THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test - Bhaskara Contest

NMTC at JUNIOR LEVEL - IX & X Standards

Saturday, 1 September, 2018

Note:

1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.

2. Diagrams are only visual aids; they are NOT drawn to scale.

3. You are free to do rough work on separate sheets.

4. Duration of the test: 2 pm to 4 pm -- 2 hours.

PART—A

Note

- Only one of the choices A. B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering; seek the guidance of the supervisor.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{2}$ mark.**

1. The value of $\dfrac{3+\sqrt{6}}{8\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}-\sqrt{27}}$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{6}$ (D) $\sqrt{18}$

Ans. (B)

Sol. $\dfrac{3+\sqrt{6}}{8\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}-3\sqrt{3}} = \dfrac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \dfrac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}} = \sqrt{3}$.

2. A train moving with a constant speed crosses a stationary pole in 4 seconds and a platform 75 m long in 9 seconds. The length of the train is (in meters)

(A) 56 (B) 58 (C) 60 (D) 62

Ans. (C)

Sol. Let the train have length ℓm and speed s m/sec.

$s = \dfrac{\ell}{4}$ ....(i)

$s = \dfrac{\ell+75}{9}$

$\dfrac{\ell}{4} = \dfrac{\ell+75}{9}$ (By using (i))

$9\ell = 4\ell + 300$

$5\ell = 300$

$\ell = 60$ m.

3. One of the factors of $9x^2 - 4z^2 - 24xy + 16y^2 + 20y - 15x + 10$ is **(Bonus)**

(A) $3x - 4y - 2z$ (B) $3x + 4y - 2z$ (C) $3x + 4y + 2z$ (D) $3x - 4y + 2z$

4. The natural number which is subtracted from each of the four numbers 17,31,25,47 to give four numbers in proportion is

(A) 1 (B) 2 (C*) 3 (D) 4

Ans. (C)

Sol. Let x be subtracted so that 17, 31, 25, 47 are in proportion.

$$\frac{17-x}{31-x} = \frac{25-x}{47-x}$$

$(17 - x)(47 - x) = (25 - x)(31 - x)$

$799 + x^2 - 64x = 775 + x^2 - 56x$

$24 = 8x$

$x = 3.$

5. The solution to the equation $5(3^x) + 3(5^x) = 510$ is **(Bonus)**

(A) 2 (B) 4 (C) 5 (D) No solution

6. If $(x + 1)^2 = x$, the value of $11x^3 + 8x^2 + 8x - 2$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (A)

Sol. $(x + 1)^2 = x$

$x^2 + x + 1 = 0$

$11x^3 + 8x^2 + 8x - 2$

$= (x^2 + x + 1)(11x - 3) + 1$

$= (0)(11x - 3) + 1 = 1.$

7. There are two values of m for which the equation $4x^2 + mx + 8x + 9 = 0$ has only one solution for x. The sum of these two value of m is **(Bonus)**

(A) 1 (B) 2 (C) 3 (D) 4

Sol. $D = 0$

$(m + 8)^2 - 4.4.9 = 0$

$m + 8 = \pm 12$

$m = 4, -20$

sum $= 4 - 20 = -16.$

8. The number of zeros in the product of the first 100 natural numbers is

(A) 12 (B) 15 (C) 18 (D) 24

Ans. (D)

Sol. $$\left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] + \left[\frac{100}{5^3}\right] + \ldots\ldots$$

$= 20 + 4 + 0 + 0 + \ldots\ldots$

$= 24$

So number of zeros is 24.

9. The length of each side of a triangle in increased by 20% then the percentage increase of area is

(A) 60% (B) 120% (C) 80% (D) 44%

Ans. (D)

Sol. Let side of Δ are a, b, c

(Δ) area $= \sqrt{s(s-a)\ (s-b)\ (s-c)}$

When each side increased by 20%

$a' = 1.2a$

$b' = 1.2b$

$c' = 1.2c$

$s' = \dfrac{1.2\ \ (a+b+c)}{2} = 1.2\ s$

(Δ') new area = $\sqrt{1.2s(1.2s-1.2a)\ \ (1.2s-1.2b)\ \ (1.2s-1.2c)}$

$= (1.2)^2\ \Delta$

$= 1.44\ \Delta$

% increase = $\dfrac{\Delta' - \Delta}{\Delta} \times 100 = 44\%$.

10. The number of pairs of relatively prime positive integers (a, b) such that $\dfrac{a}{b} + \dfrac{15b}{4a}$ is an integer is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (D)

Sol. Given a, b are relative prime positive integer such that $\dfrac{a}{b} + \dfrac{15b}{4a}$ = k (where k is an integer)

$\Rightarrow$ $a = kb - \dfrac{15b^2}{4a}$

Now as H.C.F. (a, b) = 1

so a/15

$\Rightarrow$ a {1 , 3 , 5 , 15 }

Put a = 1

$1 = kb - \dfrac{15b^2}{4}$

$4/b^2 \Rightarrow b = \{ 2 , 4 , 6 , 8 ... \}$

Similarly only b = 2 simplify

Put a = 3 $\Rightarrow$ b = 2

a = 5 $\Rightarrow$ b = 2

a = 15 $\Rightarrow$ b = 2

(a , b) = (1 , 2) , (3 , 2) , (5 , 2) , (15 , 2)

4 **Ans.**

11. The four digit number 8ab9 is a perfect square. The value of $a^2 + b^2$ is

(A) 52 (B) 62 (C) 54 (D) 68

Ans. (A)

Sol. $93^2 = 8649$

$\therefore$ a = 6, b = 4

$\therefore$ $a^2 + b^2 = 6^2 + 4^2 = 52$.

12. a, b are positive real numbers such that $\dfrac{1}{a} + \dfrac{9}{b} = 1$. The smallest value of a + b is

(A) 15 (B) 16 (C) 17 (D) 18

Ans. (B)

Sol. $\dfrac{1}{a} + \dfrac{9}{b} = 1$

$b = \dfrac{9a}{a-1}$

$b = 9 + \dfrac{9}{a-1}$

$a + b = a + 9 + \dfrac{9}{a-1} = 10 + (a - 1) + \dfrac{9}{a-1}$

for (a – 1), $\frac{9}{a-1}$

AM ≥ GM

$$\frac{a-1+\frac{9}{a-1}}{2} \geq \sqrt{(a-1)\frac{9}{a-1}}$$

$a - 1 + \frac{9}{a-1} \geq 6$

∴ $a + b \geq 10 + 6$

$a + b \geq 16.$

13. a, b real numbers. The least value of $a^2 + ab + b^2 - a - 2b$ is

(A) 1 (B) 0 (C) – 1 (D) 2

Ans. (C)

Sol. $f(a, b) = a^2 + ab + b^2 - a - 2b$

$f_a = 2a + b - 1$

$f_b = a + 2b - 2$

$f_{aa} = 2$

$f_{bb} = 2$

$f_{ab} = 1$

for stationary points we need $f_a = f_b = 0$

which gives

2a + b = 1

a + 2b = 2

a = 0, b = 1

only one point (a, b) = (0, 1)

Now check

$f_{aa} f_{bb} - f_{ab}^2$

$(2)(2) - (1)^2 = 3 > 0$

Now at (a, b) = (0, 1) $f_{aa} > 0$ and $f_{bb} > 0$

So, (a, b) = (0, 1) will given minimum value

so f(0, 1) = 0 + 0 + 1 – 0 – 2 = – 1 is the minimum value.

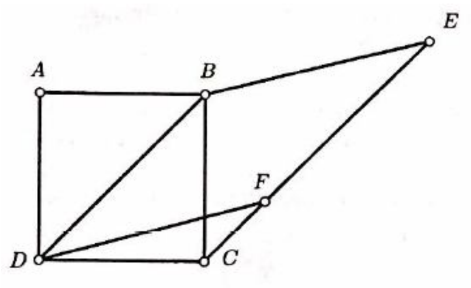
14. I is the incenter of a triangle ABC in which ∠A = 80º. ∠BIC =

(A) 120º (B) 110º (C) 125º (D) 130º

Ans. (D)

Sol. $\angle BIC = 90 + \frac{A}{2} = 90 + \frac{80}{2} = 130°.$

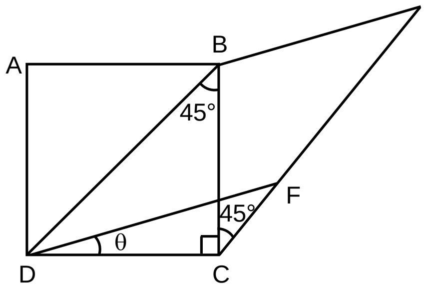
15. In the adjoining figure ABCD is a square and DFEB is a rhombus ∠CDF =



(A) 15º (B) 18º (C) 20º (D) 30º

Ans. (A)

Sol. Let side of square = 1



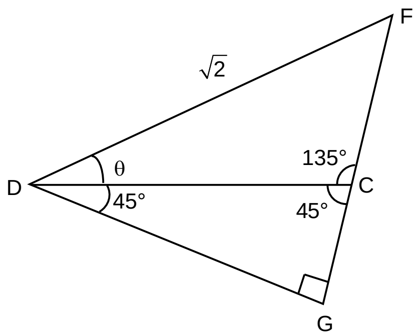
∴ side of rhombus = $\sqrt{2}$

∴ DG = GC = $\frac{1}{\sqrt{2}}$

GF = $\sqrt{\left(\sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$ = $\frac{\sqrt{3}}{2}$

CF = GF – GC = $\frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$ = $\frac{\sqrt{3}-1}{\sqrt{2}}$

Apply sine rule ΔCDF



$$\frac{\sin\theta}{\frac{\sqrt{3}-1}{\sqrt{2}}} = \frac{\sin 135°}{\sqrt{2}}$$

sin θ = $\frac{\sqrt{3}-1}{2\sqrt{2}}$ and we know sin 15° = $\frac{\sqrt{3}-1}{2\sqrt{2}}$

∴ θ = 15°.

PART - B

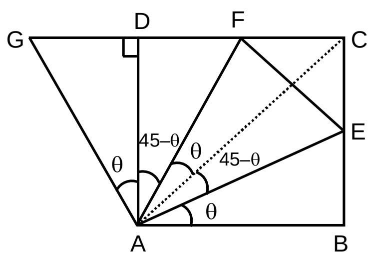
Note :

- Write the correct answer in the space provided in the response sheet
- For each correct response you get 1 mark. For each incorrect response you lose $\frac{1}{4}$ marks.

16. ABCD is a square E,F are point on BC, CD respectively and EAF = 45º. The value of $\frac{EF}{BE+DF}$ is ____________.

Ans. 1

Sol.



By ASA $\Delta ABE \cong \Delta ADG$

$AE = AG$

$BE = GD$ (CPCT)

By SAS $\Delta GAF \cong \Delta EAF$

$GF = EF$

$GD + DF = EF$

$BE + DF = EF$

$1 = \frac{EF}{BE+DF}$.

17. The average of 5 consecutive natural numbers is 10. The sum of the second and fourth of these numbers is __________.

Ans. 20

Sol. $\frac{x+x+1+x+2+x+3+x+4}{5} = 10$

$5x + 10 = 50$

$x + 2 = 10$

$x = 8$

so the number are 8, 9, 10, 11, 12

$\therefore$ 9 + 11 = 20.

18. The number of natural number n for which $n^2 + 96$ is a perfect square is _____.

Ans. 4

Sol. $n^2 + 96 = k^2$

$k^2 - n^2 = 96$

$(k - n)(k + n) = 96 \times 1$

$= 48 \times 2$

$= 24 \times 4$

$= 12 \times 8$

$= 16 \times 6$

$= 32 \times 3$

n = 23, 10, 2, 5

so number of values of n is 4.

19. n is an integer and $\sqrt{\frac{3n-5}{n+1}}$ is also an integer. The sum of all such n is _________

Ans. – 6

Sol. $\sqrt{\frac{3n-5}{n+1}} = \sqrt{3-\frac{8}{n+1}}$

$\therefore$ $n + 1 = \pm 1, 2, 4, 8.$

$n + 1 = 4 \Rightarrow n = 3$

$n + 1 = -8 \Rightarrow n = -9$

So sum of two value of n = 3 – 9 = – 6

So that $\sqrt{\frac{3n-5}{n+1}}$ is a perfect square.

20. $\frac{a}{b}$ is a fraction where a, b have no common factors other 1. b exceeds a by 3. If the numerator is increased by 7, the fraction is increased by unity. The value of a + b _________.

Ans. 11

Sol. $\frac{a}{b} = \frac{x}{x+3}$

$\frac{a+7}{b} - \frac{a}{b} = 1$

$\frac{x+7}{x+3} - \frac{x}{x+3} = 1$

$\frac{x+7-x}{x+3} = 1$

7 = x + 3

x = 4.

$\therefore \quad \frac{a}{b} = \frac{x}{x+3} = \frac{y}{4+3} = \frac{4}{7}$.

$\therefore \quad a + b = 7 + 4 = 11$

21. If $x = \sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}$, then the value of $2x^3 - 6x$ is __________.

Ans. 5

Sol. $x = \sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}$

$x^3 = 2 + \frac{1}{2} + 3\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}\right)$

$x^3 = \frac{5}{2} + 3(x)$

$2x^3 = 5 + 6x$

$2x^3 - 6x = 5$.

22. The angle of a heptagon are 160º, 135º, 185º, 140º, 125º, xº, xº. The value of x is ______.

Ans. $77\frac{1}{2}^{\circ}$

Sol. 160 + 135 + 185 + 140 + 125 + 2x = 900°

745° + 2x = 900°

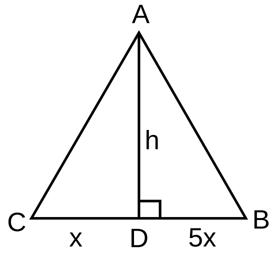
2x = 155°

$x = \left(\frac{155}{2}\right)^{\circ} = 77\frac{1}{2}^{\circ}$.

23. ABC is a triangle and AD is its altitude. If BD = 5DC, then the value of $\frac{3\left(AB^2 - AC^2\right)}{BC^2}$ is _______.

Ans. 2

Sol.

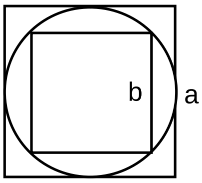


$\frac{3(AB^2 - AC^2)}{BC^2} = \frac{3[\{h^2 + 25x^2\} - \{h^2 + x^2\}]}{(6x)^2} = \frac{3\ [24x^2]}{36x^2} = 2.$

24. As sphere is inscribed in a cube that has surface area of 24 cm^2. A second cube is then inscribed within the sphere. The surface area of the inner cube (in cm^2) is _______

Ans. 8

Sol.



$6a^2 = 24$

$a = 2$

$2 = \sqrt{3}b$

$b = \frac{2}{\sqrt{3}}$

$6b^2 = \left(\frac{2}{\sqrt{3}}\right)^2 \times 6 = \frac{4}{3} \times 6 = 8.$

25. A positive integer n is multiple of 7. If $\sqrt{n}$ lies between 15 and 16, the number of possible values (s) of n is ________.

Ans. 4

Sol. $15 < \sqrt{n} < 16$

$225 < n < 256$

as n is multiple of 7 are 231, 238, 245, 252

so total 4 numbers.

26. The value of x which satisfies the equation $\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}} = \frac{7}{3}$ is ______

Ans. 20

Sol. By C and D

$\frac{2\sqrt{x+5}}{2\sqrt{x-16}} = \frac{7+3}{7-3}$

$\frac{x+5}{x-16} = \left(\frac{10}{4}\right)^2$

$\frac{x+5}{x-16} = \frac{25}{4}$

$4x + 20 = 25x - 400$

$420 = 21x$

$\Rightarrow x = \frac{420}{21} = 20.$

27. M man do a work in m days. If there had been N men more, the work would have been finished n days earlier , then the value of $\frac{m}{n} - \frac{M}{N}$ is _______.

Ans. 1

Sol.

Men	Day	Work
M	m	Mm
$M+N$	$m-n$	$(M+N)(m-n)$

$Mm = (M + N)(m - n)$

Mm = Mm – Mn + Nm –Nn

Nm – Mn = Nn ……..(i)

$\therefore \quad \frac{m}{n} - \frac{M}{N} = \frac{mN - Mn}{nN} = \frac{Nn}{Nn} = 1.$

28. The sum of the digit of a two number is 15. If the digits of the given number are reversed, the number is increased by the square of 3. The original number is __________.

Ans. 78

Sol. N = 10a + b

N′ = 10b + a

a + b = 15

$N' = N + 3^2$

10b + a = 10a + b + 9

9b – 9a = 9

b – a = 1

a = 7, b = 8

N = 78.

29. When expanded the units place of $(3127)^{173}$ is _______.

Ans. 7

Sol. Cyclicity of 7 is 4

173 = 4 × 43 + 1

so unit digit is $7^1 = 7$.

30. If a : (b + c) = 1 : 3 and c : (a + b) = 5 : 7 , then b : (c + a) is _________

Ans. $\frac{1}{2}$

Sol. $\frac{a}{b+c} = \frac{1}{3} \qquad \frac{c}{a+b} = \frac{5}{7}$

3a – b – c = 0 ………..(i)

5a + 5b – 7c = 0 ………..(ii)

by (i) × 5 – (ii) × 3, we get

15a – 5b – 5c = 0

15a + 15b –21c = 0

– – +

____

– 20b + 16c = 0

$b = \frac{16}{20}c = \frac{4}{5}c$

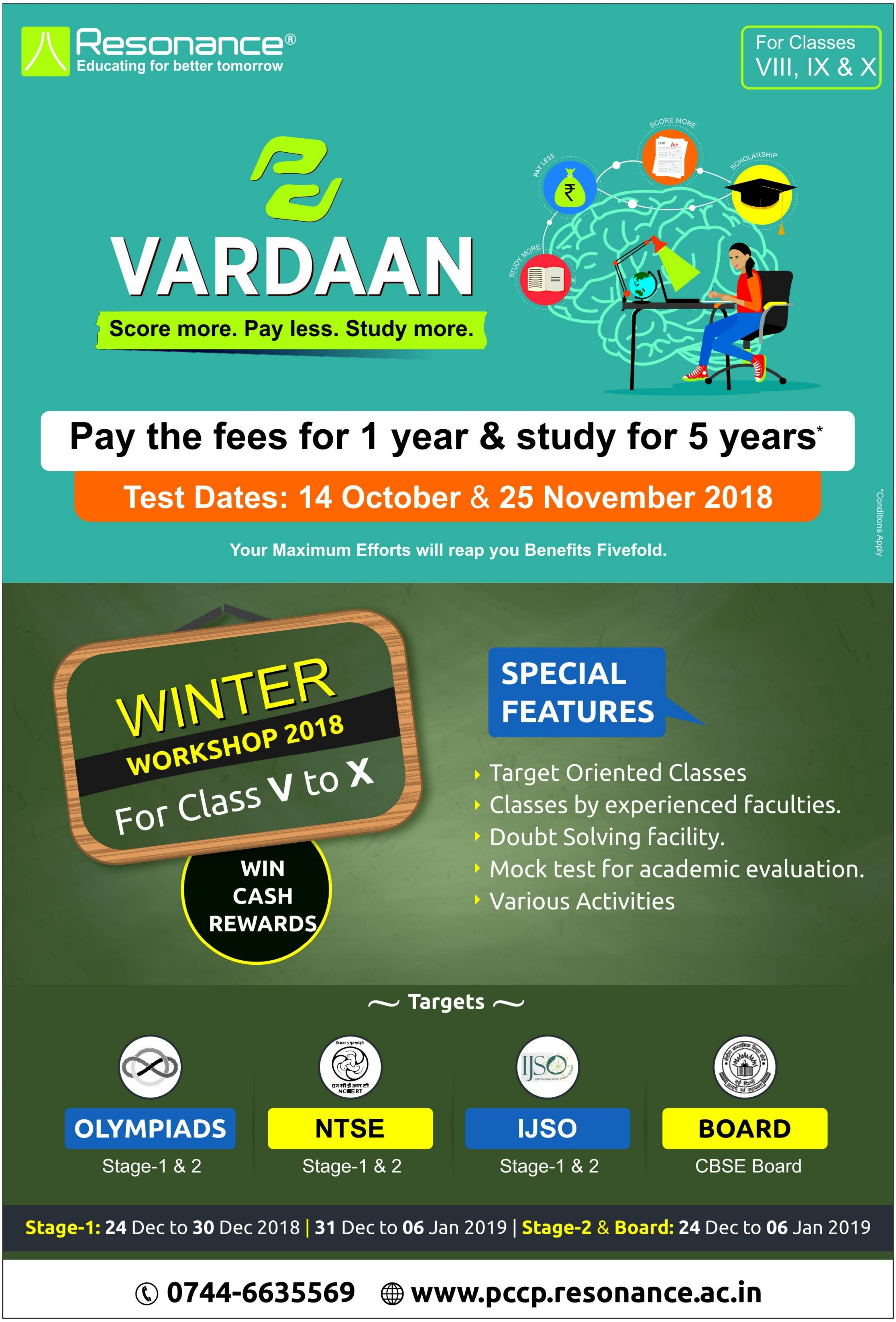
Put $b = \frac{4}{5}c$ in (i)

$3a - \frac{4}{5}c - c = 0$

$3a = \frac{9}{5}c$

$a = \frac{3}{5}c$

$\therefore \quad \frac{b}{c+a} = \frac{\frac{4}{5}c}{c + \frac{3}{5}c} = \frac{\frac{4}{5}c}{c + \frac{8}{5}c} = \frac{1}{2}.$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