Resonance®
Educating for better tomorrow

VERSION CODE
**C3**

## SUBJECT : MATHEMATICS

# KARNATAKA COMMON ENTRANCE TEST (KCET) 2019

**Date: 29 April, 2019 | Total Duration: 80 Minutes**
**Maximum Time for Answering: 70 Min. | Max. Marks: 60**

**Dos :**

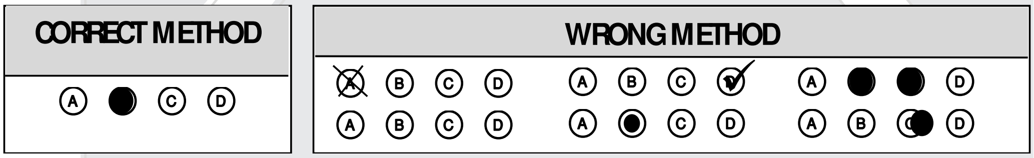
1. Once again confirm whether the CET No. and name printed on the OMR Answer Sheet and the Admission Ticket are same.
2. This question booklet is issued to you by the invigilator after the 2' bell i.e., after 2.30 pm.
3. Confirm whether the OMR Answer Sheet and the Question Paper issued to you are with same version code.
4. The Version Code and Serial Number of this question booklet should be entered on the Nominal Roll without any mistakes.
5. Compulsorily affix the complete signature at the bottom portion of the OMR answer sheet in the space provided.

**DONTs :**

1. The timing and marks printed on the OMR answer sheet should not be damaged / mutilated / spoiled.
2. The 3rd Bell rings at 2.40 pm, till then;
- Do not remove the seal present on the right hand side of this question booklet.
- Do not look inside this question booklet.
- Do not start answering on the OMR answer sheet.

**IMPORTANT INSTRUCTIONS TO CANDIDATES**

1. This question booklet contains **60** questions and each question will have one statement and four distracters. (Four different options choices.)
2. After the 3rd Bell is rung at 2.40 am, remove the seal on the right hand side of this question booklet and check that this booklet does not have any unprinted or torn or missing pages or items etc., if so, get it replaced immediately by complete test booklet by showing it to Room Invigilator. Read each item and start answering on the OMR answer sheet.
3. During the subsequent 70 minutes:
- Read each question carefully.
- Choose the correct answer from out of the four available distracters (options / choices) given under each question / statement.
- Completely darken / shade the relevant circle with a blue or black ink ballpoint pen against the question number on the OMR answer sheet.




4. Please note that even a minute unintended ink dot on the OMR answer sheet will also be recognized and recorded by the scanner. Therefore, avoid multiple markings of any kind on the OMR answer sheet.
5. Use the space provided on each page of the question booklet for Rough Work. Do not use the OMR answer sheet for the same.
6. After the last hell is rung at 3.50 pm, stop writing on the OMR answer sheet and affix your left hand thumb impression on the OMR answer sheet as per the instructions.
7. Hand over the OMR answer sheet to the room invigilator as it is.
8. After separating the top sheet (KEA copy), the invigilator will return the bottom sheet replica (Candidate's copy) to you to carry home for self-evaluation.
9. Preserve the replica of the OMR answer sheet for a minimum period of ONE year.
10. In case of any discrepancy in the English and Kannada Versions, the English version will be taken as final.

**Resonance Eduventures Ltd.**
**Corporate & Reg. Office:** CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005
**Tel.No.:** 0744-6607777, 3012100, 3012222, 6635555 | **Toll Free:** 1800 258 5555 | **Fax:** +91-022-39167222 | 08003 444 888
**Website:** www.resonance.ac.in | **E-mail:** contact@resonance.ac.in | **CIN:** U80302RJ2007PLC024029
**Toll Free : 1800 258 5555** 08003 444 888 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

**Resonance®**
**Educating for better tomorrow**

**Success at JEE Main 2019**

# HIGHEST number of CLASSROOM Students Qualified for JEE (Advanced) 2019 from any Institute of India*

# 12465*

(All are from Regular Classroom Program)

**with 8 classroom students in top 100 in JEE Main 2019**


*Based on the information collected from public domain till 7th May 2019

**Top Category Ranks in JEE Main 2019**

## AIR-2, 4, 7 (ST)
**Atin Bainada, Raja, Kuldeep Meena**

## AIR-11 (SC)
**Anshul Navphule**

**Students qualified for JEE Advanced 2019**

**8235** students from Repeaters' Batches

**4230** students from Freshers' Batches

**HIGHEST CLASSROOM GIRLS**
Student Qualified for JEE (Advanced) 2019 from any Institute of India*
**1506**

**HIGHEST CLASSROOM HINDI MEDIUM**
Students Qualified for JEE (Advanced) 2019 from any Institute of India*
**1450**

*All the above students are from Classroom Contact Program*

**For scholarship on the basis of JEE Main 2019, SMS RESO SCHO at 56677**

**ADMISSIONS OPEN FOR 2019-20**

**Classes: V to XII & XII+**
Target: JEE (Main+Advanced)
JEE (Main) | AIIMS/ NEET
Pre-foundation | Commerce & CLAT

**ResoNET Dates**
**9th & 16th June 2019**

**COURSE: VIJAY (JR)**
**FOR CLASS: XIII**
**Target: JEE (Main+Advanced) 2020**
**Course Starts from**
**10th JUNE 2019**


**Resonance Eduventures Limited**
Registered & Corporate Office: CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Rajasthan) - 324005
Tel. No.: 0744-2777777, 2777700 | CIN: U80302RJ2007PLC024029

Toll Free: **1800 258 5555**
website: www.resonance.ac.in

# MATHEMATICS

**1.** If $|3x-5| \le 2$ then

(A) $1 \le x \le \frac{7}{3}$ (B) $-1 \le x \le \frac{9}{3}$ (C) $-1 \le x \le \frac{7}{3}$ (D) $1 \le x \le \frac{9}{3}$

**Ans** **(A)**

**Sol.** $|3x-5| \le 2$

$\Rightarrow -2 \le 3x - 5 \le 2$

$\Rightarrow 3 \le 3x \le 7$

$\Rightarrow 1 \le x \le \frac{7}{3}$

**2.** A random variable 'X' has the following probability distribution

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| P(X) | $k-1$ | $3k$ | $k$ | $3k$ | $3k^2$ | $k^2$ | $k^2+k$ |

Then the value of k is

(A) $-2$ (B) $\frac{1}{10}$ (C) $\frac{1}{5}$ (D) $\frac{2}{7}$

**Ans** **(C)**

**Sol.** $\sum P_i = 1$

$$K-1+3K+k+3K+3K^2+K^2+K^2+K=1$$

$$5K^2+9K-2=0$$

$$5K^2+10K-K-2=0$$

$$5K(K+2)-1(K+2)=0$$

$$(5K-1)(K+2)=0$$

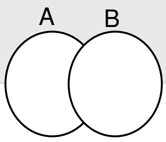
$K = \frac{1}{5}, -2$ (K = –2 is not possible).

**3.** If A and B are two events of a sample space S such that P(A) = 0.2, P(B) = 0.6 and P(A | B) = 0.5 then P(A′ | B) =

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{3}{10}$ (D) $\frac{1}{2}$

**Ans** **(D)**

**Sol.** $P(A) = 0.2,\ P(B) = 0.6$




$\Rightarrow P(A|B) = 0.5$

$$\frac{P(A \cap B)}{P(B)} = 0.5$$

$$\Rightarrow P(A \cap B) = 0.5(P(B)) = (0.5)(0.6) = 0.3$$

$$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{0.6-0.3}{0.6} = \frac{0.3}{0.6} = \frac{3}{6} = \frac{1}{2}$$

**4.** If 'X' has a binomial distribution with parameters n = 6, p and P(X = 2) = 12, P(X = 3) = 5 then P =

(A) $\frac{16}{21}$ (B) $\frac{5}{16}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$

**Ans** **Given Options are not matching**

**Sol.** n = 6

$P(x=r)={}^{n}C_{r}q^{n-r}p^{r}$

P(x = 2) = 12

${}^{6}C_{2}q^{4}p^{2}=12$ ...(1)

P(x = 3) = 5

${}^{6}C_{3}q^{3}p^{3}=5$ ...(2)

$\frac{(1)}{(2)}\Rightarrow\frac{{}^{6}C_{2}q^{4}p^{2}}{{}^{6}C_{3}q^{3}p^{3}}=\frac{12}{5}$

$\frac{15q}{20p}=\frac{12}{5}$

75q = 240 p

75(1 – p) = 240 p

75 – 75p = 240 p

75 = 315 p

$p=\frac{75}{315}=\frac{5}{21}$

**5.** A man speaks truth 2 out of 3 times. He picks one of the natural numbers in the set S = {1, 2, 3, 4, 5, 6, 7} and reports that it is even. The probability that it is actually even is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{1}{10}$

**Ans** **(B)**

**Sol.** S = {1, 2, 3, 4, 5, 6, 7}

$E_1$ = An even number is picked, $E_2$ = An odd number is picked

$P(E_1)=\frac{3}{7},\ P(E_2)=\frac{4}{7}$

E : A man reports an even number

$P(E|E_1)=\frac{2}{3}$

$P(E|E_2)=\frac{1}{3}$

Required probability = $P(E_1|E)$

$$=\frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)}$$

$$=\frac{\left(\frac{2}{3}\right)\left(\frac{3}{7}\right)}{\left(\frac{2}{3}\right)\left(\frac{3}{7}\right)+\left(\frac{1}{3}\right)\left(\frac{4}{7}\right)}=\frac{6}{6+4}=\frac{3}{5}$$

**6.** $\int_{-3}^{3} \cot^{-1} x dx =$

(A) 0 (B) 3 (C) $3\pi$ (D) $6\pi$

**Ans (C)**

**Sol.** $\int_{-3}^{3} \cot^{-1} x\, dx = \int_{-3}^{3} \cot^{-1} x.1 dx$

$= \left[\cot^{-1} x.x\right]_{-3}^{3} - \int_{-3}^{3} x\left(-\frac{1}{1+x^2}\right)dx$

$= 3 \cot^{-1} 3 - (-3) \cot^{-1} (-3) + \int_{-3}^{3} \frac{x}{1+x^2} dx$

$= 3 \cot^{-1} 3 + 3 (\pi - \cot^{-1} 3) + \frac{1}{2}\left[\log\left[1+x^2\right]\right]_{-3}^{3}$

$= 3 \cot^{-1} 3 + 3\pi - 3 \cot^{-1} 3 + \frac{1}{2}\left[\log 10 - \log 10\right]$

$= 3\pi$

**7.** $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx =$

(A) $\frac{1}{2}\tan^{-1}\sqrt{x} + C$ (B) $2\tan^{-1}\sqrt{x} + C$ (C) $2\log\left(\sqrt{x}+1\right) + C$ (D) $\tan^{-1}\sqrt{x} + C$

**Ans (B)**

**Sol.** $I = \int \frac{1}{\sqrt{x}\left[1+\left(\sqrt{x}\right)^2\right]} dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

$\therefore\ I = \int \frac{2dt}{1+t^2} = 2\tan^{-1} t + C = 2\tan^{-1}\sqrt{x} + C$

**8.** $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = A \log|x-1| + B\log|x+2| + C\log|x-3| + K,$ then A, B, C are respectively

(A) $\frac{1}{6}, \frac{1}{3}, \frac{1}{5}$ (B) $\frac{-1}{6}, \frac{-1}{3}, \frac{1}{2}$ (C) $\frac{-1}{6}, \frac{1}{3}, \frac{-1}{2}$ (D) $\frac{1}{6}, \frac{-1}{3}, \frac{1}{3}$

**Ans (B)**

**Sol.** $\int \frac{2x-1}{(x-1)(x+2)(x-3)} = A\log|x-1| + B\log|x+2| + C\log|x-3| + K$

Diff w.r.t 'x'

$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$2x - 1 = A (x + 2) (x - 3) + B (x - 1) (x - 3) + C (x - 1) (x + 2)$

Comparing constants

$-1 = -6A + 3B - 2C$

$A = \frac{-1}{6}, B = \frac{-1}{3}, C = \frac{1}{2}$

**9.** $\int_0^2 [x^2]dx =$

(A) $5+\sqrt{2}-\sqrt{3}$ (B) $-5-\sqrt{2}-\sqrt{3}$ (C) $5-\sqrt{2}-\sqrt{3}$ (D) $5-\sqrt{2}+\sqrt{3}$

**Ans** **(C)**

**Sol.** $\int_0^2 [x^2]\,dx$

$= \int_0^1 0 + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{\sqrt{4}=2} 3dx$

$= 0 + \left(\sqrt{2}-1\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(2-\sqrt{3}\right)$

$= \sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$

$= 5-\sqrt{2}-\sqrt{3}$

**10.** $\int_0^1 \sqrt{\frac{1+x}{1-x}}\,dx =$

(A) $\frac{\pi}{2}+1$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{2}-1$ (D) $\frac{\pi}{2}$

**Ans** **(A)**

**Sol.** $\int_0^1 \sqrt{\frac{1+x}{1-x}}\,dx = \int_0^1 \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}}\,dx$

$= \int_0^1 \frac{1+x}{\sqrt{1-x^2}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{1-x^2}} + \int_0^1 \frac{xdx}{\sqrt{1-x^2}}$

$= [\sin^{-1}]_0^1 + \int_0^1 \frac{xdx}{\sqrt{1-x^2}}$

$= \left(\frac{\pi}{2}-0\right) + \int_0^1 \frac{xdx}{\sqrt{1-x^2}}$

put $1 - x^2 = t \Rightarrow -2x\,dx = dt$

$\Rightarrow xdx = -\frac{dt}{2}$

at x = 0 t = 1

x = 1 t = 0

$\therefore\ I = \frac{\pi}{2} + \int_1^0 \frac{-dt}{2\sqrt{t}}$

$= \frac{\pi}{2} + \left(-\sqrt{t}\right)_1^0 = \frac{\pi}{2} + (0+1) = \frac{\pi}{2}+1$

**11.** The inverse of the matrix $\begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 3 & -5 & 5 \\ -1 & -6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

**Ans** **(D)**

**Sol.** Let $A=\begin{bmatrix}2 & 5 & 0\\ 0 & 1 & 1\\ -1 & 0 & 3\end{bmatrix}$

$|A|=2(3-0)-5(0+1)$

= 6 – 5 = 1

$\therefore |A|=1$

$\text{adj } A=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}^T=\begin{bmatrix}3 & -15 & 5\\ -1 & 6 & -2\\ 1 & -5 & 2\end{bmatrix}$

$A^{-1}=\dfrac{\text{adj } A}{|A|}=\begin{bmatrix}3 & -15 & 5\\ -1 & 6 & -2\\ 1 & -5 & 2\end{bmatrix}$

**12.** If P and Q are symmetric matrices of the same order then PQ – QP is
(A) symmetric matrix (B) skew symmetric matrix
(C) identity matrix (D) zero matrix

**Ans** **(B)**

**Sol.** Given P = P′ and Q = Q′

(PQ – QP)′ = (PQ) ′ – (QP) ′

= (Q′P′ – P′Q′)

= QP – PQ

= –[PQ – QP]

∴ (PQ – QP) is skew symmetric.

**13.** If $3A+4B'=\begin{bmatrix}7 & -10 & 17\\ 0 & 6 & 31\end{bmatrix}$ and $2B-3A'=\begin{bmatrix}-1 & 18\\ 4 & 0\\ -5 & -7\end{bmatrix}$ then B =

(A) $\begin{bmatrix}1 & -3\\ -1 & 1\\ 2 & 4\end{bmatrix}$ (B) $\begin{bmatrix}1 & 3\\ -1 & 1\\ 2 & -4\end{bmatrix}$ (C) $\begin{bmatrix}1 & 3\\ -1 & 1\\ 2 & 4\end{bmatrix}$ (D) $\begin{bmatrix}-1 & -18\\ 4 & -16\\ -5 & -7\end{bmatrix}$

**Ans** **(C)**

**Sol.** $3A+4B'=\begin{bmatrix}7 & -10 & 17\\ 0 & 6 & 31\end{bmatrix}$ ...(1)

$(2B-3A')'=(2B)'-(3A')'=2B'-3A$

$\Rightarrow 2B'-3A=\begin{bmatrix}-1 & 4 & -5\\ 18 & 0 & -7\end{bmatrix}$ ...(2)

Adding (1) and (2) we get, $6B'=\begin{bmatrix}6 & -6 & 12\\ 18 & 6 & 24\end{bmatrix}$

$\Rightarrow B'=\begin{bmatrix}1 & -1 & 2\\ 3 & 1 & 4\end{bmatrix}$ $\therefore B=\begin{bmatrix}1 & 3\\ -1 & 1\\ 2 & 4\end{bmatrix}$

**14.** If $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then | ABB′ | =

(A) – 250 (B) 250 (C) 50 (D) 100

**Ans** **(A)**

**Sol.** $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ $B' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

$|ABB'| = |A|\,|B|\,|B'| = (2-12)\times(4+1)\times(4+1)$

$= -10\times5\times5 = -250$

**15.** If the value of a third order determinant is 16, then the value of the determinant formed by replacing each of its elements by its cofactor is

(A) 48 (B) 16 (C) 96 (D) 256

**Ans** **(D)**

**Sol.** |A| = 16, O(A) = 3

|adj A| = $|A|^2$

= $16^2$

= 256

**16.** If U is the universal set with 100 elements; A and B are two sets such that n(A) = 50, n(B) = 60, n(A ∩ B) = 20 then n(A′ ∩ B′) =

(A) 20 (B) 10 (C) 40 (D) 90

**Ans** **(B)**

n(U) = 100

n(A) = 50

n(B) = 60

$n(A\cap B) = 20$

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

= 50 + 60 – 20

= 110 – 20

= 90

$n(A'\cap B') = n\left((A\cup B)'\right)$

= n(U) – n(A ∪ B)

= 100 – 90

= 10

**17.** The domain of the function f : **R** → **R** defined by $f(x) = \sqrt{x^2 - 7x + 12}$ is

(A) (–∞, 3] ∪ (4, ∞) (B) (3, 4) (C) (–∞, 3] ∪ [4, ∞) (D) (–∞, 3] ∩ [4, ∞)

**Ans** **(C)**

**Sol.** f: R → R

$f(x) = \sqrt{x^2 - 7x + 12}$

$x^2 - 7x + 12 \ge 0$

(x – 4) (x – 3) ≥ 0

$\Rightarrow x \in (-\infty, 3] \cup [4, \infty)$

**18.** If cos x = |sin x| then, the general solution is

(A) $x = 2n\pi \pm \frac{\pi}{4}, n \in Z$ (B) $x = (2n+1)\pi \pm \frac{\pi}{4}, n \in \mathbf{Z}$

(C) $x = n\pi \pm \frac{\pi}{4}, n \in \mathbf{Z}$ (D) $x = n\pi + (-1)^n \frac{\pi}{4}, n \in \mathbf{Z}$

**Ans** **(A)**

**Sol.** cos x = |sin x|

$\Rightarrow \pm \cos x = \sin x$

$\Rightarrow \tan x = \pm 1$

$x = n\pi \pm \frac{\pi}{4}, n \in \mathbf{Z}$, but cos x is positive so $x = 2n\pi \pm \frac{\pi}{4}, n \in Z$.

**19.** $\sqrt{3}\,\text{cosec}\,20° - \sec 20° =$

(A) 3 (B) 1 (C) 2 (D) 4

**Ans** **(D)**

**Sol.** $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cdot \cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\frac{1}{2}[2\sin 20° \cdot \cos 20°]}$$

$$= 4\frac{[\sin 60° \cdot \cos 20° - \cos 60° \cdot \sin 20°]}{\sin 40°} = 4\frac{\sin(60° - 20°)}{\sin 40°} = 4\frac{\sin 40°}{\sin 40°} = 4$$

**20.** If $P(n): 2^n < n!$ then the smallest positive integer for which P(n) is true, is

(A) 3 (B) 5 (C) 2 (D) 4

**Ans** **(D)**

**Sol.** P(n) ; $2^n < n!$

n = 2; $2^2 = 4$, n! = 2! = 2

$4 \not< 2$

n = 3; $2^3 = 8$, 3! = 6

$8 \not< 6$

n = 4; $2^4 = 16$, 4! = 24

16 < 24

$\therefore$ n = 4

**21.** Foot of the perpendicular drawn from the point (1, 3, 4) to the plane 2x – y + z + 3 = 0 is

(A) (0, –4, –7) (B) (–3, 5, 2) (C) (–1, 4, 3) (D) (1, 2, –3)

**Ans** **(C)**

**Sol.** The dr's of PA are $x_1 - 1$, $y_1 - 3$, $z_1 - 4$

The dr's $\vec{n}$ are 2, –1, 1

The dr's of PA and $\vec{n}$ are parallel

$$\therefore \frac{x_1 - 1}{2} = \frac{y_1 - 3}{-1} = \frac{z_1 - 4}{1} = \lambda$$

$x_1 = 2\lambda + 1$, $y_1 = -\lambda + 3$, $z_1 = \lambda + 4$

$\therefore$ A = $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ lies on 2x – y + z + 3 = 0

$\Rightarrow -4\lambda + 2 - \lambda - 3 - \lambda + 4 + 3 = 0$

$6\lambda = -6$

$\lambda = -1$

$\therefore$ P = (–1, 4, 3)

**22.** Acute angle between the line $\frac{x-5}{2} = \frac{y+1}{-1} = \frac{z+4}{1}$ and the plane 3x – 4y – z + 5 = 0 is

(A) $\sin^{-1}\left(\frac{9}{\sqrt{364}}\right)$ (B) $\sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$ (C) $\cos^{-1}\left(\frac{9}{\sqrt{364}}\right)$ (D) $\cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$

**Ans** **(D)**

**Sol.** dr's of line 2, –1, 1
dr's of normal to plane –3, 4, 1

$$\sin\theta = \left|\frac{-6-4+1}{\sqrt{4+1+1}\sqrt{9+16+1}}\right| = \left|\frac{-9}{\sqrt{156}}\right| = \frac{9}{\sqrt{156}}$$

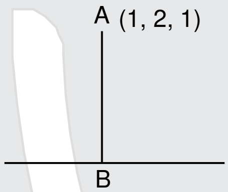
$$\theta = \sin^{-1}\left(\frac{9}{\sqrt{156}}\right) = \sin^{-1}\left(\frac{9}{2\sqrt{39}}\right) = \cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$$

**23.** The distance of the point (1, 2, 1) from the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ is

(A) $\frac{2\sqrt{5}}{3}$ (B) $\frac{20}{3}$ (C) $\frac{2\sqrt{3}}{5}$ (D) $\frac{\sqrt{5}}{3}$

**Ans** **(A)**

**Sol.** $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2} = k$




any point on the line = (2k + 1, k + 2, 2k + 3)
drs of AB : 2k, k, 2k + 2
drs of line : 2, 1, 2
AB ⊥ line ⇒ 4k + k + 4k + 4 = 0
9k = –4

$k = \frac{-4}{9}$

$$\text{Distance} = \sqrt{\frac{64}{81}+\frac{16}{81}+\frac{100}{81}} = \frac{\sqrt{180}}{9} = \frac{2\sqrt{5}}{3}$$

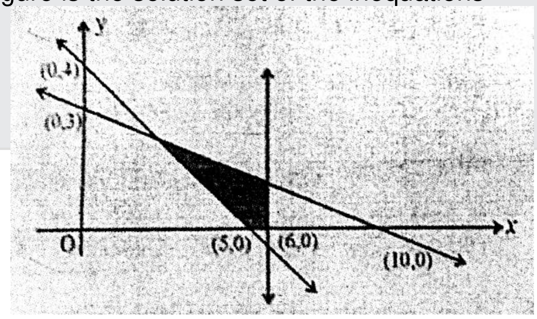
**24.** XY-plane divides the line joining the points A(2, 3, –5) and B(–1, –2, –3) in the ratio
(A) 3 : 2 externally (B) 5 : 3 externally (C) 2 : 1 internally (D) 5 : 3 internal

**Ans** **(B)**

**Sol.** Equation of xy-plane is z = 0
Ratio in which xy-plane

$$\text{divides} = -\frac{(-5)}{-3} = -\frac{5}{3}$$

**25.** The shaded region in the figure is the solution set of the inequations




(A) 4x + 5y ≥ 20, 3x + 10y ≤ 30, x ≥ 6, x, y ≥ 0 (B) 4x + 5y ≤ 20, 3x + 10y ≤ 30, x ≥ 6, x, y ≥ 0
(C) 4x + 5y ≥ 20, 3x + 10y ≤ 30, x ≤ 6, x, y ≥ 0 (D) 4x + 5y ≤ 20, 3x + 10y ≤ 30, x ≤ 6, x, y ≥ 0

**Ans** **(C)**

**26.** If $\alpha$ and $\beta$ are roots of the equation $x^2 + x + 1 = 0$ then $\alpha^2 + \beta^2$ is

(A) $\frac{-1+i\sqrt{3}}{2}$ (B) –1 (C) 1 (D) $\frac{-1-i\sqrt{3}}{2}$

**Ans** **(B)**

**Sol.** $x^2 + x + 1 = 0$

$\Rightarrow x = \omega, \beta = \omega^2$ where $\omega$ is the cube root of unity

$\alpha^2 + \beta^2 = \omega^2 + \omega^4$

$= \omega^2 + \omega$

$= -1$

**27.** The number of 4 digit numbers without repetition that can be formed using the digits 1, 2, 3, 4, 5, 6, 7 in which each number has two odd digits and two even digits is

(A) 436 (B) 454 (C) 432 (D) 450

**Ans** **(C)**

**Sol.** Given digits are 1, 2, 3, 4, 5, 6, 7.

Two even digits can be selected in ${}^3C_2$

Two odd digits can be selected in ${}^4C_2$ ways.

These selected 4 digits can be arranged in 4! ways.

$\therefore$ Total number of ways = ${}^4C_2 \cdot {}^3C_2 \cdot 4!$

$= 6 \times 3 \times 24$

$= 18 \times 24$

$= 432$

**28.** The number of terms in the expansion of $(x^2 + y^2)^{25} - (x^2 - y^2)^{25}$ after simplification is

(A) 13 (B) 50 (C) 0 (D) 26

**Ans** **(A)**

**Sol.** $(a + b)^n - (a - b)^n$ if n is odd

On simplification we get $\frac{n+1}{2}$ terms i.e., $\frac{25+1}{2} = 13$ terms

**29.** The third term of a G.P. is 9. The product of its first five terms is

(A) $3^9$ (B) $3^{12}$ (C) $3^5$ (D) $3^{10}$

**Ans** **(D)**

**Sol.** Let $\frac{a}{r^2} \cdot \frac{a}{r} \cdot a \cdot ar \cdot ar^2$ are in G.P., Given a = 9

$\therefore$ product = $9^5 = 3^{10}$

**30.** A line cuts off equal intercepts on the co-ordinate axes. The angle made by this line with the positive direction of X-axis is

(A) 90° (B) 135° (C) 45° (D) 120°

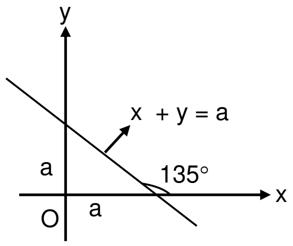
**Ans** **(B)**

**Sol.** $x + y = a$

$\frac{dy}{dx} = -1$

$\tan\theta = -1$

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \Rightarrow \theta = 135°$



**31.** The order of the differential equation $y = C_1 e^{C_2 + x} + C_3 e^{C_4 + x}$ is

(A) 2 (B) 4 (C) 1 (D) 3

**Ans (C)**

**Sol.** $y = c_1 e^{c_2 + x} + c_3 e^{c_4 + x}$

$= \left(c_1 e^{c_2} + c_3 e^{c_4}\right) e^x$

$= Ae^x$

$\frac{dy}{dx} - Ae^x = y$

$\Rightarrow$ order = 1

**32.** If $|\vec{a}| = 16,\ |\vec{b}| = 4$ then, $\sqrt{|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2} =$

(A) 8 (B) 64 (C) 4 (D) 16

**Ans (B)**

**Sol.** $\sqrt{\left(\vec{a}\times\vec{b}\right)^2 + |\vec{a}\cdot\vec{b}|^2} = \sqrt{|\vec{a}|^2|\vec{b}|^2} = |\vec{a}||\vec{b}| = (16)(4) = 64$

**33.** If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and the projection of $\vec{a}$ in the direction of $\vec{b}$ is – 2, then $|\vec{a}| =$

(A) 3 (B) 1 (C) 4 (D) 2

**Ans (C)**

$\theta = \frac{2\pi}{3}$

$\Rightarrow \frac{\vec{a}.\vec{b}}{|\vec{b}|} = -2$

$\Rightarrow \frac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{b}|} = -2 \quad \Rightarrow |\vec{a}|\cos\frac{2\pi}{3} = -2 \quad \Rightarrow |\vec{a}| \times -\frac{1}{2} = -2 \quad \Rightarrow |\vec{a}| = 4$

**34.** A unit vector perpendicular to the plane containing the vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} + 3\hat{k}$ is

(A) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (B) $\frac{-\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ (C) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (D) $\frac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

**Ans (D)**

$\hat{n} = \left(\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 1 & 3 \end{vmatrix} = 5\hat{i} - 5\hat{j} + 5\hat{k}$

$|\vec{a}\times\vec{b}| = 5\sqrt{3}$

$\therefore\ \hat{n} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ or $\hat{n} = \frac{-i+j-\hat{k}}{\sqrt{3}}$

**35.** $\left[\vec{a}+2\vec{b}-\vec{c},\ \vec{a}-\vec{b},\ \vec{a}-\vec{b}-\vec{c}\right]=$

(A) $\left[\vec{a},\ \vec{b},\ \vec{c}\right]$ (B) $3\left[\vec{a},\vec{b},\vec{c}\right]$ (C) 0 (D) $2\left[\vec{a},\vec{b},\vec{c}\right]$

**Ans** **(B)**

**Sol.** $\left(\vec{a}+2\vec{b}-\vec{c}\right)\cdot\left\{\vec{a}-\vec{b}\times\vec{a}-\vec{b}-\vec{c}\right\}$

$=\left(\vec{a}+2\vec{b}-\vec{c}\right)\cdot\left\{-\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{a}+\vec{b}\times\vec{c}\right\}$

$=\left[\vec{a}\ \vec{b}\ \vec{c}\right]-2\left[\vec{b}\ \vec{a}\ \vec{c}\right]$

$=\left[\vec{a}\ \vec{b}\ \vec{c}\right]+2\left[\ \vec{a}\vec{b}\vec{c}\right]$

$=3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

**36.** $f:\mathbf{R}\to\mathbf{R}$ and $g:[0,\infty)\to\mathbf{R}$ is defined by f(x) = $x^2$ and $g(x)=\sqrt{x}$. Which one of the following is not true?

(A) $f\circ g(-4)=4$ (B) $g\circ f(-2)=2$ (C) $g\circ f(4)=4$ (D) $f\circ g(2)=2$

**Ans** **(A)**

**Sol.** $f(x)=x^2 g(x)=\sqrt{x}$

$f\circ g(-4)=f[g(-4)]$ is not defined

**37.** If $A=\{x\,|\,x\in\mathbf{N},\ x\le 5\},\ B=\{x\,|\,x\in\mathbf{Z},\ x^2-5x+6=0\}$, then the number of onto functions from A to B is

(A) 23 (B) 32 (C) 2 (D) 30

**Ans** **(D)**

**Sol.** A = {1, 2, 3, 4, 5}

B = {2, 3}

number of onto functions from a set to a set containing 2 elements is

$=2^n-2$

= $2^5-2$ = 30

**38.** On the set of positive rationals, a binary operation * is defined by $a*b=\frac{2ab}{5}$. If $2*x=3^{-1}$ then x =

(A) $\frac{5}{12}$ (B) $\frac{125}{48}$ (C) $\frac{1}{6}$ (D) $\frac{2}{5}$

**Ans** **(B)**

**Sol.** a * e = a

$a*e=a\Rightarrow\frac{2ae}{5}=a\Rightarrow e=\frac{5}{2}$

$a*a^{-1}=e\Rightarrow\frac{2aa^{-1}}{5}=\frac{5}{2}\Rightarrow a^{-1}=\frac{25}{4a}$

$2*x=3^{-1}\Rightarrow\frac{2(2x)}{5}=\frac{25}{4(3)}$

$\Rightarrow\ x=\frac{125}{48}$

**39.** $\cos\left[2\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right]=$

(A) $\frac{3}{4}$ (B) does not exist (C) $\frac{-3}{4}$ (D) $\frac{3}{5}$

**Ans** **(C)**

**Sol.** $GE=\cos\left[\frac{\pi}{2}+\sin^{-1}\frac{3}{4}\right]=-\sin\left(\sin^{-1}\frac{3}{4}\right)=\frac{-3}{4}$

**40.** If $a+\frac{\pi}{2}<2\tan^{-1}x+3\cot^{-1}x<b$ then 'a' and 'b' are respectively.

(A) $\frac{\pi}{2}$ and $2\pi$ (B) $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ (C) 0 and $\pi$ (D) 0 and $2\pi$

**Ans** **(A)**

**Sol.** $a+\frac{\pi}{2}<\sin^{-1}x+2\cot^{-1}x+\frac{\pi}{2}<b$

$a+\frac{\pi}{2}<\frac{\pi}{2}+\cot^{-1}x+\frac{\pi}{2}<b$

$a+\frac{\pi}{2}<\pi+\cot^{-1}x<b$

$a-\frac{\pi}{2}<\cot^{-1}x<b-\pi$ [since $0<\cot^{1}x<\pi$]

$\Rightarrow a=\frac{\pi}{2}$ b = $2\pi$

**41.** If $\sqrt[3]{y}\sqrt{x}=\sqrt[6]{(x+y)^5}$, then $\frac{dy}{dx}=$

(A) x + y (B) $\frac{y}{x}$ (C) $\frac{x}{y}$ (D) x – y

**Ans** **(B)**

**Sol.** $\sqrt[3]{y}\sqrt{x}=\sqrt[6]{(x+y)^5}$

$\Rightarrow y^2x^3=(x+y)^5$

$\Rightarrow y^2x^3=y^5\left(\frac{x}{y}+1\right)^5$

$\Rightarrow \left(\frac{x}{y}\right)^3=\left(\frac{x}{y}+1\right)^5$ | $f\left(\frac{x}{y}\right)=c$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$ | $\Rightarrow \frac{dy}{dx}=\frac{y}{x}$

**42.** Rolle's theorem is not applicable in which one of the following cases?

(A) $f(x) = x^2 - x$ in [0, 1] (B) f(x) = [x] in [2.5, 2.7]

(C) $f(x) = x^2 - 4x + 5$ in [1, 3] (D) f(x) = | x | in [–2, 2]

**Ans** **(D)**

**Sol.** f(x) = |x| in [–2, 2]

is not differentiable at x = 0

**43.** The interval in which the function $f(x) = x^3 - 6x^2 + 9x + 10$ is increasing in

(A) $(-\infty, 1] \cup [3, \infty)$ (B) $(-\infty, -1] \cup [3, \infty)$

(C) $(-\infty, 1) \cup (3, \infty)$ (D) $[1, 3]$

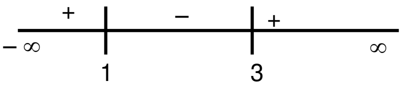
**Ans** **(A)**

**Sol.** $f(x) = x^3 - 6x^2 + 9x + 10$

$f'(x) = 3x^2 - 12x + 9$

$f'(x) = 0$

$3x^2 - 12x + 9 = 0$ (dividing by 3)




$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x - 3) - 1(x - 3) = 0$

$x = 3, 1$

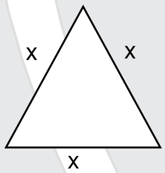
$(-\infty, 1] \cup [3, \infty)$

**44.** The sides of an equilateral triangle are increasing at the rate of 4 cm/sec. The rate at which its area is increasing, when the side is 14 cm

(A) $14\sqrt{3}\ \text{cm}^2/\text{sec}$ (B) 14 $\text{cm}^2$/sec (C) $10\sqrt{3}\,\text{cm}^2/\text{sec}$ (D) 42 $\text{cm}^2$ / sec

**Ans** **Given Options are not matching**

**Sol.** $\frac{dx}{dt} = 4\text{cm/sec}, x = 14\text{cm}$




$A = \frac{\sqrt{3}}{4}x^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\cdot 2x\frac{dx}{dt}$

$= \frac{\sqrt{3}}{2}\cdot 14 \times 4$

$= \sqrt{3}\cdot 7 \times 4$

$= 28\sqrt{3}$

**45.** The value of $\sqrt{24.99}$ is

(A) 4.899 (B) 4.897 (C) 4.999 (D) 5.001

**Ans** **(C)**

**Sol.** $\sqrt{24.99}$

$\Delta y = f(x + \Delta x) - f(x)$ $\Delta y \approx \frac{dy}{dx}\times\Delta x$

Take $f(x) = \sqrt{x}$ $\approx \frac{1}{2\sqrt{x}}\times -0.01$

Let x = 25

$\Delta x = -0.01$ $\approx -\frac{0.01}{10}$

$\therefore\ f(24.99) = \sqrt{25} + \Delta y$

$\approx 5 - 0.001$

$\approx 4.999$

**46.** $\int x^3 \sin 3x\,dx =$

(A) $\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}-\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

(B) $-\frac{x^3.\cos 3x}{3}+\frac{x^2\sin 3x}{3}-\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

(C) $-\frac{x^3\cos 3x}{3}-\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

(D) $-\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

**Ans** (D)

**Sol.** $\int x^3 \sin 3x\,dx = x^3\left[\frac{-\cos 3x}{3}\right]-3x^2\left[\frac{-\sin 3x}{9}\right]+6x\left[\frac{\cos 3x}{27}\right]-6\left[\frac{\sin 3x}{81}\right]+C$

$= -\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

**47.** The area of the region above X-axis included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$ in square units is

(A) $\frac{3}{2}-\frac{\pi}{4}$ (B) $\frac{\pi}{4}-\frac{2}{3}$ (C) $\frac{\pi}{4}-\frac{3}{2}$ (D) $\frac{2}{3}-\frac{\pi}{4}$

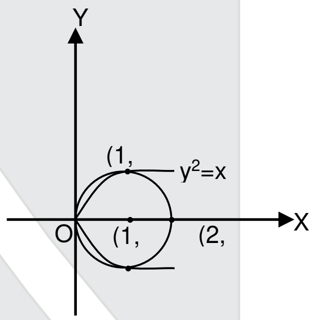
**Ans** **(B)**

**Sol.** $y^2 = x \quad \rightarrow (1)$

$x^2 + y^2 = 2x \quad \rightarrow (2)$

Equation (2) is a circle with centre (1, 0) and radius 1.

Solving (1) and (2), we get the points of intersection (0, 0) and (1, 1)




$(x-1)^2 + y^2 = 1$

$y^2 = x$

$(x-1)^2 + x = 1$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, x = 1$

area $= \int_0^1 \left\{\sqrt{1-(x-1)^2}-\sqrt{x}\right\}dx$

$= -\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + \left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^1$

$= -\frac{2}{3}+\left\{0+\frac{\pi}{4}\right\} = -\frac{2}{3}+\frac{\pi}{4}$

**48.** The area of the region bounded by y-axis. y = cos x and y = sin x; $0 \le x \le \frac{\pi}{2}$ is

(A) $\sqrt{2}$ Sq.units (B) $2-\sqrt{2}$ Sq.units

(C) $\sqrt{2}-1$ Sq.units (D) $\sqrt{2}+1$ Sq.units

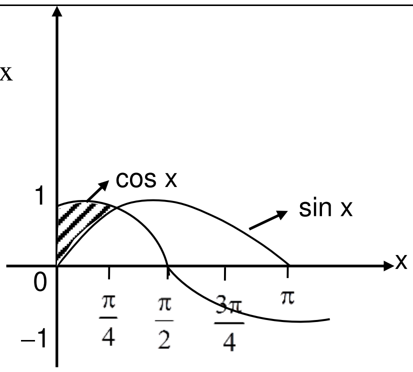
**Ans** **(C)**

**Sol.** Required area = $\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx$

$=(\sin x+\cos x)_0^{\frac{\pi}{4}}$

$=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)$

$=\left(\sqrt{2}-1\right)$ sq units




**49.** The integrating factor of the differential equation $(2x + 3y^2)\,dy = y\,dx\ (y > 0)$ is

(A) $-\frac{1}{y^2}$ (B) $\frac{1}{y^2}$ (C) $e^{\frac{1}{y}}$ (D) $\frac{1}{x}$

**Ans** **(B)**

**Sol.** $(2x + 3y^2)dy = y\,dx$

$\Rightarrow \frac{2x+3y^2}{y}=\frac{dx}{dy}$ $\Rightarrow \frac{2x}{y}+3y=\frac{dx}{dy}$ $\Rightarrow \frac{dx}{dy}-\frac{2}{y}x=3y$

If = $=e^{\int -\frac{2}{y}dy}$ $=e^{-2\log y}$ $=e^{\log y^{-2}}$ = $\frac{1}{y^2}$

**50.** The equation of the curve passing through the point (1, 1) such that the slope of the tangent at any point (x, y) is equal to the product of its co-ordinates is

(A) $2\log y=x^2+1$ (B) $2\log x=y^2+1$ (C) $2\log x=y^2-1$ (D) $2\log y=x^2-1$

**Ans** (D)

**Sol.** $\frac{dy}{dx}=xy$

$\frac{1}{y}dy=xdx$

Integrate

$\log y=\frac{x^2}{2}+c$ ...(1)

Equation (1) passing through (1, 1)

$\log(1)=\frac{1^2}{2}+C$

$0=\frac{1}{2}+C$

$C=-\frac{1}{2}$

$\log y=\frac{x^2}{2}-\frac{1}{2}$

$2\log y=x^2-1$

**51.** The constant term in the expansion of $\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$ is

(A) 2 (B) 6 (C) 0 (D) –10

**Ans** (B)

**Sol.** Take x = 0

$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix} = 1(-3)+1(3)+2(3)$ = 6

**52.** If [x] represents the greatest integer function and f(x) = x – [x] – cos x then $f'\left(\frac{\pi}{2}\right) =$

(A) 1 (B) does not exist (C) 0 (D) 2

**Ans** **(D)**

**Sol.** f(x) = x – [x] – cos x

f′(x) = 1 + sin x

$f'\left(\frac{\pi}{2}\right) = 1+1=2$

**53.** If $f(x) = \begin{cases} \dfrac{\sin 3x}{e^{2x}-1} & ; \quad x \neq 0 \\ k-2 & ; \quad x = 0 \end{cases}$ is continuous at x = 0, then k =

(A) $\frac{9}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

**Ans** **Given Options are not matching**

**Sol.** $f(x) = \begin{cases} \dfrac{\sin 3x}{e^{2x}-1} & ; \quad n \neq 0 \\ k-2 & ; \quad n = 0 \end{cases}$

Since f is continuous at x = 0

$\Rightarrow \underset{x\to 0}{\text{lt}}\ f(x) = f(0)$

$\lim_{x\to 0} \frac{\sin 3x}{e^{2x}-1} = k-2$

$\Rightarrow \lim_{x\to 0} \frac{\frac{\sin 3x}{x}}{\frac{e^{2x}-1}{x}} = k-2$

$\Rightarrow \frac{\lim_{x\to 0} \frac{\sin 3x}{x}}{\underset{x\to 0}{\text{lt}} \frac{e^{2x}-1}{x}} = k-2$

$\Rightarrow \frac{3}{2} = k-2 \quad \Rightarrow k = \frac{3}{2}+2 = \frac{7}{2} \qquad \therefore k = \frac{7}{2}$

**54.** If $f\left(x\right)=\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right]$, then $f'(0)=$

(A) log 2 (B) $\frac{4\log 2}{5}$ (C) 2 log 2 (D) $\frac{2\log 2}{5}$

**Ans** **(A)**

**Sol.** $f(x)=\sin^{-1}\left[\frac{2\cdot 2^{x}}{1+(2^{x})^{2}}\right]$

Put $2^x = \tan\theta$

$f(x)=\sin^{-1}\left[\frac{2\tan\theta}{1+\tan^{2}\theta}\right]$

$= \sin^{-1}[\sin 2\theta]$

$f(x) = 2\theta$

$f(x) = 2\tan^{-1}(2^x)$

$f'(x)=\frac{2}{1+(2^{x})^{2}}\cdot 2^{x}\log_{e}^{2}$

$f'(0)=\log_{e}2$

**55.** If $x = a\sec^2\theta$, $y = a\tan^2\theta$ then $\frac{d^2y}{dx^2}=$

(A) 1 (B) 4 (C) 2a (D) 0

**Ans** **(D)**

**Sol.** $\frac{dx}{d\theta}=2a\sec^{2}\theta\tan\theta$

$\frac{dy}{d\theta}=2a\tan\theta\sec^{2}\theta$

$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{2a\tan\theta\cdot\sec^{2}\theta}{2a\tan\theta\sec^{2}\theta}=1 \qquad \therefore \frac{d^{2}y}{dx^{2}}=0$

**56.** The eccentricity of the ellipse $9x^2+25y^2=225$ is

(A) $\frac{3}{5}$ (B) $\frac{9}{16}$ (C) $\frac{4}{5}$ (D) $\frac{3}{4}$

**Ans** **(C)**

**Sol.** $9x^2 + 25y^2 = 225$

i.e., $\frac{9x^{2}}{225}+\frac{25y^{2}}{225}=1$

$\Rightarrow \frac{x^{2}}{25}+\frac{y^{2}}{9}=1$

$\Rightarrow \frac{x^{2}}{5^{2}}+\frac{y^{2}}{3^{2}}=1$

a = 5, b = 3

$c=\sqrt{a^{2}-b^{2}}=\sqrt{5^{2}-3^{2}}=\sqrt{16}=4$

Eccentricity $e=\frac{c}{a}=\frac{4}{5}$

**57.** $\sum_{r=1}^{n}(2r-1)=x$ then $\lim_{n\to\infty}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+...+\frac{n^3}{x^2}\right]=$

(A) $\frac{1}{4}$ (B) 4 (C) $\frac{1}{2}$ (D) 1

**Ans** **(A)**

**Sol.** x = 1 + 3 + 5 +……………….. + (2n – 1)

$x = n^2$

$x^2 = n^4$

$\therefore \lim_{n\to\infty}\left[\frac{1^3+2^3+. \quad +n^3}{x^2}\right]$

$\therefore \lim_{n\to\infty}\left[\frac{\frac{n^2(n+1)^2}{4}}{n^4}\right]$

$\therefore \lim_{n\to\infty}\frac{n^2.n^2\left(1+\frac{1}{n}\right)^2}{4n^4}$ $\quad \therefore \lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^2}{4}=\frac{1}{4}$

**58.** The negation of the statement "All continuous functions are differentiable"

(A) Some continuous functions are differentiable

(B) All differentiable functions are continuous

(C) All continuous functions are not differentiable

(D) Some continuous functions are not differentiable

**Ans** **(D)**

**59.** Mean and standard deviation of 100 items are 50 and 4 respectively. The sum of all squares of the items is

(A) 256100 (B) 261600 (C) 251600 (D) 266000

**Ans** **(C)**

**Sol.** $\sigma=\sqrt{\frac{\Sigma xi^2}{n}-\left(\bar{x}^2\right)}$

$4=\sqrt{\frac{\Sigma xi^2}{100}-(50)^2}$

$16=\frac{\Sigma xi^2}{100}-2500 \Rightarrow \Sigma xi^2=251600$

**60.** Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is vowel and the other is consonant is

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{8}{9}$ (D) $\frac{3}{9}$

**Ans** **(B)**

**Sol.** 9 Letters word

vowels = 5

consonants = 4

P(1 vowel and 1 consonant) = $\frac{5C_1\times 4C_1}{9C_2}=\frac{5\times 4}{\frac{9\times 8}{2}}$