
7. A proton (mass m) accelerated by a potential difference V files through a uniform transverses magnetic field B. The field occupies a region of space by width 'd'. If 'α' be the angle of deviation of proton from initial direction of motion (see figure,) the value of sin α will be



(1) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$ (2) $Bd\sqrt{\frac{q}{2mV}}$ (3) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$ (4) $qV\sqrt{\frac{Bd}{2m}}$

Ans. **(2)**

Sol. $\sin\alpha = \frac{d}{R}$



$\sin\alpha = \frac{dqB}{mu}$

$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}$

$\therefore \left[qV = \frac{1}{2}mu^2\right]$

8. A bat moving at $10ms^{-1}$ towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound = $320\ ms^{-1}$)

(1) 8000 (2) 8424 (3) 8258 (4) 8516

Ans. **(4)**

Sol. Frequency incident on wall = $\frac{V}{V-10}f$

Reflected frequency reaching bat = $\frac{V+10}{V-10}f$ = 8516 Hz

9. If the capacitance of a nanocapacitor is measured in terms of a unit 'u' made by combining the electronic charge 'e', Bohr radius 'a_0', Planck's constant 'h' and speed of light 'c' then.

(1) $u = \frac{e^2h}{ea_0}$ (2) $u = \frac{e^2c}{ha_0}$ (3) $u = \frac{hc}{e^2a_0}$ (4) $u = \frac{e^2a_0}{hc}$

Ans. **(4)**

Sol. $[u] = [e]^a[a_0]^b[h]^c[c]^d$

$[M^{-1}L^{-2}T^{+4}A^{+2}] = [A^1T^1]^a[L]^b[ML2T^{-1}]^c[LT^{-1}]^d$

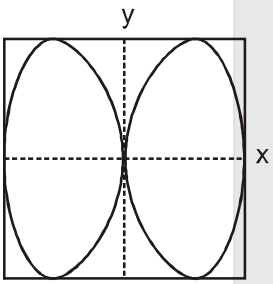
$[M^{-1}L^{-2}T^{+4}A^{+2}] = [M^cL^{b+2c+d}T^{a-c-d}A^a]$

a = 2, b = 1, c = – 1, d = – 1

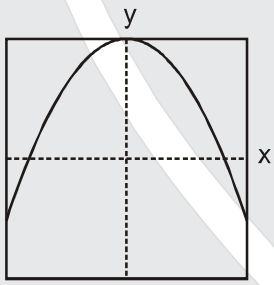
$u = \frac{e^2a_0}{hc}$

10. x and y displacements of a particle are given as x(t) = a sin ωt and y(t) = a sin 2ωt. Its trajectory will look like.

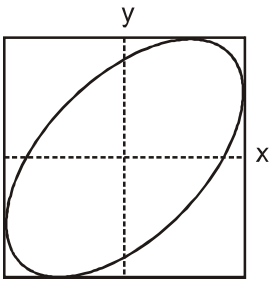
(1)



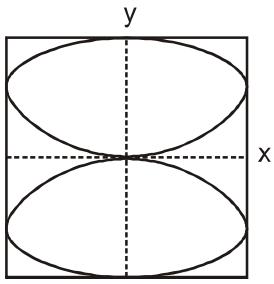
(2)



(3)



(4)



Ans. **(1)**

Sol. At t = 0, x(t) = 0 ; y(t) = 0
x(t) is a
sinusoidal function hence option(1)

At t = $\frac{\pi}{2\omega}$

x (t) = a
y (t) = 0

11. Diameter of a steel ball is measured using a vernier callipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions of its vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as :

S.No.	MS (cm)	VS divisions
1	0.5	8
2	0.5	4
3	0.5	6

If the zero error is – 0.03 cm, then mean corrected diameter is :
(1) 0.53 cm (2) 0.56 cm (3) 0.59 cm (4) 0.52 cm

Ans. **(3)**

Sol. Least count = 0.01 cm
$d_1 = 0.5 + 8 \times 0.01 + 0.03 = 0.61$ cm
$d_2 = 0.5 + 4 \times 0.01 + 0.03 = 0.57$ cm
$d_3 = 0.5 + 6 \times 0.01 + 0.03 = 0.59$ cm

Mean diameter = $\frac{0.61+0.57+0.59}{2} = 0.59$ cm

12. In an unbiased n-p junction electrons diffuse from n-regionto p-region because :
(1) electrons travel across the junction due to potential difference
(2) electron concentration in n-region is more as compared to that in p-region
(3) only electrons move from n to p region and not the vice-versa
(4) holes in p-region attract them

Ans. **(2)**

Sol. T

13. A block of mass m = 0.1 kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3ms^{-1}. the total initial energy of the spring is
(1) 0.3 J (2) 0.6 J (3) 1.5 J (4) 0.8 J

Ans. **(2)**

Sol. Apply momentum conservation

$0.1u + m(0) = 0.1(0) + m(3)$

$$\frac{1}{2}0.1u^2 = \frac{1}{2}m(3)^2$$

Solving $u = 3$

$$\frac{1}{2}kx^2 = \frac{1}{2}K\left(\frac{x}{2}\right)^2 + \frac{1}{2}(0.1)3^2$$

$$\frac{3}{4}Kx^2 = 0.9$$

$$\frac{1}{2}Kx^2 = 0.6J$$

14. In an ideal gas at temperature t, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is

(1) $\frac{1}{4}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

Ans. **(4)**

Sol. Pressure α V_{rms}^2

Force α $V_{rms}^2 \propto T$

15. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R (R << L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then :

(1) $T \propto r$ (2) $T \propto \sqrt{r}$ (3) $T \propto r^2$ (4) $T^2 \propto r^3$

Ans. **(1)**

Sol. $F = \frac{2GM}{Lr}m$

$$mr\left(\frac{2\pi}{T}\right)^2 = \frac{2GMm}{Lr}$$

$T \propto r$

16. de-Broglie wavelength of an electron accelerated by a voltage of 50 V is close to ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ Js) :

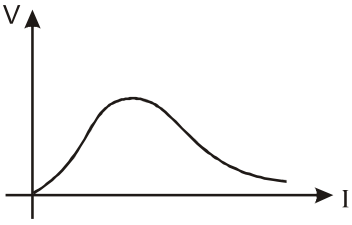
(1) 1.2 Å (2) 2.4 Å (3) 0.5 Å (4) 1.7 Å

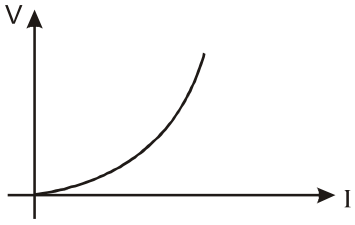
Ans. **(4)**

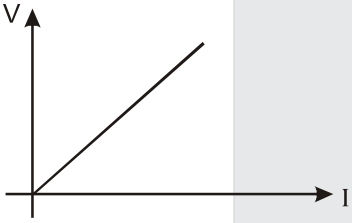
Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

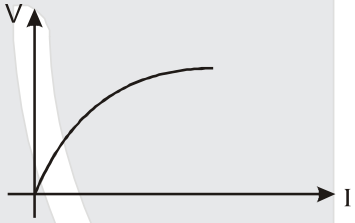
$= 1.7\text{Å}$

17. Suppose the drift velocity υ_d in a material varied with the applied electric field E as $\upsilon_d \propto \sqrt{E}$. Then V – I graph for a wire made of such a material is best given by :

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. $i = neAV_d$

$i \propto \sqrt{E}$

$i^2 \propto E$

$i^2 \propto V$

18. When current in a coil changes from 5 A to 2 A in 0.1 s, average voltage of 50V is produced. The self - inductance of the coil is :

(1) 1.67 H (2) 6H (3) 3 H (4) 0.67 H

Ans. **(1)**

Sol. $EMF = \frac{Ldi}{dt}$

$50 = L\left(\frac{5-2}{0.1\text{sec}}\right)$

$\Rightarrow L = \frac{50 \times 0.1}{3} = \frac{5}{3} = 1.67\text{H}$

19. If one were to apply Bohr model to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a magnetic field 'B', the energy of the charged particle in the nth level will be :

(1) $n\left(\frac{hqB}{4\pi m}\right)$ (2) $n\left(\frac{hqB}{\pi m}\right)$ (3) $n\left(\frac{hqB}{2\pi m}\right)$ (4) $n\left(\frac{hqB}{8\pi m}\right)$

Ans. **(1)**

Sol. $qVB = \frac{mv^2}{r}$ ....(i)

$\frac{nh}{2\pi} = mvr$ ....(ii)

multiply both $\frac{qBnh}{2\pi} = m^2v^2$

$n\frac{qBh}{4\pi m} = \frac{1}{2}mv^2$

$KE = n\left[\frac{qBh}{4\pi m}\right]$

20. A 25 cm long solenoid has radius 2 cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetization $\vec{M}$ (magnetic moment/volume), then $|\vec{M}|$ is :

(1) $3\pi\ Am^{-1}$ (2) $30000\pi\ Am^{-1}$ (3) $300\ Am^{-1}$ (4) $30000\ Am^{-1}$

Ans. **(4)**

Sol. $\vec{M}$ (mag. moment / volume) $= \frac{NiA}{A\ell}$

$= \frac{Ni}{\ell}$

$= \frac{(500)15}{25\times10^{-2}}$

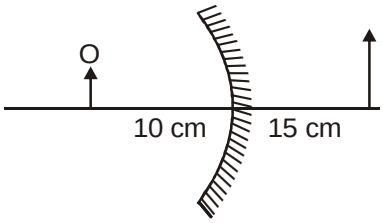
$= 30000\ Am^{-1}$

21. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be :

(1) 60 cm (2) 24 cm (3) 30 cm (4) – 24 cm

Ans. **(1)**

Sol.

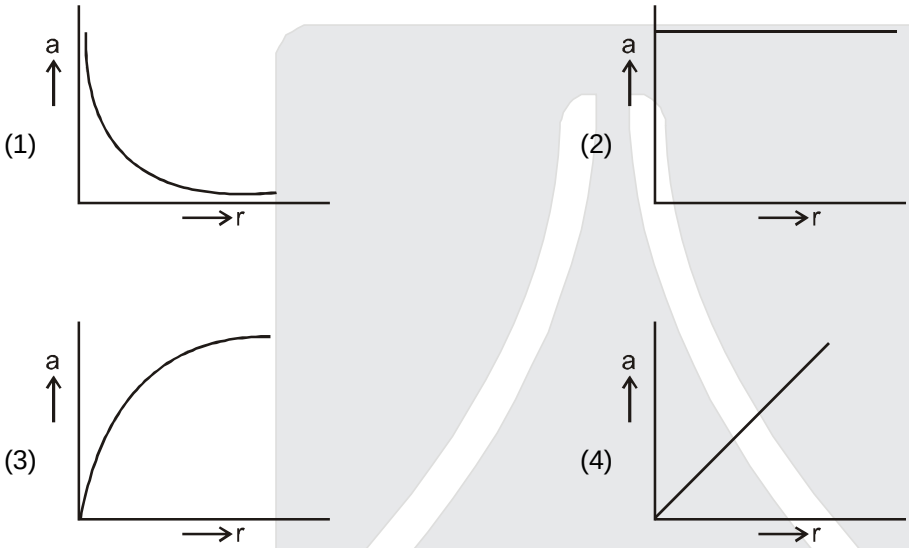


$$\frac{1}{15} + \frac{1}{(-10)} = \frac{1}{f}$$

f = –30 cm

R = – 60 cm

22. If a body moving in a circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between acceleration and radius ?



Ans. **(1)**

Sol. V = constant

$$a = \frac{V^2}{r}$$

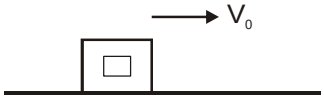
r_a = constant

23. A block of mass m = 10 kg rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed υ, that gets embedded in it, the block moves and come to stop after moving a distance of 2m on the table. If a freely falling object were to acquire speed $\frac{\upsilon}{10}$ after being dropped from height H, then neglecting energy losses and taking g = 10 ms^{-2}, the value of H is close to :

(1) 0.2 km (2) 0.3 km (3) 0.5 km (4) 0.4 km

Ans. **(Bonus)**

Sol. Momentum conservation $0.05V = 10V_0$



$0 - V_0^2 = 2(-\mu g)2$

$V_0 = \sqrt{2}$

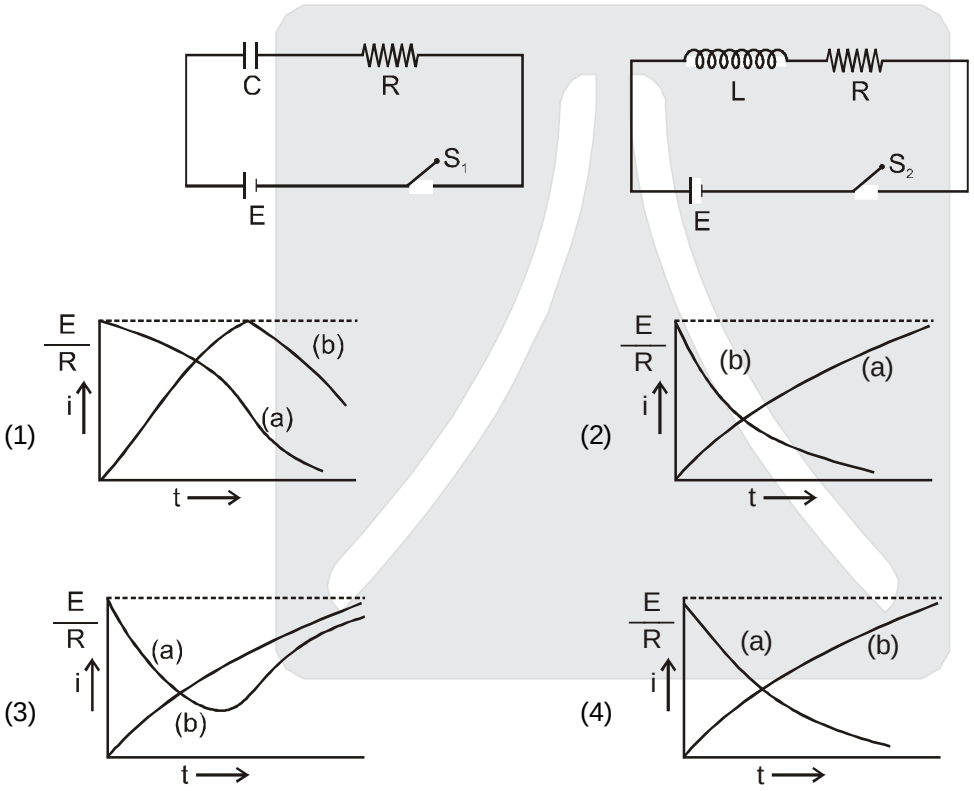
object falling from height H.

$\frac{V}{10} = \sqrt{2gH}$

H = 40 m = 0.04 km

No option match (Bonus)

24. In the circuits (a) and (b) switches S_1 and S_2 are closed at t = 0 and are kept closed for a long time. The variation of currents in the two circuits for $t \geq 0$ are roughly shown by (figures are schematic and not drawn to scale :

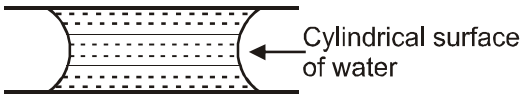


Ans. **(4)**

Sol. For capacitor circuit $i = i_0 e^{-t/RC}$

For inductor circuit $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

25. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by :



(1) $\frac{2T}{R}$ (2) $\frac{4T}{R}$ (3) $\frac{T}{4R}$ (4) $\frac{T}{2R}$

Ans. ()

Sol. Excess pressure = $T\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$

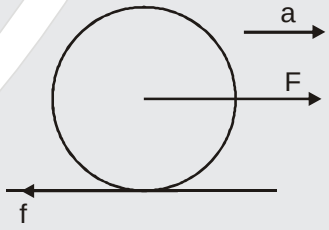
$= \frac{T}{R} \quad \because \quad \begin{pmatrix} r_1 = R \\ r_2 = 0 \end{pmatrix}$

26. A uniform solid cylindrical roller of mass 'm' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and it is rolling without slipping then the value of 'F' is :

(1) ma (2) 2 ma (3) $\frac{5}{3}$ ma (4) $\frac{3}{2}$ ma

Ans. **(4)**

Sol. ma = F – f .....(i)



$\frac{mR^2}{2}\cdot\alpha = fR$

$\frac{mR^2}{2}\frac{a}{R} = fR$

$\frac{ma}{2} = f$ .....(ii)

Put in equation (i)

$ma = F - \frac{ma}{2}$

$F = \frac{3ma}{2}$

27. An electromagnetic wave travelling in the x-direction has frequency of 2×10^{14} Hz and electric field amplitude of 27 Vm^{-1}. From the options given below, which one describes the magnetic field for this wave ?

(1) $\vec{B}(x, t) = (9 \times 10^{-8} T)\hat{j}$ $\sin\left[1.5 \times 10^{-6} x - 2 \times 10^{14} t\right]$

(2) $\vec{B}(x, t) = (9 \times 10^{-8} T)\hat{i}$ $\sin\left[2\pi(1.5 \times 10^{-8} x - 2 \times 10^{14} t)\right]$

(3) $\vec{B}(x, t) = (9 \times 10^{-8} T)\hat{k}$ $\sin\left[2\pi(1.5 \times 10^{-6} x - 2 \times 10^{14} t)\right]$

(4) $\vec{B}(x, t) = (3 \times 10^{-8} T)\hat{j}$ $\sin\left[2\pi(1.5 \times 10^{-8} x - 2 \times 10^{14} t)\right]$

Ans. **(3)**

Sol. $\omega = 2\pi \times 2 \times 10^{14}$ Hz

$B_0 = \frac{E_0}{C} = \frac{27}{3 \times 10^8} = 9 \times 10^{-8}$ Tesla

Oscillation of B can be only along $\hat{j}$ or $\hat{k}$ direction.

$\therefore$ Option (3)

28. A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to :

(1) 15° (2) 60° (3) 30° (4) 1°

Ans. **(2)**

Sol. $MP = \frac{f_0}{f_e} = \frac{150}{5} = 30$

$\theta_0 = \frac{50}{1000} = \frac{1}{20}$ rad

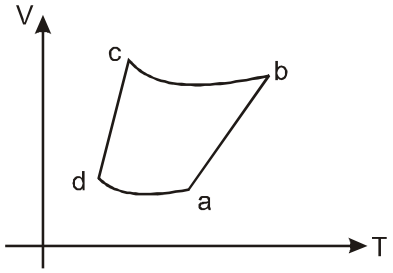
$\theta = MP \times \theta_0$

$= 30 \frac{1}{20}$

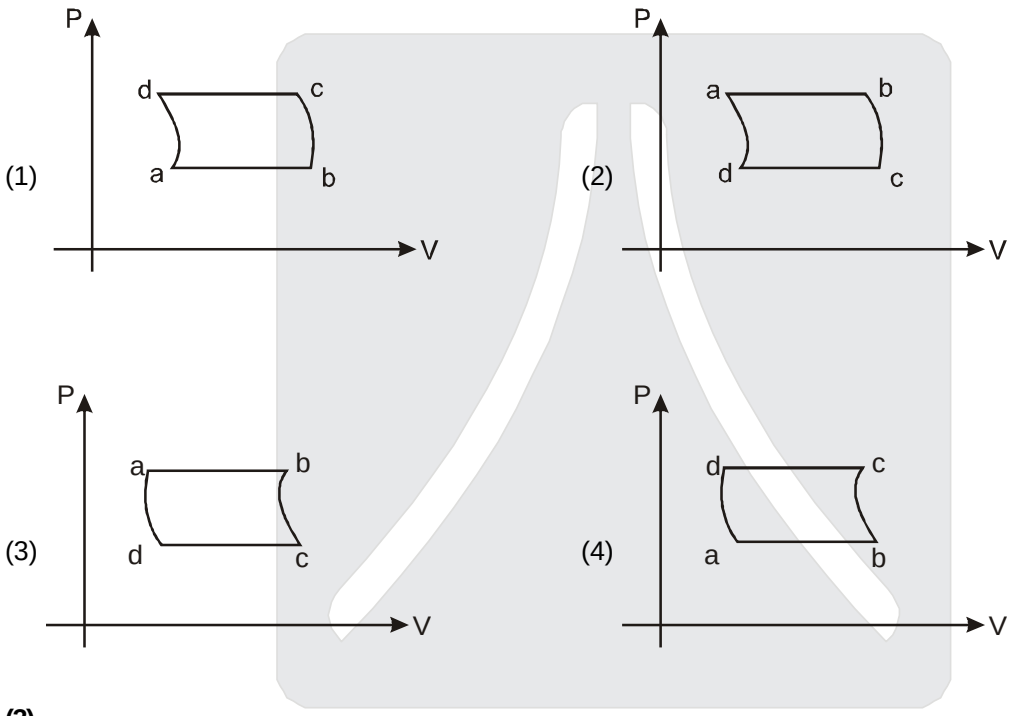
$= \frac{3}{2} = 1.5$ rad

$= 1.5 \times \frac{180}{\pi} = 86^\circ$

29. An ideal gas goes through a reversible cycle a → b → c → d has the V - T diagram shown below. Process d → a and b → c are adiabatic.



The corresponding P - V diagram for the process is (all figures are schematic and not drawn to scale) :



Ans. **(3)**

Sol. In VT graph

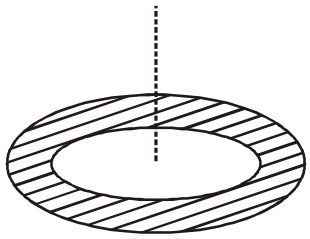
ab-process : Isobaric line passes through origin, temperature increases.

bc process : Adiabatic, pressure decreases.

cd process : isobaric, volume decreases.

da process : Adiabatic, pressure increase.

30. A thin disc of radius b = 2a has concentric hole of radius 'a' in it (see figure). It carries uniform surface charge 'σ' on it. If the electric field on its axis at height 'h' (h << a) from its centre is given as 'Ch' then value of 'C' is :



(1) $\frac{\sigma}{2a\in}$ (2) $\frac{\sigma}{4a\in_0}$ (3) $\frac{\sigma}{8a\in_0}$ (4) $\frac{\sigma}{a\in_0}$

Ans. **(2)**

Sol. Electric field due to complete disc (R = 2a)

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{\sqrt{4a^2+h^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right]$$

Electric field due to disc (R = a)

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{a}\right)$$

Electric field due to given disc.

$E = E_1 - E_2$

$$= \frac{\sigma h}{4\varepsilon_0 a}$$

option (2)

PART - B : CHEMISTRY

1. Photochemical smog consists of excessive amount of X, in addition to aldehydes, ketones, peroxy acetyl nitrile (PAN), and so forth. X is :

(1) CH_4 (2) O_3 (3*) CO_2 (4) CO

Ans. **(3)**

Sol.

2. The reaction $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$ follows first order kinetics. The pressure of a vessel containing only N_2O_5 was found to increase from 50 mm Hg to 87.5 mm Hg in 30 min. The pressure exerted by the gases after 60 min. will be (Assume temperature remains constant) :

(1) 125 mm Hg (2*) 106.25 mm Hg (3) 116.25 mm Hg (4) 150 mm Hg

Ans. **(2)**

Sol. $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

$t = 0 \quad 50 \quad 0 \quad 0$

$t = 30 \quad 50 - 2x \quad 4x \quad x$

$\Rightarrow 87.5 = 50 + 3x$

$\Rightarrow 3x = 37.5 \quad \Rightarrow \quad x = 12.5$

$\Rightarrow P_{N_2O_5}$ after 30 min $= 50 - 25 = 25$

$\Rightarrow t_{1/2} = 30$ min.

Hence after 60 min, (two half lives), $P_{N_2O_5}$ remaining $= \frac{50}{4} = 12.5$ torr.

$\Rightarrow$ Hence decrease in $P_{N_2O_5} = 50 - 12.5 = 37.5$ torr.

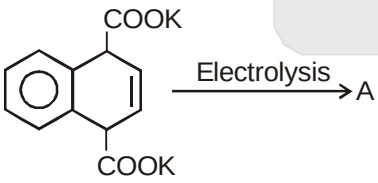
$\Rightarrow P_{NO_2} = 2 \times 37.5 = 75$ torr

$P_{O_2} = \frac{37.5}{2} = 18.75$ torr

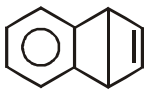
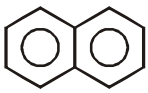
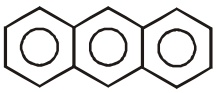
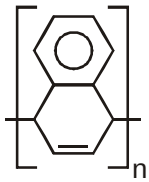
$\Rightarrow P_{total} = 12.5 + 75 + 18.75$

$= 106.25$ torr.

3.



A is :

(1)  (2*)  (3)  (4) 

Ans. (2)

Sol.

4. The correct statement on the isomerism associated with the following complex ions,
(a) $[Ni(H_2O)_5NH_3]^{2+}$, (b) $[Ni(H_2O)_4(NH_3)_2]^{2+}$ and (c) $[Ni(H_2O)_3(NH_3)_3]^{2+}$ is :
(1) (a) and (b) show geometrical and optical isomerism
(2) (b) and (c) show only geometrical isomerism
(3*) (b) and (c) show only geometrical isomerism
(4) (a) and (b) show geometrical and optical isomerism

Ans. **(3)**

Sol. (a) does not show G.I.(b) and (c) show G.I.but all isomers are optically inactive.

5. If the principal quantum number n = 6, the correct sequence of filling of electrons will be :
(1) ns→np→(n – 1)d→(n – 2)f
(2) ns→(n – 1)d→(n – 2)f→np
(3) ns→(n – 2)f→np→(n – 1)d
(4*) ns→(n – 2)f→(n – 1)d→np

Ans. **(4)**

Sol. Following Aufbau principle for filling electrons.

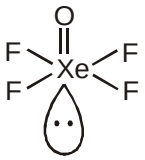
6. After understanding the assertion and reason, choose the correct option.
Assertion : In the bonding molecular orbital (MO) of H_2, electron density is increased between the nuclei.
Reason : The bonding MO is $\psi_A + \psi_B$, which shows destructive interference of the combining electron waves.
(1*) Assertion is correct, reason is incorrect.
(2) Assertion is incorrect, reason is correct.
(3) Assertion and reason are correct, but reason is not the correct explanation for the assertion.
(4) Assertion and reason are correct and reason is the correct and reason is the correct explanation for the assertion.

Ans. **(1)**

Sol. Bonding molecular orbital results in increased electron density between nuclei due to constructive interference of combining electron waves.

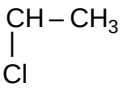
7. The geometry of $XeOF_4$ by by VSEPR theory is :
(1) pentagonal planar
(2) octahedral
(3*) square pyramidal
(4) trigonal bipyramidal

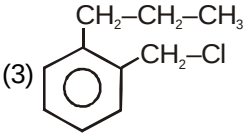
Ans. **(3)**

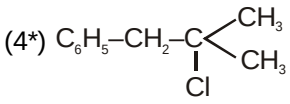
Sol.  sp^3d^2 hybridized with one position occupied by lone pair.

8. A compound A with molecular formula $C_{10}H_{13}Cl$ give a white precipitate on adding silver nitrate solution. A on reacting with alcoholic KOH gives compound B as the main product. B on ozonolysis gives C and D. C gives Cannizaro reaction but not aldol condensation. D gives aldol condensation but not Cannizaro reaction. A is :

(1) $C_6H_5–CH_2–CH_2–CH_2–CH_2–Cl$

(2) 

(3) 

(4*) 

Ans. **(4)**

Sol.

9. The correct order of thermal stability of hydroxides is :
(1) $Ba(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Mg(OH)_2$
(2) $Ba(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Mg(OH)_2$
(3) $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$
(4*) $Mg(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Ba(OH)_2$

Ans. **(4)**

Sol. Larger cation is able to stabilize polyatomic anion more than smaller cation.

10. Which of the following is not an assumption of the kinetic theory of gases ?
(1) Gas particles have negligible volume
(2) A gas consists of many identical particels which are in continual motion
(3*) At high pressure, gas particles are difficult to compress
(4) Collisions of gas particles are perfectly elastic

Ans. **(3)**

Sol. No such assumption is made by KTG.

11. In the presence of small amount of phosphorous, aliphatic carboxylic acids react with chlorine or bromine to yield a compound in which α-hydrogen has been replaced by halogen. This reaction is known as :
(1) Wolff-Kischner reaction (2) Etard reaction
(3) Rosenmund reaction (4*) Hell-Volhard-Zelinsky reaction

Ans. **(4)**

Sol.

12. In the isolation of metals, reaction process usually results in :
(1) Metal sulphide (2) metal carbonate
(3) metal hydroxide (4*) metal oxide

Ans. **(4)**

Sol. Usually calcination results in metal oxides as metal carbonates, hydroxides, all decompose to oxides.

13. In the long form of the periodic table, the valence shell electronic configuration of $5s^25p^4$ corresponds to the element present in :
(1) Group 17 and period 6 (2) Group 17 and period 5
(3) Group 16 and period 6 (4*) Group 16 and period 5

Ans. **(4)**

Sol. Valence shell number indicates period number. ns^2np^4 correspond to group 16.

14. Gaseous N_2O_4 dissociates into gaseous NO_2 according to the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
At 300 K and 1 atm pressure, the degree of dissociation of N_2O_4 is 0.2. If one mole of N_2O_4 gas is contained in a vessel, then the density of the equilibrium mixture is :
(1*) 3.11 g/L (2) 4.56 g/L (3) 1.56 g/L (4) 6.22 g/L

Ans. **(1)**

Sol. $\frac{M_{Th}}{M_{Ob}} = 1 + (2 - 1)\,\alpha = 1.2.$

$\Rightarrow$ $M_{Ob} = \frac{92}{1.2}$

and $d = \frac{PM}{RT} = \frac{1 \times 92}{1.2 \times 0.082 \times 300} = 3.116$ g/L

15. Match the polymers in column-A with their main used in column-B and choose the correct answer :

Column-A	**Column-B**
(A) Polystyrene	(i) Paints and lacquers
(B) Glyptal	(ii) Rain coats
(C) Polyvinyl chloride	(iii) Manufacture of toys
(D) Bakelite	(iv) Computer discs

(1) (A) - (ii), (B) - (i), (C) - (iii), (D)- (iv) (2*) (A) - (iii), (B) - (i), (C) - (ii), (D)- (iv)

(3) (A) - (ii), (B) - (iv), (C) - (iii), (D)- (i) (4) (A) - (iii), (B) - (iv), (C) - (ii), (D)- (i)

Ans. **(2)**

Sol. (A) - Manufacture of toys (iii)

(B) - Points & lacquers (i)

(C) - Rain coats (ii)

(D) - Complete discs. (iv)

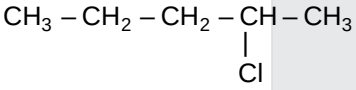
16. The optically inactive compound from the following is :

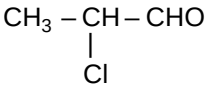
(1) 2-chloropentane (2) 2-chloropropanal

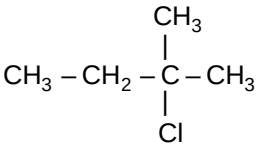
(3*) 2-chloro-2-methylbutane (4) 2-chlorobutane

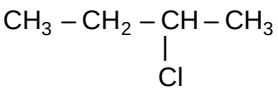
Ans. **(3)**

Sol.









17. Complex hydrolysis of starch gives :

(1*) glucose only (2) glucose and fructose in equimolar

(3) galactose and fructose in equimolar amounts (4) glucose and galactose in equimolar amounts

Ans. **(1)**

Sol. Starch is a polymer of glucose.

18. The heat of atomixation of methane and ethane are 360 kJ/mol and 620 kJ/mol, respectively. The longest wavelength of light capable of breaking the C-C bond is (A vogadro number = 6.02×10^{23}, h = 6.62×10^{-34} J s):

(1) 2.48×10^3 nm (2*) 1.49×10^3 nm (3) 2.49×10^4 nm (4) 2.48×10^4 nm

Ans. **(2)**

Sol. $CH_4(g) \longrightarrow C(g) + 4H(g)$

$\Rightarrow$ $4 \times E_{C-H} = 360$ KJ/Mol. $\Rightarrow$ $E_{C-H} = 90$ KJ/Mol.

and $C_2H_6(g) \longrightarrow 2C(g) + 6H(g)$

$\Rightarrow$ $E_{C-C} + 6 \times 90 = 620$ $\Rightarrow$ $E_{C-C} = 80$ kJ/mol

$\Rightarrow$ $N_A \times \frac{hc}{\lambda} = 80 \times 1000$ J

$$\lambda = \frac{6.02 \times 10^{23} \times 6.62 \times 10^{-34} \times 3 \times 10^8}{80000}$$

$= 14.9 \times 10^{-7}$ m $= 1.49 \times 10^{-6}$ m

$= 1.49 \times 10^3$ nm.

19. The following statements relate to the adsorption of gases on a solid surface. Identify the incorrect statement among them :

(1) On adsorption decrease in surface energy appears as heat

(2) Enthalpy of adsorption is negative

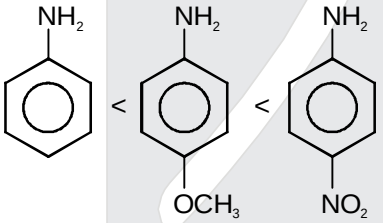
(3*) On adsorption, the residual forces on the surface are increased

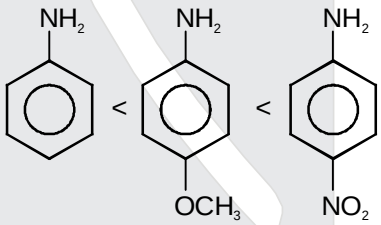
(4) Entropy of adsorption is negative

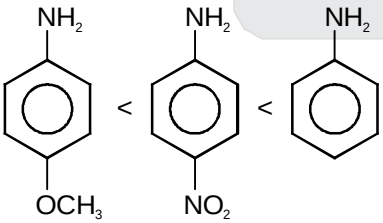
Ans. **(3)**

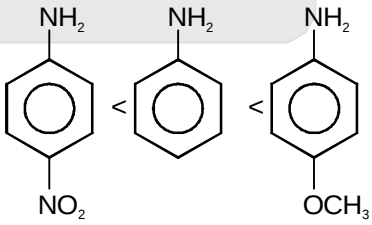
Sol. Adsorption takes place due to the presence of residual forces on the surface. After adsorption, these are decreased.

20. Arrange the following amines in the order of increasing basicity :

(1) CH_3NH_2 < 

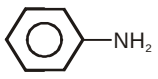
(2)  < CH_3NH_2

(3)  < CH_3NH_2

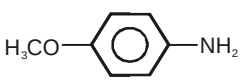
(4*)  < CH_3NH_2

Ans. **(4)**

Sol. $CH_3–NH_2$ pKa = 10.64

 pKa = 4.62

 pKa = 0.98

 pKa = 5.29

21. Permanent hardness in water cannot be cured by :
(1) Treatment with washing soda (2*) Boiling
(3) Ion exchange method (4) Calgon's method

Ans. **(2)**

Sol. Boiling can remove only temporary hardness caused by bicarbonates of Ca^{2+}, Mg^{2+}.

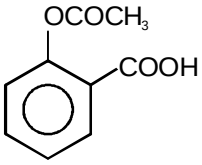
22. A solution at 20ºC is composed of 1.5 mol of benzene and 3.5 mol of toluene. If the vapour pressure of pure benzene and pure toluene at this temperature are 74.7 torr and 22.3 torr, respectively, then the total vapour pressure of the solution and the benzene mole fraction in equilibrium with it will be, respectively :
(1) 35.0 torr and 0.480 (2) 30.5 torr and 0.389
(3*) 38.0 torr and 0.589 (4) 35.8 torr and 0.280

Ans. **(3)**

Sol. $x_{Benzene} = \frac{1.5}{5} = 0.3$, $x_{toluene} = 0.7$

$\Rightarrow$ $P_T = 74.7 \times 0.3 + 0.7 \times 22.3$
$= 22.41 + 15.61$
$= 38.02$ torr

and $y_{benzene} = \frac{22.41}{38.02} = 0.589$

23. 

is used as :

(1) Antihistamine (2) Antacid (3) Insecticide (4*) Analgesic

Ans. **(4)**

Sol. Aspirin is non-narcotic analgesic.

24. The cation that will not be precipitated by H_2S in the presence of dil HCl is:
(1) Pb^{2+} (2) As^{3+} (3*) Co^{2+} (4) Cu^{2+}

Ans. **(3)**

Sol. Co^{2+} is precipitated when we have sufficient S^{2-} concentration.

25. The least number of oxyacids are formed by:
(1) Nitrogen (2*) Fluorine (3) Chlorine (4) Sulphur

Ans. **(2)**

Sol. Fluorine only forms HOF as it cannot show multiple oxidation states.

26. Which molecule/ion among the following cannot act as a ligand in complex compounds?
(1*) CH_4 (2) CN^- (3) Br^- (4) CO

Ans. **(1)**

Sol. CH_4 does not have lone pair.

27. A sample of a hydrate of barium chloride weighing 61 g was heated until all the water of hydration is removed. The dried sample weighed 52g. The formula of the hydrated salt is: (atomic mass, Ba = 137 amu, Cl = 35.5 amu)

(1) $BaCl_2 + H_2O$ (2) $BaCl_2 + 4H_2O$ (3) $BaCl_2 + 3H_2O$ (4*) $BaCl_2 + 2H_2O$

Ans. **(4)**

Sol. $BaCl_2.xH_2O \longrightarrow BaCl_2 + xH_2O.$

$m_{H_2O} = 61 - 52 = 9g$

$\Rightarrow \quad n_{H_2O} = \frac{9}{18} = \frac{1}{2}$

$m_{BaCl_2} = 52 \quad \Rightarrow \quad n_{BaCl_2} = \frac{52}{208} = \frac{1}{4} \Rightarrow$ simplest formula $= \frac{1}{4} : \frac{1}{2} = 1 : 2 \Rightarrow BaCl_2.2H_2O$

28. A variable, opposite external potential (E_{ext}) is applied to the cell $Zn|Zn^{2+}$ (1 M) || Cu^{2+} (1M) | Cu, of potential 1.1 V. When $E_{ext} < 1.1$ V and $E_{ext} > 1.1$ V respectively electrons flow from :

(1) Cathode to anode in both cases (2) cathode to anode and anode to cathode

(3) anode to cathode and cathode to anode (4*) anode to cathode in both cases

Ans. **(4)**

Sol. Electrons flow from anode to cathode always.

29. 1.4 g of an organic compound was digested according to Kjeldahl's method and the ammonia evolved was absorbed in 60 mL of M/10 H_2SO_4 solution. The excess sulphuric acid required 20 mL of M/10 NaOH solution for neutralization. The percentage of nitrogen in the compound is :

(1) 24 (2) 5 (3*) 10 (4) 3

Ans. **(3)**

Sol. Organic compound $\longrightarrow NH_3$
(1.4g)

$2NH_3 + H_2SO_4 \longrightarrow (NH_4)2SO_4$

$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

$n_{NH_3} + 20 \times \frac{1}{10} \times \frac{1}{1000} = 60 \times \frac{1}{10} \times 2 \times \frac{1}{1000}$

$n_{NH_3} = \frac{12}{1000} - \frac{2}{1000} = \frac{10}{100}$

$n_N = n_{NH_3} = 0.01 \quad \Rightarrow \quad m_N = 0.01 \times 14 = 0.14$ g $\quad \Rightarrow \quad$ % of N $= \frac{0.14}{1.4} \times 100 = 10\%.$

30. An aqueous solution of a salt X turns blood red on treatment with SCN^- and blue on treatment with $K_4[Fe(CN)_6]$. X also gives a positive chromyl chloride test. The salt X is :

(1) $CuCl_2$ (2) $Cu(NO_3)_2$ (3*) $FeCl_3$ (4) $Fe(NO_3)_3$

Ans. **(3)**

Sol. $FeCl_3$ gives chromyl chloride test,

$Fe^{3+} + SCN^- \longrightarrow$ blood red color.

and $Fe^{3+} + K_4[Fe(CN)_6] \longrightarrow \underset{\text{(blue)}}{Fe_4[Fe(CN)_6]_3}$

PART - C : MATHEMATICS

1. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is :

(1) 1880 (2) 1120 (3) 1240

(4) 1960

Ans. **(3)**

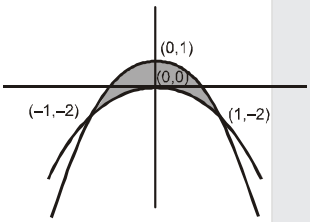
Sol. $\frac{15\times16\times31}{6} = 1240$

2. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to :

(1) $\frac{1}{3}$ (2) $\frac{3}{4}$ (3) $\frac{3}{5}$ (4*) $\frac{4}{3}$

Ans. **(4)**

Sol. $\left.\begin{matrix} y+2x^2=0 \\ y+3x^2=1 \end{matrix}\right\}$ are parabola



point of intersection of these two curves are (1, – 2) & (–1, –2)

Area bounded by these two curves $= 2\int_0^1 \{(1-3x^2)-(-2x^2)\}\,dx$

$= 2\int_0^1 (1-x^2)\,dx$

$= 2\left|x-\frac{x^3}{3}\right|_0^1$

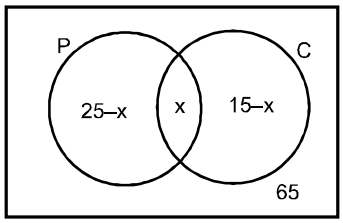
$= 2\left(\frac{2}{3}-0\right)$

$= \frac{4}{3}$

3. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements :
(a) 5% families own both a car and a phone
(b) 35% families own either a car or a phone
(c) 40,000 families live in the town
Then,
(1) Only (b) and (c) are correct
(2) Only (a) and (c) are correct
(3*) All (a), (b) and (c) are correct
(4) Only (a) and (b) are correct

Ans. **(3)**

Sol.



$65 + 25 - x + x + 15 - x = 100 \Rightarrow 105 - x = 100 \Rightarrow x = 5$

and $\frac{5}{100} \times 40000 = 2000$
family live in the town.

4. If y(x) is the solution of the differential equation $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to :
(1) 2
(2*) 0
(3) –1
(4) 1

Ans. **(2)**

Sol. $(x + 2)\frac{dy}{dx} = (x + 2)^2 - 13 \qquad \frac{dy}{dx} = (x + 2) - \frac{13}{x+2}$

$y = \frac{x^2}{2} + 2x - 13\,\ell n(x + 2) + C$ at $x = 0, y = 0 \Rightarrow c = 13\,\ell n2$

$y = \frac{x^2}{2} + 2x - 13\,\ell n|x + 2| + 13\,\ell n2$

Now $y(-4) = 8 - 8 - 13\ell n|-4 + 2| + 13\ell n2 = 0$

5. $\lim\limits_{x\to 0} \frac{e^{x^2} - \cos x}{\sin^2 x}$ is equal to :

(1) 2
(2) 3
(3) $\frac{5}{4}$
(4*) $\frac{3}{2}$

Ans. **(4)**

Sol. $\lim\limits_{x\to 0} \frac{\left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + ...\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...\right)}{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...\right)^2}$

$= \frac{3}{2}$

6. If Roll's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = \frac{1}{2}$, then $2b + c$ equals :

(1) 1 (2) 2 (3*) –1 (4) –3

Ans. **(3)**

Sol. $f(1) = f(-1)$

$2 + b + c = -2 + b - c \Rightarrow c = -2$

$f'(x) = 6x^2 + 2bx + c$

$= 6\left(\frac{1}{2}\right)^2 + 2b\left(\frac{1}{2}\right) + c$

$= \frac{3}{2} + b = c = 0 \Rightarrow b = \frac{1}{2}$

Now $2b + c = 1 - 2 = -1$

7. If the tangent to the conic, $y - 6 = x^2$ at (2, 10) touches the circle, $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β); then (α, β) is :

(1) $\left(-\frac{7}{17}, \frac{6}{17}\right)$ (2) $\left(-\frac{6}{17}, \frac{10}{17}\right)$ (3) $\left(-\frac{4}{17}, \frac{1}{17}\right)$ (4*) $\left(-\frac{8}{17}, \frac{2}{17}\right)$

Ans. **(4)**

Sol. $y' = 2x$ at (2, 10), $y' = 4$

tangent $y - 10 = 4(x - 2)$

$\Rightarrow y = 4x + 2 \Rightarrow 4x - y + 2 = 0$

Pass $(\alpha, \beta) \Rightarrow 4\alpha - \beta + 2 = 0 \Rightarrow \beta = 4\alpha + 2$ ..... (1)

and $2x + 2y\,y' + 8 - 2\,y' = 0$

$y' = \frac{2x + 8}{2 - 2y} = \frac{2\alpha + 8}{2 - 2\beta} = 4$ ..... (2)

from 1 & 2 we get $\alpha = \frac{-8}{17}$, $\beta = \frac{2}{17}$

8. Let X be a set containing 10 elements and P(X) be its power set. If A and B are picked up at random from P(X), with replacement, then the probability that A and B have equal number of elements, is :

(1) $\frac{(2^{10} - 1)}{2^{20}}$ (2) $\frac{{}^{20}C_{10}}{2^{10}}$ (3) $\frac{(2^{10} - 1)}{2^{10}}$ (4*) $\frac{{}^{20}C_{10}}{2^{20}}$

Ans. **(4)**

Sol. Total number of subsubsets of set $X = 2^{10} = 1024$

number of subsets with one element $= {}^{10}C_1$

Number of subsets with two elements $= {}^{10}C_2$

:

:

Number of subsets with 10 elements $= {}^{10}C_{10}$

A & B are taken from P(X) from 2^{10} subsets so total ways $= 2^{10}, 2^{10}$

Number of ways such that A and B have equal number of elements =

$({}^{10}C_0)^2 + ({}^{10}C_1)^2 + ({}^{10}C_2)^2 + ... + ({}^{10}C_{10})^2$

$= {}^{20}C_{10}$

Probability $= \frac{{}^{20}C_{10}}{2^{10}.2^{10}}$

9. The distance, from the origin, of the normal to the curve, $x = 2\cos t + 2t\sin t$, $y = 2\sin t - 2t\cos t$ at $t = \frac{\pi}{4}$, is :

(1) 4 (2) $\sqrt{2}$ (3*) 2 (4) $2\sqrt{2}$

Ans. **(3)**

Sol. $\frac{dx}{dt} = -2\sin t + 2\sin t + 2t\cos t = 2t\cos t$

$\frac{dy}{dt} = 2\cos t - 2\cos t + 2t\sin t = 2t\sin t$

so $\frac{dy}{dx} = \tan t$

slope of normal $= -\frac{1}{\tan t}$

$m|_{t=\frac{\pi}{4}} = -1$

equation of normal $a + t = \frac{\pi}{4}$

$y - \sqrt{2} + \frac{\pi}{2\sqrt{2}} = -1\left(x - \sqrt{2} - \frac{\pi}{2\sqrt{2}}\right)$

$x + y - 2\sqrt{2} = 0$

distance of normal form

$= \left|\frac{-2\sqrt{3}}{\sqrt{1+1}}\right| = 2$

10. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is **not** correct ?

(1) $A^3 - I = A(A - I)$ (2) $A^3 + I = A(A^3 - I)$ (3*) $A^2 + I = A(A^2 - I)$ (4) $A^4 - I = A^2 + I$

Ans. **(3)**

Sol. $A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

$A^3 = -A$

$A^4 = -A^2 = I$

$A^5 = A$

Now

(1) $A^3 - I = -A - I$

$A(A - I) = A^2 - A = -I - A$

(2) $A^3 + I = -A + I$

$A(A^3 - I) = A(-A - I) = -A^2 - A = I - A$

(3) $A^2 + I = -A(I + I) = -2A$

$A^2 + I \neq A(A^2 - I)$

(4) $A^4 - I = I - I = 0$

$A^2 + I = -I + I = 0$

11. The contrapositive of the statement "If it is raining, then I will not come", is :

(1) If I will come, then it is raining (2) If I will not come, then it is not raining

(3) If I will not come, then it is raining (4*) If I will come, then it is not raining

Ans. **(4)**

Sol. Let p : It is raining

q : I will not come

contrapositive of $p \rightarrow q$

is $\sim q \rightarrow \sim p$

$\Rightarrow$ If I will come then if is not raining

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is equal to:

(1*) $\sqrt{55}$ (2) $\sqrt{51}$ (3) $\sqrt{43}$ (4) $\sqrt{37}$

Ans. **(1)**

Sol. $|\vec{a} + \vec{b}| = \sqrt{3}$

$\Rightarrow |\vec{a} + \vec{b}|^2 = 3$

$\Rightarrow 1 + 1 + 2.1.1 \cos\theta = 3$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$ ..... (1)

$\because |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a} . \vec{b}|^2$

$\Rightarrow |\vec{a} + \vec{b}|^2 = 1 - \left(\frac{1}{2}\right)^2$

$\Rightarrow |\vec{a} + \vec{b}|^2 = \frac{\sqrt{3}}{2}$

$|\vec{c}| = |\vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})|^2$

$= (\vec{a} + 2\vec{b} + 3|\vec{a} \times \vec{b}|).(\vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b}))$

$= |\vec{a}|^2 + 4\vec{a} . \vec{b} + 9|\vec{a} \times \vec{b}|^2 + 4|\vec{b}|^2$

$= 1 + 4 + 4\left(\frac{1}{2}\right) + 9\left(\frac{3}{4}\right)$

$= 7 + \frac{27}{4} = \frac{55}{4}$

$|\vec{c}| = \frac{\sqrt{55}}{2} \Rightarrow 2|\vec{c}| = \sqrt{55}$

13. An ellipse passes through the foci of the hyperbola, $9x^2 - 4y^2 = 36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. if the product of eccentricities of the two conics is $\frac{1}{2}$, then which of the following points **does not** lie on the ellipse ?

(1) $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$ (2) $(\sqrt{13}, 0)$ (3) $\left(\frac{1}{2}\sqrt{13}, \frac{\sqrt{3}}{2}\right)$ (4) $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

Ans. (3)

Sol. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

focii are $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$

eccentricity of hyperbola is $e_H = \frac{\sqrt{13}}{2}$ $\Rightarrow$ eccentricity of ellipse is $e_E = \frac{1}{\sqrt{13}}$

Let equation ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\Rightarrow e^2_E = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{13} = 1 - \frac{b^2}{a^2}$ $\Rightarrow \frac{b}{a} = \sqrt{\frac{12}{13}}$ ....(1)

Ellipse passes through $(\pm\sqrt{13}, 0)$ $\Rightarrow \frac{13}{a^2} = 1$ $\Rightarrow a^2 = 13$ $\Rightarrow b = \sqrt{12}$

Equation of ellipse $\equiv \frac{x^2}{13} + \frac{y^2}{12} = 1$

which is passes through $(\sqrt{13}, 0)$, $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$ and $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

14. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point P(0, h) meet the x-axis at points A and B. If the area of ΔAPB is minimum, then h is equal to :

(1) $4\sqrt{2}$ (2) $4\sqrt{3}$ (3) $3\sqrt{2}$ (4) $3\sqrt{3}$

Ans. (1)

Sol. Equation of tangent from (0, h) to the circle is $y - h = m(x - 0)$
$y = mx + h$ touch the circle

$\Rightarrow \frac{h}{\sqrt{1+m^2}} = 4 \Rightarrow h = 4\sqrt{1+m^2}$ $\Rightarrow y = \pm\sqrt{\left(\frac{h^2}{16} - 1\right)}x + h$

Area of ΔPAB is $= \frac{1}{2}(h)\left(\frac{8h}{\sqrt{h^2-16}}\right) = \frac{4h^2}{\sqrt{h^2-16}}$

$\Delta = \frac{2h^2}{\sqrt{h^2-16}} \Rightarrow \frac{d\Delta}{dh} = \frac{2\left(\sqrt{h^2-16}(4h) - 2h^2 \cdot \frac{h}{\sqrt{h^2-16}}\right)}{h^2-16} = 0 \Rightarrow h = 4\sqrt{2}$

15. The largest value of r for which the region represented by the set $\{\omega \in C/|\omega - 4 - i| \le r\}$ is contained in the region
represented by the set $\{z \in C/ |z-1| \le |z+i|\}$, is equal to :

(1) $\frac{3}{2}\sqrt{2}$ (2) $\sqrt{17}$ (3) $2\sqrt{2}$ (4) $\frac{5}{2}\sqrt{2}$

Ans. **(4)**

Sol. $|\omega - 4 - i| \le r \Rightarrow$ circle centre (4, 1) radius = r
$|z - 1| \le |z + i| \Rightarrow$ straight line $y = -x$

$$\therefore \quad \text{maximum } r = \frac{4+1}{\sqrt{1+1}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

16. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) :

(1) form an acute angled triangle. (2*) lie on a straight line
(3) form an obtuse angled triangle (4) form a right angled triangle.

Ans. **(2)**

Sol. $AB = \sqrt{(1-0)^2 + \left(3 - \frac{3}{8}\right)^2} = \frac{\sqrt{10}}{3}$

$BC = \sqrt{(82-1)^2 + (30-3)^2} = 27\sqrt{10}$

$CA = \sqrt{(82-0)^2 + \left(30 - \frac{8}{3}\right)^2} = \frac{82\sqrt{10}}{3}$

Clearly AB + BC = CA $\therefore$ A,B,C are collinear

17. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to :

(1) 7775 (2) 7785 (3) 7770 (4) 7780

Ans. **(4)**

Sol. Given $= \sum_{r=16}^{30}(r^2 - r - 6) = \sum_{r=1}^{30}(r^2 - r - 6) - \sum_{r=1}^{15}(r^2 - r - 6)$

$$= \left(\sum_{r=1}^{30} r^2 - \sum_{r=1}^{15} r^2\right) - \left(\sum_{r=1}^{30} r - \sum_{r=1}^{15} r\right) - 6(30 - 15)$$

$= 8215 - 345 - 90 = 7780$

18. If the points (1, 1, λ) and (–3, 0, 1) are equidistant from the plane, 3x + 4y – 12z + 13 = 0, then λ satisfies the equation :

(1) $3x^2 - 10x + 21 = 0$ (2) $3x^2 + 10x - 13 = 0$

(3) $3x^2 - 10x + 7 = 0$ (4) $3x^2 + 10x + 7 = 0$

Ans. **(3)**

Sol. $|3 + 4 - 12\lambda + 13| = |-9 + 0 - 12 + 13|$

$\Rightarrow \quad |20 - 12\lambda| = 8$

$\Rightarrow \quad 12\lambda - 20 = \pm 8$

$\Rightarrow \quad 12\lambda = 20 \pm 8$

$\Rightarrow \quad \lambda = 1, \frac{7}{3}$

$\lambda = 1$ or $\lambda = \frac{7}{3} \quad \Rightarrow \quad x^2 - \left(1+\frac{7}{3}\right)x + \frac{7}{3} = 0$

$3x^2 - 10x + 7 = 0$

19. In a ΔABC, $\frac{a}{b} = 2+\sqrt{3}$ and $\angle C = 60^\circ$. Then the ordered pair $(\angle A, \angle B)$ is equal to :

(1) $(45^\circ, 75^\circ)$ (2) $(75^\circ, 45^\circ)$ (3) $(105^\circ, 15^\circ)$ (4) $(15^\circ, 105^\circ)$

Ans. **(3)**

Sol. $\frac{a}{b} = \frac{2+\sqrt{3}}{1} \quad \therefore \quad \frac{a+b}{a-b} = \frac{3+\sqrt{3}}{\sqrt{3}+1} = \sqrt{3}$

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{1}{\sqrt{3}}\cot 30^\circ = 1$

$\frac{A-B}{2} = 45 \quad \Rightarrow \quad A + B = 120 \quad A = 105^\circ, B = 15^\circ$

20. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative is :

(1) – 8 (2) –1 (3) $-2\sqrt{2}$ (4) $-16\sqrt{2}$

Ans. (1)

Sol. $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$

$\Rightarrow xyz + 2 - y - x - z \geq 0$

$\Rightarrow xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$

put $(xyz)^{1/3} = t$

$\Rightarrow t^3 - 3t + 2 \geq 0$

$\Rightarrow (t-1)(t^2 + t - 2) \geq 0$

$\Rightarrow (t-1)^2(t+2) \geq 0$

– | + | –

–2 1

$\Rightarrow t \geq -2$

$\Rightarrow (xyz)^{1/3} \geq -2$

$\Rightarrow xyz \geq -8$

21. If the shortest distance between the lines $\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}$, $(\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value of α is :

(1) $\frac{32}{19}$ (2) $\frac{19}{32}$ (3) $-\frac{16}{19}$ (4) $-\frac{19}{16}$

Ans. **(1)**

Sol. Any plane $x + y + z\ (2x - y + z + 3) = 0$

$\Rightarrow$ $(2\lambda + 1)x + (1 - \lambda)y + (1 + \lambda)z + 3\lambda + 1 = 0$

parallel to given line if

$\alpha\ (2\lambda + 1) - 1(1 - \lambda) + 1.(1 + \lambda) = 0$

$\Rightarrow$ $\alpha = \frac{-2\lambda}{2\lambda+1}$ ....(1)

by (1)

Also $\frac{|2\lambda+1-(1-\lambda)+0+3\lambda+1|}{\sqrt{(2\lambda+1)^2+(1-\lambda)^2+(1+\lambda)^2}}=\frac{1}{\sqrt{3}}$

$\Rightarrow$ $\lambda = 0, -\frac{32}{102}$ $\alpha = 0$, or $\alpha = \frac{32}{19}$

22. Let the sum of the first three terms of an A.P. be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is :

(1) 29.5 (2) 26.5 (3) 28 (4) 31

Ans. **(1)**

Sol. $10 + (10 + d) + (10 + 2d) = 39$ $\Rightarrow$ $d = 3$

$t_n = 10 + (n - 1)3 = 3n + 7$

Also $(3n + 7) + (3n - 3 + 7) + (3n - 9 + 7) = 178$

$\Rightarrow$ $n = 14$

$\therefore$ median$= \frac{t_7 + t_8}{2} = \frac{28+31}{2} = 29.5$

23. Let L be the line passing through the point P(1, 2) such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is line perpendicular to L and passing through the point (–2, 1), then the point of intersection of L and L_1 is

(1*) $\left(\frac{4}{5},\frac{12}{5}\right)$ (2) $\left(\frac{3}{5},\frac{23}{10}\right)$ (3) $\left(\frac{3}{10},\frac{17}{5}\right)$ (4) $\left(\frac{11}{20},\frac{29}{10}\right)$

Ans. (1)

Sol. Line L is $2x + y = 4$

Line L_1 is $x - 2y = -4$

intersection point is $\left(\frac{4}{5},\frac{12}{5}\right)$

24. For $x > 0$, let $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then $f(x) + f\left(\frac{1}{x}\right)$ is equal to

(1) $\frac{1}{4}\log x^2$ (2) $\frac{1}{4}(\log x)^2$ (3) $\log x$ (4*) $\frac{1}{2}(\log x)^2$

Ans. (4)

Sol. $f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\log t}{t}dt = \frac{(\log x)^2}{2}$

25. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is

(1) $x^2 + y^2 - 3x - 9y = 0$ (2) $x^2 + y^2 + 3x + 9y = 0$
(3*) $x^2 + y^2 - 3x + 9y = 0$ (4) $x^2 + y^2 + 3x - 9y = 0$

Ans. (3)

Sol. $x^2 + y^2 - 30x + \lambda(y + 3x) = 0$

centre $\equiv \left[-\frac{3\lambda - 30}{2}, -\frac{\lambda}{2}\right]$

centre lies on $y + 3x = 0$

$\Rightarrow \lambda = 9$

circles is $x^2 + y^2 - 3x + 9y = 0$

26. A factor is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is Rs. 54 and per day mean wage of all the worker is Rs. 60, then per day mean wage of the night shift workers (in Rs.) is

(1) 75 (2) 69 (3) 66 (4) 74

Ans. (4)

Sol. $\frac{70x + 30y}{100} = 60 \Rightarrow 3y = 600 - 7x \Rightarrow 3y = 600 - 378\ (x = 54) \Rightarrow y = \frac{222}{3} = 74$

27. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

(1) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$ (2*) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$ (3) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}} + C$ (4) $4\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}} + C$

Ans. (2)

Sol. $\int \frac{dx}{(x+1)^2\left(\frac{x-2}{x+1}\right)^{5/4}} \Rightarrow \frac{x-2}{x+1} = t \Rightarrow \frac{1}{(x+1)^2}dx = \frac{dt}{3}$

$= \int \frac{dt}{3t^{5/4}} = \frac{-4}{3t^{1/4}} = -\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$

28. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation

(1) exists and is equal to 1 (2) exists and is equal to $-\frac{1}{2}$

(3*) exists and is equal to $\frac{1}{2}$ (4) does not exist

Ans. (3)

Sol. $2 + 3i + 2 - 3i + \alpha = \frac{9}{2}$

$\alpha = \frac{9}{2} - 4 = \frac{1}{2}$

29. If $f(x) = 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x > 1$, then $f(5)$ is equal to

(1) $\frac{\pi}{2}$ (2) $\tan^{-1}\left(\frac{65}{156}\right)$ (3) $4\tan^{-1}(5)$ (4*) π

Ans. (4)

Sol. $2\tan^{-1}x + \sin^{-1}\sin(2\tan^{-1}x)$

$2\tan^{-1}x + \pi - 2\tan^{-1}x$

$= \pi$

30. If the coefficients of the three successive terms in the binomial expansion of $(1 + x)^n$ are in the ratio $1 : 7 : 42$, then the first of these terms in the expansion is

(1) 6^{th} (2) 7^{th} (3) 8^{th} (4) 9^{th}

Ans. (2)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$\frac{r}{n-r+1} = \frac{1}{7}$

$\Rightarrow 8r = n + 1$

$\frac{r+1}{n-r} = \frac{1}{6}$

$\Rightarrow 7r = n - 6$

$\Rightarrow r = 7$

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