THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
Screening Test - Kaprekar Contest
NMTC at SUB JUNIOR LEVEL - VII & VIII Standards
Saturday, 31 August, 2019

Note:

1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.

2. Diagrams are only visual aids; they are NOT drawn to scale.

3. You are free to do rough work on separate sheets.

4. Duration of the test: **2 hours.**

PART—A

Note

- Only one of the choices A. B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering; seek the guidance of the supervisor.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{2}$ mark.**

1. If 4921 × D = ABBBD, then the sum of the digits of ABBBD × D is _________

(A) 19 (B) 20 (C) 25 (D) 26

Sol. **(A)**

4-digit no. (4921) is multiplied by a single digit no. (D) & result is five digit no., so definitely D > 2

So by hit & trial we put the values of D from 3 to 9.

at D = 7

4921 × 7 = 34447 (ABBBD)

So A = 3, B = 4 , D = 7

Now ABBBD (34447) × 7 = 241129

Sum of digits = 2 + 4 + 1 + 1 + 2 + 9 = 19

2. What is the 2019th digit to the right of the decimal point, in the decimal representation of $\frac{5}{28}$?

(A) 2 (B) 4 (C) 8 (D) 7

Sol. **(C)**

$\frac{5}{28} = \cdot 17\overline{857142}$

$\Rightarrow$ 2019 = 2 + 336 × 6 + 1 [2 for 17 & 336 pairs of 6 repeating number]

2019th digit from right side to decimal is first digit in repetition

So correct answer is 8

3. If X is a 1000 digit number, Y is the sum of its digits, Z the sum of the digits of Y and W the sum of the digits of Z, then the maximum possible value of W is

(A) 10 (B) 11 (C) 12 (D) 22

Sol. (B)

$X \to 1000$ digit no.

If all digit are '9' so that

maximum sum of digit of 'X' is 9000

So maximum value of Y is 9000

But for maximum sum of digit of Y is 35 for number (8999)

So Z is maximum 35.

Now for maximum sum of digit of Z is 11 for number 29.

So W = 11.

Practical example: if $X = \underbrace{99....9}_{333\text{ times}}\ \underbrace{000...0}_{666\text{ times}}\ 2$

Sum of digit of X = Y = 2999

Sum of digit of Y = Z = 29

Sum of digit of Z = W = 11

4. Let x be the number 0.000.......001 which has 2019 zeroes after the decimal point. Then which of the following numbers is the greatest?

(A) 10000 + x (B) $10000 \cdot x$ (C) $\frac{10000}{x}$ (D) $\frac{1}{x^2}$

Sol. (D)

$x = \cdot 000 \ldots. 01 = 10^{-2020}$

From option (A) = $10000 + x = 1000 + 10^{-2020} = 10000\cdot\underbrace{000....01}_{2019\text{ times}}$

From option (B) = $10000 \times x = 10^4 \times 10^{-2020} = 10^{-2016}$

From option (C) = $\frac{10000}{x} = \frac{10^4}{10^{-2020}} = 10^{2024}$

From option (D) = $\frac{1}{x^2} = \frac{1}{\left(10^{-2020}\right)^2} = 10^{4040}$

So from options $\frac{1}{x^2}$ is greatest.

5. If $\begin{array}{r} ABC \\ +\ \ CBA \\ \hline DEDD \end{array}$ then the number of possible values for A, B, C, D, E satisfying this equation where A, B, C, D and E are distinct digits is

(A) 6 (B) 5 (C) 4 (D) 3

Sol. (C)

$\begin{array}{r} ABC \\ +\ \ CBA \\ \hline DEDD \end{array}$ $D \to$ Must be 1 .

Means C + A = 1 or 11

$\therefore$ Sum of B + B is even & D is 1.

So possible values of B is 0 or 5.

But if we take B as '0' so there is no carry forward & Sum of A & C, did not get different digit from D.

So B must be 5.

Sum is convert into $\begin{array}{r} A\,5\,C \\ C\,5\,A \\ \hline 1211 \end{array}$

Now possible pairs of (A,C) are (3,8) (4,7)(7,4) (8,3)

So total 4 possible solutions are there.

6. In a 5 × 5 grid having 25 cells, Janani has to enter 0 or 1 in each cell such that each sub square grid of size 2 × 2 has exactly three equal numbers. What is the maximum possible sum of the numbers in all the 25 cells put together?

(A) 23 (B) 21 (C) 19 (D) 18

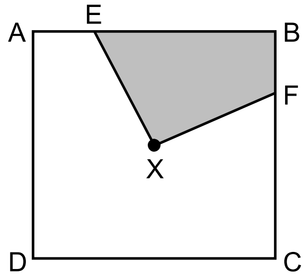
Sol. **(B)**

Required answer

1	1	1	1	1
1	0	1	0	1
1	1	1	1	1
1	0	1	0	1
1	1	1	1	1

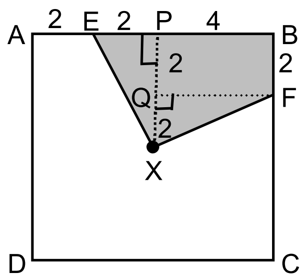
So sum of all no's is 21.

7. ABCD is a square. E is one fourth of the way from A to B and F is one fourth of the way from B to C. X is the centre of the square. Side of the square is 8 cm. Then the area of the shaded region in the figure in cm^2 is



(A) 14 (B) 16 (C) 18 (D) 20

Sol. **(B)**

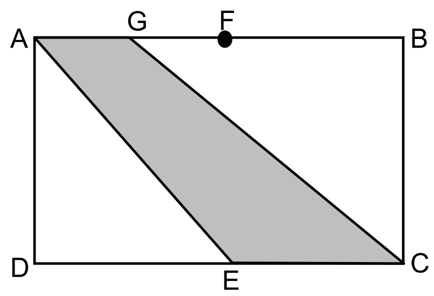


Draw perpendicular XP from X to AB & FQ on PX.

So required area = Area of Δ XPE + Area of ΔFQX + Area of quadrilateral PBFQ.

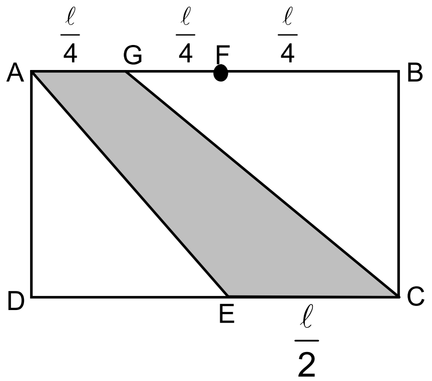
$= \frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2 \times 4 + 4 \times 2$ = 4 + 4 + 8 = 16

8. ABCD is a rectangle with E and F are midpoints of CD and AB respectively and G is the mid-point of AF. The ratio of the area of ABCD to area of AECG is



(A) 4 : 3 (B) 3 : 2 (C) 6 : 5 (D) 8 : 3

Sol. **(D)**



Let length of rectangle AB = CD = ℓ

& breadth of rectangle AD = BC = b

So area of trapezium AECG = $\frac{1}{2}\left(\frac{\ell}{2}+\frac{\ell}{4}\right)\times b = \frac{3}{8}\ell b$

$$\frac{\text{Area of ABCD}}{\text{Area of AECG}} = \frac{\ell b}{\frac{3}{8}\ell b} = \frac{8}{3} \ (8:3)$$

9. If
$$\begin{array}{r} R\ A\ T \\ +\ M\ A\ T \\ +\ V\ A\ T \\ \hline F\,L\,A\,T \end{array}$$
and each alphabet represents a different digit, what is the maximum possible value of FLAT?

(A) 2450 (B) 2405 (C) 2305 (D) 2350

Sol. **(A)**

T should be 0 or 5

But if we take T as '5' sum is 15 & 1 carry is forward and sum of 3 'A' and '1' never give unit digit 'A'

So T must be '0'

Now again possible values of A are '0' & 5 but alphabet represents different digits so A is 5.

For maximum value of FLAT, we take maximum value of R, M & V as 9, 8 & 7, but sum is 25 & 5 is repeat. So by taking R, M & V as 9, 8 & 6. We get maximum value of FLAT.

$$\begin{array}{r} 9\ 5\ 0 \\ 8\ 5\ 0 \\ 6\ 5\ 0 \\ \hline 2\ 4\ 5\ 0 \end{array} \qquad \text{FLAT} \Rightarrow 2450$$

10. How many positive integers smaller than 400 can you get as a sum of eleven consecutive positive integers?

(A) 37 (B) 35 (C) 33 (D) 31

Sol. (D)

Number are $1 + 2 + 3 + \ldots\ldots\ldots + 11 = 66$

$2 + 3 + 4 + \ldots\ldots\ldots + 12 = 77$

$3 + 4 + 5 + \ldots\ldots\ldots + 13 = 88$

So number are 66,77,88…..

These number are multiple of 11 from 6th multiple.

So largest number which is multiple of 11 & less than 400 is 396.

396 is 36th multiple of 11.

So required no's are $36 - 5 = 31$ (5 for first 5 multiples)

11. Let x, y and z be positive real numbers and let $x \geq y \geq z$ so that $x + y + z = 20.1$. Which of the following statements is true ?

(A) Always $xy < 99$ (B) Always $xy > 1$ (C) Always $xy \neq 75$ (D) Always $yz \neq 49$

Sol. (D)

$x + y + z = 20.1$

In option (A)

If we take $x = y = 10$ & $z = .1$

$xy = 100 > 99$

So option (A) is wrong

In option (B) if we take, $x = 20.050$, $y = 00.049$, $z = .001$

$xy = 0.98245 < 1$

So option (B) is also wrong

In option (C) If we take $x = 15$, $y = 5$, & $z = .1$

$xy = 75$

Option (C) is also wrong

In option (D)

Minimum value of $x = \dfrac{20.1}{3} = 6 \cdot 7$

maximum value of $z = \dfrac{20.1}{3} = 6 \cdot 7$

If $x = 6.7$ & $z = 6.7$ y is also 6.7

So maximum product of $yz = 6.7 \times 6.7 = 44.89$

So yz is never equal to 49.

Option (D) is correct.

12. A sequence $[a_n]$ is generated by the rule, $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$. Given $a_1 = 2$ and $a_2 = 4$, then sum of the first 2019 terms of the sequence is given by

(A) 8 (B) 2692 (C) – 2692 (D) – 8

Sol. (A)

$a_1 = 2$, $a_2 = 4$

$a_3 = a_2 - a_1 = 4 - 2 = 2$

$a_4 = a_3 - a_2 = 2 - 4 = -2$

$a_5 = a_4 - a_3 = -2 - (2) = -4$

$a_6 = a_5 - a_4 = -4 - (-2) = -2$

$a_7 = a_6 - a_5 = -2 - (-4) = 2$

$a_8 = a_7 - a_6 = 2 - (-2) = 4$

So pattern of no's are 2,4,2,–2, –4, –2, 2,4,2,–2,–4,–2 ……………… repeated after 6 numbers

Sum of 6 number = 2 + 4 + 2 + (–2) + (–4) + (–2) = 0

2019 = 2016 + 3

(336 × 6)

So sum of first 2016 terms = 0

Sum of first 2019 terms = 2 + 4 + 2 = 8

13. There are exactly 5 prime numbers between 2000 and 2030. Note: 2021 = 43 × 47 is not a prime number. The difference between the largest and the smallest among these is

(A) 16 (B) 20 (C) 24 (D) 26

Sol. **(D)**

Prime no's between 2000 & 2030 are

2003, 2011, 2017, 2027, 2029

Difference between 2029 & 2003 is

2029 – 2003 = 26

14. Which of the following geometric figures is possible to construct?

(A) A pentagon with 4 right angled vertices

(B) An octagon with all 8 sides equal and 4 angles each of measure 60° and other four angles of measure 210°

(C) A parallelogram with 3 vertices of obtuse angle measures.

(D) A hexagon with 4 reflex angles.

Sol. **(B)**

⇒ From Option (A)

Sum of all interior angles of pentagon is 540

If 4 angles are right angle so remaining fifth angle is 540 – 360 = 180º

& 180º at vertex is not possible so option (A) is incorrect

⇒ From option (B) Sum of all interior angles of octagon is 1080.

So according to given information sum of all angles is 4 × 60 + 4 × 210 = 1080

So this is possible

⇒ From option (C) sum of two adjacent angles of parallelogram is 180º.

So both angles are not possible to obtuse.

So 3 angles are obtuse is also not possible

So this option is also wrong.

⇒ From option (D) Sum of all interior angles of hexagon is 720.

If 4 angles are reflex than sum of interior angles is greater than 720º. So this option is also not possible.

15. If $y^{10} = 2019$, then

(A) $2 < y < 3$ (B) $1 < y < 2$ (C) $4 < y < 5$ (D) $3 < y < 4$

Sol. **(A)**

$2^{10} = 1024$ & $3^{10} = 59049$

$2^{10} < 2019 < 3^{10}$

So y is lie between 2 & 3

$2 < y < 3$

PART - B

Note :

- Write the correct answer in the space provided in the response sheet
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{4}$ marks.

16. A sequence of all natural numbers whose second digit (from left to right) is 1, is written in strictly increasing order without repetition as follows: 11, 21, 31, 41, 51, 61, 71, 81, 91, 110, 111,... Note that the first term of the sequence is 11. The third term is 31, eighth term is 81 and tenth term is 110. The 100th term of the sequence will be ___________?

Sol. **(1100)**

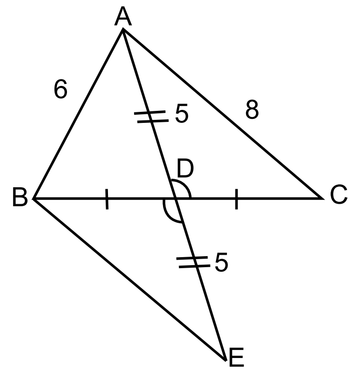
11, 21,31	….	91	{9}
110,111,112	…	119	{10}
210, 211,	…	219	{10]
: :		:	
: :		:	
910 911		919	{10}

Total 99 no's upto 919
So next 100 no is 1100

17. In ΔABC, AB = 6 cm, AC = 8 cm, median AD = 5 cm. Then, the area of ΔABC in cm^2 is ________.

Sol. **(24)**

Extend AD to E such that AD = DE = 5



BC = DC
DE = AD
$\angle DBE = \angle ADC$

So by SAS rule

$\Delta BDE \cong \Delta CDA$

by CPCT

BE = CA = 8

So Ar of (ΔBDE) = Ar of (ΔCDE)

add Ar (ΔABD) on both sides

Ar (ΔBDE) + Ar (ΔABD) = Ar (ΔABD) + (ΔADC)

So Ar (ΔABE) = Ar (ΔABC) … (1)

In ΔABE AB = 6
BE = 8
AE = 10

$\therefore$ $10^2 = 8^2 + 6^2$, so by converse of Pythagoras Theorem

$\angle ABE = 90^o$

So Ar (ΔABC) = Ar(ΔABE) = $\frac{1}{2} \times 6 \times 8 = 24\,cm^2$

18. Given a, b, c are real numbers such that 9a + b + 8c = 12 and 8a + 12b + 9c = 1. Then $a^2 - b^2 + c^2$ = __________.

(Bonus)

Questions is wrong correct question is

Given a, b, c are real numbers such that 9a + b + 8c = 12 and 8a – 12b – 9c = 1. Then $a^2 - b^2 + c^2$ = _________.

Sol.

$9a + 8c = 12 - b$ … (1)

$8a - 9c = 1 + 12b$ … (2)

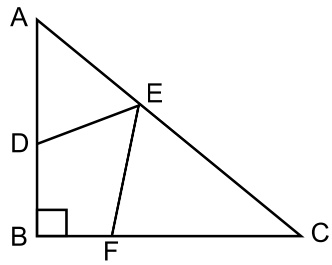
add both equation after squaring

$(9a + 8c)^2 + (8a - 9c)^2 = (12 - b)^2 + (1 + 12b)^2$

$145(a^2 + c^2) = 145(b^2 + 1)$

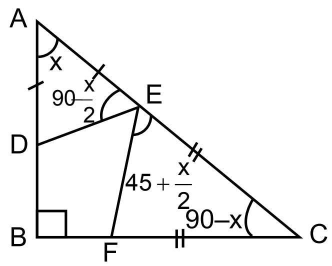
$a^2 - b^2 + c^2 = 1$

19. In the given figure, ΔABC is a right angled triangle with $\angle ABC = 90°$. D, E, F are points on AB, AC, BC respectively such that AD = AE and CE = CF. Then, $\angle DEF$ = ______ (in degree).



Sol. **(45)**

Let $\angle A = x$



So $\angle ADE = \angle AED = \frac{1}{2}(180 - x) = 90 - \frac{x}{2}$

& $\angle C = 90 - x$

& $\angle CEF = \angle EFC = \frac{180 - (90 - x)}{2} = \frac{x + 90}{2} = 45 + \frac{x}{2}$

So $\angle DEF = 180 - \angle AED - \angle CEF$

$180 - \left(90 - \frac{x}{2}\right) - \left(45 + \frac{x}{2}\right) = 45^o$

20. Numbers of 5-digit multiples of 13 is ___________.

Sol. **(6923)**

Smallest 5-digit no which is multiple of 13 is

= 10010 = 770 × 13

Largest 5-digit no. which is multiple of 13 is

= 99996 = 7692 × 13

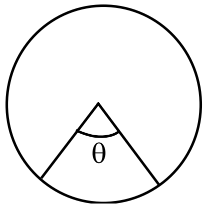
So number of five digit multiples of 13 is

7692 – 770 + 1 = 6923

21. The area of a sector and the length of the arc of the sector are equal in numerical value. Then the radius of the circle is __________.

Sol. **(2)**

Area of sector = length of are of sector



$$\frac{\theta}{360}\times \pi r^2 = \frac{\theta}{360}\times 2\pi r$$

r = 2

22. If a, b, c, d are positive integers such that $a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d}}} = \frac{43}{30}$, then d is __________.

Sol. **(4)**

$$\frac{43}{30} = 1+\frac{13}{30} = 1+\cfrac{1}{\cfrac{30}{13}} = 1+\cfrac{1}{2+\cfrac{4}{13}} = 1+\cfrac{1}{2+\cfrac{1}{\cfrac{13}{4}}} = 1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}}$$

So a = 1, b = 2, c = 3 , d = 4

23. A teacher asks 10 of her students to guess her age. They guessed it as 34, 38, 40, 42, 46, 48, 51, 54, 57 and 59. Teacher said "At least half of you guessed it too low and two of you are off by one. Also my age is a prime number". The teacher's age is __________.

Sol. **(47)**

Age of teacher is greater than 46.
Again according to questions two of them are off by one.
So there are two possibilities 47 & 58
47 from (46 & 48)
58 from (57 & 59)
But 58 is not a prime so age of teacher in 47.

24. The sum of 8 positive integers is 22 and their LCM is 9. The number of integers among these that are less than 4 is __________.

Sol. **(7)**

L.C.M. is 9, so all numbers are from 1,3 & 9. If we take 2 times 9 sum as 22 is not possible so, 9 will come only one time & remaining 7 numbers are from 1 & 3. So 4 times 1 & 3 times 3 will come.
1 + 1 + 1 + 1 + 3 + 3 + 3 + 9 = 22
So numbers less than 4 is 7

25. The number of natural numbers $n \le 2019$ such that $\sqrt[3]{48n}$ is an integer is __________.

Sol. **(3)**

$n \le 2019$

$\sqrt[3]{48n}$ = integer

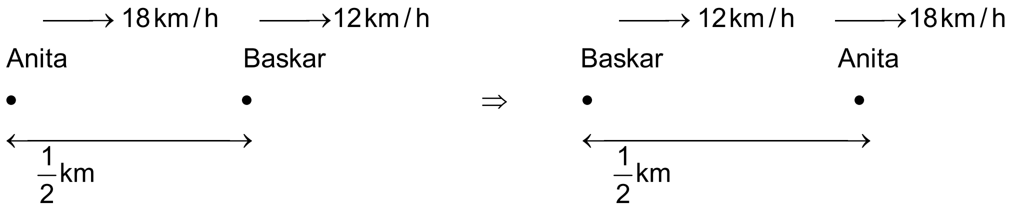
$\sqrt[3]{2^4 \times 3 \times n} = \text{integer}$

$2\times\sqrt[3]{2\times 3n} = \text{integer}$

So n should be multiple of $2^2 \times 3^2 = 36$ for integer value.
So, possible 'n' are
$= 2^2 \times 3^2 = 36 < 2019$
$= 2^2 \times 3^2 \times 2^3 = 288 < 2019$
$= 2^2 \times 3^2 \times 3^3 = 972 < 2019$
$= 2^2 \times 3^2 \times 4^3 = 2304 > 2019$ (reject)
So only three values of n are possible 36, 288, 972

26. Anita is riding her bicycle at the rate of 18 km/h. When Anita is riding her bicycle on a straight road, she sees Basker skating at the rate of 12 km/h in the same direction, $\frac{1}{2}$ km in front of her. Anita overtakes him and can see him in her rear view mirror until he is $\frac{1}{2}$ km behind her. The total time in seconds that Anita can see Baskar is ____________.

Sol. **(600)**



Relative speed = 18 – 12 = 6km/h = $6 \times \frac{5}{18} m/s$

Relative distance = $\frac{1}{2} + \frac{1}{2} = 1 km = 1000$ m

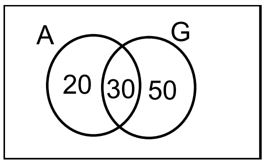
So time = $\frac{\text{distance}}{\text{speed}} = \frac{1000}{6 \times \frac{5}{18}} = 600\,\text{sec}$

27. In a room, 50% of the people are wearing gloves, and 80% of the people are wearing hats. The minimum percentage of people in the room wearing both a hat and a glove is __________.

Sol. **(30)**

Minimum % of people in the room wearing both a hat & a gloves is

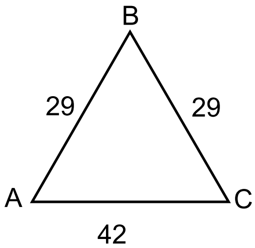
50 + 80 – 100 = 30%



28. In $\triangle ABC$, AB = BC = 29 and AC = 42 cm. The, area of $\triangle ABC$ = ________ cm^2.

Sol. **(420)**

$s = \frac{a+b+c}{2} = \frac{29+29+42}{2} = 50$



By Heron's formula

Area of $\triangle ABC$ = $\sqrt{s(s-a)(s-a)(s-c)}$

$= \sqrt{50 \times 8 \times 21 \times 21} = 20 \times 21 = 420 \text{ cm}^2$

29. The smallest integer larger than the perimeter of any triangle with two sides of length 10 and 20 units is ____________.

Sol. **(60)**

Sum of two sides of any triangle is greater than third side

So than third side < 10 + 20

third side < 30

so perimeter < 60

Smallest integer greater than perimeter is 60.

30. The number of perfect cubes that lie between $2^9 + 1$ and $2^{18} + 1$ is __________.

Sol. **(56)**

$(2^3)^3 + 1$ and $(2^6)^3 + 1$

$8^3 + 1$ and $64^3 + 1$

Numbers lies between 9, 10, 11, ..., 64

Total perfect cubes number 64 – 8 = 56.