GAUSS CONTEST - FINAL - PRIMARY
Classes V & VI
AMTI - Saturday, 3rd November_2018.

Instructions:

1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

---

1. Write down all the ten digit numbers whose digital sum is 2. (The digital sum of a number is the sum of the digits of the number. The digital sum of 4022 is 4 + 0 + 2 + 2 is 8). Find the sum of all the 10 digit numbers with digital sum 2.

Sol. Number of 10 digits numbers with digital sum 2 can be formed by taking 2 as one of its digits ar using two 1's as two digits.

Case – I :

2 as one of its digits only one number can be formed.

2 0 0 _ _ _ _ _ _ 0

1 possible.

Case – II :

Using two 1's

1 1 0 0 0 0 _ _ _ 0
1 0 1 0 0 0 _ _ _ 0
1 0 0 1 0 _ _ _ _ 0
↓
1 0 0 0 _ _ _ _ 1

Total 9 possible case

Total number of numbers = 10.

Sum of these numbers =

20000 0
11000 0
101000.....0
⋮
100 1

11111111111

2. The sum of the 3-digit numbers 35a and 4b7 is divisible by 36. Find all possible pairs (a, b).

Sol. $(35a + 4b7) \div 36$

$(300 + 50 + a + 400 + 10b + 7) \div 36$

$(700 + 57 + 10b + a) \div 36$

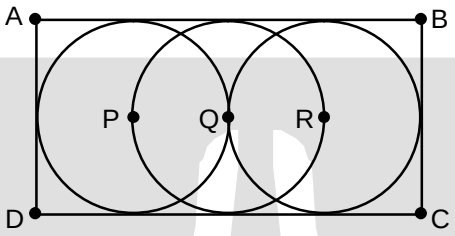
$(756 + 10b + a + 1) \div 36$

$10b + a = 35$

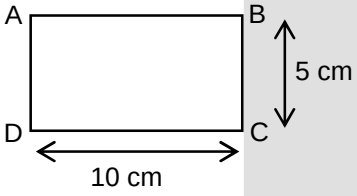
$10b + a = 71.$

$(a, b) \rightarrow (5, 3), (1, 7)$

3. Three congruent circles with centres P, Q and R, are tangent to the sides of rectangle ABCD as shown. The circle with centre at Q has diameter 5 cm and passes through the points P and R. Find the area of the rectangle ABCD.



Sol. area of ABCD = $10 \times 5 = 50\ cm^2$.



4. A lucky year is one in which at least one date, when written in the form day/month/year, has the following property. The product of the month times the day equals the last two digits of the year. For example, 1944 is a lucky year because it has the date 11/4/44 where $11 \times 4 = 44$. From 1951 to 2000 how many years are not lucky ? Given proper explanation for your answer.

Sol. Month × Date = {51, 52, 53, ..., 99} & 0

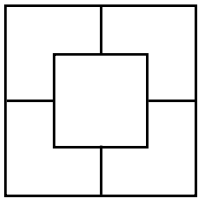
Month $\rightarrow$ {1, 2, 3,....., 12}

Date $\rightarrow$ {1, 2, 3,....., 31}

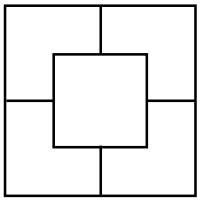
years which are not lucky are 2000, 1953, 1958, 1959, 1961, 1962, 1967, 1971, 1973, 1974, 1979, 1982, 1983, 1986, 1989, 1994, 1997

Total 17 years.

5. The area of each of the four congruent L-shaped regions of this 100 cm by 100 cm square is 3/16 of the total area. How many centimeters long is the side of the centre square ?



Sol.



Area of outer square = $100 \times 100 = 10000$ cm^2

Area of L-shaped region = $3 \times 625 = 1875$ cm^2

Area of 4 L-shaped region $4 \times 1875 = 7500$ cm^2

Area of centre square = $10000 - 7500$ $cm^2 = 2500$

Side of centre square = $\sqrt{2500} = 50$ cm.

6. For any positive integer n, s(n) is the sum of the digits of n. What is the minimum value of $\frac{n}{s(n)}$ when (1) $10 \le n \le 99$ and (2) $100 \le n \le 999$.

Sol. $\frac{n}{S(n)} \rightarrow$ minimum, where S(n) is sum of digits of the number n

n should be smallest and S(n) should be greatest

(i) $10 \le n \le 99$

S(n) = {1, 2, 3,....., 18}

$$\frac{99}{18} > \frac{89}{17} > \frac{79}{16} > \frac{69}{15} > \frac{59}{14} > \frac{49}{13} > \frac{39}{12} > \frac{29}{11} > \boxed{\frac{19}{10}} < \frac{18}{9} < \frac{17}{8} .....$$

(ii) $100 \le n \le 999$.

$$\frac{999}{27} > \frac{899}{26} > \frac{799}{25} > \frac{699}{24} > \frac{599}{23} > \frac{499}{22} > \frac{399}{21} > \frac{299}{20} > \boxed{\frac{199}{19}} < \frac{189}{18} < \frac{179}{17} < .....$$

7. A 122 digit number is obtained by writing the 2 digit numbers 39 to 99 i.e., 39404142434445.....96979899. You have to remove 61 digits from this number in such a way that the remaining digits in that order form the largest number possible. (For example in 15161718 if we remove the four 1's we get the number 5678, but if we remove 1, 5, 1 and the 1 after 6, we get 6718. This will be the largest number possible in this case.) What will be the first 10 digits of the largest number obtained ?

Sol. 122 digit number is

3940414243444546474849505 1 9899

In this first we will remove 3 so that number will start from 9

Now we will remove

404142..... 484

so that number continue from 9 i.e.,

995051525354555657585960 61 .....

Now remove 50515253 585

Remaining digits 99960616263.....8999

Similarly removing 6061..... 686

Remaining number 9999707172.....8999

Till now we removed 58 digits remaining 3 digits which should be removal 0, 1, 2.

Final number 999977773747757677 78.....99

First 10 digits

9, 9, 9, 9, 7, 7, 7, 7, 3, 7

8. Given the numbers 2, 4, 8, 10, 14 and 16 : a % b is defined as the remainder when the ordinary product a • b is divided by 18. Find the % product of every pair of these numbers including the product of a number with itself. Fill in the table given below.

(1) Find 2%2%2%....%2, where we find the % product of fifteen 2's.

(2) Find 8%8%8%.....%8 where we have ten 8's

%	2	4	8	10	14	16
2						
4						
8				8		
10						
14		2				
16						

Sol. 2%4 = 8
2%8 = 16
2%10 = 2
2%14 = 10
2%16 = 14
4%8 = 14
4%10 = 4
4%14 = 2
4%16 = 10
8%10 = 8
8%14 = 4
8%16 = 2
10 %14 = 14
10 %16 = 16
14%16 = 8
2%2 = 4
4 % 4 = 16
8 % 8 = 10
10%10 = 10
14%14 = 16
16%16 = 4.

(i) 2%2%2%2.....2%2
2%2 = 4
4%2 = 8
8%2 = 16
16%2 = 14
14%2 = 10
10%2 = 2
2%2 = 4
Repeated
So, ans = 8.

(ii) 8% 8% 8 % 8.....
8% 8 = 10
10% 8 = 8
8 % 8 = 10
% product of ten 8's = 10.

%	2	4	8	10	14	16
2	4	8	16	2	10	14
4	8	16	14	4	2	10
8	16	14	10	8	4	2
10	2	4	8	10	14	16
14	10	2	4	14	16	8
16	14	10	2	16	8	4

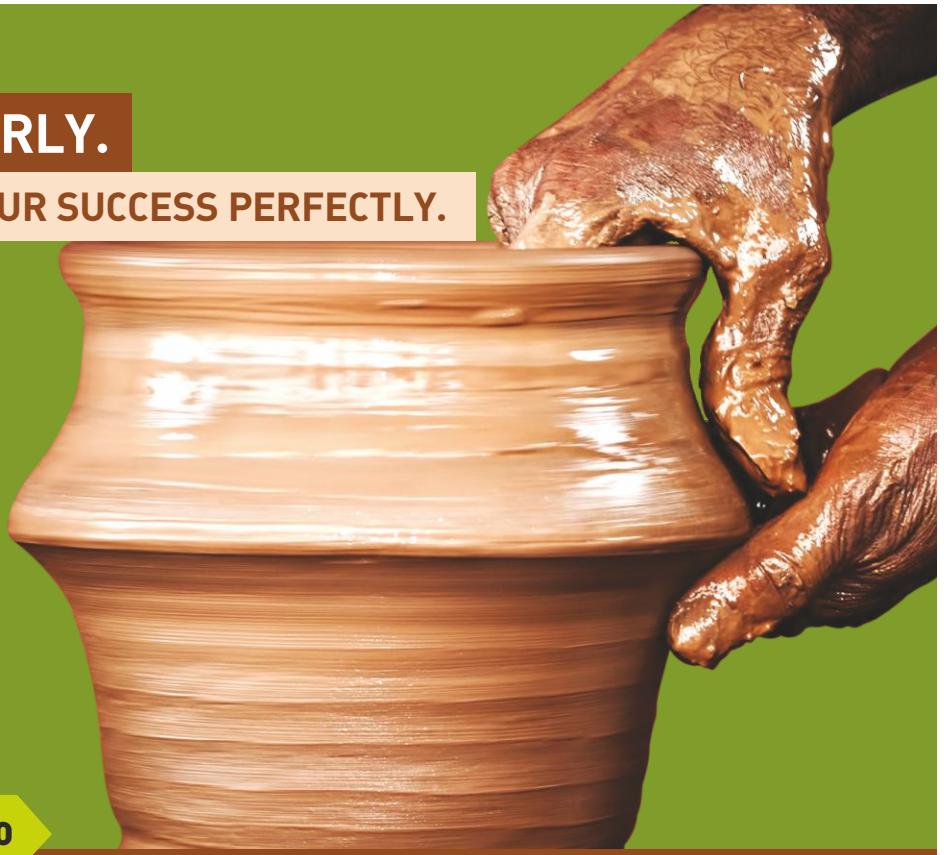
Resonance®
Educating for better tomorrow

POWER OF PCCP DIVISION

Reso FAST®
Be FAST to be the FIRST

START EARLY.

SHAPE YOUR SUCCESS PERFECTLY.



ADMISSION OPEN 2019-20

PRE-FOUNDATION CAREER CARE PROGRAM

FOR CLASS: V, VI, VII, VIII, IX & X

TEST DATES

25 Nov 2018

9, 23 & 30 Dec 2018

SCHOLARSHIP

Upto 90*%

TARGETS

- ✓ NTSE ✓ IJSO
- ✓ OLYMPIADS
- ✓ BOARDS

Resonance Eduventures Limited

PCCP Head Office: CG Tower-2, Plot No. A-51 (A), IPIA, Behind City Mall, Jhalawar Road, Kota (Rajasthan) - 324005

✆ 0744-6635569 🌐 www.pccp.resonance.ac.in ✉ pccp@resonance.ac.in