Resonance®
Educating for better tomorrow

NATIONAL BOARD FOR HIGHER MATHEMATICS
AND
HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

REGIONAL MATHEMATICAL OLYMPIAD, 2019
(All Region)

QUESTION PAPER WITH SOLUTION

Sunday, October 20, 2019 | Time: 1 PM – 4 PM

Resonance®
Educating for better tomorrow

SUCCESS AT JEE ADVANCED 2019

TOTAL SELECTIONS

5162

1 or 2 Yearlong Classroom: **3473** | Distance Learning & e-Learning: **1689**
Kota Classroom : **2245** | All Study Centres (Classroom): **1228**

AIR 73
Ananjan Nandi
Classroom Student since Class XI

AIR 80
Tamajit Banerjee
Classroom Student since Class XII

List of all our selected students is available on our official website

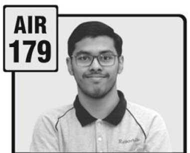
HIGHEST* CLASSROOM GIRL STUDENTS SELECTED	HIGHEST* CLASSROOM HINDI MEDIUM STUDENTS SELECTED
353	**277**

AIR 127 — **SAHASRA RANJAN**, Classroom Student since class XII

AIR 145 — **ANUBHAV KALYANI**, Classroom Student since class XI

AIR 161 — **ANGIKAR GHOSAL**, Classroom Student since class XII

AIR 168 — **KRITIN SHARMA**, Classroom Student since class XI



AIR 179 — **ATREYA GOSWAMI**, Classroom Student since class XI

AIR 192 — **SAPTARSHI DASGUPTA**, Classroom Student since class XI

AIR 212 — **ATUR GUPTA**, Classroom Student since class VIII

AIR 216 — **SHUBHANKAR**, Classroom Student since class X

AIR 225 — **AMAN GUPTA**, Classroom Student since class IX

AIR 237 — **KANISHK SINGHAL**, Classroom Student since class VIII

AIR 247 — **RUPINDER GOYAL**, Classroom Student since class XI

BEST RANKS IN CATEGORIES

SC — AIR 3 — **ANSHUL NAVPHULE**, Classroom Student since class VII

ST — AIR 4 — **ATIN BAINADA**, Classroom Student since class XI

OBC NCL — AIR 11 — **SAHASRA RANJAN**, Classroom Student since class XII

GEN EWS — AIR 21 — **SOUMIL SARAWGI**, Classroom Student since class XI

*Based on the information collected from public domain till 17th June, 2019, 1:00 PM

Top 100 AIRs - Other Categories from Classroom Programs

Gen - EWS	21, 22, 23, 37, 42, 43, 54, 90, 94
OBC - NCL	11, 34, 40, 56, 72, 73, 76
SC	3, 11, 30, 31, 36, 37, 53, 64, 72, 92, 94, 100
ST	4, 10, 13, 18, 21, 22, 30, 37, 43, 53, 68, 70, 74, 83, 89, 90, 91, 94

Top 100 AIRs
Distance Learning Program

18	42	48	54
58	61	90	98

All from General Category

JNV Bundi Result Highlight

HIGHEST* SELECTION RATIO
amongst any JNV across India

84% 84 students selected out of 100 students appeared

Toll Free: 1800 258 5555 | Website: www.resonance.ac.in | Email: contact@resonance.ac.in

Time : 3 hours (समय : 3 घंटे) **October 20, 2019** **Total marks (अधिकतम अंक) : 102**

Instruction निर्देश :

1. Calculators (in any form) and protractors are not allowed.

कैलकुलेटर (किसी भी रूप में) या चांदा लाने की अनुमति नहीं है।

2. Rulers and compasses are allowed.

रूलर एवं प्रकार लाने की अनुमति है।

3. Answer all the questions. Draw neat Geometry diagrams.

सभी प्रश्नों के जवाब दें।

4. All questions carry equal marks. Maximum marks 102

सभी प्रश्न बराबर अंकों के हैं। अधिकतम अंक : 102

5. Answerer to each question should start on a new page, clearly indicate the question number.

हर प्रश्न का हल नए पन्ने से शुरू करें। प्रश्न संख्या का साफ–साफ उल्लेख करें।

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1. Suppose x is a nonzero real number such that both x^5 and $20x + \frac{19}{x}$ are rational numbers. Prove that x is a rational number.

मान लो कि x ऐसी अशून्य वास्तविक संख्या है जिसके लिए x^5 व $20x + \frac{19}{x}$ दोनों ही परिमेय संख्याएँ है। सिद्ध करो कि x भी एक परिमेय संख्या है।

Sol. Let $20x + \frac{19}{x} = \alpha$ $(\alpha \in Q)$

$\Rightarrow$ $20x^2 - \alpha x + 19 = 0$ $\Rightarrow$ $x^2 = px + q$ where $p = \frac{\alpha}{20}$ & $q = \frac{-19}{20}$ hence $p, q \in Q$.

Now $x^5 = x(px + q)^2 = x\{p^2x^2 + 2pqx + q^2\} = p^2x(px + q) + 2pq(px + q) + q^2x$

$= p^3(px + q) + (3p^2q + q^2)x + 2pq^2 = (p^4 + 3p^2q + q^2)x + p^3q + 2pq^2 = \lambda$(say)

Given that x^5 is rational

so $x = \frac{\lambda - p^3q - 2pq^2}{p^4 + 3p^2q + q^2} \in Q$

Hence x is also a rational number.

Hindi. माना $20x + \frac{19}{x} = \alpha$ $(\alpha \in Q)$

$\Rightarrow$ $20x^2 - \alpha x + 19 = 0$ $\Rightarrow$ $x^2 = px + q$ जहाँ $p = \frac{\alpha}{20}$ तथा $q = \frac{-19}{20}$ अतः $p, q \in Q$.

अब $x^5 = x(px + q)^2 = x\{p^2x^2 + 2pqx + q^2\} = p^2x(px + q) + 2pq(px + q) + q^2x$

$= p^3(px + q) + (3p^2q + q^2)x + 2pq^2 = (p^4 + 3p^2q + q^2)x + p^3q + 2pq^2 = \lambda$(माना)

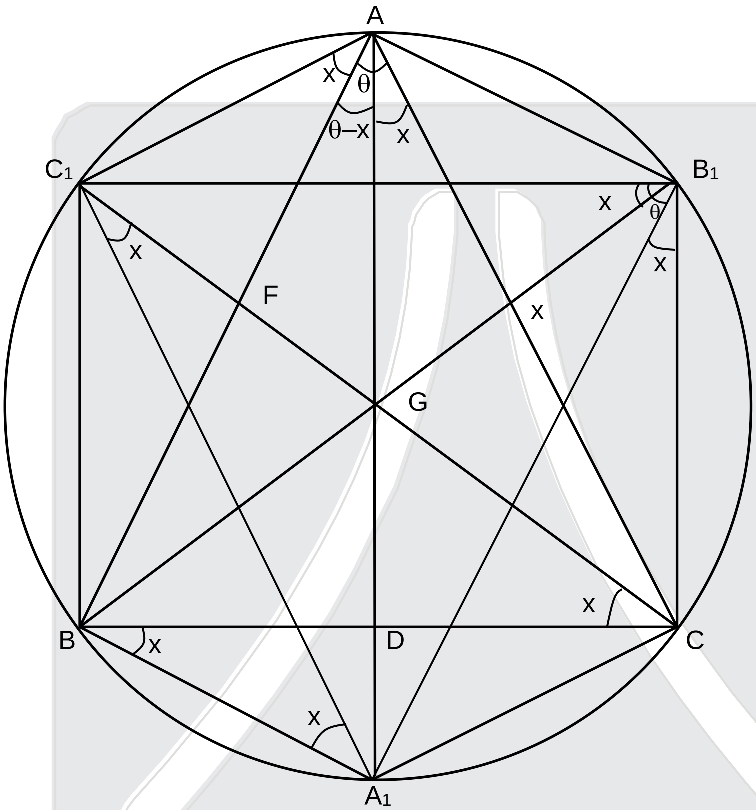
दिया गया है कि x^5 परिमेय है

इसलिए $x = \frac{\lambda - p^3q - 2pq^2}{p^4 + 3p^2q + q^2} \in Q$

x परिमेय संख्या भी है।

2. Let ABC be a triangle with circumcircle Ω and let G be the centroid of triangle ABC. Extend AG, BG and CG to meet the circle Ω again in A_1, B_1 and C_1, respectively. Suppose $\angle BAC = \angle A_1B_1C_1$, $\angle ABC = \angle A_1C_1B_1$ and $\angle ACB = \angle B_1A_1C_1$. Prove that ABC and $A_1B_1C_1$ are equilateral triangles.
मान लो कि ABC एक त्रिभुज है जिसका परिवृत Ω है और मान लो कि G त्रिभुज ABC का केंद्रक है। रेखाखंड AG, BG व CG विस्तार करने पर वृत Ω से क्रमशः A_1, B_1 व C_1 में पुनः मिलते है। मान लो कि $\angle BAC = \angle A_1B_1C_1$, $\angle ABC = \angle A_1C_1B_1$ व $\angle ACB = \angle B_1A_1C_1$ है। सिद्ध करो कि ABC व $A_1B_1C_1$ समबाहु त्रिभुज है।

Sol.



Let G is centroid of ΔABC

Let BG intersect AC at E,

AG intersect BC at D,

CG intersect AB at F.

AB_1, B_1C, CA_1, A_1B, BC_1 and C_1A.

Proof : $\angle BAC = \angle C_1B_1A_1 = \theta$ and $\angle A_1AC = x$

Now $\angle BAA_1 = \theta - x$

$\angle C_1AA_1 = \angle C_1B_1A_1 = \theta$

$\Rightarrow$ $\angle C_1AB = \angle C_1AA_1 - \angle BAA_1 = x$

$\Rightarrow$ $\angle C_1AB = \angle C_1B_1B = \angle C_1CB = \angle C_1A_1B = x$

$\angle A_1B_1C = \angle A_1AC = \angle AC_1C = \angle A_1BC = x$

Now $\angle C_1CB = x = \angle CBA_1$

$\Rightarrow$ A_1B is parallel to CC_1

Similarly B_1C is parallel to AA_1

and AC_1 is parallel to BB_1

Now in ΔAC_1C, mid point of AC is E

Now EG is parallel to AC_1

$\Rightarrow$ G is mid point of chord CC_1

Similarly G is mid point of chords BB_1 & A_1A_1 also.

$\Rightarrow$ G is circumcentre of ΔABC also

$\Rightarrow$ ΔABC is equilateral

because ΔABC is similarly with $\Delta B_1C_1A_1 \Rightarrow$ $\Delta A_1B_1C_1$ is also equilateral.

माना कि G त्रिभुज ΔABC का केन्द्रक है

माना BG, AC को E पर प्रतिच्छेद करता है

AG, BC को D पर प्रतिच्छेद करता है

CG, AB को F पर प्रतिच्छेद करता है

AB_1, B_1C, CA_1, A_1B, BC_1 और C_1A.

उत्पत्ति : $\angle BAC = \angle C_1B_1A_1 = \theta$ और $\angle A_1AC = x$

अब $\angle BAA_1 = \theta - x$

$\angle C_1AA_1 = \angle C_1B_1A_1 = \theta$

$\Rightarrow$ $\angle C_1AB = \angle C_1AA_1 - \angle BAA_1 = x$

$\Rightarrow$ $\angle C_1AB = \angle C_1B_1B = \angle C_1CB = \angle C_1A_1B = x$

$\angle A_1B_1C = \angle A_1AC = \angle AC_1C = \angle A_1BC = x$

अब $\angle C_1CB = x = \angle CBA_1$

$\Rightarrow$ A_1B, CC_1 के समान्तर है

इसी प्रकार B_1C, AA_1 के समान्तर है

तथा AC_1, BB_1 के समान्तर है

अब ΔAC_1C में AC का मध्य बिन्दु E है

अब EG, AC_1 के समान्तर है

$\Rightarrow$ G, जीवा CC_1 का मध्य बिन्दु है

इसी प्रकार G, जीवाओं BB_1 और A_1A_1 का मध्य बिन्दु भी है

$\Rightarrow$ G, त्रिभुज ΔABC का परिकेन्द्र भी है

$\Rightarrow$ ΔABC समबाहु त्रिभुज है।

क्योकि ΔABC, $\Delta B_1C_1A_1$ के समरूप है $\Rightarrow$ $\Delta A_1B_1C_1$ इसलिए यह समबाहु त्रिभुज है

3. Let a, b, c be positive real numbers such that a + b + c = 1. Prove that $\frac{a}{a^2+b^3+c^3}+\frac{b}{b^2+c^3+a^3}+\frac{c}{c^2+a^3+b^3}\le\frac{1}{5abc}$

मान लो कि a, b, c ऐसी धनात्मक वास्तविक संख्याएँ हैं जिनके लिए a + b + c = 1 सिद्ध करो कि $\frac{a}{a^2+b^3+c^3}+\frac{b}{b^2+c^3+a^3}+\frac{c}{c^2+a^3+b^3}\le\frac{1}{5abc}$

Sol. To prove

$$\frac{a}{a^2+b^3+c^3}+\frac{b}{b^2+c^3+a^3}+\frac{c}{c^2+a^3+b^3}\le\frac{1}{5abc}$$

$$\frac{a}{a^2(a+b+c)+b^3+c^3}+\frac{b}{b^2(a+b+c)+c^3+a^3}+\frac{c}{c^2(a+b+c)+a^3+b^3}\le\frac{1}{5} \quad \text{........(1)}$$

Now $$\frac{a^3+b^3+c^3+a^2b+a^2c}{5}\ge(a^7b^4c^4)^{1/5}$$

$$\frac{1}{a^3+b^3+c^3+a^2b+a^2c}\le\frac{1}{5(a^7b^4c^4)^{1/5}}$$

$$\frac{a^2bc}{a^3+b^3+c^3+a^2b+a^2c}\le\frac{a^2bc}{5(a^7b^4c^4)^{1/5}}$$

$$\frac{a^2bc}{a^3+b^3+c^3+a^2b+a^2c}\le\frac{a^{3/5}b^{1/5}c^{1/5}}{5} \quad \text{........(2)}$$

Similarly $$\frac{b^2ac}{b^3a+b^3+cb^2+c^3+a^3}\le\frac{a^{1/5}b^{3/5}c^{1/5}}{5} \quad \text{........(3)}$$

$$\frac{abc^2}{c^2a+c^2b+c^3+a^3+b^3}\le a^{1/5}b^{1/5}c^{3/5} \quad \text{........(4)}$$

add (2), (3) and (4) we get

$$\frac{a^2bc}{a^3+a^2b+a^2c+b^3+c^3}+\frac{b^2ac}{b^3a+b^3+b^2c+c^3+a^3}+\frac{c^2ab}{c^2a+c^2b+c^3+a^3+b^3}$$

$$\leq \frac{a^{3/5}b^{1/5}c^{1/5}+a^{1/5}b^{3/5}c^{1/5}+a^{1/5}b^{1/5}c^{3/5}}{5} \qquad \text{........(5)}$$

Now without less of generally assume $a \geq b \geq c$

then $a^{3/5} \geq b^{3/5} \geq c^{3/5}$

$a^{1/5} \geq b^{1/5} \geq c^{1/5}$

$a^{1/5} \geq b^{1/5} \geq c^{1/5}$

$\Rightarrow \quad a^{3/5}\, b^{1/5}\, c^{1/5} + b^{3/5}\, c^{1/5}\, a^{1/5} + c^{3/5}\, a^{1/5}\, b^{1/5} \leq a^{3/5}\, a^{1/5}\, a^{1/5} + b^{3/5}\, b^{1/5}\, b^{1/5} + c^{3/5}\, c^{1/5}\, c^{1/5}$

$\Rightarrow \quad a^{3/5}\, b^{1/5}\, c^{1/5} + b^{3/5}\, c^{1/5}\, a^{1/5} + c^{3/5}\, a^{1/5}\, b^{1/5} \leq 1$ .... (6)

Using (5) & (6) we get

$$\frac{a^2bc}{a^3+a^2b+a^2c+b^3+c^3}+\frac{b^2ac}{b^3a+b^3+b^2c+c^3+a^3}+\frac{c^2ab}{c^2a+c^2b+c^3+a^3+b^3} \leq \frac{1}{5}$$

4. Consider the following 3 × 2 array formed by using the numbers 1, 2, 3, 4, 5, 6 : $\begin{pmatrix} a_{11}, a_{12} \\ a_{21}, a_{22} \\ a_{31}, a_{32} \end{pmatrix} = \begin{pmatrix} 1, & 6 \\ 2, & 5 \\ 3, & 4 \end{pmatrix}$.

Observe that all row sums equal, but the sum of the squares is not same for each row. Extend the above array to a 3 × k array $(a_{1j})_{3\times k}$ for a suitable k, adding more columns. Using the numbers 7, 8, 9,...... 3k such that

$$\sum_{j=1}^{k} a_{1j} = \sum_{j=1}^{k} a_{2j} = \sum_{j=1}^{k} a_{3j} \text{ and } \sum_{j=1}^{k} (a_{1j})^2 = \sum_{j=1}^{k} (a_{2j})^2 = \sum_{j=1}^{k} (a_{3j})^2$$

संख्या 1, 2, 3, 4, 5, 6 से निर्मित निम्न 3 × 2 सारणी को लो: $\begin{pmatrix} a_{11}, a_{12} \\ a_{21}, a_{22} \\ a_{31}, a_{32} \end{pmatrix} = \begin{pmatrix} 1, & 6 \\ 2, & 5 \\ 3, & 4 \end{pmatrix}$

देखो कि सभी पंक्तियों का योग बराबर है, पर हर पंक्ति में वर्गो का योग बराबर नहीं है। इस सारणी में, किसी एक उपयुक्त k के लिए, संख्याओं 7, 8, 9,...... 3k का प्रयोग करके अन्य स्तम्भ जोड़ो और इसे एक 3 × k सारणी $(a_{1j})_{3\times k}$ में ऐसे बदलो जिससे कि:

$$\sum_{j=1}^{k} a_{1j} = \sum_{j=1}^{k} a_{2j} = \sum_{j=1}^{k} a_{3j} \text{ और } \sum_{j=1}^{k} (a_{1j})^2 = \sum_{j=1}^{k} (a_{2j})^2 = \sum_{j=1}^{k} (a_{3j})^2$$

Sol. We observe that

$(k + 5)^2 - (k + 4)^2 = (k + 1)^2 - k^2 + 8$ and $(k + 5)^2 - (k + 3)^2 = (k + 2)^2 - k^2 + 12$

$\Rightarrow \quad (k + 5)^2 + k^2 = (k + 1)^2 + (k + 4)^2 + 8 = (k + 2)^2 + (k + 3)^2 + 12$

for k = 1, we have $(1^2 + 6^2) = (2^2 + 5^2) + 8 = (3^2 + 4^2) + 12$

Let us write first two columns as $\begin{pmatrix} 1^2 & 6^2 \\ 2^2 & 5^2 \\ 3^2 & 4^2 \end{pmatrix}$ to have sums as $\alpha + 12 + \alpha + 4, \alpha$ rowwise

new let us write next two columns in such a way that we get sums as $\beta, \beta + 12, \beta + 4$ and then further next two columns to get sums $\gamma + 4, \gamma, \gamma + 12$ so that we get sum in each column as $\alpha + \beta + \gamma + 16$

In that way, then matrix would be $\begin{pmatrix} 1 & 6 & 9 & 10 & 14 & 17 \\ 2 & 5 & 7 & 12 & 15 & 16 \\ 3 & 4 & 8 & 11 & 13 & 18 \end{pmatrix}$

Here sum of numbers in each row is 57 and sum of squares of numbers in each row is 703 so we can chose k = 6

Hindi. हम देखते है कि

$(k + 5)^2 - (k + 4)^2 = (k + 1)^2 - k^2 + 8$ और $(k + 5)^2 - (k + 3)^2 = (k + 2)^2 - k^2 + 12$

$\Rightarrow \quad (k + 5)^2 + k^2 = (k + 1)^2 + (k + 4)^2 + 8 = (k + 2)^2 + (k + 3)^2 + 12$

k = 1 के लिए, $(1^2 + 6^2) = (2^2 + 5^2) + 8 = (3^2 + 4^2) + 12$

माना प्रथम दो स्तभों को $\begin{pmatrix} 1^2 & 6^2 \\ 2^2 & 5^2 \\ 3^2 & 4^2 \end{pmatrix}$ के रूप मे लिखा जा सकता है जिसका एक पंक्ति में $\alpha + 12 + \alpha + 4, \alpha$

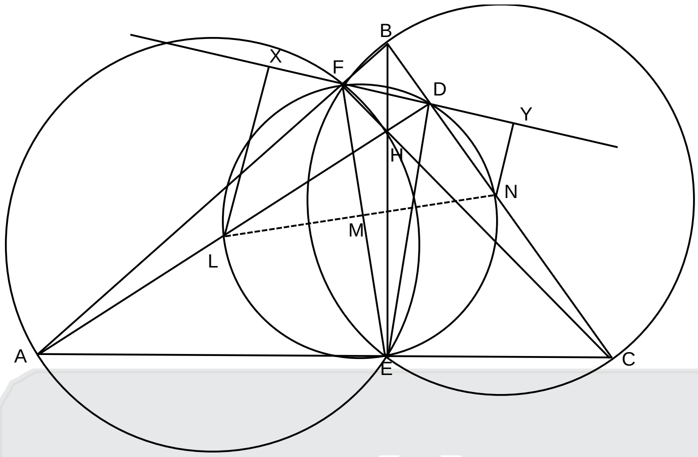
अब दो स्तम्भ इस प्रकार लिख सकते है कि $\beta, \beta + 12, \beta + 4$ तथा पुनः अगले दो स्तभों को $\gamma + 4, \gamma, \gamma + 12$ के योगफल के रूप मे लिखा जा सकता है। इस प्रकार प्रत्येक स्तम्भ मे योगफल $\alpha + \beta + \gamma + 16$ के रूप में होगा तब आव्यूह $\begin{pmatrix} 1 & 6 & 9 & 10 & 14 & 17 \\ 2 & 5 & 7 & 12 & 15 & 16 \\ 3 & 4 & 8 & 11 & 13 & 18 \end{pmatrix}$ होगी।

अतः प्रत्येक पंक्ति मे संख्याओं का योगफल 57 है, तथा प्रत्येक पंक्ति में संख्याओं के वर्गो का योगफल 703 है। अतः हम k = 6 चुन सकते है।

5. In an acute angled triangle ABC, let H be the orthocentre, and let D, E, F be the feet of altitudes from A, B, C to the opposite sides, respectively. Let L, M, N be midpoints of segments AH, EF, BC, respectively. Let X, Y be feet of altitudes from L, N on to the line DF. Prove that XM is perpendicular to MY.

मान लो कि किसी न्युनकोण त्रिभुज ABC में H लम्ब–केन्द्र है, व D, E, F वह बिंदु है जिस पर क्रमशः A, B, C से लम्ब सामने वाली भुजा (आधार) से मिलते है। मान लो कि L, M, N क्रमशः रेखाखंड AH, EF, BC के मध्य बिंदु है। मान लो कि बिंदु X, Y से रेंखा DF पर लम्ब उससे क्रमशः X, Y पर मिलते है। सिद्ध करो कि XM रेखा XY से लम्ब है।

Sol.



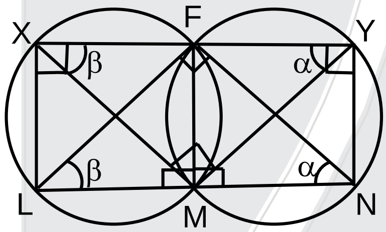
Quadrilateral AFHE and BFEC are cyclic quadrilateral with centres L and N respectively

$\Rightarrow$ LN is perpendicular to common chord (FE) of circumcircle of AFHE and circumcircle BFEF

Also LN passes through midpoint(M) of EF

Now circumcircle ΔDEF also passes through L and N, where LN is diameter of circle (because $\angle$LDN = 90°) $\Rightarrow$ $\angle$LFN = 90°

Now quadrilateral XFML and FMNY are also cyclic quadrilateral.



Let $\angle FNM = \angle FYM = \alpha$ and $\angle FXM = \angle FLM = \beta$

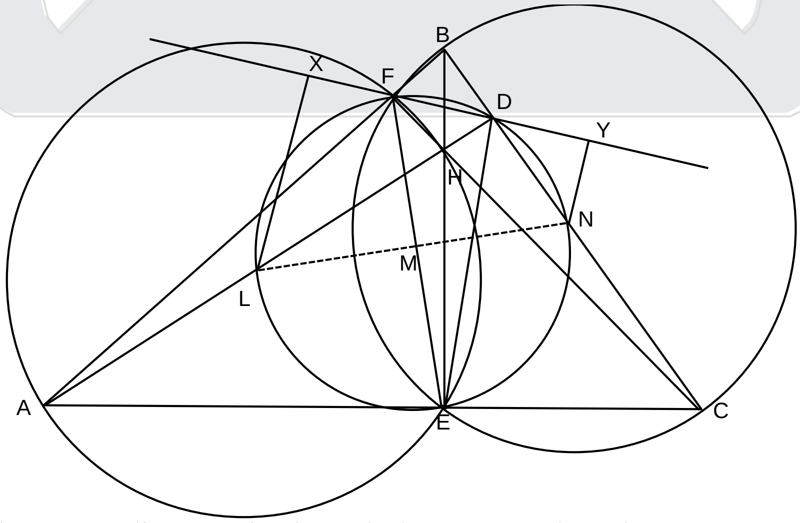
Now $\angle FLN + \angle FNL = 180° - \angle LFN$

$\Rightarrow$ $\alpha + \beta = 180° - 90° = 90°$ $\Rightarrow$ $\angle MXY + \angle MYX = 90°$

$\Rightarrow$ $\angle XMY = 180° - \angle MXY - \angle MYX = 180° - 90° = 90°$

$\Rightarrow$ Hence Prove

Hindi.



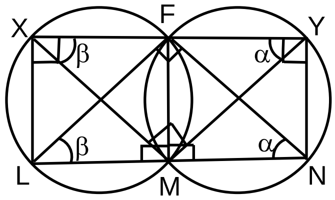
चतुर्भुज AFHE और BFEC निर्देशांक चतुर्भुज है जिनके केन्द्र क्रमशः L और N है।

$\Rightarrow$ LN, परिवृत्त AFHE और परिवृत्त BFEF की उभयनिष्ठ जीवा (FE) के लम्बवत है।

तथा LN, EF के मध्य बिन्दु (M) से गुजरता है।

अब ΔDEF का परिगत वृत्त, L और N से गुजरता है जहां LN वृत्त का व्यास है (क्योंकि $\angle LDN = 90°$) $\Rightarrow$ $\angle LFN = 90°$

अब चतुर्भुज XFML और FMNY भी चक्रीय चतुर्भुज है।



माना $\angle FNM = \angle FYM = \alpha$ and $\angle FXM = \angle FLM = \beta$

अब $\angle FLN + \angle FNL = 180° - \angle LFN$

$\Rightarrow \quad \alpha + \beta = 180° - 90° = 90° \quad \Rightarrow \quad \angle MXY + \angle MYX = 90°$

$\Rightarrow$ अतः $\angle XMY = 180° - \angle MXY - \angle MYX = 180° - 90° = 90°$

6. Suppose 91 distinct positive integers greater than 1 are given such that there are at least 456 pairs, among them which are relatively prime. Shon that one can find four integers a, b, c, d among them such that gcd (a, b) = gcd (b, c) = gcd (c, d) = gcd (d, a) = 1.

मान लो कि 91 अलग–अलग 1 से बड़े ऐसे धनात्मक पूर्णाक दिए गए है कि उनमें कम से कम 456 जोडे ऐसे है जो असहभाज्य है। सिद्ध करो कि इनमें ऐसे चार पूर्णाक a, b, c, d मिलेंगे जिनके लिए gcd (a, b) = gcd (b, c) = gcd (c, d) = gcd (d, a) = 1 (यहाँ gcd माने म. स. या महतम समापवर्तक)

Sol. Let numbers are $\{n_1, n_2, n_3, \ldots\ldots n_{91}\} = A$

Let numbers coprime with n_i are m_i

Number of unordered pairs (x_1, y_1) from set A coprime with n_1 are ${}^{m_1}C_2 = \dfrac{m_1(m_1-1)}{2}$

similarly number of unordered (x_2, y_2) from set A coprime with n_2 are $\dfrac{m_2(m_2-1)}{2}$

…….. and so on.

Now, if $\gcd(n_1, n_2) = \gcd(n_2, n_3) = \gcd(n_3, n_4) = \gcd(n_4, n_1) = 1$ then

in $\sum_{i=1}^{91} \dfrac{m_i(m_i-1)}{2}$ the pair (n_2, n_4) comes two times.

Let us assume no four numbers exist in set A such that these are coprime in cyclic order. Then in

$\sum_{i=1}^{91} \dfrac{m_i(m_i-1)}{2}$, all pair (x_k, y_k) are distinct

But $\quad \sum_{i=1}^{91} \dfrac{m_i^2}{2} - \sum_{i=1}^{91} \dfrac{m_i}{2} \le \dfrac{91 \times 90}{2} \quad \Rightarrow \quad \dfrac{\sum_{i=1}^{91} m_i^2}{91} \le \dfrac{\sum m_i}{91} + 90$

Because $\quad \dfrac{\sum_{i=1}^{91} m_i^2}{91} \ge \left(\dfrac{\sum_{i=1}^{91} m_i}{91}\right)^2$, hence

$$\left(\dfrac{\sum_{i=1}^{91} m_i}{91}\right)^2 \le \left(\dfrac{\sum_{i=1}^{91} m_i}{91}\right) + 90 \Rightarrow \dfrac{\sum_{i=1}^{91} m_i}{91} \le 10 \quad \Rightarrow \quad \dfrac{\sum_{i=1}^{91} m_i}{2} \le 455$$

$\Rightarrow$ number of coprime pairs ≤ 455
But number of coprime pairs ≥ 456, hence our assumption is failed.
$\Rightarrow$ four numbers a, b, c, d exist in set A which are coprime in cyclic order.

Hindi. माना कि संख्याएँ $\{n_1, n_2, n_3, \ldots.. n_{91}\} = A$ है
माना n_i और m_i सहअभाज्य संख्याएँ है।

समुच्चय A से n_1 के साथ सहअभाज्य अयुग्मित युग्म (x_1, y_1) की संख्या ${}^{m_1}C_2 = \frac{m_1(m_1-1)}{2}$

इसी प्रकार समुच्चय A, n_2 के सहअभाज्य अक्रमित युग्म (x_2, y_2) की संख्या $\frac{m_2(m_2-1)}{2}$

.... और तब.

अब यदि $\gcd(n_1, n_2) = \gcd(n_2, n_3) = \gcd(n_3, n_4) = \gcd(n_4, n_1) = 1$

in $\sum_{i=1}^{91} \frac{m_i(m_i-1)}{2}$, (n_2, n_4) दो बार आएगा।

माना कि कोई भी चार संख्याएं समुच्चय A मे विद्यमान नहीं है कि ये सभी चक्रीय क्रम मे सहअभाज्य है। तब

$\sum_{i=1}^{91} \frac{m_i(m_i-1)}{2}$ में (x_k, y_k) सभी युग्म भिन्न–भिन्न है।

परन्तु $$\sum_{i=1}^{91} \frac{m_i^2}{2} - \sum_{i=1}^{91} \frac{m_i}{2} \le \frac{91 \times 90}{2} \quad \Rightarrow \quad \frac{\sum_{i=1}^{91} m_i^2}{91} \le \frac{\sum m_i}{91} + 90$$

क्योकि $$\frac{\sum_{i=1}^{91} m_i^2}{91} \ge \left(\frac{\sum_{i=1}^{91} m_i}{91}\right)^2$$, अतः

$$\left(\frac{\sum_{i=1}^{91} m_i}{91}\right)^2 \le \left(\frac{\sum_{i=1}^{91} m_i}{91}\right) + 90 \Rightarrow \frac{\sum_{i=1}^{91} m_i}{91} \le 10 \quad \Rightarrow \quad \frac{\sum_{i=1}^{91} m_i}{2} \le 455$$

$\Rightarrow$ सह अभाज्य युग्मों की संख्या ≤ 455
जो कि विरोधाभास है।
$\Rightarrow$ चार संख्याएं a, b, c, d समुच्चय A विद्यमान है जो कि ये सभी चक्रीय क्रम मे सह अभाज्य है।

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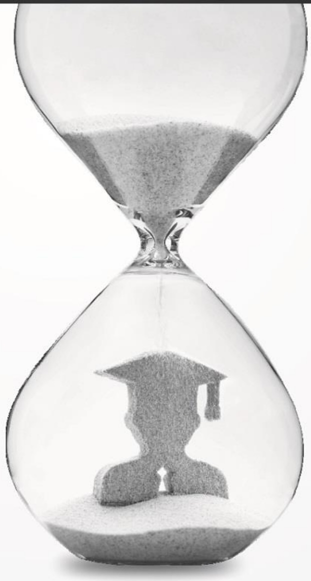

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