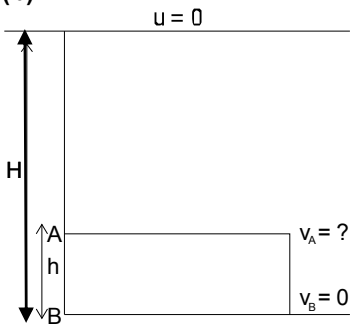
# HINTS & SOLUTIONS

**1. (b)**




Velocity at point A

$V_A^2 = u^2 + 2as$

Here u = 0

a = g

S = (H–h)

$V_A^2 = 0 + 2g\,(H–h)$

$V_A = \sqrt{2g(H-h)}$

Now for point A and point B

$V_B^2 = V_A^2 = 2ah$

$0 = \left(\sqrt{2g(H-h)}\right)^2 + 2ah$ Here $\left(V_B = 0 \text{ \& } V_A = \sqrt{2g(H-h)}\right)$

2ah = 2g (H–h)

So $a = g\left(\frac{H-h}{h}\right)$

$a = g\left(\frac{H}{h}-1\right)$

**2. (a)**

Given $\lambda_1 - \lambda_2 = 2$ m……(i)

And $\frac{n_2}{n_1} = 1.4$

$n_1 = \frac{1000}{20} = 50$ Hz.

So $n_2 = 1.4 \times 50 = 70$ Hz

Now from equation (i)

$\lambda_1 - \lambda_2 = 2$

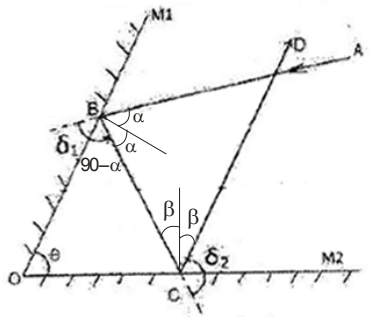
$\frac{v}{n_1} - \frac{v}{n_2} = 2$

$$\frac{v}{50}-\frac{v}{70}=2$$

$$v\left[\frac{20}{3500}\right]=2$$

$v = 350$ m/s

**3.** **(d)**




Deviation through $M_1$

$\delta_1 = 180 - 2\alpha$.....(i)

Deviation through $M_2$

$\delta_2 = 180 - 2\beta$.....(ii)

so total deviation

$\delta = \delta_1 + \delta_2 = 360 - 2(\alpha+\beta)$.....(iii)

Now from $\underline{\Delta BOC}$

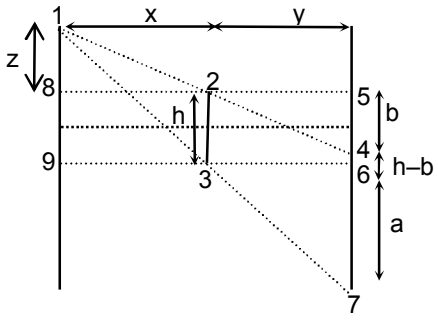
$\theta + 90 - \alpha + 90 - \beta = 180$

$\theta = \alpha + \beta$.....(iv)

From equation (iii) and (iv)

$\delta = 360 - 2\theta$

**4.** **(c)**




From $\Delta$ 182 & $\Delta$ 254

$\frac{x}{y} = \frac{z}{b}$ ..... (1)

From $\Delta$ 193 & $\Delta$ 367

$\frac{x}{y} = \frac{z+h}{a}$ ..... (2)

From Equ. (1) & (2) $b = \frac{zy}{x}, a = \frac{(z+h)y}{x}$

Height of image = a + h – b

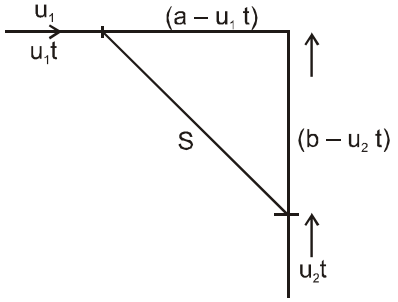
$= h + \frac{(z+h)y}{x} - \frac{zy}{x} = h + \left[\frac{y}{x}(z+h-z)\right]$

$= h + \frac{yh}{x} = h\left[1+\frac{y}{x}\right]$

Since h , y & x are constant so height of image will always be same.

**5.** **(a)**

Let instant is t then distance travel by particle $P_1$ is $u_1t$ and distance travel by particle $P_2$ is $u_2t$




So, relative separation

$S = \sqrt{(a-u_1t)^2 + (b-u_2t)^2}$

$S = \sqrt{a^2 + u_1^2t^2 - 2a\,u_1t + b^2 + u_2^2t^2 - 2b\,u_2t}$

$S = \left(a^2 + b^2 + \left(u_1^2 + u_2^2\right)t^2 - 2\left(au_1 + bu_2\right)t\right)^{\frac{1}{2}}$

So option (a) is correct

**6.** **(c)**

Let x kg mass of fuel burn per hour there for $\frac{x}{3600}$ kg/sec.

Now efficiency = $\frac{\text{output}}{\text{input}}$

$0.25 = \frac{25\times10^3 w}{\frac{x}{3600}\times17200\times10^3\times4.2}$

$x \cong 5$ kg per hour

Now electric energy generated per tonne of fuel burnt

$= \frac{17200\times10^6\times0.25\times4.2}{3.6\times10^6} = 5000$ Kwh

**7.** **(b)**

$m = \frac{f}{f-u}$

For first case when object is at 25 cm

$m = \frac{f}{f-(-25)}$ .... (1)

Now for second case when object is at 40 cm

$m' = \frac{f}{f-(-40)}$ ... (2)

m = 4m'

$$4m' = \frac{f}{f-(-25)} \quad \ldots (3)$$

(3) ÷ (2)

$$4 = \frac{f}{f+25} \times \frac{f+40}{f}$$

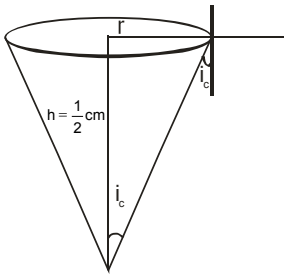
4f + 100 = f + 40

3f = 60

f = 20 cm

**8.** **(b)**

We know for TIR $\sin i_c = \frac{1}{\mu}$




From figure TIR $\sin i_c = \frac{r}{\sqrt{r^2+h^2}}$

So $\frac{r}{\sqrt{r^2+h^2}} = \frac{1}{\mu}$

$r^2 \mu^2 = r^2 + h^2$

$r^2 (\mu^2 - 1) = h^2$

$$r = \frac{h}{\sqrt{\mu^2-1}} = \frac{\frac{1}{2}}{\sqrt{(1.5)^2-1}} = 0.447 \text{ cm}$$

**9.** **(c)**

$$\alpha = \frac{L'-L}{L(T'-T)}$$

So unit of a is $K^{-1}$ So option (C) is correct

**10.** **(a)**

Since time period is 2 sec

So time taken in 60 oscillation is 2 min

Now soldier complete 110 beats in 2 min. therefore 55 beats per min which is less than 60 so it is bradycardia

Now $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$2 = 2\pi\sqrt{\frac{\ell}{g}}$$

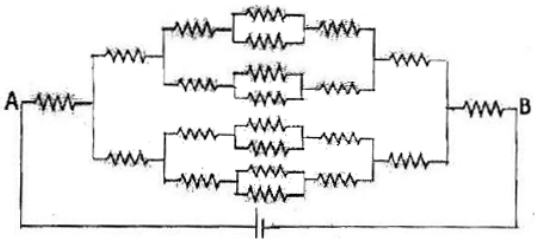
$$4 = 4\pi^2 \frac{\ell}{g}$$

$\ell = 1$m

That means length is 1 m and symptom is brady-cardia

So option (a) incorrect

**11. (b)**




$R_{eq}$ between A and B

$= \frac{29R}{8}$

So minimum value of R, for integral value for equivalent resistance between A and B must be $8\Omega$

**12. (b)**

In water

$$\frac{\rho_0}{\rho_L} = \frac{V_i}{V} \Rightarrow \frac{\rho_0}{1} = \frac{5/8V}{V} \Rightarrow \rho_0 = \frac{5}{8}$$

For salt solution .

$$\frac{5/8}{1.12} = \frac{V_i}{V}$$

so $\frac{V_o}{V} = 1 - \frac{V_i}{V}$

$$\frac{V_o}{V} = 1 - \frac{5/8}{1.12} = 0.44$$

**13. (b)**

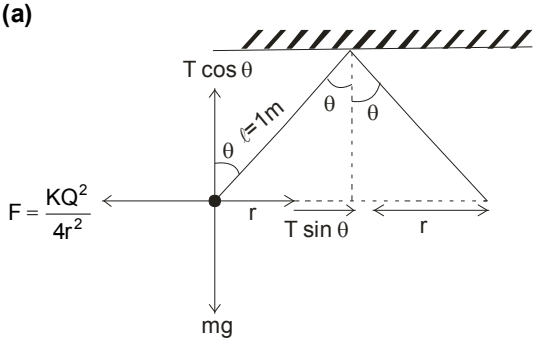
$$F = kg\frac{M}{sec^2}$$

Unit of mass = $\frac{F sec^2}{m} = \frac{F}{V} sec$

$$= \frac{10}{100} \times \frac{1}{100}$$

$$= 10^{-3} \text{ kg}$$

**14. (a)**




$$T \sin\theta = \frac{KQ^2}{4r^2}$$

and $T \cos\theta = mg$

$$\tan\theta = \frac{KQ^2}{4r^2 mg}$$

So $\tan\theta = \dfrac{KQ^2}{4\sin^2\theta\, gm}$ $\quad \because \sin\theta = \dfrac{r}{1}$

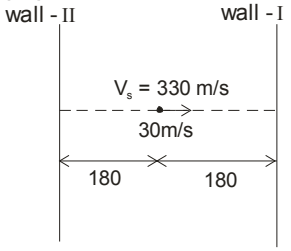
$\tan\theta \sin^2\theta = \dfrac{KQ^2}{4\, gm}$ $\quad r = \sin\theta$

$\theta$ is maximum . so g is minimum

15. **(c)**

According to fraunhaffer good absorbers are good radiators and bad absorbers are bad radiators so black body is good absorber and good radiator.

16. **(b*)**




reflection from wall – I

$360 - 30t_1 = 330\, t_1$

$t_1 = 1$ sec

$720 + 30\, t_2 = 330\, t_2$

$t_2 = 2.4$ sec

$1080 - 30\, t_3 = 330\, t_3$

$t_3 = \dfrac{1080}{360} = 3$ sec

reflection from wall - II

$360 + 30\, t_1 = 330\, t_1$

$t_1 = 1.2$ sec

$720 - 30\, t_2 = 330\, t_2$

$t_2 = 2$ sec

$1080 + 30\, t_3 = 330\, t_3$

$t_3 = 3.6$ sec

Answer May be (b)

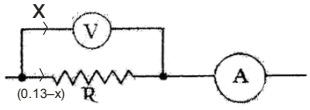
17. **(b)**

(i) Since potential at surface of solid and hollow sphere are $\dfrac{Kq}{r}$ so option I is incorrect .

(ii) A charged body can attract another uncharged body so it is correct.

(iii) Two electric line of force never intersect each other so this is incorrect.

18. **(b)**




Let current through V be 'x'

So current through R is (0.13 – x)

Now potential difference in parallel combination must be same.

So 117 = R (013–x) ……..(i)

And V = IR

117 = 9000 x

so $x = \frac{117}{9000}$

using this value in equation (i)

$117 = R\left(0.13 - \frac{117}{9000}\right)$

or $R = 1000\ \Omega$

19. **(b)**

20. **(c)**

Because volume is minimum at 4ºC so volume will increase at temperature greater than or less than 4ºC.

21. **(d)**

Because number of protons are same in both, but number of electrons are more in $p^{3-}$, so there will be greater repulsion between electrons in $p^{3-}$

22. **(b)**

Given

Molarity = 0.5 molar

M = 0.5 M

Volume = 4 litre

Mass of substance (x) = 240 gm

Molarity (M) = $\frac{\text{given Mass of substance}}{\text{Molecular mass of substance} \times \text{volume}}$

$M = \frac{x}{MW \times V}$

$0.5 = \frac{240}{MW \times 4}$

$MW = \frac{240}{0.5 \times 4}$

$= \frac{600}{5} = 120$ gm/mole

23. **(c)**

Density (d) = 1.28 g/cc = $\frac{m}{1ml}$ $m = 1.28 \times 10^3$

$M = 4.2 = \frac{w}{98 \times 1000} \times 1000$

$H_2SO_4$ (w) = 4.2 × 98 g of 1000 $cm^3$

$\left(\frac{m}{wl}\right)\% = \frac{4.2 \times 98 \times 10}{1.28 \times 10^3} \times 100 = 32\ \%$

24. **(a)**

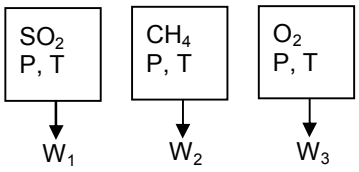
| Element | % composition | Atomic mass | Relative no.od after | Simple atomic ratio | Simplets whole No. Atomic ratio |
|---|---|---|---|---|---|
| X | 60 | 10 | $\frac{60}{10} = 6$ | $\frac{6}{2} = 2$ | 3 |
| Y | 40 | 20 | $\frac{40}{20} = 2$ | $\frac{2}{2} = 1$ | 1 |

Simplet formula of this compound = $X_3$ Y

25. **(c)**
Given number of Nitrogen atom = $4.095 \times 10^{24}$
Mole = $\frac{\text{given number of atom}}{\text{Avogadro number}}$
Mole of Nitrogen atom = $\frac{4.095\times10^{24}}{6.023\times10^{23}}$ = 6.80 mole
Now we know that
1 mole nitrogen gas has 2 mole of nitrogen atom
So mole of nitrogen gas = $\frac{\text{mole of Nitrogen atom}}{2} = \frac{6.80}{2}$ = 3.4 mole

26. **(d)**
Due to absence of gravity

27. **(c)**




$n_{SO_2} = n_{CH_4} = n_{O_2}$

$\frac{W_1}{64} = \frac{W_2}{16} = \frac{W_3}{32}$

$64\times\frac{W_1}{64} = 64\times\frac{W_2}{16} = 64\times\frac{W_3}{32}$

$W_1 : 4\ W_2 : 2W_3$
If $W_2 = x$
then $W_1 : W_2 : W_3$
4x : x : 2x
4 : 1 : 2

28. **(b)**
Observation of student 'Q' is correct because sulhpur dioxide is soluble in water, then water will rushed in jar.

29. **(b)**
$Al_2(SO_4)_3 \rightarrow 2Al^{3+} + 3SO_4^{2-}$
1 mole $\rightarrow$ 27 × 2 gm
54 gm $Al^{3+}$ ions will be formed by = 1 mole $Al_2(SO_4)_3$
1 gm $Al^{3+}$ ions will be formed by = $\frac{1}{54}$ mole $Al_2(SO_4)_3$
0.17 gm $Al^{3+}$ ion will be formed by = $\frac{0.17}{54}$ mole $Al_2(SO_4)_3$
= 0.00314 $Al_2(SO_4)_3$
We know $Al_2(SO_4)_3 + 3Ba(OH)_2 \rightarrow \underset{\text{ppt}}{3BaSO_4} + \underset{\text{ppt}}{2Al(OH)_3}$
From stochiometry
$\rightarrow$ 1mole $Al_2(SO_4)_3$ = 3 mole $BaSO_4$
0.00314 mole $Al_2(SO_4)_3$ will produce = 3 × 0.00314 mole $BaSO_4$
= 3 × 0.00314 × 233.3 gm $BaSO_4$ = 2.20 gm $BaSO_4$
$\rightarrow$ 1 mole $Al_2(SO_4)_3$ will produce = 2 mole $Al(OH)_3$ = 2 × 78 gm $Al(OH)_3$
0.00314 mole $Al_2(SO_4)_3$ will produce = 2 × 78 × 0.00314 gm $Al(OH)_2$
= 0.48 g gm $Al(OH)_3$
$\rightarrow$ Now the total mass of precipitate = mass of $BaSO_4$ + 0.489 $Al(OH)_3$ = 2.68984 gm precipitate
$\approx$ = 2.7 gm precipitate

30. **(b)**
Bobby added $Na_3PO_4$ which is basic in nature, remaining all salts are either neutral or acidic.
So, $Na_3PO_4$ will increase the pH of soil which is suitable for growing the daffodil

31. **(a)**
Because ionic compound has high melting point then co-valent compounds.

32. **(a)**
When $CO_2$ is KOH them reaction is
$2KOH_{(aq)} + H_2CO_3 \rightarrow K_2CO_3 + 2H_2O$
1 mole of $H_2CO_3$ is required for 2 mol of KOH
Given data
$\Rightarrow$ 1000 ml solution contain 0.1 mol KOH
1 ml solution contain $\frac{0.1}{1000}$
then 5 ml contain = $\frac{0.1}{1000} \times 5 = 5 \times 10^{-4}$ mol
If 2 mole KOH dissolve $\rightarrow$ 1 mol $H_2CO_3$
then 1 mol KOH dissolve $\rightarrow \frac{1}{2}$ mol $H_2CO_3$
$5 \times 10^{-4}$ mol KOH dissolve $\rightarrow \frac{1}{2} \times 5 \times 10^{-4}$ mol
$2.5 \times 10^{-4}$ mol $H_2CO_3$
$C + O_2 \rightarrow CO_2 + H_2O \rightarrow H_2CO_3$
1 mole C gives 1 mol $CO_2$ and 1 mol $CO_2$ gives 1 mol $H_2CO_3$
So requirement of C is $2.5 \times 10^{-4} \times 12 = 30 \times 10^{-4}$ g
In mg = $30 \times 10^{-4} \times 10^3$ mg
= $3 \times 10^{-3} \times 10^3$
= 3 mg

33. **(a)**
Given both contain equal number molecule
So in (a) option
mole of $SO_2$ = $\frac{1120}{22400} = \frac{1}{20}$ = 0.05 mole
Mole of $NH_3$ = $\frac{0.85}{17}$ = 0.05 mole
Equal moles or equal volume of all gases at NTP contains equal number of molecules.

34. **(a)**
$Fe + CuSO_4 \rightarrow FeSO_4 + Cu$
7g + 21 gm 8.6 g
Acc. to Law of conservation of mass. 28–8.6 = 19.40 gm

35. **(a)**
If solutions are same in nature
$N_1V_1 + N_2V_2 = N_RV_R$
$V_R$ = 2 Ltr ($V_1 + V_2$ = 2 Ltr)
If $V_1 = x$
$V_2 = (2 - x)$
$\Rightarrow 0.5 \times x + 0.1 (2 - x) = 2 \times 0.2$
$\Rightarrow 0.5 x + 0.2 - 0.1 x = 0.4$
$\Rightarrow 0.4 x = 0.2$
$x = \frac{0.2}{0.4} = 0.5$
$x = 0.5 = V_1 = 0.5$ Ltr
$V_2 = 2–0.5 = 1.5$ Ltr

36. **(c)**

Oxalic acid – 6.39 g

$(COOH)_2 . 2H_2O \rightarrow$ gram molecular mass = 126 gm

Mole of oxalic acid = $\frac{6.3}{126} = \frac{1}{20}$ = 0.05 mol

In 100 ml water = 0.05 mol of oxalic acid present

Now In 25 ml of water, amount of oxalic acid

will be = $\frac{0.05}{100} \times 25 = \frac{0.05}{4}$ mol

250 ml solution contains = 0.0125 mol of oxalic acid

then 10 ml contains = $\frac{0.0125}{250} \times 10 = \frac{0.0125}{25}$ = 0.0005 mol

So, mole of NaOH will be required = 0.0005 × 2

Mass of NaOH = 0.001 × 40 = 0.04 gm

Weight of NaOH in ppm = 0.04 × 1000 = 40

37. **(d)**

Iso-octane improve the quality of petrol because it has more branches.

38. **(a)**

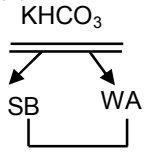
$2K\overset{+5}{Br}O_3 + 12H^+ 10e^- \rightarrow \overset{0}{Br_2} + 6H_2O + 2K^+$

Total change in oxidation number of $KBrO_3$ = valency factor = 5

Eq. Wt. of $KBrO_3 = \frac{\text{Molecular wt. of } KBrO_3}{\text{Valency factor}}$

Eq. Wt. $KBrO_3 = \frac{M}{5}$

39. **(c)**




For basic salt solution colour would be blue

Universal indicator colour for different pH range

| pH | Type of solution | Colour |
|---|---|---|
| < 3 | Strong acid | Red |
| 3–6 | Weak acid | Orange or yellow |
| 7 | Neutral | Green |
| B–11 | Weak basic | Blue |
| > 11 | Strong base | Violet of indigo |

40. **(c)**

(a) Chalcocite = $Cu_2S$

(b) Magnetite = $Fe_3O_4$

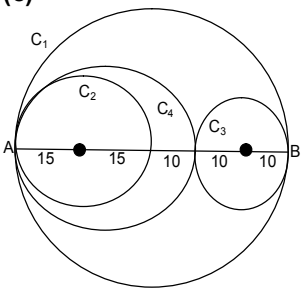
(c) Calamine = $ZnCO_3$

(d) Galena = PbS

Option (c) is correct

calamine is a ore of zinc

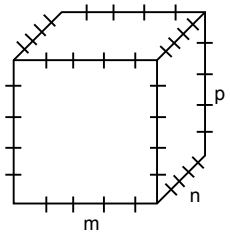
41. **(c)**




Diameter of $C_4$ = 15 + 15 + 10 = 40

Radius = 20 cm

42. **(d)**




$$\frac{m(m+1)}{2} \times \frac{n(n+1)}{2} \times \frac{p(p+1)}{2}$$

$$\frac{5\times6}{2} \times \frac{5\times6}{2} \times \frac{5\times6}{2}$$

15 × 15 × 15

= 3375

43. **(d)**

$n^2 (n^2 – 1) (n^2 – n – 2)$

$n^2 (n^2 – 1) (n^2 – 2n + n – 2)$

$n^2 (n^2 – 1) (n – 2) (n + 1)$

$n^2 (n – 1) (n + 1) (n – 2) (n + 1)$

$(n – 2) (n – 1) n^2 (n + 1)^2$ (after factorization)

$\Rightarrow$ (n – 2) (n – 1) (n ) (n + 1) is the product of four consecutive number which is always divisible by 4! = 24.

$\Rightarrow$ Now in this product of $(n – 2) (n – 1) n^2 (n + 1)^2$ there are two perfect squares of n and n + 1 in which one of them should be even so it should be a multiple of 2. and square of 2 is 4.

$\Rightarrow$ So (n – 2) (n – 1) (n) (n + 1) is divisible by 24 and $(n – 2) (n – 1) (n)^2 (n + 1)^2$ should be divsible by 48.

44. **(c, d)**

12 through

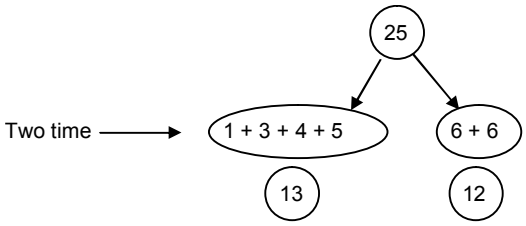
one number (3 times) 4 number (2 times) one number (1 time)

1, 2, 3, 4, 5, 6

each number constant at least one time

$$\frac{6\times7}{2} = 21$$

Required sum = 46 – 41 = 25




6 – three times

1, 3, 4, 5 $\rightarrow$ two times

2 $\rightarrow$ one time

at least number is 2.

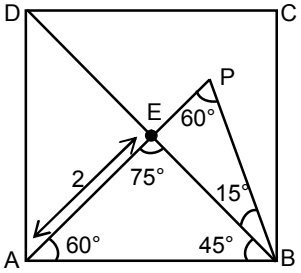
One more case is possible

2 + 3 + 4 + 6 + (5 + 5) = 25

so at least number is 1.

45. **(a)**

In $\Delta AEB$ by sine Rule




$$\frac{\sin 45}{AE} = \frac{\sin 75}{AB}$$

$$\frac{\frac{1}{\sqrt{2}}}{2} = \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{AB}$$

$AB = \sqrt{3}+1$

Area of square ABCD = $AB^2 = \left(\sqrt{3}+1\right)^2$

$= 3 + 1 + 2\sqrt{3} = 4 + 2\sqrt{3}$.

46. **(c, d)**

Let $n = 2^5$

$\therefore 9 \times n = 2^5 \times 3^2$

Number of divisor = (5 + 1) (2 + 1)

$= 6 \times 3 = 18$

Let $n = 75 = 3 \times 5^2$

$\therefore 9 \times n = 3^3 \times 5^2$

Number of divisor = (3 + 1) (2 + 1)

$= 4 \times 3 = 12$

47. **(Bonus)**

$$x = \frac{2ab}{b^2+1}$$

$$\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$\Rightarrow \frac{\left(\sqrt{a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\sqrt{(a+x)(a-x)}}{(a+x)-(a-x)}$$

$$\Rightarrow \frac{a+x+a-x-2\sqrt{a^2-x^2}}{2x}$$

$$\Rightarrow \frac{2a-2\sqrt{a^2-x^2}}{2x}$$

$$\Rightarrow \frac{a-\sqrt{a^2-\left(\frac{2ab}{b^2+1}\right)^2}}{\frac{2ab}{b^2+1}}$$

$$\Rightarrow \left(a-\sqrt{a^2-\frac{4a^2b^2}{b^2+1}}\right)\times\frac{b^2+1}{2ab}$$

$$\Rightarrow a\left[\frac{b^2+1-\sqrt{(b^2+1)^2-4b^2}}{b^2+1}\right]\times\frac{b^2+1}{2ab}$$

$$\Rightarrow \frac{b^2+1-\sqrt{b^4+1+2b^2-4b^2}}{2b}$$

$$\Rightarrow \frac{b^2+1-\sqrt{(1-b^2)^2}}{2b} \Rightarrow \frac{b^2+1-1+b^2}{2b} = b$$

48. **(b)**

Let $n_1$ and $n_2$ are number of sides of polygon them

$$n_1+\frac{n_2(n_2-3)}{2} = 103$$

$2n_1 + n_2^2 - 3n_2 = 206$ .....(i)

$$\frac{n_1(n_1-3)}{2} + n_2 = 80$$

$n_1^2 - 3n_1 + 2n_2 = 160$ .....(ii)

multiply equation (i) by (ii) by and equation (ii) by 3 and add.

$4n_1 + 2n_2^2 - 6n_2 + 3n_1^2 - 9n_1 + 6n_2 = 412 + 480$

$2n_2^2 + 3n_1^2 - 5n_1 = 892$ .....(iii)

and again multiply (i) by (iii) and equation (ii) by 2 and add.

$2n_1^2 + 3n_2^2 - 5n_2 = 938$ .....(iv)

subtract equation (iii) from (iv)

$-n_1^2 + n_2^2 - 5n_2 + 5n_1 = 46$

$(n_2^2 - n_1^2) - 5(n_2 - n_1) = 46$

$(n_2 - n_1)(n_2 + n_1 - 5) = 46$

Factors of 46 are

$1 \times 46$

$2 \times 23$

$23 \times 2$

$46 \times 1$

**In case – 1**

$(n_2 - n_1)$ and $n_2 + n_1 - 5 = 46 \quad \Rightarrow \quad n_2 + n_1 = 5$

After solving

$n_2 = 26$ and $n_1 = 25$

But given condition of diagonals are not satisfy.

**In case – 2**

$n_2 - n_1 = 2$

$n_1 + n_2 - 5 = 23 \quad \Rightarrow \quad n_1 + n_2 = 28$

After solving

$n_1 = 13, n_2 = 15$

**In case - 3**

$n_2 - n_1 = 23$

$n_1 + n_2 - 5 = 2, n_1 + n_2 = 7$

$n_2 = 15$ and $n_1 = -8$

not possible.

**In case - 4**

$n_2 - n_1 = 46$ and $n_1 + n_2 - 5 = 1$

$\Rightarrow \quad n_1 + n_2 = 6.$

$n_1 = -20, n_2 = 26.$

not possible

Hence answer is 28.

49. **(b)**

Let number of Red & Yellow balls are respectively.

ATQ

$r - 1 = \frac{1}{7}(r - 1 + y) \Rightarrow 7r - 7 = r - 1 + y$

$6r - y = 6$ ...(1)

$r = \frac{1}{6}(r + y - 1)$

$\Rightarrow 6r = r + y - 1$

$5r - y = -1$ ....(2)

(1) – (2)

$6r - y = 6$

$5r - y = -1$

$- \quad + \quad +$

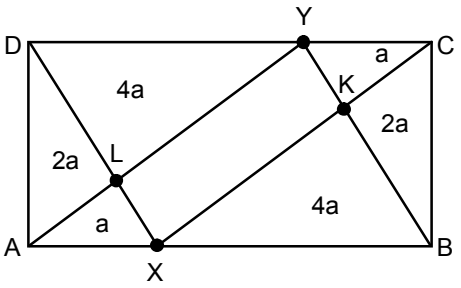
$r = 7$

$\therefore y = 36$

Number of balls = $r + y = 7 + 36 = 43$

Sum of digit = $4 + 3 = 7$

50. **(b)**




$\Delta KYC \sim \Delta KBX$

$$\frac{KY}{KB} = \frac{KC}{KX} = \frac{YC}{XB} = \frac{1}{2}$$

Let ar $\Delta KYC = a$

$$\frac{ar\ \Delta KYC}{ar\ \Delta KCB} = \frac{KY}{KB} = \frac{1}{2}$$

$$\frac{a}{ar\ \Delta KCB} = \frac{1}{2}$$

ar $\Delta KCB = 2a$

$$\frac{ar\ \Delta KCB}{ar\ \Delta KBX} = \frac{CK}{KX} = \frac{1}{2}$$

$$\frac{2a}{ar\ \Delta KBX} = \frac{1}{2}$$

ar $\Delta KBX = 4a$

$$\frac{ar\ \Delta XBC}{ar\ ABC} = \frac{BX}{AB} = \frac{2}{3}$$

$$\frac{6a}{ar\ \Delta ABC} = \frac{2}{3}$$

$\Rightarrow$ ar $\Delta ABC = 9a$.

ar Rect ABCD = 2ar $\Delta ABC = 2\ (9a) = 18a$

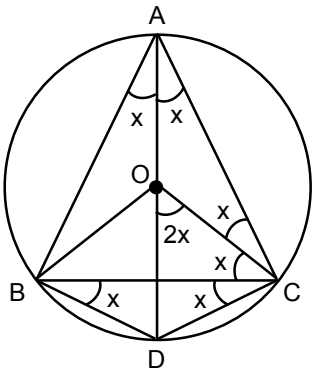
ar XKYL = ar ABCD – [a + $4^a$ + $2^a$ + a + $4^a$ + 2a]

= $18^a$ – [14a] = 4a

$$\frac{ar\ XKYL}{ar\ ABCD} = \frac{4a}{18a} = \frac{2}{9} = \frac{m}{n}$$

$\therefore$ m + n = 2 + 9 = 11.

51. **(a)**




$\angle BAD = \angle DAC = x$ (AD is angle bisector)

$\angle BCD = \angle BAD = x$

$\angle BAC = \angle ACB = 2x$ (equilateral triangle)

$\angle DOC = 2 \times DAC = 2x$

In $\Delta DOC \quad \Rightarrow \quad OD = DC$

In $\Delta BDC \quad \Rightarrow \quad BD = DC$

and $\quad BD + DC = 4$

$\quad BD = DC = 2$

$\quad OD = DC = 2$

Diameter $= 2 \times 2 = 4$m.

52. **(c)**

$T_m = \frac{1}{n}$ , $T_n = \frac{1}{m}$

$T_{mn} = A + (mn - 1)D$ .

$\frac{1}{n} = A + (m - 1)D$ ..... (1)

$\frac{1}{m} = A + (n - 1)D$ .....(2)

equ. (1) – equ.(2)

$\frac{1}{n} - \frac{1}{m} = (m - 1)D - (n - 1)D$

$\frac{m-n}{nm} \Rightarrow D(m - 1 - n + 1)$

$\frac{m-n}{nm} \Rightarrow D(m - n)$

$D = \frac{1}{nm}$

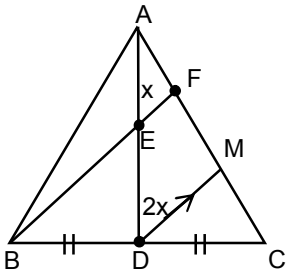
from equ. (1)

$\frac{1}{n} = A + (m - 1) \times \frac{1}{nm}$

$A = \frac{1}{nm}$

$T_{nm} = \frac{1}{nm} + (mn - 1)\,1 - \frac{1}{nm} = 1$

53. **(c)**




Construction draw DM || BF

In $\Delta$ BCF, D is mid point of BC & DM || BF

FM = MC $\Rightarrow \frac{FM}{MC} = \frac{1}{1}$

In $\Delta$ ADM

EF || DM , $\frac{AE}{ED} = \frac{1}{2}$

therefore $\frac{AF}{FM} = \frac{1}{2}$

therefore AF : FM : MC $\Rightarrow$ 1 : 2 : 2

$\frac{AF}{FC} = \frac{1}{4}$

54. **(a)**

$px^2 + qx + r = 0$

$\sin^2\theta + \cos^2\theta = -\frac{q}{p}$

$\sin\theta\cos\theta = \frac{r}{p}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{q^2}{p^2}$

$1 + 2\frac{r}{p} = \frac{q^2}{p^2}$

$\frac{p+2r}{P} = \frac{q^2}{p^2}$

$p(p + 2r) = q^2$

$p^2 + 2pr = q^2$

$p^2 - q^2 + 2pr = 0$

55. **(b)**

a(n – 2), a(n),a(n + 3) are in AP

$\frac{(n-2-2)\times 180}{n-2}$ , $\frac{(n-2)\times 180}{n}$ , $\frac{(n+3-2)\times 180}{n+3}$

$\frac{(n-4)\times 180}{n-2}$ , $\frac{(n-2)\times 180}{n}$ , $\frac{(n+1)\times 180}{n+3}$

from the property of A.P.

$\frac{\frac{(n-4)\times 180}{n-2} + \frac{(n+1)\times 180}{n+3}}{2} = \frac{(n-2)\times 180}{n}$

$\frac{(n-4)\times 180}{n-2} + \frac{(n+1)\times 180}{n+3} = \frac{2(n-2)\times 180}{n}$

$180\left(\frac{n-4}{n-2} + \frac{n+1}{n+3}\right) = 180\ \frac{(n-2)\times 2}{n}$

$\frac{(n-4)(n+3)+(n+1)(n-2)}{(n-2)(n+3)} = \frac{2n-4}{n}$

$\frac{n^2+3n-4n-12+n^2-2n+n-2}{n^2+3n-2n-6} = \frac{2n-4}{n}$

$\frac{2n^2-2n-14}{n^2+n-6} = \frac{2n-4}{n}$

$2n^3 - 2n^2 - 14n = (2n - 4)(n^2 + n - 6)$

$2n^3 - 2n^2 - 14n = 2n^3 + 2n^2 - 12n - 4n^2 - 4n + 24$

$2n = 24$

$n = 12$

Sum of the digits = 1 + 2 = 3

56. **(b)**

Let numbers are

$\frac{a}{d^2}, \frac{a}{d}$, a, ad, $ad^2$

So, A.T.Q.

$\frac{a}{d^2}+\frac{a}{d}$ + a + ad + $ad^2$ = 24

$a\left\{\frac{1+d+d^2+d^3+d^4}{d^2}\right\} = 24$ and $\frac{1+d+d^2+d^3+d^4}{d^2} = \frac{24}{a}$ .....(i)

$\frac{d^2}{a}+\frac{d}{a}+\frac{1}{a}+\frac{1}{ad}+\frac{1}{ad^2} = 6$

$\frac{1+d+d^2+d^3+d^4}{ad^2} = 6$

From equation (i)

$\frac{24}{a\times a} = 6$

$a^2 = 4$

$a = \pm 2$

Product of the terms = $\frac{a}{d^2}\times\frac{a}{d}\times a \times ad \times ad^2 = a^5 = (\pm 2)^5 = \pm 32$.

Answer is 32.

57. **(Bonus)**

491 × 25b is divisible by 36 means divisible by 4 and 9.

$\Rightarrow$ 491 not a divisible by 4 in any case of a.

Hence 25b should be divisible by4.

So possible values of b are 2 & 6.

**Case-1** If b = 2 , 252 is also divisible by 9.

Hence 252 is divisible by 36.

So possible value of a is 10 (from 0 to 9)

ordered pairs = 10

**Case-2** If b = 6, 252 is not divisible by 9.

So 4a1 $\Rightarrow$ divisible by 9.

Possible value of a is only 4.

Ordered pairs = 1.

Tortal ordered pairs of (a , b) are = 10 +1 = 11

58. **(a)**

$\sqrt{\underbrace{111...1}_{2018}-\underbrace{222...2}_{1009}}$

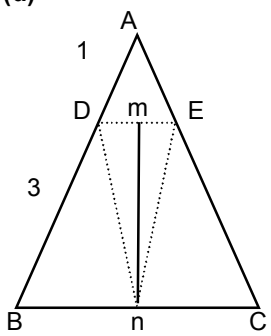
$= \sqrt{\frac{1}{9}\times\underbrace{9999...9}_{2018}-\frac{2}{9}\times\underbrace{9999...9}_{1009}}$

$= \sqrt{\frac{10^{2018}-1}{9}-\frac{2}{9}(10^{1009}-1)}$

$= \sqrt{\frac{10^{2018}-2\times10^{1009}+1}{9}}$

$$= \sqrt{\left(\frac{10^{1009}-1}{3}\right)^2} = \frac{10^{1009}-1}{3}$$

59. **(d)**




$\Delta ADE \sim \Delta ABC$

Hence $\frac{\text{Area}(ADE)}{\text{Area}(ABC)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Let area of $\Delta$ ABC = 16x, then area of $\Delta$ ADE = x

Area of BCFE = 16x – x = 15x

Now in trapezium BCFE

Area ($\Delta$DMN) = Area (MEN)

& Area (BDN) = Area (ENC) {$\because$ M & N are mid points of DE & BC respectively}

Hence Area (BNMD) = Area (NCEM) = $\frac{15x}{2}$

So $\frac{\text{Area}(BNMD)}{\text{Area}(ABC)} = \frac{\frac{15x}{2}}{16x} = \frac{15}{32}$

60. **(d)**

$\left[\frac{100}{1}\right], \left[\frac{100}{2}\right], \left[\frac{100}{3}\right], \ldots\ldots\ldots, \left[\frac{100}{20}\right]$

After solving values are

100, 50, 33, 25, 20, 16, 14, 12, 11, 10, 9, 8, 7, 7, 6, 6, 5, 5, 5, 5

Distinct integers are = 15.