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INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN PHYSICS 2013-2014

Date of Examination : 24th November 2013

Time 9.30 to 11.30 Hrs

Q. P. Code No. 1 ONE 3 THREE 7 SEVEN

INSTRUCTION TO CANDIDATES

1. In addition to this question paper, you are given answer sheet for part A and an answer paper for part B.

2. On the answer sheet for part A, fill up all the entries carefully in the space provided, **ONLY In BLOCK CAPITALS**.
Incomplete / incorrect / carelessly filled information may disqualify your candidature.

3. On part A answer sheet, use only **BLUE of BLACK BALL PEN** for making entries and marking answer.

4. Part A has two parts. In part A1 (Q.No. 1 to 40) each question has **Four** alternatives, out of which only one is correct. Choose the correct alternative and mark a cross (X) in the corresponding box on the answer sheet.

For example,

Q.	a	b	c	d
22		X		

Part A2 (Q.Nos. 41 to 50) has four alternatives in each question, but any number of these (4, 3, 2, or 1) may be correct. You have to mark **ALL** correct alternatives and mark a cross (X) for each, like

Q.	a	b	c	d
44		X		X

5. For **Part A1**, each correct answer gets 3 marks; wrong answer gets a penalty of 1 mark. For **Part A2** full marks are 6 for each question, you get them when **ALL** correct answer only are marked.

6. Any rough work should be done only on the sheets provided with part B answer paper.

7. Use of nonprogrammable calculator is allowed.

8. No candidate should leave the examination hall before the completion of the examination.

9. After submitting your answer papers, read the instructions regarding evaluation given on next page.

10. Attempt the examination honestly. Any dishonestly will disqualify you.

PLEASE DO NOT MAKE ANY MARKS OTHER THAN (X) IN THE SPACE PROIDED ON THE ANSWER SHEET OF PART A.

Answer sheets are evaluated with the help of a machine. Hence, **CHANGE OF ENTRY IS NOT ALLOWED.**

Scratching or overwriting may result in wrong score.
DO NOT WRITE ANYTHING ON THE BACK SIDE OF PART A ANSWER SHEET.

INSTRUCTIONS TO CANDIDATES:

1. The answers / solutions to this question paper will be available on our website- **www.iapt.org.in** by 3 rd Decembet 2013.

EVALUATION PROCEDURE (NSEP) :

2. Part 'A' Answers of ALL the candidates are examined.

3. Part B is evaluated of only those students who get marks above a certain' "cut oft" marks in Part A. (e.g. NSEP Total marks for Part A are 180. Students getting (say) 100 or more than 100 marks In Part A are identified and their Part B is evaluated. Thus "cut off" marks are 100 in this example.)

4. PART 'B' IS NOT EVALUATED OF ALL THE CANDIDATES.

CERTIFICATES & AWARDS

Following certificates are awarded by the I.A.P.T. to students successful in NSEP.
i) Certificate for "Centre Top 10%" students on the basis of marks in part A only.
ii) Merit certificates to statewise Top 1% students on the basis of (A+B) marks.
iii) Merit certificate and a prize in the form of a book to Nationwise Top 1% students based on (A+B) marks.

5. **Result sheets and the "centre top 10%"** certificates of NSEP are dispatched to the Professor in charge of the centre. Thus you will get your marks from the Professor in charge of your centre by January 2014 end.

6. TOP 300 (or so) students are called for the next examination -Indian National Physics Olympiads (INPhO). Individual letters are sent to these students ONLY.

7. Gold medals will be awarded to TOP 35 students in this entire process.

8. No queries will be entertained in this regard.

INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN PHYSICS 2013-2014

Total Time : 120 Minutes (A-1, A-2 & B) Code - 137

PART A **MARKS = 180**

N.B. : Physical constants are given at the end

SUB-PART A-1

(ONLY ONE OUT OF FOUR OPTIONS IS CORRECT)

1. A certain quantity of oxygen $\left(\gamma = \frac{7}{5}\right)$ is compressed isothermally until is pressure is doubled (P_2). The gas is then allowed to expand adiabatically until is original volume is restored. Then the final pressure (P_3) in terms of initial pressure (P_1) is :

(a) $P_3 = 0.55\ P_1$ (b) $P_3 = 0.76\ P_1$ (c) $P_3 = 0.68\ P_1$ (d) $P_3 = 2.55\ P_1$

Sol. (b)

$P_0V_0 = (2P_0)\ V'$ (isothermal process)

$$V' = \frac{V_0}{2} \Rightarrow (2P_0)\left(\frac{V}{2}\right)^{\gamma} = P_3(V_0)^{\gamma}$$

$$\frac{2P_0V_0^{\gamma}}{2^{\gamma}} = P_3^{1}$$

$$P_3 = \frac{2^{1-\frac{7}{5}}}{2^{\frac{-2}{5}}} P_0$$

$$P_3 = \frac{P_0}{2^{0.4}} = \frac{P_0}{(2^2)^5} = \frac{P_0}{(4)^{1/5}}$$

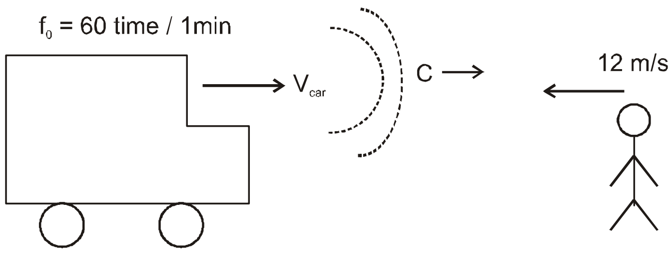
$$P_3 = \frac{P_0}{(4)^{1/5}} = 0.757\ P_0\ .$$

2. A car fitted with a device which transmits sound 60 times per minute. There is no wind and speed of sound in still air is 345 m/s. If you hear the sound 68 times per minute when you are moving towards the car with a speed of 12 m/s, the speed of the car must be nearly.

(a) 20.0 m/s towards you (b) 30.0 m/s towards you
(c) 10.0 m/s away from you (d) 10.0 m/s towards you

Sol. (b)

C = 345 m/s f_{ob} = 68 time/1min



$$f' = \frac{C+12}{C-V_2}\ f_0$$

$$\frac{68}{60} = \frac{345+12}{345-V_2}$$

$$345 - V_2 = 357 \times \frac{60}{68}$$

$$V_2 = 345 - \frac{357\times 60}{68} = 345 - 315 = 30 \text{ m/s}$$

3. A 43 m long rope of mass 5.0 kg joins two rock climbers. One climber strikes the rope and the second one feels the effect 1.4 s later. What is the tension in the rope ?

(a) 110 N (b) 301 N (c) 215 N (d) 154 N

Sol. (a)

$$V_{wave} = \frac{43}{7} \times \text{m/s}$$

$$V_{wave} = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow T = (V_{wave})^2\mu$$

$$= \left(\frac{43\times 5}{7}\right)^2 \times \frac{5}{43} \simeq 109.69 \text{ N}$$

4. Two cats in a house mew at each other with sound intensities 5×10^{-9} Wm^{-2} and 9×10^{-6} Wm^{-2}. By how many decides is the louder sound above the other ?

(a) ~13 dB (b) ~23 dB (c) ~33 dB (d) ~43 dB

Sol. (c)

$$d\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$\beta_2 - \beta_2 = 10\log\left(\frac{I_2}{I_1}\right)$$

$$= 10 \log\left(\frac{9\times 10^{-6}}{5\times 10^{-9}}\right)$$

$$= 30 \log\left(\frac{9}{5}\right) = 30 \log (1.8)$$

5. An electron orbiting around a nucleus has angular momentum L. The magnetic field produced by the electron at the centre of the orbit can be expressed as :

(a) $B = (\mu_0 e / 8\pi m r^3)L$ (b) $B = (\mu_0 e / 4\pi m r^3)L$

(c) $B = (\mu_0 e / \pi m r^3)L$ (d) $B = (e / 4\pi\varepsilon_0 m r^3)L$

Sol. (b)

$$\frac{B}{L} = \frac{\mu_0}{4\pi}\frac{\frac{eV}{r^2}}{mVr} \qquad L = m\,V\,r$$

$$\frac{B}{L} = \frac{\mu_0}{4\pi}\frac{eV}{r^2}\times\frac{1}{mVr} \qquad \Rightarrow \qquad B = \frac{\mu_0}{4\pi}\ \frac{L}{r} = \frac{\mu_0 eL}{4\pi m r^3}$$

6. Suppose a radioactive nucleus A disintegrates at origin into a nuclesus B with the emission of a positron (e^+) and a neutrino (v) such that the positron and the neutrino move at right angles to each other and carry momentum 2×10^{-22} kg ms^{-1} (+Y-axis) and 5×10^{-23} kg ms^{-1} (+X-axis) respectively. Then the nucleus.

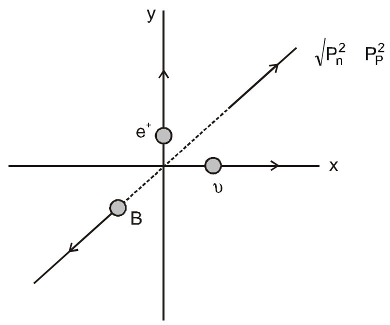
(a) A recoils with the momentum 2.86×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. +X-axis

(b) A recoils with the momentum 2.06×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. –X-axis

(c) B recoils with the momentum 2.86×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. +X-axis

(d) B recoils with the momentum 2.06×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. –X-axis

Sol. **(d)**



$P_P = 5 \times 10^{-23}$ kg m/s $= \frac{5}{10} \times 10^{-22}$

$P_n = 2 \times 10^{-22}$ kg m/s

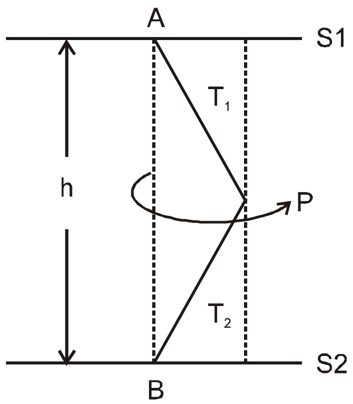
$$P_B = \sqrt{(2)^2 + \left(\frac{1}{2}\right)^2} \times 10^{-22}$$

$$= \sqrt{\frac{17}{2}} \times 10^{-22} = 2.06 \times 10^{-22}$$

$$\tan\theta = \frac{P_P}{P_n} = 4$$

$$\theta = \tan^{-1} 4$$

7. Two identical strings with fixed ends separated by height h have their other ends tied to a body P of mass m as shown in figure. When the body rotates with uniform angular speed $2\sqrt{2g/h}$ in a horizontal plane about the vertical axis the ratio of tensions (T_1/T_2) in the string is :



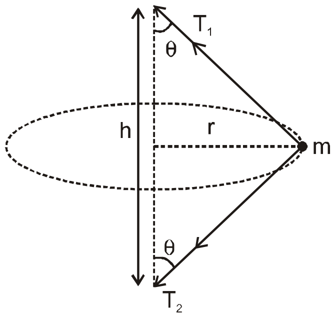
(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

Sol. **(b)**

$T_1 \cos\theta = mg + T_2 \cos\theta$

$(T_1 - T_2)\ \cos\theta = mg$

$(T_1 \sin\theta + T_2 \sin\theta) = m\omega^2 r$



$$\frac{(T_1 + T_2)}{(T_1 - T_2)} \frac{\sin\theta}{\cos\theta} = \frac{m\omega^2 r}{mg}$$

$$\frac{T_1 + T_2}{T_1 - T_2} \tan\theta = \frac{\omega^2 r}{g}$$

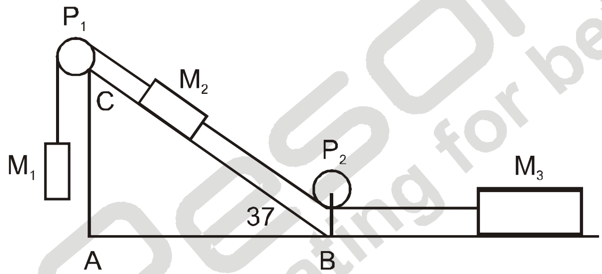
$$\frac{T_1 + T_2}{T_1 - T_2} \tan\theta = \frac{\omega^2 r}{g} \times \frac{h}{2} \tan\theta$$

$$\frac{T_1 + T_2}{T_1 - T_2} = \frac{h}{2g} \times \frac{4 \times 2g}{h}$$

$T_1 + T_2 = 4T_1 - 4T_2$

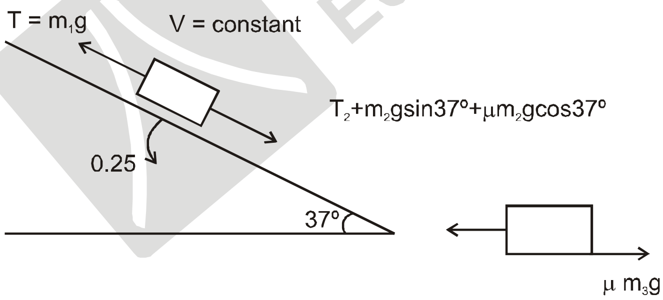
$5T_2 = 3T_1 \qquad \Rightarrow \frac{T_1}{T_2} = \frac{5}{3}$

8. Two identical bodies M_2 and M_3 each of 4.0 kg are tied to a massless inextensible string which is made to pass around pulleys P_1 and P_2 as shown in figure. Angle ABC = 37º. The coefficient of kinetic friction between the bodies and the surface on which they slide is 0.25. If the body M_1 moves done with uniform speed, neglecting the masses and friction of pulleys. M_1 =



(a) 36.8 kg (b) 9.8 kg (c) 4.2 kg (d) 2.1 kg

Sol. **(c)**



Acceleration = 0

$m_1g = m_2g + \sin 37 + \mu m_2 g \cos 37 - \mu m_3 g$

$$= 4 \times 10 \left[\frac{3}{5} + \frac{1}{4} \times \frac{4}{5} - \frac{1}{4}\right]$$

$$= 40 \left[\frac{12}{20} + \frac{4}{20} - \frac{5}{20}\right]$$

m = 4.2 kg

9. A particle of mass 0.2 kg moves along a path given by the relation : $\vec{r} = 2\cos\omega t\,\hat{i} + 3\sin\omega t\,\hat{j}$. Then the torque on the particle about the origin is :

(a) $\sqrt{13}\,\hat{k}\,\text{Nm}$ (b) $\sqrt{\frac{2}{3}}\,\hat{k}\,\text{Nm}$ (c) $\sqrt{\frac{3}{2}}\,\hat{k}\,\text{Nm}$ (d) $0\hat{k}$

Sol. **(d)**

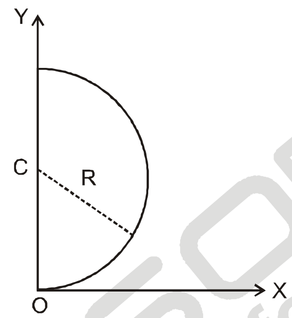
$L = m\left[\vec{r} \times \vec{V}\right]$

$V = -2\omega\sin\omega t\,\hat{i} + 3\omega\cos\omega t\,\hat{j}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\cos\omega t & 3\sin\omega t & 0 \\ -2\omega\sin\omega t & 3\omega\cos\omega t & 0 \end{vmatrix} \begin{array}{l} = \hat{R}\left[6\omega\cos^2\omega t + 6\omega\sin^2\omega t\right] \\ = 6\omega\hat{k}\,\text{constant} \end{array}$$

L = 0

10. A bead of mass 5.0 g can move without friction on a piece of wire bent in the form of a semicircular ring of radius 0.10 m, as shown in the adjacent figure. This ring can freely rotate about the vertical axis OY. At what height will the bead stay above the ground level OX, if this semicircular are revolves with angular velocity 10.63 rad/s ?



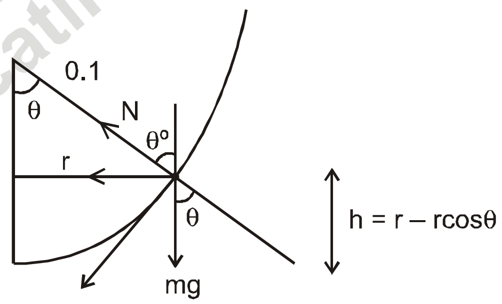
(a) 0.013 m (b) 0.087 m (c) 0.027 m (d) 0.073 m

Sol. **(a)**

$N\sin\theta = mr\omega^2$

$N\cos\theta = mg$

$\tan\theta = \dfrac{r\omega^2}{g}$



$\tan\theta = \dfrac{0.1\sin\theta(10.63)^2}{10}$

$\dfrac{1}{\cos\theta} = \dfrac{(10.63)^2}{100}$

$\cos\theta = \left(\dfrac{10}{10.63}\right)^2 = 0.885$

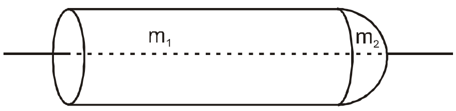
h = 0.1 [1 – 0.885] = 0.01

11. One of the flat surfaces of a cylinder (radius r and length *l*) and the flat surface of hemisphere are cemented together. The cylinder and the hemisphere are made of the same material. The combined mass of the system is M. The moment of inertia of this system about an axis coinciding with the axis of cylinder is:

(a) (1/10) M $\{5r^2 + 4l^2\}$ (b) (1/10) Mr^2 $\{(15l + 8r) / (3l + 2r)\}$

(c) (1/10) Mr^2 $\{(3l + 4r) / (3l + 2r)\}$ (d) (1/10) M $\{(5r^3 l + 4l^2r^2) / (5r + 4l)\}$

Sol. **(b)**



$$I = \frac{M_1 r^2}{2} + \frac{2}{5}M_2 r^2 \qquad \text{............(i)}$$

Now, $\frac{M_1}{M_2} = \frac{3}{2}\frac{\ell}{r}$ and $M_1 + M_2 = M$

$$M_1 + \frac{2rM_1}{3\ell} = M$$

$$M_1\left[\frac{3\ell + 2r}{3\ell}\right] = M$$

$$M_1 = \left[\frac{3\ell \times M}{3\ell + 2r}\right],\ M_2 = \left(\frac{2r}{3\ell + 2r}\right)M\ .$$

From equation (i)

$$I = \frac{M_1 r^2}{2} + \frac{2}{5}M_2 r^2$$

$$I = \frac{r^2}{2}\frac{3\ell M}{3\ell + 2r} + \frac{2}{5}r^2.\frac{2r}{(3\ell + 2r)}M$$

$$I = Mr^2\left[\frac{3\ell}{2(3\ell + 2r)} + \frac{4r}{5(3\ell + 2r)}\right]$$

$$I = Mr^2\left[\frac{15\ell + 4r}{10(3\ell + 2r)}\right]$$

12. The radius of cross section of a long pipe varies gradually as $r = r_0 e^{-\alpha x}$, where x is the distance from the pipe inlet and a = 0.4 m^{-1} is a constant. Then the ratio of Reynolds number for the cross sections separated by a distance 8.0 m is :

(a) 24.5 (b) 28.5 (c) 2.45 (d) 2.85

Sol. **(a)**

$$R = \frac{\rho V r}{2\eta} = \frac{\rho}{2\eta} \qquad \frac{V\pi r^2}{\pi r} = \frac{\rho AV}{2\pi\eta r}$$

here AV = volume flow rate.

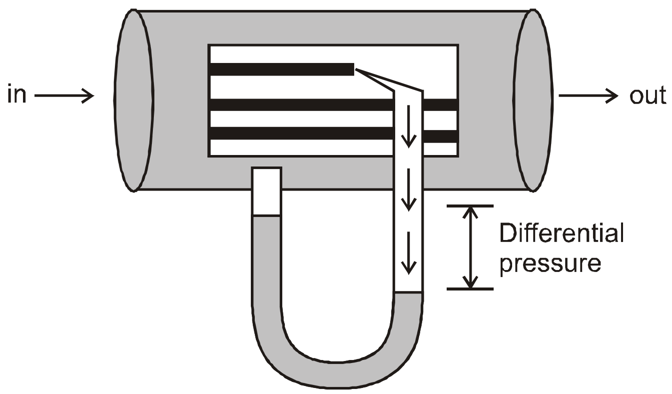
$$R_1 = \left(\frac{\rho AV}{2\pi\eta}\right)\frac{1}{\eta}$$

$$R_2 = \left(\frac{\rho AV}{2\pi\eta}\right)\left(\frac{1}{r^2}\right)$$

$$\frac{R_2}{R_1} = \frac{r_1}{r_2} = \frac{r_0 e^{-\alpha(0)}}{r_0 e^{-\alpha(8)}}$$

$$\frac{R_2}{R_1} = e^{8\alpha} = e^{3.2} = 24.5$$

13. The Pitot tube shown in the figure is used to measure fluid flow velocity in a pipe of cross sectional area S. It was invented by a French engineer Henri Pitot in the early 18th century. The volume of the gas flowing across the section of the pipe per unit time is (The difference in the liquid columns is Δh, ρ_0 and ρ are the densities of liquid and the gas respectively)



(a) $Q = 2S\sqrt{\frac{\Delta h \rho_0 g}{\rho}}$ (b) $Q = S\sqrt{\frac{2\Delta h \rho_0 g}{\rho}}$ (c) $Q = S\sqrt{\frac{\Delta h g \rho_0}{\rho}}$ (d) $Q = S\sqrt{\frac{2\Delta h g \rho}{\rho_0}}$

Sol. **(b)**

$$\frac{1}{2}\rho v^2 = \Delta p = \rho_0 g \Delta h$$

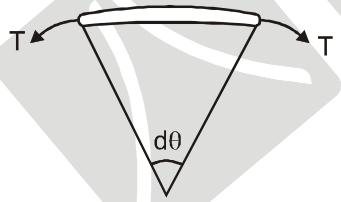
$$v = \sqrt{\frac{2\rho_0 g \Delta h}{\rho}}$$

$$Q = S\sqrt{\frac{2\Delta h \rho_0 g}{\rho}}$$

14. A thin ring has a radius R, density ρ and Young's modulus Y. The ring is rotated in its own plane about an axis passing through its centre with angular velocity ω. Then the small increase in its radius is :

(a) $dR = \frac{\rho\omega^2 R^3}{Y}$ (b) $dR = \frac{3\rho\omega^2 R^3}{Y}$ (c) $dR = \frac{6\rho\omega^2 R^3}{Y}$ (d) $dR = \frac{\rho\omega^2 R^3}{2Y}$

Sol. **(a)**



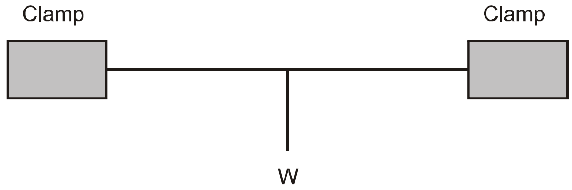
$$2T.\sin\frac{d\theta}{2} = dm\omega^2 R$$

$\Rightarrow$ $T.d\theta = \rho A r d\theta\ \omega^2 R$

$\Rightarrow$ $T = \rho A R^2 \omega^2$

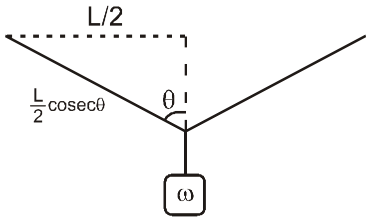
$\therefore$ $\frac{dR}{R} = \frac{T}{AY}$ $\Rightarrow$ $dR = \frac{\rho A R^3 \omega^2}{AY} = \frac{\rho \omega^2 R^3}{Y}$.

15. A uniform metal wire is clamped by chuck nuts at the two ends as shown in the adjacent figure. The wire has cross section area A, length ℓ and density ρ. A weight W is suspended from the midpoint of the wire. Then the vertical displacement δ, through which the midpoint moves down is given by :



(a) $\delta = \frac{\ell}{2}\left(\frac{W}{AY}\right)^{1/2}$ (b) $\delta = \frac{\ell}{2}\left(\frac{W}{AY}\right)^{1/3}$ (c) $\delta = \frac{\ell}{3}\left(\frac{W}{AY}\right)^{1/2}$ (d) $\delta = \frac{\ell}{4}\left(\frac{W}{AY}\right)^{1/3}$

Sol. **(b)**



$$T = YA\frac{\Delta L}{(L/2)}$$

$$T = \frac{YA}{L/2}.\frac{L}{2}(\text{cosec}\theta - 1)$$

and $2T\cos\theta = \omega$

$\Rightarrow$ $2YA(\text{cosec}\theta - 1).\cos\theta = \omega$

$\Rightarrow$ $\frac{\cos\theta}{\sin\theta} - \cos\theta = \frac{\omega}{2YA}$ $\Rightarrow$ $\frac{8}{L/2} - \frac{8}{\sqrt{\frac{L^2}{4} + 8^2}} = \frac{\omega}{2YA}$

$\Rightarrow$ $\frac{8}{L/2}\left(1 - \left(1 + \frac{48^2}{L^2}\right)^{-1/2}\right) = \frac{\omega}{2YA}$ $\Rightarrow$ $\frac{8}{L/2}\frac{28^2}{L^2} = \frac{\omega}{2YA}$

$\Rightarrow$ $8^3 = \frac{\omega}{8YA}$ $\Rightarrow$ $8 = \frac{L}{2}\left(\frac{\omega}{YA}\right)^{1/3}$.

16. Two bodies of mass M_1 and M_2 are kept separated by a distance d. The potential at the point where the gravitational field produced by them is zero, is :

(a) $-\frac{G}{d}\left(M_1 + M_2 + 2\sqrt{M_1M_2}\right)$ (b) $-\frac{G}{d}\left(M_1M_2 + 2\sqrt{M_1 + M_2}\right)$

(c) $-\frac{G}{d}\left(M_1 - M_2 + 2\sqrt{M_1M_2}\right)$ (d) $-\frac{G}{d}\left(M_1M_2 - 2\sqrt{M_1 + M_2}\right)$

Sol. **(a)**

$\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2}$ $\Rightarrow$ $\frac{d-x}{x} = \sqrt{\frac{M_2}{M_1}}$

$\Rightarrow$ $\frac{d}{x} = 1 + \sqrt{\frac{M_2}{M_1}}$ $\Rightarrow$ $x = \frac{\sqrt{M_1}d}{\sqrt{M_1} + \sqrt{M_2}}$

$$\therefore \qquad U_{grav} = \frac{-G\left(\sqrt{M_1}+\sqrt{M_2}\right)\sqrt{M_1}}{d} + \frac{-G\left(\sqrt{M_1}+\sqrt{M_2}\right)\sqrt{M_2}}{d}$$

$$= -\frac{G}{d}\left(M_1 + \sqrt{M_1M_2} + \sqrt{M_1 + M_2} + M_2\right)$$

$$= -\frac{G}{d}\left(M_1 + M_2 + 2\sqrt{M_1M_2}\right)$$

17. A ball dropped on ground from a height of 1.00 m rises to a height of 75 cm on rebounce. When thrown down from the same height with a velocity of 2.0 m/s, it would rise to (assume $g = 10\ m/s^2$)

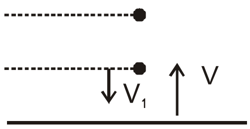
(a) 80 cm (b) 90 cm (c) 85 cm (d) 95 cm

Sol. **(b)**

$$e = \frac{\sqrt{2g\left(\frac{3}{4}h\right)}}{\sqrt{2gh}} = \frac{\sqrt{3}}{2}$$

$$mgh = \frac{1}{2}\,mV^2$$

$$V = \sqrt{2gh}$$



$V_2 = eV_1$ $\qquad V_1^2 = (2)^2 + 2g\ (1) = 4 + 20 = 24$

$$= \frac{\sqrt{3}}{2} \times \sqrt{24}$$

$$V_2 = \sqrt{18} = \sqrt{2gh'}$$

$$\Rightarrow \frac{18}{2\times10} = h'$$

$h^{11} = 0.9$ m

18. A metallic body of material with density of $8000\ kg/m^3$ has a cavity inside. A spring balance shown its mass to be 10.0 kg in air and 7.5 kg when immersed in water. The ratio of the volume of the cavity to the volume of the material of the body must be

(a) 2/5 (b) 1/2 (c) 1 (d) 3/4

Sol. **(c)**

$(V - V_C)\ 8000 = 100$

$(V - V_C) = (8000 - 1000)V = 7.5,$

$1000\ V = 2.5$

$$\left(\frac{V - V_C}{V}\right) 8 = \frac{4}{1}$$

$$\left(\frac{V - V_C}{V}\right) = \frac{1}{2} \qquad 2V - 2V_C = V$$

$$V = 2V_C$$

$$V_C = \frac{V}{2}$$

$$\frac{V_C}{V - V_C} = \frac{\frac{V}{2}}{\left(V - \frac{V}{2}\right)} = 1$$

19. In a steel factory it is found that to maintain M kg of iron in the molten state at its melting point an input power P watt is required. When the power source is turned off, the sample completely solidifies in time t second. The latent heat of fusion of iron is

(a) $\frac{2Pt}{M}$ (b) $\frac{Pt}{2M}$ (c) $\frac{Pt}{M}$ (d) $\frac{PM}{t}$

Sol. **(c)**

$Pt = ML \quad \Rightarrow \quad L = \frac{Pt}{M}$

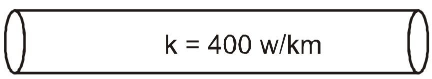
20. A LASER source of heat of power 1.2 W is placed very close to one end of a rod of cross section area 3 cm^2 and thermal conductivity 400 W/mK. The length of the rod (L) required to maintain a temperature difference of 10ºC across its ends is (assume that all power emitted by the source falls on the rod)

(a) 1.5 m (b) 2.2 m (c) 1.8 m (d) 1 m

Sol. **(d)**

P = 1.2w

A = 3 cm^2



$\Delta\theta$ = 10ºC

$$\frac{d\theta}{dt} = kA\left(\frac{d\theta}{\Delta x}\right)$$

$$1.2 = 400 \times (3 \times 10^{-4}) \times \frac{10}{2}$$

$$L = \frac{400 \times 3 \times 10^{-3}}{12} \times 10$$

$= 10^3 \times 10^{-3} = 1$ m

21. The temperature at which average de Broglie wavelength of helium atom becomes 0.5 nm is

(a) 6.6 K (b) 7.1 K (c) 279.6 K (d) 280.1 K

Sol. **(a)**

$\lambda = \frac{h}{\sqrt{2mE_0}}$ and $E_0 = \frac{3}{2}kT$

Approximate : T = 6.6 K.

22. A dielectric slab is introduced between the plates of a capacitor. If the charge on the capacitor is q and the magnitude of the induced charge on the dielectric surface is q', then

(a) q' < q (always) (b) q' > q (always) (c) q' = q (always) (d) q' = 0

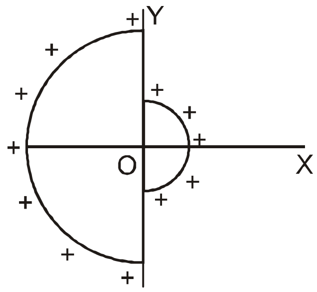
Sol. **(a)**

23. When two ends of a spring are pulled apart increasing its length it produces force equal to kx at its ends. At a point 1/4 of its length from one end the force is

(a) 0.25 kx (b) 0.75 kx (c) 1.0 kx (d) 0.5 kx

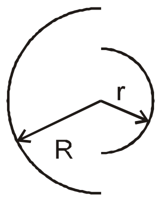
Sol. **(c)**

24. Two semicircular wires of radius 20 cm and 10 cm have a common centre at the origin O as shown in the figure. Assume that both the wires are uniformly charged and have an equal charge of 0.70 nC each. The magnitude of electric field at the common centre of curvature O of the system is



(a) 100 V/m (b) 301 V/m (c) 401 V/m (d) 501 V/m

Sol. **(b)**



$$E_{Net} = 2K\left[\frac{Q}{\pi r^2} - \frac{Q}{\pi R^2}\right]$$

$$= \frac{2KQ}{\pi}\left[\frac{1}{r^2} - \frac{1}{R^2}\right]$$

$$= \frac{2\times 9\times 10^9 \times 0.7\times 10^{-9}}{3.14\times 75}$$

= 300.96 V/m

25. An electron has its path unchanged when it is passing through a region of electric field (E = 3.4 × 10^4 V/m) and a magnetic field (B = 2 × 10^{-3} Wb/m^2) both perpendicular to each other. However, on switching off the electric field, the electrons move along a circular path. What is the radius of circular path ?

(a) 4.82 m (b) 4.82 × 10^{-3} m (c) 4.82 × 10^{-2} m (d) None of the above

Sol. **(c)**

E = VB

3.4 × 10^4 = V × 2 × 10^{-3}

V = 1.7 × 10^7

$$R = R = \frac{mV}{qB} = \frac{9.1\times 10^{-31}\times 1.7\times 10^7}{1.6\times 10^{-19}\times 2\times 10^{-3}}$$

= 4.83 × 10^{-2} m

26. A 1000 μF capacitor fully charged to 250 V discharges through a resistance wire embedded in a thermally insulated block of specific heat 2.5 × 10^2 J $kg^{-1}K^{-1}$ and mass of 0.01 kg. How much is the increases in the temperature of the block?

(a) 12.5 K (b) 8.5 K (c) 7.0 K (d) 15.5 K

Sol. **(a)**

$$U_i = \frac{1}{2} \times 10^{-3} \times (250)^2$$

$$\Delta\theta = \frac{\frac{1}{2}(25)^2 \times 10^{-1}}{2.5\times 10^2 \times 10^{-2}} = \frac{25}{2} = 12.5k$$

27. Two bodies A and B hanging in air are tied to the ends of a string which passes over a frictionless pulley. The masses of the string and the pulley are negligible and the masses of two bodies are 2 kg and 3 kg respectively. (Assume $g = 10\ m/s^2$). Body A moves upwards under a force equal to

(a) 30 N (b) 24 N (c) 10 N (d) 4 N

Sol. (d)

$$a = \left(\frac{3-2}{3+2}\right)g = 2cm^{-2}$$

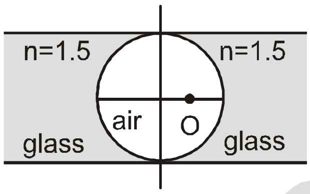
$$F_{net}\ A = T - 2g = 2 \times a = 4N$$

28. An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B. The combination forms a real image three times the size of the object. If lens B is concave with a focal length 30 cm, then focal length of lens A is

(a) 10 cm (b) 7.5 cm (c) 6 cm (d) –6 cm

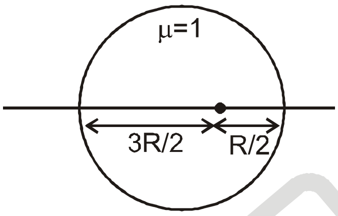
Sol. (c)

29. Two concave refracting surfaces of equal radii of curvature face each other in air as shown in figure. A point object O is placed midway between the centre and one of the poles. Then the separation between the images of O formed by each refracting surface is



(a) 11.4 R (b) 1.14 R (c) 0.114 R (d) 0.0114 R

Sol. (c)



event - 1 $$\frac{3}{2v_1} - \frac{1}{\left(\frac{-3R}{2}\right)} = \frac{\frac{3}{2}-1}{-R}$$

$$\frac{3}{2v_1} + \frac{2}{3R} = \frac{-1}{2R} \Rightarrow \frac{3}{2v_1} = \frac{-1}{2R} = \frac{-2}{3R} = \frac{-7}{6R} \Rightarrow V_1 = \frac{-9R}{7}$$

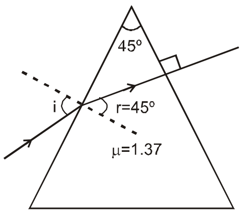
event - 2 $$\frac{3}{2v_2} - \frac{1}{\left(-\frac{R}{2}\right)} = \frac{\frac{3}{2}-1}{-R} \Rightarrow \frac{3}{2v_2} + \frac{2}{R} = \frac{-1}{2R} \Rightarrow \frac{3}{2v_2} = \frac{-1}{2R} - \frac{2}{R} = \frac{-5}{2R}$$

$$v_2 = -\frac{3R}{5}$$

30. A ray of white light falls on an isosceles prism at such an angle that the red light leaves the prism perpendicular to the other face of the prism. Find angle of deviation if the refractive index of the prism for red light is 1.37 and refracting angle of prism is 45º

(a) 20º37' (b) 28º37' (c) 35º37' (d) 30º37'

Sol. **(d)**



sin i = μ sin r

sin i = (1.37) $\left(\frac{1}{\sqrt{2}}\right)$

sin i = 0.97

i = $\sin^{-1}$ (0.97)

δ = $\sin^{-1}$ (0.97) – 45º

= 30.37[1]

31. The voltage between the terminals of a battery is 6.00 V. When a wire is connected across its terminals it falls to 5.6 V. If one more identical wire is connected between the terminals then it will fall to

(a) 4.80 V (b) 5.15 V (c) 5.25 V (d) 5.80 V

Sol. **(c)**

r = $\left(\frac{\varepsilon - V}{V}\right)R$

Let after connecting the identical wire between terminals potential dropis V

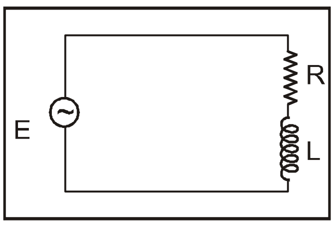
$\Rightarrow \quad \left(\frac{6-5.6}{6}\right)R = \left(\frac{6-V}{V}\right)\frac{R}{2} \quad \Rightarrow \quad V = 5.25$ Volt

32. Water is siphoned out from a tank at a higher level into another of identical size 2.0 m below. The length of the siphon tube is 4.0 m and each of its ends is below the water surface by 10 cm. In the upper tank the water level is at 1.00 m and in the lower one it is at 50 cm from the bases of the respective tanks. Water through siphon rises in 40 cm of the length of the tube to a level which is 20 cm higher than the water level in the upper tank before it begins to flow down. Assume the values of atmospheric pressure and acceleration due to gravity to be 103.4 kPa and 10 m/s^2. The pressure at the cross section of the tube at the highest point is

(a) 101.9 kPa (b) 105.4 kPa (c) 107.4 kPa (d) 109.1 kPa

Sol. **(a)**

33. The impedance of the RL circuit given in the adjacent figure is expressed by the relation $Z^2 = A^2 + B^2$. Then the dimensions of AB are



(a) $[M^1L^2I^{-2}T^{-3}]$ (b) $[M^2L^4I^{-4}T^{-6}]$ (c) $[M^1L^{-1}I^{-2}T^{-3}]$ (d) $[M^{-1}L^{-2}I^2T^4]$

Sol. **(b)**

$$[AB] = \left[\frac{V}{I}\right]^2$$

$$= \left[\frac{M^1L^2T^{-2}}{I^2T^1}\right]^2 = [M^2L^4T^{-6}I^{-4}]$$

34. A micrometer screw gauge with pitch of 0.5 mm and 50 divisions on circular scale is used to measure the diameter of a thin wire. Initially when the gap is closed the line of fourth division coincides with the reference line. Three readings show 46^{th}, 48^{th} and 44^{th} division coinciding with the reference line which is beyond 0.5 mm of the main scale. The (best) measured value is

(a) 0.46 mm (b) 0.94 mm (c) 0.92 mm (d) 1.00 mm

Sol. **(c)**

Zero error = 4 × ε

Reading = 0.5 mm + 48 × $\frac{0.5}{50}$

= 0.5 mm + 0.48 mm

= 098 mm

Actual value = 0.98 mm – 4ε = 0.92 mm.

35. In a meter bridge experiment the resistance to be measured is connected in the right gap and a known resistance in the left gap has value of 50 $\pm$ 0.2 Ω when the null point is judged to be at 60 $\pm$ 0.2 cm. The students notes that the ends of the bridge wire are not at 0.0 cm and 100.0 cm of the scale and makes a guess that they may be somewhere within 0.2 cm beyond the scale ends. The value of the unknown resistance should be expressed as

(a) 33.33 ± 1Ω (b) 75 ± 1 Ω (c) 75.0 ± 0.9 Ω (d) 33.4 ± 0.5 Ω

Sol. **(d)**

$$\frac{40\times 50}{60} - \frac{100}{3}$$

$$= \pm\frac{|\Delta\ell_1|}{\ell_1} \pm \frac{\Delta\ell_2}{\ell_1} \pm \frac{\Delta R}{R}$$

$$\Delta x = x\left[\frac{0.2}{60} + \frac{0.2}{40} + \frac{0.2}{50}\right]$$

$$= \left(\frac{100}{3} \pm 0.4\right)\Omega \ .$$

36. A body A revolves along a circular orbit close to the earth's surface. Body B oscillates along an imaginary straight tunnel drilled through the earth, whereas another body C through a similar longest tunnel. Let T_A, T_B and T_C be the corresponding periods of revolution or oscillation, then

(a) $T_A > T_B = T_C$ (b) $T_A > T_C > T_B$ (c) $T_A = T_B = T_C$ (d) $T_A < T_B = T_C$

Sol. **(c)**

$$T_A = \frac{R^{3/2}}{\sqrt{GM}} = 2\pi\sqrt{\frac{R}{g}}$$

$$T_B = 2\pi\sqrt{\frac{R}{g}}$$

$$T_C = 2\pi\sqrt{\frac{R}{g}}$$

37. The vibrations of a string of length 100 cm and fixed at both ends are represented by the equation : $y = 2 \sin (\pi x / 10) \cos (50 \pi t)$. Then the equations of the component waves whose superposition gives the above vibrations are

(a) $2 \sin (\frac{\pi x}{10} + 50 \pi t) + 2 \sin (\frac{\pi x}{10} - 50 \pi t)$ (b) $\sin (\frac{\pi x}{10} + 50 \pi t) - \sin (\frac{\pi x}{10} - 50 \pi t)$

(c) $\sin (\frac{\pi x}{10} + 50 \pi t) + \sin (\frac{\pi x}{10} - 50 \pi t)$ (d) $2 \sin (\frac{\pi x}{10} + 50 \pi t) + 2 \sin (\frac{\pi x}{10} + 50 \pi t)$

Sol. **(c)**

38. Hot coffee in a mug cools from 90ºC t o 70ºC in 4.8 minutes. The room temperature is 20ºC. Applying Newton's law of cooling the time needed to cool it further by 10ºC should be nearly

(a) 4.2 min (b) 3.8 min (c) 3.2 min (d) 2.4 min

Sol. **(c)**

$$\frac{\Delta\theta}{\Delta t} = k(\theta_{avg} - \theta_s)$$

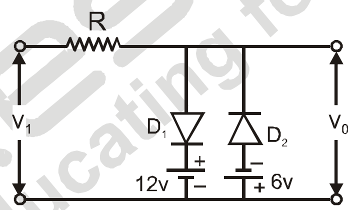
90ºC $\rightarrow$ 70ºC $\therefore$ $\frac{20}{4.8} = k(80 - 20)$

70ºC $\rightarrow$ 60ºC $\therefore$ $\frac{10}{t} = k(65 - 20)$

$\Rightarrow$ $\frac{20}{4.8} \times \frac{t}{10} = \frac{60}{45}$

$\Rightarrow$ $t = \frac{4}{3} \times 2.4 = 3.2$ min

39. A sinusoidal voltage of amplitude 15 V is connected between the input terminals of the circuit shown in the figure. Assume that the diodes are ideal. In the output waveform



(a) positive peaks of the input will be clipped ot +12V and negative peaks will be clipped to –6 V
(b) positive peaks of the input will be clipped to +6V and negative peaks will be clipped to – 12 V
(c) positive peaks of the input will be clipped to +12V and negative peaks will be clipped to –12 V
(d) positive peaks of the input will be clipped to +6V and negative peaks will be clipped to –6 V

Sol. **(a)**

For positive cycle D_1 – forward
D_2 – reverse

So, N_0 = +12

For negative cycle D_1 – reverse
D_2 – forward

So, v_0 = –6 volt

40. Correlate a physicist among P, Q, R to an appropriate physicist among L, M, N. Let us pay a tribute to them.

P. Louis de Broglie — L. Davison and Germer
Q. Max Plank — M. James Clerk Maxwell
R. Christian Huygen — N. Arthur H. Compton

(a) P & N, Q & M, R & L
(b) P & L, Q & N, R & M
(c) P & L, Q & M, R & N
(d) P & M, Q & L, R & N

Sol. **(b)**

$\lambda = \frac{h}{p}$ De Brogle

Justification was given by Davison and Germer. According to huygen light is wave Maxwell also used wave concept of light. Max Plank and Compton used photon concept.

SUB- PART A-2

In question 41 to 50 any number of options (1 or 2 or 3 or all 4) may be correct. You are to identify all of them correctly to get 6 marks. Even if one answer identified is incorrect or one correct answer is missed, you get zero score.

41. In a circuit carrying an alternating current
(a) magnetic field around it oscillates with the frequency of the current
(b) heat is generated with double the frequency of the magnetic field
(c) voltage across the circuit is proportional to the rate of change of magnetic flux around it
(d) the current always lags in phase behind the applied voltage.

Ans. **(a,b)**

42. The stress exerted by vehicles on the central part of a bridge with convex curvature will :
(a) be more than that at the ends
(b) same as that at the ends
(c) be less than that at the ends
(d) decrease in proportion to the square of the speed of the vehicle

Ans. **(a)**

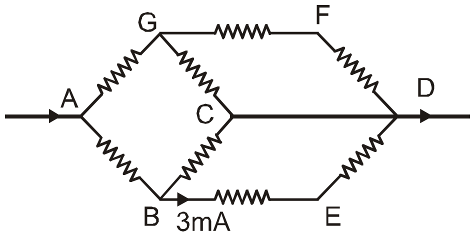
43. A cylindrical vessel filled with water is connected by a narrow pipe at its bottom to another identical empty vessel. Then :
(a) potential energy of water is proportional to the square of the height of its level
(b) $3/4^{th}$ of the potential energy is lost when the water flow stops
(c) half the potential energy is lost when the levels in both the vessels are same
(d) loss in potential energy is equal to the rise in the thermal energy

Ans. **(a,c,d)**

44. The deviation produced by a prism depends upon
(a) angle of incidence on face of prism
(b) refracting angle of prism
(c) refractive index of prism
(d) wavelength of light used

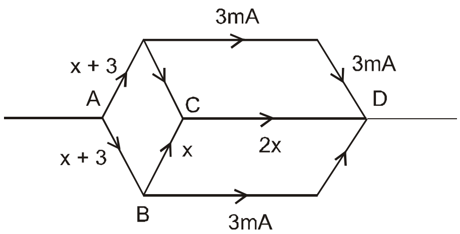
Ans. **(a,b,c,d)**

45. Consider nine identical resistances arranged as shown in the figure. In this arrangement electric current enters at node A and leaves from node D. Let V_{AD} = 5 volt and I_{BE} = 3 mA. Therefore :



(a) I_{AB} = 5 mA
(b) each resistance is (5000/11)Ω
(c) effective resistance between A and D is 500Ω
(d) power dissipation along the path BCD is (100/11)mW

Sol. **(a,b,c,d)**



(x + 3 + 3 + 3) r = 5
(x + 9) r = 5
x + 2x = 6 ...(1)
3x = 6 $\Rightarrow$ x = 2 mA ...(2)

$\therefore$ 11r = 5 $\Rightarrow$ r = $\frac{5}{11 \times 10^{-3}} = \frac{5000}{11}\Omega$

$\therefore$ x + 3 = 2 + 3 = 5mA
R (5 + 5) 10^{-3} = [x + 3 + 3 + 3] 10^{-3} r
R 10 = (x + 9) r
R (10) = (2 + 9) r

R = $\frac{11}{10}$r $\quad \therefore$ R = $\left(\frac{11}{10}\right)\left(\frac{5000}{11}\right) = 500\,\Omega$

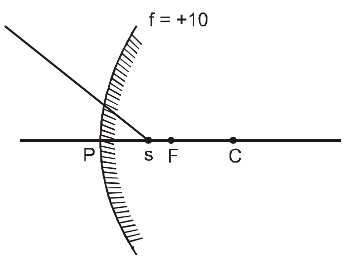
$(P)_{BCD} = x^2r + 4x^2r = 5x^2r$

$= 5\left(\frac{2}{1000}\right)^2 \left(\frac{5000}{11}\right)$

$= \frac{5 \times 4}{1000 \times 1000} \times \frac{5000}{11} = \frac{100}{1000 \times 11}$ W $= \frac{100}{11}$mW

46. Mark the correct statement(s) of the following
(a) A convex mirror forms virtual images for all positions of object
(b) A concave mirror forms real images for all positions of the object
(c) A concave mirror can form a virtual magnified image
(d) The magnification produced by a convex mirror is always less than unity

Sol. **(c)**



u = +5
V = ?, f = +10

$$\frac{1}{V}+\frac{1}{x}=\frac{1}{f}$$

$$\frac{1}{V}+\frac{1}{5}=\frac{1}{10}$$

$$\frac{1}{V}=\frac{1}{10}-\frac{1}{5}=\frac{1}{10}-\frac{2}{10}=-\frac{1}{10}$$

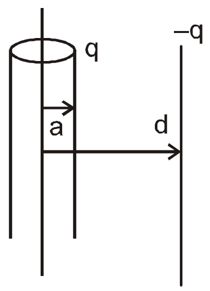
V = –10

$$m=-\frac{V}{x}=-\frac{-10}{+5}=2$$

47. A solid cylindrical conductor of radius a and charge q is coaxial with a cylindrical shell of negligible thickness, radius b(>a) and charge –q. The capacitance of this cylindrical capacitor per unit length is proportional to :

(a) log (b/a) (b) log (a/b) (c) $[\log (b/a)]^{-1}$ (d) 1/ [logb – loga]

Sol. **(c,d)**



$$E=\frac{2k\lambda}{r},\ V=2\,2k\lambda\ \ell n\left(\frac{b}{a}\right)$$

$$C=\frac{\lambda}{2k\lambda\,\ell n\left(\frac{b}{a}\right)}$$

$$=\frac{4\pi\varepsilon_0}{2\left[\ell n\left(\frac{b}{a}\right)\right]}$$

48. Mark the correct statement(s) of the following :

(a) In case of liquids the boiling point increases with pressure for all liquids
(b) In case of solids the melting point decreases with pressure for all solids
(c) In case of ice the melting point decreases with pressure
(d) In case of ice the melting point increases with pressure

Sol. **(a,c)**

49. The electric potential (in volt) in a region along the x-axis varies with x according to the relation $V(x) = 5 + 4x^2$, where x is in m. Therefore :

(a) the potential difference between the points x = 1 and x = –3 is 32 V

(b) force experienced by a charge of 1C placed at x = –1 m is 8N

(c) force experienced by the above mentioned charge is along the positive x-axis

(d) a uniform electric field exists in this region along the x-axis

Sol. **(a,b,c)**

$V(x) = 5 + 4x^2$

$V(1) = 5 + 4 = 9$, $V(-3) = 5 + 4(9) = 41$

$V(-3) - V(1) = 41 - 9 = 32$ V

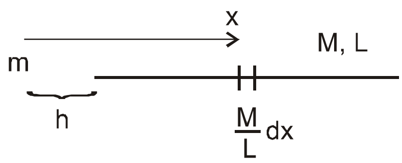
$$E = -\frac{dV}{dx} = -(0 + 8x) = -8x = 8$$

$\therefore \quad F = 8 \times 1 = 8$ N $(\rightarrow)$

50. A homogeneous bar of length L and mass M is situated at a distance h from a particle of mass m as shown. The gravitational force exerted by the bar on the particle varies inversely as :

(a) $(L - h)^2$ (b) $(h + L/2)^2$ (c) $h(h + L)$ (d) h^2 if L << h

Sol. **(c,d)**



$$F = \int_{x=h}^{h+L} \frac{G\left(\frac{M}{L}dx\right)m}{x^2} = \frac{GMm}{L}\left[-\frac{1}{x}\right]^{h+L}$$

$$= \frac{GMm}{L}\left[\frac{1}{x}\right]_{h+L}^{h} = \frac{GMm}{L}\left(\frac{1}{h} - \frac{1}{h+L}\right)$$

$$= \frac{GMm}{L}\left(\frac{h+L-h}{h(h+L)}\right) = \frac{GMmL}{L}\ \frac{1}{h(h+L)}$$

$$= \frac{GMm}{h(h+L)}$$

$$F \alpha \frac{1}{h(h+L)}, \quad F \alpha \frac{1}{h^2} \text{ if } L << h$$

PART B **MARKS : 60**

All questions are compulsory. **All questions carry equal marks.**

1. A particle is moving in positive x-direction with its velocity varying as $v = \alpha\sqrt{x}$. Assume that at t = 0 the particle was located at x = 0. Determine (i) the time dependence of velocity (ii) acceleration and (iii) the mean velocity of the particle averaged over the time that the particle takes to cover the first s meters of the path.

Sol. V = $\alpha\sqrt{x}$

squaring both side

$V^2 = \alpha^2 x \qquad \Rightarrow \qquad V^2 = 2\left(\frac{\alpha^2}{2}\right)x$

$\therefore$ Acceleration is constant = $\frac{\alpha^2}{2}$

u = 0

(i) $\therefore$ V = u + at

$= \frac{\alpha^2}{2}t$

(ii) acceleration (a) = $\frac{\alpha^2}{2}$

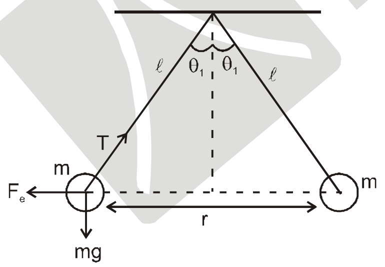
(iii) V = $\alpha\sqrt{s}$

$\therefore$ average velocity = $\frac{u+v}{2} = \frac{\alpha\sqrt{s}}{2}$

2. Two identical metal spheres of density p having equal and similar charges are supported from a common point by means of a silk thread of length ℓ and negligible mass. The two threads make an angle $2\theta_1$ with each other when in equilibrium in air. When the same system is immersed in a dielectric liquid of density σ, then the angular separation changes to $2\theta_2$.

Then find (i) the relative permittivity (ε_r) of the liquid in terms of $\rho,\sigma,\theta_1,\theta_2$. (ii) In case of the angular separation remains unchanged even on immersing the system in the dielectric liquid find the expression for ε_t.

Sol. Let charge on object is q.



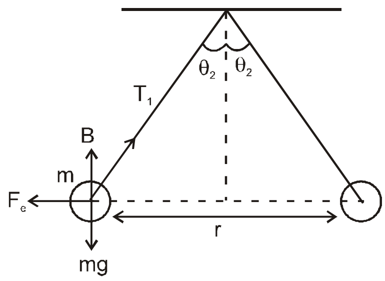
$T \sin\theta_1 = F_e = \frac{kq^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$ ......(i)

$T \cos\theta_1 = mg$ ....(ii)

$r = 2\ell \sin\theta_1$ ....(iii)

Now if immered in liquid

Fe' is new eelctrostatic force B is up thrust T_1 is new tension.



$T_1 \sin \theta_2 = Fe = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q^2}{r^2}$ ....(iv)

$T_1 \cos \theta_2 = mg - B$ .....(v)

$r' = 2\ell \sin \theta_2$ ....(vi)

Dividing (i) & (ii)

$$\tan \theta_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2\ell \sin\theta_1)^2 mg} \quad .....(vii)$$

$$\text{similarity } \tan\theta_2 = \frac{1}{4\pi\varepsilon_0 \in_r}\frac{q^2}{(2\ell \sin\theta_2)^2(mg-B)} \quad ....(viii)$$

Dividing (vii) & (viii)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_r \sin^2\theta_2(mg-B)}{\sin^2\theta_1 mg}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \varepsilon_r \frac{\sin^2\theta_2}{\sin^2\theta_1}\left(1-\frac{V\sigma g}{V\rho g}\right)$$

$$\varepsilon_r \frac{\sin^2\theta_2}{\sin^2\theta_1}\left(1-\frac{\sigma}{\rho}\right)$$

$$\frac{\sin^2\theta_1 \tan\theta_1}{\sin^2\theta_2 \tan\theta_2} = \varepsilon_r\left(1-\frac{\sigma}{\rho}\right) = \varepsilon_r\left(\frac{\rho-\sigma}{\rho}\right)$$

$$\varepsilon_r = \frac{\sin^2\theta_1 \tan\theta_1}{\sin^2\theta_1 \tan\theta_{2\,r}}\left(\frac{\rho}{\rho-\sigma}\right)$$

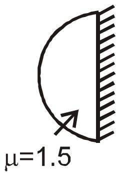
if $\theta_1 = \theta_2$

$$\varepsilon_r = \varepsilon_r = \frac{\rho}{\rho-\sigma}$$

3. (a) The plane side of a thin planoconvex lens is silvered so that the lens acts as a concave mirror of focal length 40 cm. The material of lens has refractive index 1.5. Determine the radius of curvature of the curved surface of the lens.

(b) Light falls on one end of a cylindrical glass rod at an angle α. Determine the smallest refractive index that the glass may have so that the light after entering the rod does not leave it through its curved surface irrespective of the value of α.

Sol. (a)



f = 40 cm

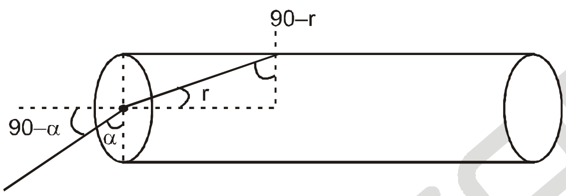
$$\frac{1}{f}=\frac{1}{f_m}-\frac{2}{f_\ell} \quad \Rightarrow \quad \frac{1}{-40}=\frac{-2}{f_\ell}$$

$$\Rightarrow \quad f_\ell = 80 \text{ cm, now } \frac{1}{f}=(n_{rel}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_\ell}=(1.5-1)\left(\frac{1}{R}-0\right) \quad \Rightarrow \quad \frac{1}{80}=0.5\left(\frac{1}{R}\right)$$

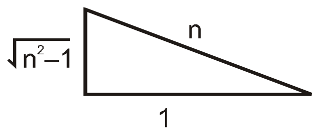
$$\Rightarrow \quad R = 0.5 \times 80 = 40 \text{ cm}$$

(b)



$$\frac{\sin(90-r)}{\sin 90^o}=n \quad \Rightarrow \quad \frac{\cos\alpha}{\sin r}=n$$

$$\frac{\sin(90-r)}{\sin 90^o}=\frac{1}{n} \quad \Rightarrow \quad \cos r = \frac{1}{n}$$



$$\text{now } \frac{\cos\alpha}{\sin r}=n \quad \Rightarrow \quad \frac{\cos\alpha}{\frac{\sqrt{n^2-1}}{n}}=n$$

$$\Rightarrow \quad \sqrt{n^2-1} = \cos\alpha$$

for minimum n $\alpha \to 0$

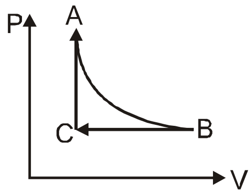
$$\therefore \quad \sqrt{n^2-1} = 1 \quad \Rightarrow \quad n^2 - 1 = 1$$

$$\Rightarrow \quad n^2 = 2$$

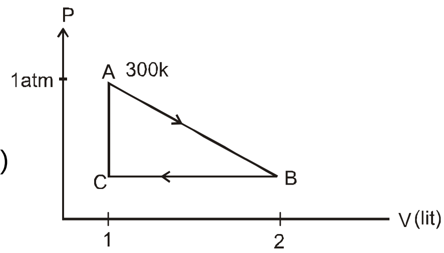
$$\Rightarrow \quad n = \sqrt{2}$$

4. A cyclic process in indicated in the following PV-diagram. In the initial state (A) temperature, pressure and volume of the system are 300 K, 1atm and 1000 cc. In the first process (AB), the adiabatic expansion increases the volume to 2000 cc. This is followed by an isobaric compression to reduce the volume of gas to 1000 c. The gas is brought to initial state by isobaric process. The system is 1 mole of a monoatomic gas.

(a) Find the pressure and temperature at state B and C.
(b) Calculate the work done in adiabatic, isobaric, isobaric processes.
(c) Calculate the total work done in cyclic process and amount of energy supplied to the system
(d) Calculate the efficiency of cyclic process



Sol. (a)



$$C_v = \frac{3R}{2}, \qquad C_p = \frac{5R}{2}$$

$$\gamma = \frac{5}{3}$$

$$P_A V_A^{\gamma} = P_B V_B^{\gamma} \qquad \Rightarrow \qquad 1\,(1) = P_B\, 2^{5/3}$$

$$\Rightarrow \qquad P_B = \frac{1}{2^{5/3}} \text{ atm}$$

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$\left(\frac{V_A}{V_B}\right)^{\frac{5}{3}-1} = \frac{T_B}{T_A} \qquad \Rightarrow \qquad T_B = (300)\left(\frac{1}{2}\right)^{2/3}$$

$$T_B = \frac{300}{2^{2/3}}\,k$$

$$\text{(b) } W = \frac{P_A V_A - P_B V_B}{\gamma - 1} = \frac{(10^5)(10^{-3}) - \frac{10^5}{2^{5/3}}(2\times 10^{-3})}{\frac{5}{3}-1} = \frac{100 - 100\left(\frac{1}{2^{2/3}}\right)}{\frac{2}{3}}$$

$$J = 150\left[1 - \frac{1}{2^{2/3}}\right] J$$

$$W_{BC} = -\left(\frac{10^5}{2^{5/3}}\right)(10^{-3})$$

$$J = -\frac{100}{2^{5/3}}\,J$$

$$W_{CA} = 0$$

(c) Total heat energy givven to system

$\Delta Q = n\, C_V\, (T_A - T_C) + n\, C_P\, (T_C - T_B)$

$$\frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C}$$

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}, \qquad \frac{V_B}{V_C} = \frac{T_B}{T_C}$$

$$\therefore \qquad \frac{2}{1} = \left(\frac{300}{2^{2/3}}\right)\frac{1}{T_C}$$

$$T_C = \frac{1}{2} = \left(\frac{300}{2^{2/3}}\right)K$$

$$\therefore \qquad \Delta Q = \eta\frac{3R}{2}\left(300 - \frac{300}{2\times 2^{2/3}}\right)$$

$$+\eta\frac{5R}{2}\left(\frac{300}{2\times 2^{2/3}} - \frac{300}{2^{2/3}}\right)$$

$$= \eta\frac{3R}{2}300\left(1 - \frac{1}{2^{5/2}}\right)\eta\frac{5R300}{2(2^{2/3})}\left(\frac{1}{2}\right)J$$

$\Delta Q = \Delta W$

$$\text{(d)} \quad \eta = \frac{\eta\frac{3R300}{2}\left(1 - \frac{1}{2^{5/2}}\right) - \eta\frac{5R300}{2(2^{2/3})(2)}}{\eta\frac{3R}{2}300\left(1 - \frac{1}{2^{5/2}}\right)} = 0.233$$

(We don't consider n = 1)

5. A 750 Hz, 20 V source is connected to a resistance of 100 Ω, a capacitance of 1.0 μF and an inductance of 0.18 H in series. Calculate the following quantities :

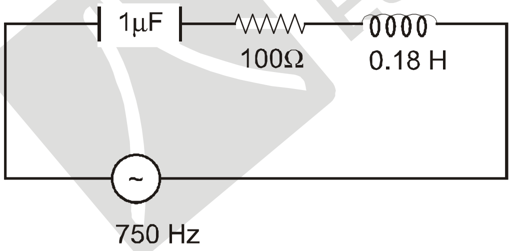
(a) Impedence of the circuit

(b) Draw an impedence diagram with suitable scale

(c) Power factor

(d) The time in which the resistance will get heated by 10ºC, provided that the thermal capacity of resistance = 2 J/ºC

Sol.



$$z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\omega = 2n\pi = 2(750)\pi = 1500\pi$$

$$X_L = \omega_L = (1500\pi)\,(0.18) = 848.5\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{10^6}{1500\pi} = 212.12\,\Omega$$

$$z = \sqrt{(636.4)^2 + (100)^2}$$

$= 100\sqrt{(6.364)^2 + 12}$

$= 100 \times 6.44 \cong 644\,\Omega$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{848.5 - 212.12}{100}$

= 6.36

(C) $\cos\phi = \dfrac{R}{z} = \dfrac{100}{644} = 0.155$

(B) impedence is constant as n is constant

(D) $i_{rms} = \dfrac{\varepsilon_{rms}}{z} = \dfrac{20}{644}\,A$

$H = (i_{rms})^2 Rt = \left(\dfrac{20}{644}\right)^2 (100)t$

$(ms)\ (\Delta\theta) = \left(\dfrac{20}{644}\right)^2\ (100)\ t$

$(2)\ (10) = \left(\dfrac{20}{644}\right)^2\ (100)\ t$

$t = \left(\dfrac{644}{20}\right)^2 \times \dfrac{1}{100} \times 20$

t = 207.36 sec.

Physical constants you may need –

1. Charge on electron $e = 1.6 \times 10^{-19}$ C
2. Mass of an electron $m_e = 9.1 \times 10^{-31}$kg
3. Universal gravitational constant $G = 6.67 \times 10^{-11}$ N m^2/kg^2
4. Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N m^2
5. Gas constant R = 8.31 J/K mol
6. Planck constant $h = 6.62 \times 10^{-34}$ Js
7. Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/m^2 k^4
8. Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K