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51st NMTC-

Code - I

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA (Regd.)

Screening Test – Ramanujan Contest

QUESTION PAPER WITH SOLUTION & ANSWER KEY

Date: 31st August, 2019 | Duration: 2 Hours

National Mathematics Talent Competitions (NMTC)

Resonance Eduventures Ltd.

CORPORATE & REG. OFFICE : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005
Reg. Office : J-2, Jawahar Nagar, Main Road, Kota (Raj.)-324005 | **Ph. No.:** +91-744-3192222 | **FAX No. :** +91-022-39167222
Ph.No. : +91-744-3012222, 6635555 | **To Know more :** sms **RESO** at **56677**
Website : www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN : U80302RJ2007PLC024029**

» *Instructions to Candidates* «

1. Do not open the Question booklet until you are told to do so.

2. **a)** This is a multiple choice and fill in the blanks test. In multiple choices each Question is followed by four answer (A), (B), (C) and (D) Only one of them it correct. When you have decided on your choice, shade the corresponding letter in the box against the question number. For example, if you decide that for a question, the correct response is (A), shade of the box marked A (Eg.[] on A in the box shading that box.
   b) In the case of fill in the blanks questions write the correct answer in the space provided.

3. Each question carries 1 marks.

4. a) If you are unable to solve a problem, it is better not to answer the question. Avoid guessing. Since you are penalized for each wrong answer.
   b) ½ mark will be deducted for wrong answers in part A
   c) ¼ mark will be deducted for wrong answers in part B

5. Be certain that you understand thoroughly the coding system for your answer sheet. If you are not sure, ask your supervisor to clarify it.

6. You are permitted to use rough paper. No other aid, like the instrument box, calculator, etc, are permitted.

7. Diagrams are not drawn to scale. They are intended as aids only.

8. Before commencing to write your answers. Fill in the details of your bio-data (Name, School/address etc.,) in the appropriate places in the response sheet. If you are writing in a centre other than your school write the agency through whom you write the exam. Eg,., AMTI for open quota.

9. After completion, return only the response sheet. The question-booklet and the rough-worksheets may be retained by you or as directed by your supervisor.

10. When your supervisor instructs you to begin, you have 120 minutes if working time.

NOTE:

1. **Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.**
2. **Diagrams are only visual aids; they are NOT drawn to scale.**
3. **You are free to do rough work on separate sheets.**
4. **Duration of the test: 2 pm to 4 pm (2 hours).**

PART-A

Note:

- Only one the choices A, B, C, D is correct for each question. Shade that alphabet of your choice in the response sheet. (If you have any doubt in the method of answering, seek the guidance of your supervisor).
- For each correct response your get 1 mark; **for each incorrect response you lose ½ mark.**

1. Ram and Shyam play table tennis with Ram's chance of winning a game being 3/5 and Shyam's 2/5. The winner gets 1 point and loser 0 points. The match terminates when one player has 2 points more than the other. The probability of Ram winning the game at exactly the end of 6^{th} game, not before, is

(A) $\frac{364}{15625}$ (B) $\frac{1296}{15625}$ (C) $\frac{432}{3125}$ (D) $\frac{2592}{15625}$

Ans. **(B)**

Sol. WL WL WW

WL LW WW

LW WL WW

LW LW WW

$\Rightarrow$ Probability is $4 \times \left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2 = \frac{1296}{15625}$ $\Rightarrow$ Option (B) is correct

2. Thirty volunteers are distributed to three poling booths. Each booth must have at least one and all must have different number of volunteers allotted. Then the number of ways of allocating volunteers is :

(A) 406

(B) 496

(C) 378

(D) None of these

Ans. **(D)**

Sol. $x + y + z = 30,\ 1 \le x < y < z$

x	y	z	
1	2	27	
	3	26	
	:	:	13 ways
	14	15	
2	3	25	
	4	24	
	:	:	11 ways
	13	15	
3	4	23	
	5	22	
	:	:	10 ways
	13	14	
	:	:	
8	9	13	2 ways
	10	12	
9	10	11	1 ways

Total ways = $\angle 3$ (13 + 11 + 10 + 8 + 7 + 5 + 4 + 2 + 1) = 6(61) = 366

3. The number of values of a for which the function $f(x) = \cos 2x + 2a(1 + \cos x)$ has a minimum value $\frac{1}{2}$ is :

(A) 0 (B) 1 (C) 2 (D) 3

Ans. **(B)**

Sol. $f(x) = 2\cos^2 x - 1 + 2a + 2a\cos x$

$= 2\cos^2 x + 2a\cos x + 2a - 1$

$\min f(x) = \frac{1}{2} \quad \Rightarrow \min(2\cos^2 x + 2a\cos x + 2a - 1) = \frac{1}{2}$

$\Rightarrow \min(2t^2 + 2at + 2a - 1) = \frac{1}{2}$ where $t \in [-1, 1]$

Case-I $-1 \le \frac{-2a}{4} \le 1$ then min value $= -\frac{(4a^2 - 4 \times 2(2a-1))}{4 \times 2} = \frac{1}{2}$

$\Rightarrow$ a = 1, 3 (rejected)

Case-II $\frac{-2a}{4} > 1$ then min value is $2 + 2a + 2a - 1 = \frac{1}{2}$

$\Rightarrow 4a = -\frac{1}{2} \quad \Rightarrow \quad a = -\frac{1}{8}$ (rejected)

Case-III $\frac{-2a}{4} < -1$ then min value is $2 - 2a + 2a - 1 = \frac{1}{2}$

$\Rightarrow \quad a \in \phi$

$\Rightarrow \quad a \in \{1\}$

Option (B) is correct

4. Let $f(x) = \frac{x}{\sqrt{x^2 - 1}}$. If $f^2(x) = f(f(x))$, $f^3(x) = f(f^2(x))$,….., $f^{n+1}(x) = f(f^n(x))$, then $f^{2019}(\sqrt{2})$ is :

(A) 1 (B) 0 (C) $\sqrt{2}$ (D) not define

Ans. **(C)**

Sol.

$$f(x) = \frac{x}{\sqrt{x^2 - 1}}$$

$$f(f(x)) = \frac{f(x)}{\sqrt{(f(x))^2 - 1}} = \frac{\frac{x}{\sqrt{x^2-1}}}{\sqrt{\frac{x^2}{x^2-1} - 1}} = x$$

$f^3(x) = f(f(f(x)) = f(x)$

$f^4(x) = f^2(x) = x$

Similarly $f^{2019}(x) = f(x) = \frac{x}{\sqrt{x^2 - 1}}$

$f^{2019}(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2 - 1}} = \sqrt{2}$ Ans.

5. The area of the curve enclosed by $|x - 2\sqrt{2}| + |y - \sqrt{5}| = 2$ is :

(A) 16 (B) 12 (C) 8 (D) 4

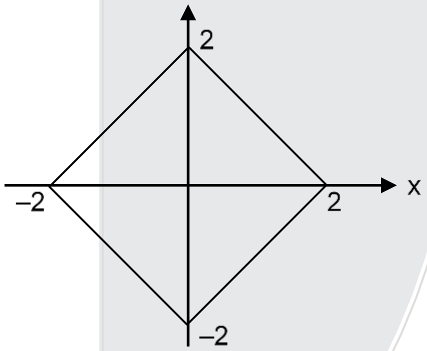
Ans. **(C)**

Sol.

Area of $|x - 2\sqrt{2}| + |y - \sqrt{5}| = 2$

is equivalent to area of $|x| + |y| = 2$

which is rhombus



Hence area = $4\left(\frac{1}{2}\times2\times2\right) = 8$

6. Let a be an irrational number. How many lines through the point (a, 2a) contain at least two points with both coordinates rational ?

(A) Infinitely many (B) At least two but finitely many

(C) Only one (D) None

Ans. **(C)**

Sol. If two points with rational coordinates lies on line then slope of line is rational.

So $\frac{k-2\alpha}{h-\alpha}$ = ratio = c {where (h, x) is point lies on line}

$\Rightarrow k - 2\alpha = hc - c\,\alpha$

$\Rightarrow k - hc = (2 - c)\,\alpha \qquad \Rightarrow c = 2$

Because line has unique slope

$\Rightarrow$ there exist exactly one line

Option (C) is correct

7. Suppose $A, A_2, \ldots\ldots, A_{33}$ be 33 sets each containing 6 elements and $B_1, B_2, \ldots\ldots, B_n$ be n sets each with 8 elements. if $U_{i=1}^{33} A_i = U_{i=1}^{n} B_i = S$ and if each element of S occurs exactly 9 times in $A_1, \ldots.$ A_2, A_{33} and exactly 4 times in $B_1, \ldots.\ B_2, B_n$, then n is :

(A) 22 (B) 33 (C) 12 (D) 11

Ans. **(D)**

Sol. Number of element in set S equal to $\frac{33\times 6}{9} = \frac{n\times 8}{4} \Rightarrow n = 11$

Option (D) is correct

8. Let a, b and c be real numbers such that $2a^2 - bc - 9a + 10 = 0$ and $4b^2 + c^2 + bc - 7a - 8 = 0$. Then the set of real values that a can take is given by

(A) [1, 4.2] (B) $(-\infty, 1) \cup (4.2, \infty)$ (C) (1, 4.2) (D) [1, 4.2)

Ans. **(A)**

Sol. $(2b - c)^2 + 5bc - 7a - 8 = 0$ …(i)

$2a^2 - bc - 9a + 10 = 0$ …..(ii)

$5\text{ (ii)} + \text{(i)} = 10a^2 + (2b - c)^2 - 52a + 42 = 0$

$10a^2 - 52a + 42 \le 0$

$(a - 1)(10a - 42) \le 0$

$a \in [1, 4.\ 2]$

9. Let g(x) = $\left[\frac{1}{\text{cosec}(x)}\right]$, then the range of g(x) is (Z is the set of integers)

(A) Z (B) Z – {0} (C) {0} (D) {0, 1, –1}

Ans. **(D)**

Sol. $g(x) = [\sin x]$ so range of g(x) is –1, 0, 1 $\because$ $-1 \le \sin x \le 1$

10. The ordered pair of numbers (x, y) satisfy both the equations $x + y = 3$ and $x^5 + y^5 + 162 = 0$. Then

(A) There are 5 pairs of real solutions

(B) there are four pairs of real solutions

(C) The are two pairs of real and two pairs of non-real solutions

(D) All four pairs are non-real solutions

Ans. **(D)**

Sol. $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4) + 162 = 0$

$x^4 + y^4 + x^2 y^2 - xy(x^2 + y^2) + 54 = 0$

$(x^2 + y^2)^2 - x^2 y^2 - xy(x^2 + y^2) + 54 = 0$

$x^2 + b^2 + 2xy = 9$

$(9 - 2xy)^2 - x^2y^2 - xy(9 - 2xy) + 54 = 0$

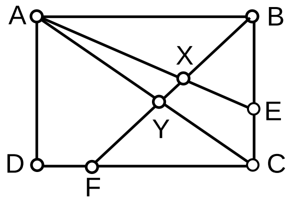
$81 + 4x^2y^2 - 36xy - x^2 y^2 + 2x^2 y^2 - 9xy + 54 = 0$

$5x^2 y^2 - 45xy + 135 = 0$

$xy = \frac{9 \pm \sqrt{81-108}}{2}$ $xy = \frac{9 \pm 3\sqrt{3}\,i}{2}$ $x + y = 3$

all four pair non real solution

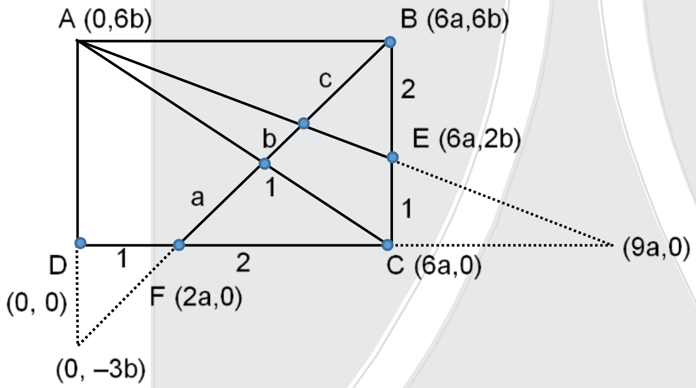
11. In a rectangle ABCD, point E lies on BC such that $\frac{BE}{EC}$ = 2 and point F lies on CD such that $\frac{CF}{FD}$ = 2. Lines AE and AC intersect BF at X and Y respectively. If FY : YX : XB = a : b : c, are relatively prime positive integers, then the minimum value of a + b + c is :



(A) 60 (B) 65 (C) 70 (D) 72

Ans. **(B)**

Sol.



$\frac{c}{a+b} = \frac{6}{7}$ $\Rightarrow 76 = 6a + 6b$ ….(i)

$\frac{b+c}{a} = \frac{6}{4} = \frac{3}{2}$ $\Rightarrow 2b + 2c = 3a$ ….(ii)

$\Rightarrow \frac{a}{26} = \frac{b}{9} = \frac{c}{30}$ $\Rightarrow a = 26, b = 9, c = 30$ $\Rightarrow a + b + c = 65$

12. Rita takes a train home at 4 : 00, arriving at the station at 6:00 Every day, driving the same rate, rate, her husband meets her at the station at 6:00. On day she takes the train an hour early and arrives at 5:00. Her husband leaves home to meet her at the usual time, so Rita begins to walk home. he meets her on the way and hey reach home 20 minutes earlier than usual. The number of minutes Rita was walking before she met her husband on the way is :

(A) 20 (B) 40 (C) 50 (D) 60

Ans. **(C)**

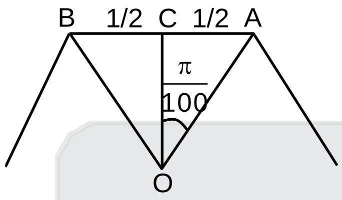
Sol. Obvious

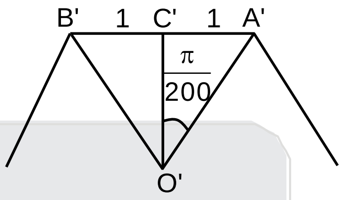
13. A regular polygon has 100 sides each of length . A another regular polygon has 200 sided each of length 2. When the area of the larger polygon is divided by the area of the smaller polygon, the quotient is closest to the integer

(A) 2 (B) 4 (C) 8 (D) 16

Ans. **(D)**

Sol.





$\frac{AC}{OC} = \tan\frac{\pi}{100}$ $\qquad$ $\frac{A'C'}{O'C'} = \tan\frac{\pi}{200}$

$\Rightarrow$ $OC = \frac{1}{2\tan\frac{\pi}{100}}$ $\qquad$ $O'C' = \frac{1}{\tan\frac{\pi}{200}}$

Area of polygon 1 $\qquad$ Area of polygon 2

$= 100 \times \frac{1}{2} \times 1 \times \frac{1}{2\tan\frac{\pi}{100}}$ $\qquad$ $= 200 \times \frac{1}{2} \times 2 \times \frac{1}{\tan\frac{\pi}{200}}$

$$\frac{\text{Area of polygon2}}{\text{Area of polygon1}} = \frac{200\cos\frac{\pi}{200}}{\sin\frac{\pi}{200}} \times \frac{\sin\frac{\pi}{100}}{25\cos\frac{\pi}{100}} = \frac{16\cos\frac{\pi}{200} \times \sin\frac{\pi}{200} \times \cos\frac{\pi}{200}}{\sin\frac{\pi}{200} \times \cos\frac{\pi}{100}}$$

$$= \frac{8\left(1+\cos\frac{\pi}{100}\right)}{\cos\frac{\pi}{100}} = 8 + 8\sec\frac{\pi}{100} \simeq 8 + 8 = 16$$

14. The function f satisfies f(f(x)) = f(x + 2) – 3 for all integers x. If f(1) = 4 ; f(4) = 3, then f(5) equals

(A) 3 (B) 6 (C) 9 (D) 12

Ans. **(D)**

Sol. f(f(x)) = f(x + 2) – 3 ……(i)

at x = 1, f(f(1)) = f(3) –3, f(1) = 4, f(4) = 3

$\Rightarrow$ 3 = f(3) – 3 $\Rightarrow$ f(3) = 6

Put x = 3 f(f(3)) = f(5) –3

f(5) = f(6) + 3 …(ii)

Put x = 4, f(f(4)) = f(6) – 3

f(3) = f(6) – 3

6 = f(6) – 3] f(6) = 9 from (ii) f(5) = 9 + 3 = 12 Ans.

15. If x and y are positive real numbers such that x+ y = 1, then maximum value of $xy^4 + x^4y$ is

(A) $\frac{1}{16}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

Ans. **(B)**

Sol. $x > 0, y > 0$

$x + y = 1 \Rightarrow$ A.M. $\geq$ G.M. $\Rightarrow \frac{x+y}{2} \geq (xy)^{1/2}$

$\Rightarrow xy \leq \frac{1}{4}$ .....(i)

$= \frac{1}{4}\left(\frac{1}{4}\right) = \frac{1}{16}$

$P = xy^4 + x^4y = xy(x^3 + y^3) = xy((x + y)^3 - 3xy(x + y))$

$= xy(1 - 3xy)$

Let xy = t

$p = t - 3t^2$

$p = t - 3t^2$

$p = -3\left(t^2 - \frac{t}{3}\right) = -3\left(\left(t - \frac{1}{6}\right)^2 - \frac{1}{36}\right)$

PART-B

Note:

- Write the correct answer in the space provided in the response sheet.
- For each correct response your get 1 mark; **for each incorrect response you lose ¼ mark.**

16. Consider all 4 element subsets of the set A = {1, 2, 3, …. 8}. Each of these subsets has a greatest element. The arithmetic mean of the greatest elements of these 4 element subsets is …….

Ans. **7.2**

Sol. Total subsets having 4 elements equals to $^8C_4 = 70$

Total subsets among there subsets having largest element as 8 equals to $^7C_3 = 35$

similarly subsets among there subsets having largest element as 7 equals to $^6C_3 = 20$

and so on.

Average of these elements equal to

$$\frac{8\times{}^7C_3+7\times{}^6C_3+6\times{}^5C_3+5\times{}^4C_3+4\times{}^3C_3}{^8C_3}$$

$$=\frac{280+140+60+20+4}{70}=\frac{504}{70}=\frac{72}{10}=7.2$$

17. The number of times the digit occurs in the result of 1 + 11 + 111 + ….. + 111 …..111 (100digits) is ……..

Ans. **11**

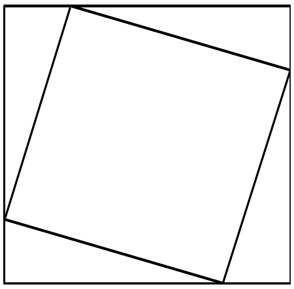
Sol. sum is

(123456790) (123456790) ……..(123456790) (1234567890)

there are 11 brackets

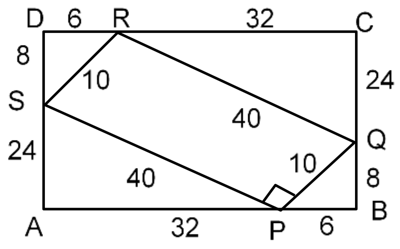
so 1 comes 11 times

18. In a 38 × 32 rectangle ABCD, points P, Q, R, S are taken on the sides AB, BC, CD, DA respectively such that the lengths AP, BQ, CR and DS are integers and PQRS is rectangle. The largest possible area of PQRS is ……..



Ans. **400**

Sol. Possible figure is

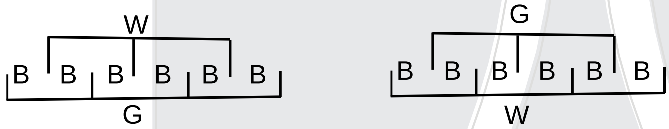


Area of PQRS = 40 × 10 = 400

19. 6 blue, 7 green and 10 white balls are arranged in row such that every blue ball is between and green and a white ball. Moreover, a white ball and a green ball must not be next to each other. The number of such arrangements is ……..

Ans. **1980**

Sol.



Total ways = ${}^6C_2\ {}^9C_3 + {}^9C_2\ {}^6C_3 = 1980$

20. Let us call a sum of integers a cool sum if the first and last terms are 1 and each term differs from its neighbours by at most . For example, 1 + 2 + 2 + 3 + 3 + 2 + 1 and 1 + 2 + 3 + 4 + 3 + 2 + 1 are cool sums. The minimum number of terms required to write 2019 as a cool sum is ……

Ans. **89**

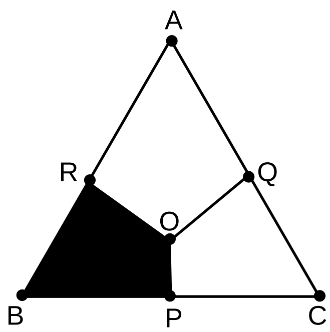
Sol. Sequence is

1 + 2 + 3 + ……..+ 19 + 20 + 21 + 22 + 23 + …..

+ 39 + 39 + 40 + 41 + 42 + 43 + 44 + 44 + 43 + 42 + …..+3+2+1

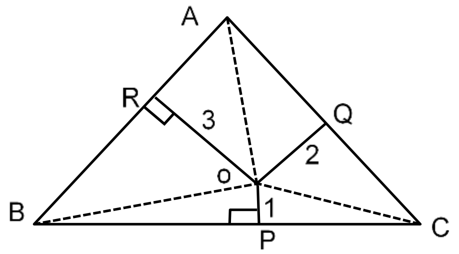
So minimum number of terms is 89

21. O is a point inside an equilateral triangle ABC. The perpendicular distance OP, OQ, OR to the sides of the triangle are in the ratio OP : OQ : OR = 1 : 2 : 3. If $\frac{\text{Area of quadrilateral OPBR}}{\text{Area of triangle ABC}} = \frac{a}{b}$, where a, b are co-prime positive integers, then a + b equals ………



Ans. **47**

Sol.



$$\frac{1}{2} \times s \times 1 + \frac{1}{2} \times s \times 2 + \frac{1}{2} \times s \times 3 = \frac{\sqrt{3}}{4} s^2$$

$$\frac{1}{2} \times s(6) = \frac{\sqrt{3}}{4} s^2$$

$$s = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

Let $\angle$ OBP = θ

$\sin\theta = \frac{OP}{OB}$ $\Rightarrow$ OB = cosec θ

Similarly OB = 3 cosec (60 – θ)

$$\Rightarrow \frac{1}{\sin\theta} = \frac{3}{\sin(60-\theta)}$$

$\Rightarrow 3 \sin\theta = \frac{\sqrt{3}}{2} \cos\theta - \frac{\sin\theta}{2}$ $\Rightarrow \tan\theta = \frac{\sqrt{3}}{7}$

$\sin\theta = \frac{\sqrt{3}}{\sqrt{52}}$ $\Rightarrow BP = \frac{7}{\sqrt{3}}, OB = \frac{\sqrt{52}}{\sqrt{3}}$

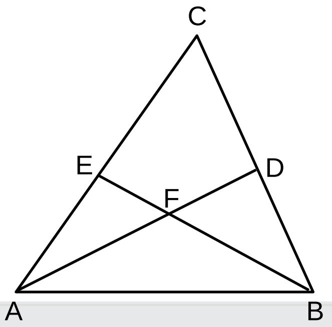
$$RB = \frac{5}{\sqrt{3}}$$

Area of quadrilateral BPOR = $\frac{1}{2} \times 1 \times \frac{7}{\sqrt{3}} + \frac{1}{2} \times 3 \times \frac{5}{\sqrt{3}} = \frac{11}{\sqrt{3}}$

Area of ΔABC = $\frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 = 12\sqrt{3}$

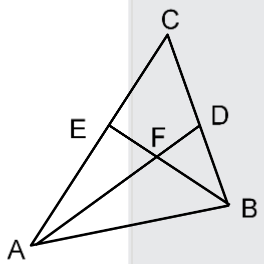
$$\frac{a}{b} = \frac{11/\sqrt{3}}{12\sqrt{3}} = \frac{11}{36} \Rightarrow a + b = 47$$

22. In ΔABC, AB = 6, BC = 7 and CA = 8. Point D lies on BC and AD bisects $\angle BAC$. Point E lies on AC and BE bisects $\angle ABC$. If the bisectors intersect at F, then the ratio AF : FD = ...



Ans. **2 : 1**

Sol.



AD bisect $\angle BAC$ $\Rightarrow$ BD : DC = AB : AC = 6 : 8

= 3 : 4

and length of BC = 7

so BD = 3, CD = 4

and also BF bisect $\angle ABD$

$\Rightarrow$ AF : FD = AB : BD

= 6 : 3

= 2 : 1 Ans.

23. Let a, b, c be real numbers such that the polynomial $f(x) = x^3 + ax^2 + x + 10$ has three distinct roots and each root of f(x) is also a root of the polynomial $h(x) = x^4 + x^3 + bx^2 + 13x + c$. The h(1) = ...

Ans. **40**

Sol. Let

$$\left(x+\frac{c}{10}\right)(x^3+ax^2+x+10)=x^4+x^3+bx^2+13x+c$$

$\Rightarrow$ compering coefficient $\frac{c}{10} + a = 1, \frac{c}{10} + 10 = 13$

$\Rightarrow$ c = 30 and a = – 2

and $\frac{ac}{10} + 1 = b \quad \Rightarrow b = -5$

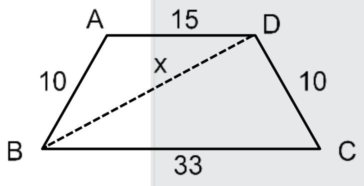
so $h(x) = x^4 + x^3 - 5x^2 + 13x + 30$

$\therefore$ h(1) = 1 + 1 – 5 + 13 + 30 = 40 Ans.

24. In quadrilateral ABCD, AB = 10, BC = 33, CD = 10 and DA = 15. If BD is an integer then BD = ….

Ans. **24**

Sol.



Difference of two side is less than third side

$\Rightarrow$ in ΔABD

15 – 10 < x

5 < x

and sum 10 +15 > x $\Rightarrow$ x < 25

also in ΔBCD, 33 – 10 < x $\Rightarrow$ x > 23

so BD = 24

25. For each positive integer n let $f(n) = n^4 - 3n^2 + 9$. Then the sum of all f(n) which are prime is ……

Ans. **20**

Sol. $f(n) = n^4 - 3n^2 + 9$

$= n^4 + 6n^2 - 9n^2 + 9$

$= (n^2 + 3)^2 - (3n)^2$

$= (n^2 + 3n + 3)(n^2 - 3n + 3)$

f(n) is prime $\Rightarrow n^2 - 3n + 3 = 1 \quad \Rightarrow n = 1, 2$

f(1) = 7, f(2) = 13

sum of values = 7 + 13 = 20

26. 13 boys are sitting in a row in a theatre. After the intermission, they return and are seated such that either they occupy the same seat or the adjacent seat in such a way that it differs from the original arrangement. The number of ways this is possible is ……

Ans. **376**

Sol. Let S_{13} is number of ways when S_{13} occupied by either B_{13} or B_{12} after intermission

$\Rightarrow S_{13} = S_{12} + S_{11}$

$= 2S_{11} + S_{10}$

$= 3S_{10} + 2S_9$

$= 5S_9 + 3S_8$

$= 8S_8 + 5S_7$

$= 13S_7 + 8S_6$

$= 21S_6 + 13S_5$

$= 34S_5 + 21S_4$

$= 55S_4 + 34S_3$

$= 89S_3 + 55S_2$

$= 89(3) + 55(2) = 267 + 110 = 377$

Required ways = 377 – 1 = 376

27. $A_1\ A_2\ A_3 \ldots\ldots A_{15}$ is a 15 sided regular polygon. The number of distinct equilateral triangles in the plane of the polygon, with exactly two of their vertices from the set $\{ A_1, A_2, A_3 \ldots\ldots A_{15}\}$ is ….

Ans. **195**

Sol. Number of distinct equilateral

triangle = ${}^{15}C_2 \times 2 = 210$

for these triangles $A_1A_6A_{11}$, $A_2A_7A_{12}$, $A_3A_8A_{13}$, $A_4A_9A_{14}$, $A_5A_{10}A_{15}$ we have more 5 × 3 triangles so subtract 15 so required triangle 210 – 15 = 195

28. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its roots, the product of its roots, and the sum of its coefficients are all equal. If the y intercept of the graph $y = P(x)$ is 2 then b = …………..

Ans. **–11**

Sol. $P(x) = x^3 + ax^2 + bx + c = 0$ has Roots α, β, γ

given $\dfrac{\alpha+\beta+\gamma}{3} = \alpha\beta\gamma = 1 + a + b + c$

$\Rightarrow \dfrac{-a}{3} = -c = 1 + a + b + c$

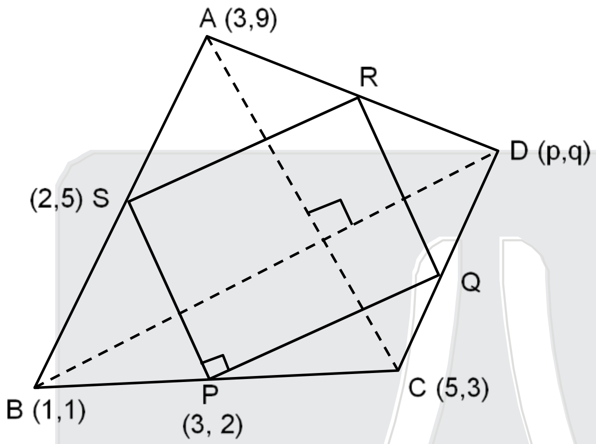
and at x = 0, y = 2 $\quad \Rightarrow c = 2$

gives a = 6, b = – 11 Ans.

29. ABCD is a quadrilateral in the first quadrant where A = (3, 9), B = (1, 1), C = (5, 3) and D = (p, q). The quadrilateral formed by joining the midpoints of AB, BC, CD and DA is a square. Then p + q =

Ans. **10**

Sol.



$BD \perp AC$

$\Rightarrow \quad \left(\frac{q-1}{p-1}\right)(-3) = -1 \quad \Rightarrow \quad 3q - p = 2$

AC = 2PS

BD = 2PQ

PS = PQ $\Rightarrow$ AC = BD $\Rightarrow$ $40 = (p-1)^2 + (q-1)^2$

$\Rightarrow \quad 40 = (3q-3)^2 + (q-1)^2 \Rightarrow q = -1, 3 \Rightarrow q = 3 \Rightarrow p = 7 \Rightarrow p + q = 10$

30. The product of four positive integers a, b, c and d is 9! The number a, b, c, d satisfy ab + a + b = 1224, bc + b + c = 549 and cd + c + d = 351. The a + b + c + d = ……

Ans. **108**

Sol. $abcd = \angle 9 = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

given ab + a + b = 1224 $\Rightarrow (1 + a)(1 + b) = 49 \times 25 = 1225$

and bc + b + c = 549 $\Rightarrow (1 + b)(1 + c) = 25 \times 22 = 550$

and cd + c + d = 351 $\Rightarrow (1 + c)(1 + d) = 352 = 22 \times 16$

So 1 + a = 49, 1 + b = 25, 1 + c = 22, 1 + d = 16

a = 48, b = 24, c = 21, d = 15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