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PAPER-1 (B.E./B. TECH.) OF JEE (MAIN)

JEE (MAIN) 2018

CBT TEST PAPER

(WITH SOLUTION & ANSWER KEY)

DATE : 15-04-2018

SUBJECT : PHYSICS, CHEMISTRY, MATHEMATICS

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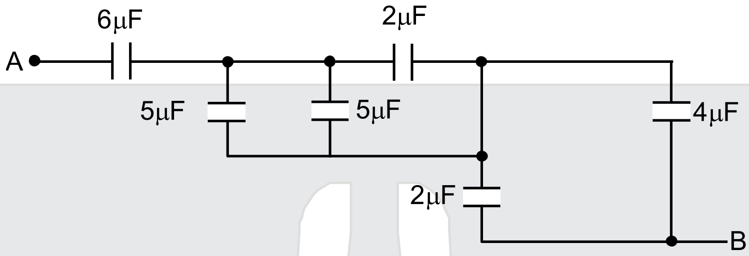
PHYSICS

Straight Objective Type

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

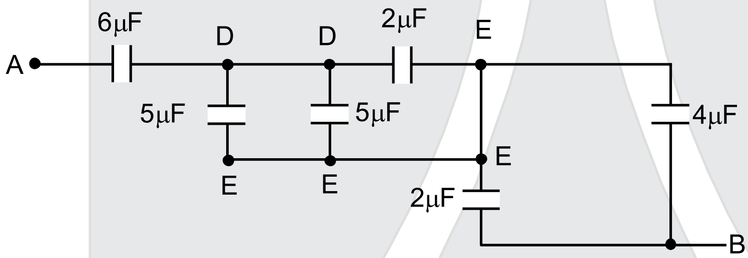
1. The equivalent capacitance between A and B in the circuit given below is :



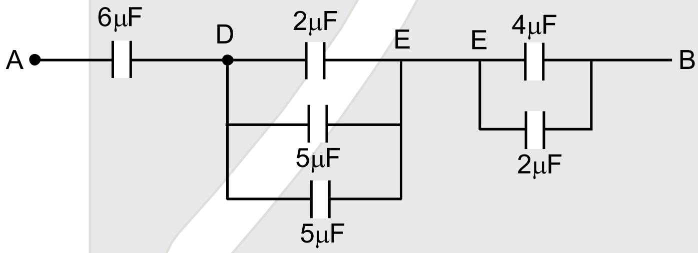
(1) 3.6μF (2) 2.4μF (3) 4.9μF (4) 5.4μF

Ans. **(2)**

Sol.

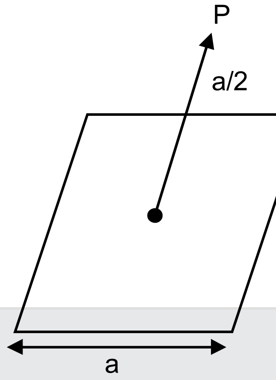


Simplified circuit



$$\frac{1}{c_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12} \quad \Rightarrow \quad C_{eq} = \frac{12}{5} = 2.4\mu F$$

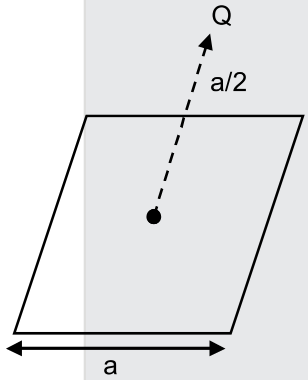
2. A charge Q is placed at a distance. The electric flux through the square the surface is a/2 above the centre of the square surface of edge a as shown in the figure.



(1) $\frac{Q}{6\in_0}$ (2) $\frac{Q}{2\in_0}$ (3) $\frac{Q}{3\in_0}$ (4) $\frac{Q}{\in_0}$

Ans. **(1)**

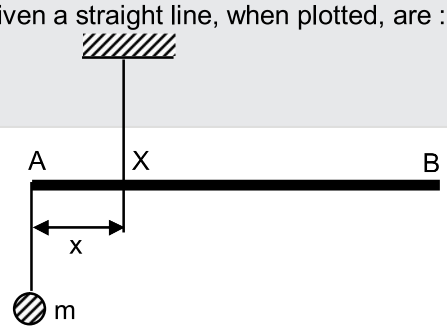
Sol.



charged particle can be Considered at centre of a cube of side a, and given surface represents its one side.

So flux $\phi = \frac{Q}{6\varepsilon_0}$

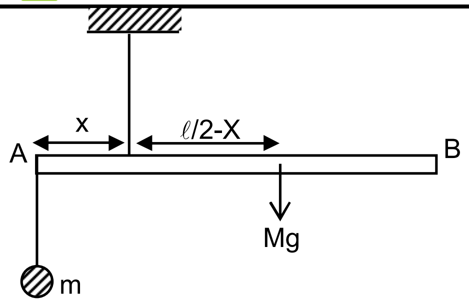
3. A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m,x) values is recorded. The appropriate variables that given a straight line, when plotted, are :



(1) m,x^2. (2) m,$\frac{1}{x^2}$ (3) m,$\frac{1}{x}$ (4) m,x

Ans. **(3)**

Sol.



Balancing torque w.r.t. point of suspension

$mg\,x = Mg\left(\frac{\ell}{2} - x\right)$

$mx = M\frac{\ell}{2} - Mx$

$m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$

$y = \alpha\frac{1}{x} - C$ equation of a straight line

4. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is :

(1) 34eV (2) 20eV (3) 79eV (4) 109eV

Ans. **(3)**

Sol. Energy required to remove e^- from singly ionized helium atom = 54.4 eV

Energy required to remove e^- form helium atom = x ev given 54.4 eV = 2.2x $\Rightarrow$ x = 24.73 eV

Energy required to ionize helium atom = 79.12 eV

5. A solution containing active cobalt $^{60}_{27}Co$ having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If 1 cm^3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body ? (1Ci = 3.7 × 10^{10} decays per second and at t = 10 hrs $e^{-\lambda t}$ = 0.84)

(1) 4 liters (2) 6 liters (3) 5 liters (4) 7 liters

Ans. **(3)**

Sol. Let total volume of blood is v, initial activity A_0 = O.Q μci its activity at time t = A = A0$e^{-\lambda t}$ activity of x volume $A^1 = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$

$V = x\left(\frac{A_0}{A^1}\right)e^{-\lambda t}$

$V = (1cm^3)\left(\frac{8\times10^{-7}\times3.7\times10^{10}}{\frac{300}{60}}\right)(0.84)$

= 4.97 × 10^3 cm^3 = 4.97 liter

6. In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by :

β is current gain, I_B, I_C and I_E are respectively base, collector and emitter currents.

(1) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$ (2) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_E}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$ (3) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta\frac{R_L}{R_{BE}}$ (4) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_E}, \beta^2\frac{R_L}{R_{BE}}$

Ans. **(1)**

Sol. From NCERT

Current gain $\beta = \frac{\Delta I_C}{\Delta I_B}$

Voltage gain $A_v = \frac{\Delta V_{CE}}{R_{BE}\Delta I_B} = B\frac{R_L}{R_{BE}}$

Power gain $A_p = \beta Av = \beta^2\frac{R_L}{R_{BE}}$

7. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$. and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is :

(1) $+\frac{Gm}{6R}$ (2) $-\frac{GMm}{2R}$ (3) $-\frac{GMm}{6R}$ (4) $\frac{GMm}{2R}$

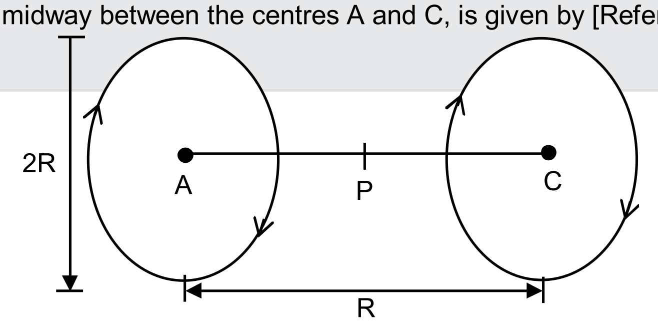
Ans. **(3)**

Sol. $E_i = -\frac{GMm}{2R}$

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R} = -\frac{4GMm}{6R} = -\frac{2Mm}{3R}$$

$$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3} + \frac{1}{2}\right) = -\frac{GMm}{6R}$$

8. A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by [Refer to figure given below]:



(1) $\frac{8N\mu_0 I}{5^{1/2}R}$ (2) $\frac{4N\mu_0 I}{5^{3/2}R}$ (3) $\frac{4N\mu_0 I}{5^{1/2}R}$ (4) $\frac{8N\mu_0 I}{5^{3/2}R}$

Ans. **(4)**

Sol. $B = 2\left(\dfrac{\mu_0 NIR^2}{2\left(R^2+\dfrac{R^2}{4}\right)^{3/2}}\right) = \dfrac{\mu_0 NIR^2}{\dfrac{5^{3/2}}{8}} = \dfrac{8\mu_0 NI}{5^{3/2}R}$

9. A thin uniform tube is bent into a circle of radius r in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_2 ($\rho_1 > \rho_2$), fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is :

(1) $\theta = \tan^{-1} \dfrac{\pi}{2}\left(\dfrac{\rho_1+\rho_2}{\rho_1-\rho_2}\right)$

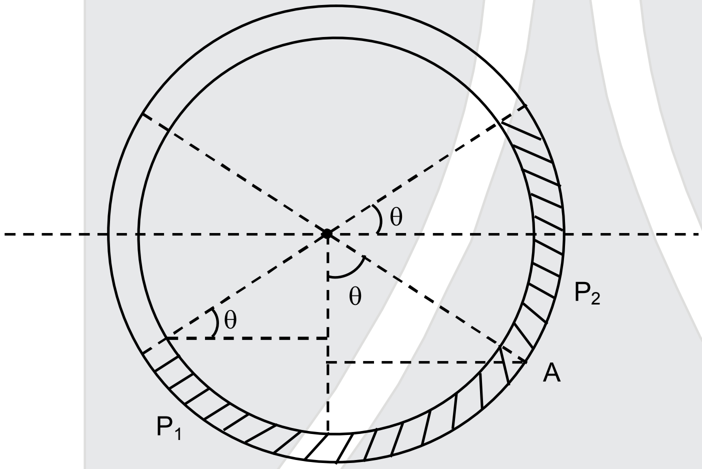
(2) $\theta = \tan^{-1}\left[\dfrac{\pi}{2}\left(\dfrac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)\right]$

(3) $\theta = \tan^{-1} \dfrac{\pi}{2}\left(\dfrac{\rho_2}{\rho_1}\right)$

(4) $\theta = \tan^{-1} \pi\left(\dfrac{\rho_1}{\rho_2}\right)$

Ans. (2)

Sol.



equating pressure at point A

ρ_1, g R $(\cos\theta - \sin\theta) = \rho_2 gR(\sin\theta + \cos\theta)$

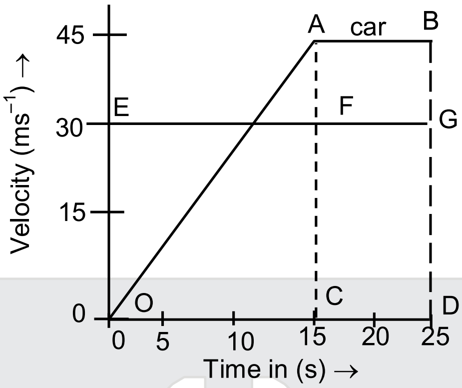
$\dfrac{\rho_1}{\rho_2} = \dfrac{\sin\theta+\cos\theta}{\cos\theta-\sin\theta} = \dfrac{\tan\theta+1}{1-\tan\theta}$

$\rho_1 - \rho_1\tan\theta = \rho_2 + \rho_2\tan\theta$

$(\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$

$\theta = \tan^{-1}\left(\dfrac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$

10. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively.



(1) 112.5 m and 22.5 s
(2) 337.5 m and 25 s
(3) 225.5 m and 10 s
(4) 112.5 m and 15 s

Ans. **(1)**

Sol. Distance travelled by car in 15 sec = $\frac{1}{2}(45)\ (15) = \frac{675}{2}\text{m}$, Distance traveled by scooter in 15 seconds = 30 × 15 = 450

Let car catches scooter in time t;

$$\frac{675}{2} + 45(t-15) = 30t$$

337.5 + 45t – 675 = 30t ⇒ 15t = 337.5 ⇒ t = 22.5 sec

11. A monochromatic beam of light has a frequency $v = \frac{3}{2\pi} \times 10^{12}\text{Hz}$ and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form the magnetic field is :

(1) $\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

$\cos\left[10^4 \frac{(\hat{i}-\hat{j})}{\sqrt{2}}.\vec{r} - (3\times10^{12})t\right]$

(2) $\frac{E_0}{C}\hat{k}$

$\cos\left[10^4 \frac{(\hat{i}+\hat{j})}{\sqrt{2}}.\vec{r} + (3\times10^{12})t\right]$

(3) $\frac{E_0}{C}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$

$\cos\left[10^4 \frac{(\hat{i}+\hat{j})}{\sqrt{2}}.\vec{r} + (3\times10^{12})t\right]$

(4) $\frac{E_0}{C}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

$\cos\left[10^4 \frac{(\hat{i}+\hat{j})}{\sqrt{2}}.\vec{r} + (3\times10^{12})t\right]$

Ans. (3)

Sol. $\hat{E}\times\hat{B}$ should give direction of wave propagation

$\Rightarrow \hat{K}\times\hat{B} \parallel \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

option – 1 $\quad \hat{K}\times\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)=\frac{\hat{j}-(-\hat{i})}{\sqrt{2}}=\frac{\hat{i}+\hat{j}}{\sqrt{2}} \,||\, \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

option -2 and 4 does not satisfy this.

wave propagation vector $\hat{K}$ should be along $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

So correct option is 3.

12. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest point on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is :

(1) 6 (2) 10^{-2} (3) 2 (4) 0.6

Ans. (3)

Sol. $F_1=\frac{GM_e m}{r_1^2} \qquad F_2=\frac{GM_e M_s}{r_2^2}$

$\Delta F_1 = -\frac{2GM_e m}{r_1^3}\Delta r_1 \qquad \Delta F_2 = -\frac{2GM_e M_s}{r_2^3}\Delta r_2$

$\frac{\Delta F_1}{\Delta F_2}=\frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s\Delta r_2}=\left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

using $\Delta r_1 = \Delta r_2 = 2R_{earth}$

$m = 8 \times 10^{22}$ kg

$M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km

$r_2 = 150 \times 10^6$ km

we get $\frac{\Delta F_1}{\Delta F_2}=2$

13. A planoconvex lens becomes an optical system of 28cm focal length when its plane surface is silvered and illuminated from left to right as shown in fig-A

If the same lens is instead silvered on the curved surface and illuminated from other side as in fig. B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is:

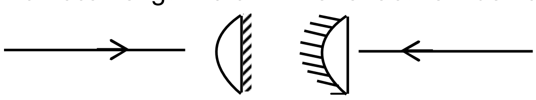
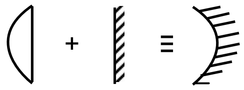


Fig .A Fig .B

(1) 1.55 (2) 1.50 (3) 1.75 (4)1.51

Ans. (1)

Sol. Case-1

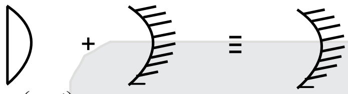


$\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)$ $\quad$ f =–28

P = 2P$_1$ + P$_2$

$$\frac{1}{28}=2\left(\frac{\mu-1}{R}\right)$$

Case-2



$\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)$ $\quad f_2=-\frac{R}{2}$ $\quad$ f=–10cm

$$P = 2P_1 + P_2 \Rightarrow \frac{1}{10}=2\left(\frac{\mu-1}{2}\right)+\frac{2}{R}$$

$$\frac{1}{10}=\frac{1}{28}+\frac{2}{R}$$

$$\frac{2}{R}=\frac{1}{10}-\frac{2}{28}=\frac{18}{280}$$

$$R=\frac{280}{9}\text{ cm}$$

$$\frac{1}{28}=2\left(\frac{\mu-1}{280}\right)9$$

$$\mu-1=\frac{5}{9}$$

$$\mu=1+\frac{5}{9}=\frac{14}{9}=1.55$$

14. One mole of an ideal monatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27°C. The done on the gas will be :

(1) 300R (2) 300R In 2 (3) 300 In 6 (4) 300R In 7

Ans. **(2)**

Sol. Work done on gas = nRT $\ell n\left(\frac{p_f}{p_i}\right)$ = R(300) ℓn(2) = 300Rℓn2

15. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding) :

(1) 150m (2) 100m (3) 75m (4) 160m

Ans. **(4)**

Sol. $S=\frac{u^2}{2a}$

$$\frac{S_1}{S_2}=\frac{u_1^2}{u_2^2}\Rightarrow S_2=\left(\frac{u_2}{u_1}\right)^2 S_1=(2)^2(40)=160\text{ m}$$

16. A carnot's engine works as a refrigerator between 250K and 300K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is

(1) 772 J (2) 420 J (3) 2100 J (4) 2520 J

Ans. **(2)**

Sol. Efficiency = $1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W}$

$\Rightarrow \quad 1 - \frac{250}{300} = \frac{W}{Q_2 + W}$

$W = \frac{Q_2}{5} = \frac{500 \times 4.2}{5} J = 420J$

17. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is :

(1) 0.4300 cm (2) 0.3150 cm (3) 0.0430 cm (4) 0.2150 cm

Ans. **(4)**

Sol. Least Count = $\frac{0.25}{5 \times 100}$ cm = 5×10^{-4} cm

Reading = 4×0.05cm + $30 \times 5 \times 10^{-4}$ cm

= (0.2 + 0.0150) cm = 0.2150 cm

18. The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be :

(1) 15 (2) 20 (3) 8 (4) 10

Ans. **(4)**

Sol. If modulating frequency is 15 KHz then band width of one channel = 30 kHz

No of channels accommodate = $\frac{300kHz}{30kHz} = 10$

19. An ideal capacitor of capacitance 0.2 μF is charged to a potential difference of 10 V. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance 0.5 mH. The current at a time when the potential difference across the capacitor is 5V is :

(1) 0.15 A (2) 0.17 A (3) 0.34 A (4) 0.25 A

Ans. **(2)**

Sol. Using energy conservation

$\frac{1}{2} \times 0.2 \times 10^{-6} \times 10^2 + 0 = \frac{1}{2} \times 0.2 \times 10^{-6} \times 5^2 + \frac{1}{2} \times 0.5 \times 10^{-3} I^2$

$I = \sqrt{3} \times 10^{-1} A = 0.17A$

20. Light of wavelength 550 nm falls normally on a slit of width 22.0×10^{-5} cm. The angular position of the second minima from the central maximum will be (in radians) :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

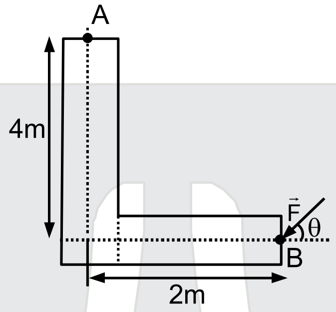
Ans. **(2)**

Sol. If angular position of 2^{nd} maxima from central maxima is θ then

$$\sin\theta = \frac{3\lambda}{2a} = \frac{3\times550\times10^{-9}}{2\times22\times10^{-7}}$$

$$\theta \simeq \frac{\pi}{8} \text{ rad}$$

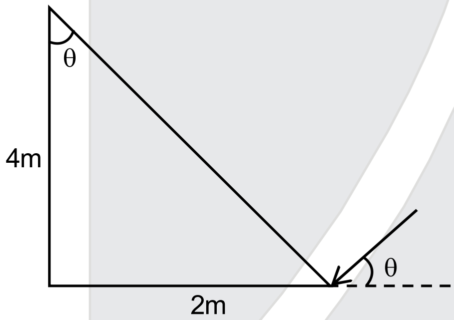
21. A force of 40 N acts on a point B at the end of an L-shaped object as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by :



(1) $\tan\theta = 4$ (2) $\tan\theta = \frac{1}{4}$ (3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = 2$

Ans. **(3)**

Sol. Moment of force will be maximum when line of action of force is perpendicular to line AB.



$$\tan\theta = \frac{2}{4} = \frac{1}{2}$$

22. A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe ? (Speed of sound in air is 340 ms^{-1})

(1) 220 cm (2) 200 cm (3) 190 cm (4) 180 cm

Ans. **(2)**

Sol. Organ pipe will have frequency either 255 or 257 Hz

Using 255Hz

$$255 = \frac{3V}{2\ell} \qquad \ell = \frac{3\times340}{2\times255} \text{ m}$$

ℓ = 200 cm.

23. A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at x = 0, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is **correct**?

(1) The total energy of the system is $\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$.

(2) The new equilibrium position is at a distance $\frac{2qE}{k}$ from x = 0.

(3) The new equilibrium position is at a distance $\frac{qE}{2k}$ from x = 0.

(4) The total energy of the system is $\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$.

Ans. **(1)**

Sol. Equilibrium position will shift to point where resultant force = 0

$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$

Energy $\frac{1}{2}m\omega^2\left[A^2 + \left(\frac{qE}{k}\right)^2\right] = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$

24. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficients of kinetic friction between the object and the incline is :

(1) $\sqrt{1-\frac{1}{n^2}}$ (2) $1 - \frac{1}{n^2}$ (3) $\frac{1}{2-n^2}$ (4) $\sqrt{\frac{1}{1-n^2}}$

Ans. **(2)**

Sol. Time taken to slide along smooth surface

$s = \frac{1}{2}g\sin 45° t_1^2$

$t_1 = \sqrt{\frac{2\sqrt{2}s}{g}}$

Time taken to slide along rough surface

$S = \frac{1}{2}(g\sin 45° - \mu g\cos 45°)t_2^2$

$t_2 = \sqrt{\frac{2\sqrt{2}s}{g(1-\mu)}}$

$t_2 = nt_1$

$\frac{2\sqrt{2}s}{g(1-\mu)} = n^2 \times \frac{2\sqrt{2}s}{g} \Rightarrow 1 - \mu = \frac{1}{n^2} \Rightarrow \quad \mu = 1 - \frac{1}{n^2}$

25. The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is :

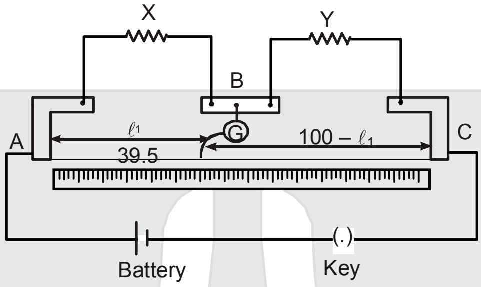
(1) $\frac{3}{2}\alpha$ (2) $\frac{2}{3}\alpha$ (3) α (4) $\frac{5}{2}\alpha$

Ans. **(1)**

Sol. $\frac{\Delta s}{s} = 2 \times \frac{\Delta r}{r}$ $\frac{\Delta v}{v} = 3 \times \frac{\Delta r}{r}$

$\frac{\Delta v}{v} = \frac{3}{2}\alpha$

26. In a meter bridge as shown in the figure it is given that resistance Y = 12.5 Ω and that the balance is obtained at a distance 39.5 cm from end A (by Jockey J). After interchanging the resistances X and Y a new balance point is found at a distance l_2 from end A. What are the values of X and l_2?



(1) 19.15 Ω and 39.5 cm (2) 8.16 Ω and 60.5 cm
(3) 8.16 Ω and 39.5 cm (4) 19.15 Ω and 60.5 cm

Ans. **(2)**

Sol. For a balanced meter bridge

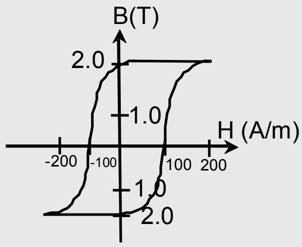
y × 39.5 = x × (100 – 39.5)

$x = \frac{12.5 \times 39.5}{60.5} = 8.16\Omega$

when x & y are interchanged ℓ_1 and $(100 - \ell_1)$ will also interchange

ℓ_2 = 60.5 cm.

27. The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is :



(1) 2 mA (2) 20 μA (3) 1 mA (4) 40 μA

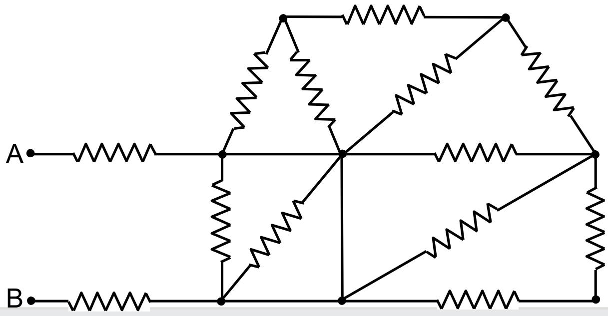
Ans. **(3)**

Sol. Coercivity of Ferro magnet H = 100 A/m

nI = 100

$I = \frac{100}{10^5} = 1mA$

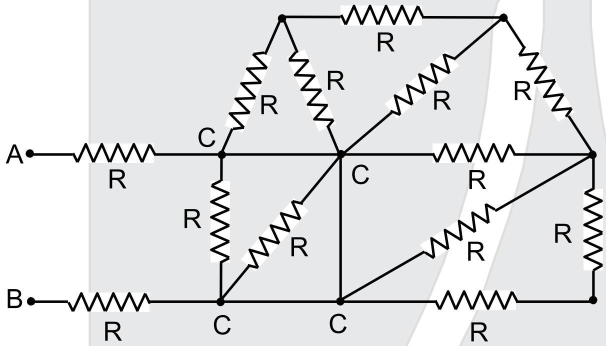
28. In the given circuit all resistances are of value of R ohm each. The equivalent resistance between A and B is :



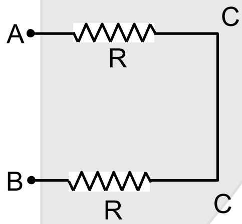
(1) $\frac{5R}{2}$ (2) 3R (3) $\frac{5R}{3}$ (4) 2R

Ans. **(4)**

Sol.

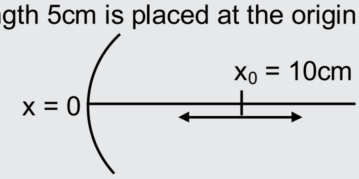


$\equiv$



$R_{eq} = 2R$

29. A particle is oscillating on the x-axis with an amplitude 2cm about the point x_0 = 10cm with a frequency ω. A concave mirror of focal length 5cm is placed at the origin (see figure).



Identify the correct statements?

(A) The image executes periodic motion

(B) The image executes non-periodic motion

(C) The turning points of the image are asymmetric w.r.t. the image of the point at x = 10 cm.

(D) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$ cm.

(1) (B, C) (2) (A, C, D) (3) (A, D) (4) (B, D)

Ans. **(2)**

Sol. When object is at 8 cm

$$V_1 = \frac{f \times u}{u - f} = -\frac{40}{3}\text{cm}$$

When object is at 12 cm

$$V_2 = -\frac{60}{7}\text{cm}$$

Separation = $|V_1 - V_2| = \frac{100}{21}$ cm

So A, C and D are correct

30. Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2 their de Broglie wavelength in the frame of reference attached to their centre of mass is :

(1) $\frac{1}{\lambda_{CM}} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (2) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ (3) $\lambda_{CM} = \lambda_1 = \lambda_2$ (4) $\lambda_{CM} = \left(\frac{\lambda_1 + \lambda_2}{2}\right)$

Ans. **(2)**

Sol. Momentum of each electron $\frac{h}{\lambda_1}\hat{i}$ & $\frac{h}{\lambda_2}\hat{j}$

Velocity of centre of mass

$$V_{cm} = \frac{h}{2m\lambda_1}\hat{i} + \frac{h}{2m\lambda_2}\hat{j}$$

Velocity of 1st particle about centre of mass

$$V_{1cm} = \frac{h}{2m\lambda_1}\hat{i} - \frac{h}{2m\lambda_2}\hat{j}$$

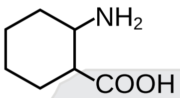
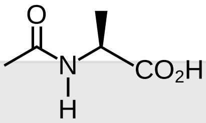
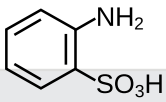
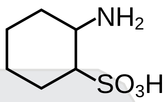
$$\lambda_{cm} = \frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2} + \frac{h^2}{4\lambda_2^2}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

CHEMISTRY

Straight Objective Type

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Which of the following will not exist in zwitter ionic form at pH = 7 ?

(1) 
(2) 
(3) 
(4) 

Ans. (2)

Sol. The N atom of amide is not basic.

2. A sample of $NaClO_3$ is converted by heat to NaCl with a loss of 0.16 g of oxygen. The residue is dissolved in water and precipitated as AgCl. The mass of AgCl (in g) obtained will be : (Given: Molar mass of AgCl = 143.5 g mol^{-1})

(1) 0.35 (2) 0.54 (3) 0.41 (4) 0.48

Ans. (4)

Sol. $2NaClO_3 \xrightarrow{\Delta} 2NaCl + 3O_2$

0.16 g

$$\frac{n_{NaCl}}{2} = \frac{n_{O_2}}{3}$$

$$n_{NaCl} = \frac{0.16}{32} \times \frac{2}{3} = \frac{1}{200} \times \frac{2}{3} = \frac{1}{300}$$

$NaCl \longrightarrow AgCl$

POAC of Cl

$1 \times n_{NaCl} = 1 \times n_{AgCl}$

$$\frac{1}{300} = n_{AgCl}$$

$$\text{Weight of AgCl} = \frac{1}{300} \times [108 + 35.5] = \frac{1}{300} \times 143.5$$

$= 0.48$ g

3. For which of the following reactions, ΔH is equal to ΔU ?

(1) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$ (2) $2HI(g) \rightarrow H_2(g) + I_2(g)$

(3) $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$ (4) $2NO_2(g) \rightarrow N_2O_4(g)$

Ans. (2)

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$2HI(g) \rightarrow H_2(g) + I_2(g)$ $\Delta n_g = (1 + 1) - 2 = 0$

4. N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetics. After 50 minutes, the pressure inside the vessel increases from 50 mm Hg to 87.5 mm Hg. The pressure of the gaseous mixture after 100 minute at constant temperature will be :

(1) 136.25 mm Hg (2) 106.25 mm Hg (3) 175.0 mm Hg (4) 116.25 mm Hg

Ans. (2)

Sol.

$$N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$$

t = 0	50	0	0
t = 50 min.	$50-p_1$ = 25	$2p_1$	$\frac{p_1}{2}$
t = 100 min.	$50-p_2$ = 12.5	$2p_2$	$\frac{p_2}{2}$

$$50 - p_1 + 2p_1 + \frac{p_1}{2} = 87.5$$

$$50 + \frac{3p_1}{2} = 87.5$$

$$\frac{3p_1}{2} = 37.5$$

$$p_1 = \frac{37.5 \times 2}{3} = 25$$

50 minute is half life period

for 100 minute (2 half life)

$$50 - p_2 = 12.5$$

$p_2 = 37.5$ mm of Hg

Total pressure at 100 minute

$$= 50 - p_2 + 2p_2 + \frac{p_2}{2}$$

$$= 50 + \frac{3p_2}{2} = 50 + \frac{3}{2} \times 37.5$$

$$= 50 + 56.25$$

= 106.25 mm of Hg

5. In the molecular orbital diagram for the molecular ion, N_2^+, the number of electrons in the σ_{2p} molecular orbital is :

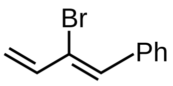
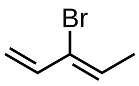
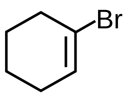
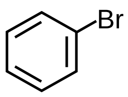
(1) 0 (2) 2 (3) 3 (4) 1

Ans. (4)

Sol. $N_2^+ \rightarrow \sigma_{1s^2}, \sigma^*_{1s^2}, \sigma_{2s^2}, \sigma^*_{2s^2}, [\pi^2_{2p_x} = \pi^2_{2p_y}]\sigma^1_{2p_z}$

Number of electron in σ_{2p_z} is 1

6. Which of the following will most readily give the dehydrohalogenation product ?

(1)  (2)  (3)  (4) 

Ans. (1)

Sol. Here dehydrohalogenation goes by E1cb and most stable carbanion formation is favoured in A.

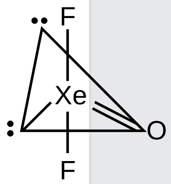
7. Identify the pair in which the geometry of the species is T-shape and square-pyramidal, respectively :

(1) ICl_2^- and ICl_5 (2) IO_3^- and $IO_2F_2^-$ (3) ClF_3 and IO_4^- (4) $XeOF_2$ and $XeOF_4$

Ans. (4)

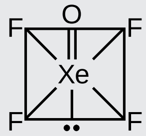
Sol.

$XeOF_2$

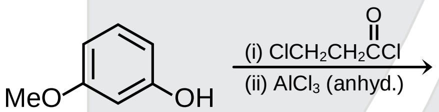


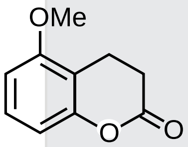
T-Shape

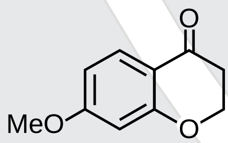
$XeOF_4$

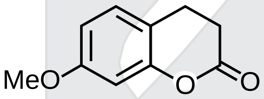


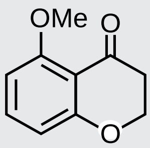
8. The major product of the following reaction is :



(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. The reactant undergoes acylation first followed by substitution Intramolecular.

9. $H-N \overset{(I)}{\cdots\cdots} N \overset{(II)}{\cdots\cdots} N$

In hydrogen azide (above) the bond orders of bonds (I) and (II) are :

	(I)	(II)		(I)	(II)
(1)	< 2	> 2	(2)	> 2	> 2
(3)	> 2	< 2	(4)	< 2	< 2

Ans. (1)

Sol. $H-\ddot{N}=\overset{\oplus}{N}=\overset{\ominus}{N}$ or $H-\ddot{N}-N\equiv N$

10. For Na^+, Mg^{2+}, F^- and O^{2-} ; the correct order of increasing ionic radii is :

(1) $O^{2-} < F^- < Na^+ < Mg^{2+}$ (2) $Na^+ < Mg^{2+} < F^- < O^{2-}$

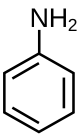
(3) $Mg^{2+} < Na^+ < F^- < O^{2-}$ (4) $Mg^{2+} < O^{2-} < Na^+ < F^-$

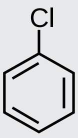
Ans. (3)

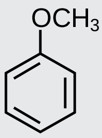
Sol. Isoelectronic series : $Mg^{2+} < Na^+ < F^- < O^{2-}$

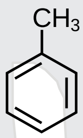
When negative charge increase, increase the radius of ion.

11. The increasing order of nitration of the following compounds is :


(a)


(b)


(c)


(d)

(1) (a) < (b) < (d) < (c) (2) (a) < (b) < (c) < (d)

(3) (b) < (a) < (c) < (d) (4) (b) < (a) < (d) < (c)

Ans. (1)

Sol. Here the aniline is the least reactive due to formation of anilinium ion in acidic medium.

12. The correct match between items of List-I and List-II is :

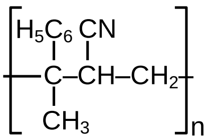
	List-I		List-II
(A)	Coloured impurity	(P)	Steam distillation
(B)	Mixture of o-nitrophenol and p-nitrophenol	(Q)	Fractional distillation
(C)	Crude Naphtha	(R)	Charcoal treatment
(D)	Mixture of glycerol and sugars	(S)	Distillation under reduced pressure

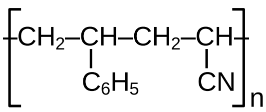
(1) (A)-(R), (B)-(S), (C)-(P), (D)-(Q) (2) (A)-(P), (B)-(S), (C)-(R), (D)-(Q)

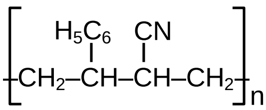
(3) (A)-(R), (B)-(P), (C)-(Q), (D)-(S) (4) (A)-(R), (B)-(P), (C)-(S), (D)-(Q)

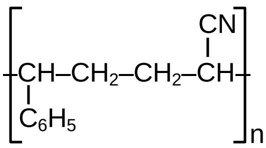
Ans. (3)

13. The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is :

(1) 

(2) 

(3) 

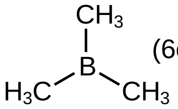
(4) 

Ans. (2)

14. Which of the following is a Lewis acid ?

(1) PH_3 (2) NF_3 (3) NaH (4) $B(CH_3)_3$

Ans. (4)

Sol. 
(6e⁻) (Lewis Acid)

15. Which of the following statements about colloids is **False** ?

(1) When silver nitrate solution is added to potassium iodide solution a negatively charged colloidal solution is formed.

(2) Freezing point of colloidal solution is lower than true solution at same concentration of a solute.

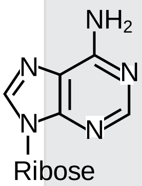
(3) Colloidal particles can pass through ordinary filter paper.

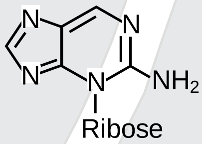
(4) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated.

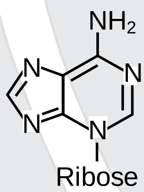
Ans. (2)

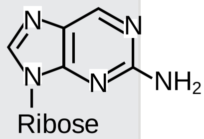
Sol. Freezing point of colloidal solution is higher than true solution at same concentration of a solute.

16. Which of the following is the correct structure of Adenosine ?

(1) 

(2) 

(3) 

(4) 

Ans. (1)

17. The correct combination is :

(1) $[NiCl_4]^{2-}$ – square-planar ; $[Ni(CN)_4]^{2-}$ – paramagnetic

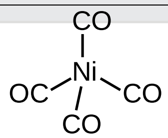
(2) $[Ni(CN)_4]^{2-}$ – tetrahedral ; $[Ni(CO)_4]^{2-}$ – paramagnetic

(3) $[NiCl_4]^{2-}$ – paramagnetic ; $[Ni(CO)_4]$ – tetrahedral

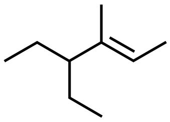
(4) $[NiCl_4]^{2-}$ – diamagnetic ; $[Ni(CO)_4]$ – square-planar

Ans. (3)

Sol.

$[NiCl_4]^{2-}$	$[Ni(CO)_4]$
sp^3	sp^3
Paramagnetic (2 unpaired electron)	
$Ni^{2+} \rightarrow [Ar]3d^8, 4s^0, 4p^0$	$Ni(O) \rightarrow [Ar]3d^8, 4s^2, 4p^0$
Cl^- (W.F.L.) (No pairing)	CO is S.F.L. $[Ar]\ 3d^{10}\ \underbrace{4s^0, 4p^0}_{sp^3}$ (Tetrahedral)

18. The IUPAC name of the following compound is :

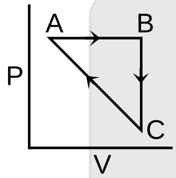


(1) 3-ethyl-4-methylhex-4-ene (2) 4,4-diethyl-3-methylbut-2-ene

(3) 4-methyl-3-ethylhex-4-ene (4) 4-ethyl-3-methylhex-2-ene

Ans. (4)

19. An ideal gas undergoes a cyclic process as shown in Figure.



$\Delta U_{BC} = -5$ kJ mol^{-1}, $q_{AB} = 2$ kJ mol^{-1}

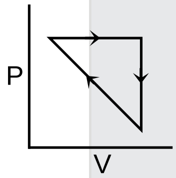
$W_{AB} = -5$ kJ mol^{-1}, $W_{CA} = 3$ kJ mol^{-1}

Heat absorbed by the system during process CA is :

(1) – 5 kJ mol^{-1} (2) + 5 kJ mol^{-1} (3) 18 kJ mol^{-1} (4) – 18 kJ mol^{-1}

Ans. (2)

Sol.



AB $\rightarrow$ isobaric

BC $\rightarrow$ Isochoric

CA $\rightarrow$ not defined

$\Delta U_{AB} = q + W$

$= 2 - 5 = -3$

$\Delta U_{ABC} = \Delta U_{AB} + \Delta U_{BC}$

$= -3 - 5 = -8$ kJ

$\Delta U_{CBA} = +8$

$= Q + W$

$8 = Q + 3$

$Q = +5$ kJ

20. Ejection of the photoelectron from metal in the photoelectric effect experiment can by stopped by applying 0.5 V when the radiation of 250 nm is used. The work function of the metal is :

(1) 4 eV (2) 5.5 eV (3) 4.5 eV (4) 5 eV

Ans. (3)

Sol. $\lambda = 250$ nm $= 2500$ Å

$$E = \frac{hc}{\lambda} = \frac{12400}{2500} = 4.96 \text{ eV}$$

KE = stopping potential = 0.5 eV

$E = W_0 + \text{K.E.}$

$4.96 = W + 0.5$

$W_0 = 4.46 \approx 4.5$ eV

21. In graphite and diamond, the percentage of p-characters of the hybrid orbitals in hybridization are respectively :

(1) 33 and 25 (2) 67 and 75 (3) 50 and 75 (4) 33 and 75

Ans. (2)

Sol.

Graphite	Diamond
sp^2 hybridisation	sp^3 hybridisation
$\% P = \frac{2}{3} \times 100 = 67\ \%$	$\% P = \frac{3}{4} \times 100 = 75\ \%$

22. When an electric current is passed through acidified water, 112 mL of hydrogen gas at N.T.P was collected at the cathode in 965 seconds. The current passed, in ampere, is :

(1) 2.0 (2) 0.1 (3) 0.5 (4) 1.0

Ans. (4)

Sol. Cathode

$2e^- + 2H_2O \longrightarrow H_2 + 2OH^-$ $\qquad (\text{v.f.})_{H_2} = 2$

$$\text{mole} = \frac{i \times t}{\text{v.f.} \times 96500}$$

$$\frac{112}{22400} = \frac{i \times 965}{2 \times 96500}$$

$$\frac{1}{2} = \frac{i}{2}$$

i = 1 amp

23. The minimum volume of water required to dissolve 0.1 g lead(II) chloride to get a saturated solution (K_{sp} of $PbCl_2 = 3.2 \times 10^{-8}$; atomic mass of Pb = 207 u) is :

(1) 1.798 L (2) 0.36 L (3) 17.95 L (4) 0.18 L

Ans. (4)

Sol. $(K_{sp})_{PbCl_2} = 32 \times 10^{-9}$

$$PbCl_2 \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{2s}{2Cl^-}$$

$K_{sp} = [Pb^{2+}][Cl^-]^2$

$K_{sp} = 4s^3 = 32 \times 10^{-9}$

$s^3 = 8 \times 10^{-9}$

$s = 2 \times 10^{-3}$ M

$$\frac{w}{M.w.} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

$$\frac{0.1}{278} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

$$V_L = \frac{0.1 \times 1000}{278 \times 2} = 0.18 \text{ L}$$

24. In which of the following reactions, an increase in the volume of the container will favour the formation of products ?

(1) $4NH_3\,(g) + 5O_2\,(g) \rightleftharpoons 4NO\,(g) + 6H_2O\,(\ell)$

(2) $2NO_2\,(g) \rightleftharpoons 2NO\,(g) + O_2\,(g)$

(3) $3O_2\,(g) \rightleftharpoons 2O_3\,(g)$

(4) $H_2\,(g) + I_2\,(g) \rightleftharpoons 2HI\,(g)$

Ans. (2)

Sol. Volume $\uparrow$ P $\downarrow$ reaction proceed in which direction where number of gases mole increases.

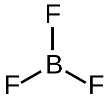
$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$ $\Delta n_g = (2 + 1) - 2 = 1$

25. The decreasing order of bond angles in BF_3, NH_3, PF_3 and I_3^- is :

(1) $I_3^- > BF_3 > NH_3 > PF_3$
(2) $BF_3 > I_3^- > PF_3 > NH_3$
(3) $BF_3 > NH_3 > PF_3 > I_3^-$
(4) $I_3^- > NH_3 > PF_3 > BF_3$

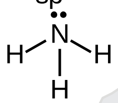
Ans. (1)

Sol.



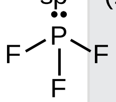
Bond angle = 120º

sp^2



Bond angle = 107º

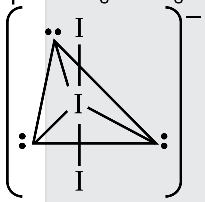
sp^3 (1 lone pair)



When central atom size ↑
Bond angle ↓

sp^3 $NH_3 > PF_3$

I_3^-



Bond angle 180º

sp^3d

Bond Angle $PF_3 < NH_3 < BF_3 < I_3^-$

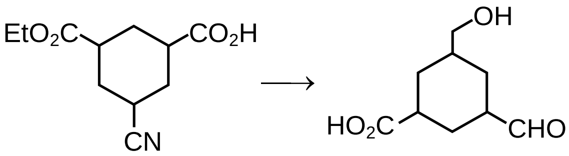
26. Which of the following arrangements shows the schematic alignment of magnetic moments of antiferromagnetic substance ?

(1) ↑ ↓ ↓ ↓ ↓ ↑
(2) ↑ ↑ ↑ ↑ ↑ ↑
(3) ↑ ↑ ↓ ↑ ↑ ↓
(4) ↑ ↓ ↑ ↓ ↑ ↓

Ans. (4)

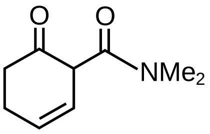
Sol. Substances which are expected to possess para-magnetism or ferro-magnetism on the basis of unpaired electrons but actually they possess zero net magnetic moment are called anti ferromagnetic substance.

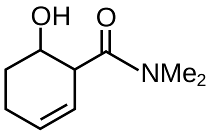
27. The reagent(s) required for the following conversion are :

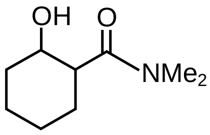


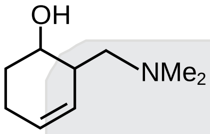
(1) (i) $NaBH_4$ (ii) Raney Ni/H_2 (iii) H_3O^+
(2) (i) $LiAlH_4$ (ii) H_3O^+
(3) (i) B_2H_6 (ii) DIBAL-H (iii) H_3O^+
(4) (i) B_2H_6 (ii) $SnCl_2/HCl$ (iii) H_3O^+

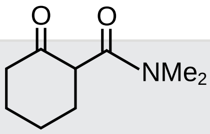
28. The main reduction product of the following compound with $NaBH_4$ in methanol is :



(1) 

(2) 

(3) 

(4) 

Ans. (1)

29. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X' and 'Y' and the oxidation state of Xe are respectively :

(1) $XeOF_4$ (+6) and XeO_3 (+6)

(2) $XeOF_2$ (+4) and XeO_3 (+6)

(3) $XeOF_4$ (+6) and XeO_2F_2 (+6)

(4) XeO_2F_2 (+6) and XeO_2 (+4)

Ans. (3)

Sol. $XeF_6 + H_2O \xrightarrow[\text{hydrolysis}]{\text{Partial}} \overset{+6}{XeOF_4}$

$XeF_6 + H_2O \xrightarrow[\text{hydrolysis}]{\text{Partial}} \overset{+6}{XeO_2F_2}$

30. A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white precipitate is obtained which does not dissolve in dilute nitric acid. The anion is :

(1) CO_3^{2-} (2) SO_4^{2-} (3) S^{2-} (4) Cl^-

Ans. (4)

Sol. $Cl^- + Na^+ \longrightarrow \underset{\text{(Neutral)}}{NaCl}$ (SASB salt) $\xrightarrow{AgNO_3}$ AgCl (white ppt) (not dissolve in dilute HNO_3)

MATHEMATICS

PART– C

Straight Objective Type (सीधे वस्तुनिष्ठ प्रकार)

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

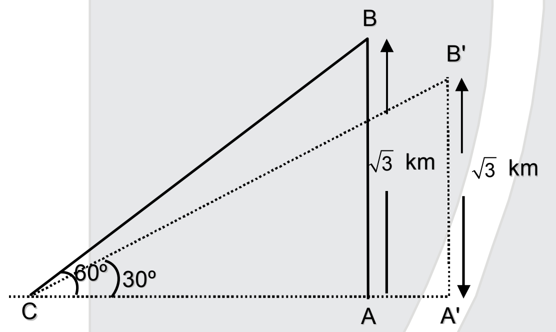
इस खण्ड में **30 बहु–विकल्पी प्रश्न** हैं। प्रत्येक प्रश्न के 4 विकल्प (1), (2), (3) तथा (4) हैं, जिनमें से **सिर्फ एक सही** है।

1. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60º from a point on the ground. If, after five seconds , its elevation from the same point, is 30º, then the speed (in km/hr) of the aeroplane, is :

(1) 720 (2) 1500 (3) 750 (4) 1440

Ans. **(4)**

Sol.



Let from point C the angle of elevation of plane at B is 60º and after 5 seconds it reach at B'

In ΔABC $AC = \sqrt{3}\cot 60º = 1$

In $\Delta CA'B'$ $A'C = \sqrt{3}\cot 30º = 3$

Hence distance AA' = 2km

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{2}{\frac{5}{60\times 60}} = \frac{2\times 60\times 60}{5} = \frac{7200}{5} = 1440 \text{ km/hr}$$

2. A box 'A' contains 2 white , 3 red and 2 black balls. Another box 'B' contains 4 white , 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly, selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is :

(1) $\frac{7}{8}$ (2) $\frac{9}{16}$ (3) $\frac{7}{16}$ (4) $\frac{9}{32}$

Sol. Probability that box A is selected $P(A) = \frac{1}{2}$

Probability that box B is selected $P(B) = \frac{1}{2}$

E be event that one ball is white while the other is red

P(E) =P(A) . P(E/A) + P(B) P(E/B)

$$= \frac{1}{2}\left[\frac{2.3}{^7C_2}+\frac{4.2}{^9C_2}\right] = \frac{1}{2}\left[\frac{6}{21}+\frac{8}{36}\right] = \frac{1}{2}\left[\frac{2}{7}+\frac{2}{9}\right] = \frac{16}{63}$$

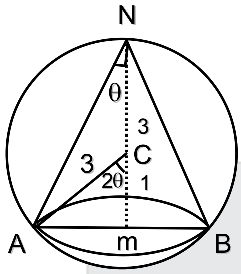
$$P(B/E) = \frac{P(B)P(E/B)}{P(E)} = \frac{1/9}{16/63} = \frac{7}{16}$$

3. If a right circular cone, having maximum volume , is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is :

(1) $8\sqrt{2}\pi$ (2) $6\sqrt{2}\pi$ (3) $8\sqrt{3}\pi$ (4) $6\sqrt{3}\pi$

Ans. **(3)**

Sol.



$V = \frac{1}{3}\pi r^2 h$

where r is radius and h is height of coin

$\Rightarrow V = \frac{1}{3}\pi (3\sin 2\theta)^2 (3 + 3\cos 2\theta)$

$= 72\pi \sin^2\theta \cos^4\theta$

$\frac{dv}{d\theta} = 72\pi [2\sin\theta \cos^5\theta - 4\sin^3\theta \cos^3\theta] = 0 \Rightarrow \tan^2\theta = \frac{1}{2}$

V_{max} if $\tan\theta = \frac{1}{\sqrt{2}}$

Hence curved surface area $S = \pi r\ell$

$= \pi r \sqrt{(3+3\cos 2\theta)^2 + (3\sin 2\theta)^2}$

$= \pi (3\sin 2\theta)\sqrt{36\sin^2\theta} = 18\pi (2\sin\theta \cos^2\theta) = 36\pi.\frac{1}{\sqrt{3}}.\frac{2}{3} = \frac{24\pi}{3} = 8\sqrt{3}\pi$

4. If β is one of the angles between the normals to the ellipse $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to :

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{2}$

Ans. **(3)**

Sol. $\frac{x^2}{9} + \frac{y^2}{3} = 1$

Normal at $(3\cos\theta, \sqrt{3}\sin\theta)$ is

$3\sec\theta . x - \sqrt{3}\,\mathrm{cosec}\theta\, y = 6$ ……..(i)

normal at $(-3\sin\theta, \sqrt{3}\cos\theta)$ is

$-3\mathrm{cosec}\theta . x - \sqrt{3}\sec\theta\, y = 6$ ……..(ii)

Angle between normal is β

$\Rightarrow \tan\beta = \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right| = \left|-\frac{\sqrt{3}}{2\sin\theta\cos\theta}\right| \Rightarrow \tan\beta = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

5. If $\left(f\dfrac{x-4}{x+2}\right) = 2x + 1$, $(x \in R - \{1, -2\})$, then $\int f(x)dx$ is equal to : (where C is a constant of integration)

(1) $12 \log_e |1 - x| - 3x + C$
(2) $-12 \log_e|1 - x| - 3x + C$
(3) $12 \log_e |1 - x| + 3x + C$
(4) $-12 \log_e |1 - x| + 3x + C$

Ans. **(2)**

Sol. $f\left(\dfrac{x-4}{x+2}\right) = 2x + 1$

$\Rightarrow f(x) = 2\left\{1-3\left(\dfrac{x+1}{x-1}\right)\right\}+1$

$= 3 - \dfrac{6x+6}{x-1} = \dfrac{-3x-9}{x-1}$

$\Rightarrow f(x) = \dfrac{3(x+3)}{(1-x)}$

$\Rightarrow \int f(x)dx = 3\int\left(\dfrac{x+3}{1-x}\right)dx = 3\int\dfrac{4-(1-x)}{1-x}dx = 3\left\{\int\dfrac{4}{1-x}dx - \int dx\right\}$

$= 3\{-4\ell n|1-x-x| + C = -12\ell n|1-x| - 3x + C$

6. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is :

(1) $4\sqrt{2}$
(2) 20
(3) $2\sqrt{5}$
(4) $2\sqrt{7}$

Ans. **(3)**

Sol. $x^2 + (2-\lambda)x \times + (10-\lambda) = 0$

Let roots are α & β

$\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (\lambda - 2)^3 - 3(10 - \lambda)(\lambda - 2)$

$= \lambda^3 - 6\lambda^2 + 12\lambda - 8 - 3(10\lambda - \lambda^2 - 20 + 2\lambda)$

$= \lambda^3 - 3\lambda^2 - 24\lambda + 52$

$\dfrac{dz}{d\lambda} = 3\lambda^2 - 6\lambda - 24$ where $= \alpha^3 + \beta^3 = 0$

$\Rightarrow \lambda^2 - 2\lambda - 8 = 0$

$\Rightarrow (\lambda - 4)(\lambda + 2) = 0$

$\Rightarrow \lambda = -2, 4$

$\dfrac{d^2z}{d\lambda^2} = 12\lambda - 6$

$\dfrac{d^2z}{d\lambda^2}(\lambda = -2) < 0 \Rightarrow \alpha^3 + \beta^3$ max if $\lambda = -2$

$\dfrac{d^2z}{d\lambda^2}(\lambda = 4) > 0 \Rightarrow \alpha^3 + \beta^3$ min. if $\lambda = 4$

$\Rightarrow$ Equation is $x^2 - 2x + 6 = 0$ $\begin{cases} 1+\sqrt{5}\,i \\ 1-\sqrt{5}\,i \end{cases}$

$|\alpha - \beta| = 2\sqrt{5}$

7. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3 , then the equation of the common tangent to the two parabolas is :

(1) $3(x + y) + 4 = 0$ (2) $8(2x + y) + 3 = 0$ (3) $x + 2y + 3 = 0$ (4) $4(x + y) + 3 = 0$

Ans. **(4)**

Sol. Equation two parabola are $y^2 = 3x$ and $x^2 = 3y$

Let equation of tangent to $y^2 = 3x$ is $y = mx + \frac{3}{4m}$

is also tangent to $x^2 = 3y$

$\Rightarrow x^2 = 3mx + \frac{9}{4m}$

$\Rightarrow 4mx^2 - 12m^2x - 9 = 0$ have equal roots

$\Rightarrow D = 0$

$\Rightarrow 144\, m^4 = 4\,(4m)\,(-9)$

$\Rightarrow m^4 + m = 0 \Rightarrow m = -1$

Hence common tangent is $y = -x - \frac{3}{4}$

$4(x + y) + 3 = 0$

8. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then

$\lim\limits_{x\to 0} \frac{f'(x)}{x}$

(1) does not exist (2) exists and is equal to – 2

(3) exists and is equal to 0 (4) exists and is equal to 2.

Ans. **(2)**

Sol. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$= -x^2 \cos x + \tan x \,.\, x^2$

$= x^2 (\tan x - \cos x)$

$\Rightarrow \lim\limits_{x\to 0} \frac{f'(x)}{x} = \lim\limits_{x\to 0} \frac{2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)}{x}$

$= \lim\limits_{x\to 0} 2\,(\tan x - \cos x) + x(\sec^2 x + \sin x)$

$= -2$

9. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}\pi$ (3) 0 (4) $\frac{3}{16}\pi$

Ans. **(2)**

Sol. $I = \int_{-\pi/2}^{\pi/2} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ ....(i)

Use proerties $\int_0^b f(x)dx = \int_a^b f(a+b-x)dx$

$= \int_{-\pi/2}^{\pi/2} \sin^4 x\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right)dx$ ....(ii)

by (i) + (ii)

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} 2\sin^4 xdx$

$\Rightarrow I = 2\int_0^{\pi/2} \sin^4 xdx =$

$= x.\ \frac{3.1}{4.2}.\frac{\pi}{2} = \frac{3\pi}{8}$

10. n-digit number are formed using only three digit 2,5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is :

(1) 9 (2) 7 (3) 8 (4) 6

Ans. **2**

Sol. n-digit number are formed using only three digits
2, 5 and 7 with repetition is = 3^n
$3^6 < 900$
$3^7 > 900$
so n = 7

11. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinates axes at the distinct points A and B, then the locus of the mid point of AB is :

(1) $4x^2 - y^2 + 16x^2y^2 = 0$ (2) $x^2 - 4y^2 + 16x^2y^2 = 0$
(3) $x^2 - 4y^2 - 16x^2y^2 = 0$ (4) $4x^2 - y^2 - 16x^2y^2 = 0$

Ans. **(3)**

Sol. Let tangent drawn at point (x, y) to the hyperbola $4y^2 = x^2 + 1$ is : $4yy_1 = xx_1 + 1$

This tangent intersect co-ordinate axes at A and B respectively then A $\left(-\frac{1}{x_1},0\right)$ and B $\left(0,\frac{1}{4y_1}\right)$

Let mid point is M (h,k) then of AB

$2h = -\frac{1}{x_1} \Rightarrow x_1 = -\frac{1}{2h}$ ....(i)

$2k = \frac{1}{4y_1} \Rightarrow y_1 = \frac{1}{8k}$ ....(ii)

Since point $P(x_1, y_1)$ lies on the hyperbola so

$4y_1^2 = x_1^2 + 1$

from (i) & (ii)

$$4\left(\frac{1}{8k}\right)^2 = \left(-\frac{1}{2h}\right)^2 + 1 \Rightarrow \frac{1}{16k^2} = \frac{1}{4h^2} + 1$$

$4h^2 = 16k^2 (1 + 4h^2)$

$x^2 = 4y^2 + 16x^2y^2$

$x^2 - 4y^2 - 16x^2y^2 = 0$

$x^2 - 4y^2 - 16x^2y^2 = 0$

locus of M

12. If tan A and tanB are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A + B) - 10\sin(A + B).\cos(A + B) - 25\cos^2(A + B)$ is :

(1) – 25 (2) 10 (3) – 10 (4) 25

Ans. **(1)**

Sol. Since tanA and tanB are roots of the equation $3x^2 - 10x - 25 = 0$

so $\tan A + \tan B = \frac{10}{3}$

$\tan B.\tan B = -\frac{25}{3}$

$$\therefore \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B} = \frac{10/3}{1 + \frac{25}{3}} = \frac{10}{28} = \frac{5}{14}$$

$$= \text{so } \sin(A + B) = \frac{5}{\sqrt{221}} \text{ and } \cos(A + B) = \frac{14}{\sqrt{227}}$$

$\therefore 3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B)$

$$= 3 \times \frac{25}{221} - \frac{10 \times 5 \times 14}{221} - 25 \times \frac{14^2}{221} = \frac{25}{221}(3 - 28 - 196) = -25$$

13. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1 & , \quad x \in [0,1] \\ 0 & , \quad \text{otherwise} \end{cases}$

If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is :

(1) $\frac{e^2 - 1}{e^3}$ (2) $\frac{1}{2e}$ (3) $\frac{e^2 + 1}{2e^4}$ (4) $\frac{e^2 - 1}{2e^3}$

Ans. **(4)**

Sol. $\frac{dy}{dx} + 2y = f(x)$ is a linear differential equation

If $= e^{\int 2dx} = e^{2x}$

solution of the above equation is

$y.e^{2x} = \int f(x).e^{2x}dx + C$

$y(x) = e^{-2x}\int_0^x f(x)e^{2x}dx + ce^{-2x}$

$y(0) = 0 \Rightarrow C = 0$

$\Rightarrow y(x) = e^{-2x}\int_0^x f(x)e^{2x}dx$

$$y(3/2) = e^{-3}\left[\int_0^1 e^{2x}dx + \int_1^{3/2} 0.dx\right] = \frac{e^{-3}}{2}[e^2 - 1] = \frac{e^2 - 1}{2e^3}$$

14. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval :

(1) $[10, \infty)$ (2) $(-\infty, -10]$ (3) $(-10,0)$ (4) $(0, 10)$

Ans. **(4)**

Sol. If b is the first term and r is the common ratio of an infinite G.P. then sum is 5

$$5 = \frac{b}{1-r}]$$

$$1 - r = \frac{b}{5}$$

$$r = 1 - \frac{b}{5}$$

$$r = \frac{5-b}{5}$$

$$\therefore -1 < r < 1$$

$$\therefore -1 < \frac{5-b}{5} < 1$$

$$-5 < 5 - b < 5$$

$$-5 < 5 - b < 5$$

$$-10 < -b < 0$$

$$0 < b < 10$$

$$b \in (0,10)$$

15. Consider the following two binary relations on the set A = {a, b, c} :

$R_1 = \{(c,a), (b,b), (a,c), (c,c), (b,c), (a,a)\}$ and $R_2 = \{(a,b), (b,a), (c,c), (c,a), (a,a), (b,b), (a,c)\}$.

Then :

(1) R_2 is symmetric but it is not transitive

(2) both R_1 and R_2 are not symmetric

(3) both R_1 and R_2 are transitive.

(4) R_1 is not symmetric but it is transitive

Ans. **(1)**

Sol. $R_1 \in (b, c)$ but $R_1 \notin (c,b)$

Example R_1 is not symmetric

in R_1 ; $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

So R_1 is not transitive

R_2 is symmetric

is R_2 ; $(b,a) \in R_2$ and $(a,c) \in R_2$ but $(b,c) \notin R_2$

So R_2 is not transitive

16. A circle passes through the points (2,3) and (4,5). If its centre lies on the line, y – 4x + 3 = 0, then its radius is equal to :

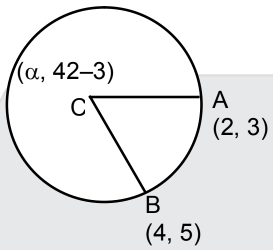
(1) $\sqrt{5}$ (2) $\sqrt{2}$ (3) 1 (4) 2

Ans. **(4)**

Sol. Let centre of circle is $c(\alpha.\beta)$

it lies is line $y - 4x + 3 = 0$ $B = 4\alpha - 3$

$\therefore c(\alpha, 4\alpha{-}3)$



$\therefore$ CA = CB

$(\alpha{-}2)^2 + (4\alpha{-}6)^2 = (\alpha{-}4)^2 + (4\alpha{-}8)^2$

$-4\alpha + 4 - 48\alpha + 36 = -8\alpha + 16 - 64\alpha + 64$

$(64 + 8 - 4 - 48)\alpha = 80 - 40$

$\alpha = \dfrac{40}{20} = 2$

c(2,5)

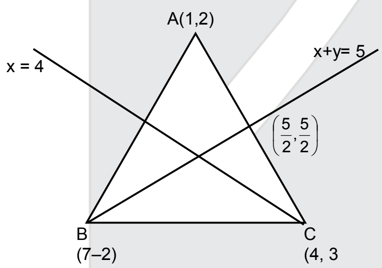
$\therefore r = 2$

17. In a triangle ABC, coordinates of A are (1,2) and the equations of the medians through B and C are respectively, x + y = 5 and x = 4. Then area of ΔABC (in sq. units) is :

(1) 12 (2) 4 (3) 9 (4) 5

Ans. **(3)**

Sol.



$\therefore$ Area of $\Delta = \dfrac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \dfrac{1}{2}[18]$ **= 9 sq.unit.**

18. The set of all $\alpha \in R$, for which $w = \dfrac{1 + (1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and Re $z \neq 1$, is :

(1) {0} (2) $\left\{0, \dfrac{1}{4}, -\dfrac{1}{4}\right\}$ (3) equal to R (4) an empty set

Ans. **(1)**

Sol. $\frac{1+(1-8\alpha)z}{1-z}+\frac{1+(1-8\alpha)\bar{z}}{1-\bar{z}}=0$

$1-\bar{z}+(1-8\alpha)z-(1-8\alpha)+1-z+(1-8\alpha)\bar{z}-(1-8\alpha)=0$

$2-(z+\bar{z})+(1-8\alpha)(z+\bar{z})-2+16\alpha=0$

$\Rightarrow$ $8\alpha(z+\bar{z})=16\alpha$

$\Rightarrow$ $z+\bar{z}=2$ or $\alpha=0$

For all $z\in C$ we have $\alpha=0$ $\Rightarrow$

19. If $x_1, x_2, \ldots\ldots\ldots, x_n$ and $\frac{1}{h_1},\frac{1}{h_2},\ldots\ldots,\frac{1}{h_n}$ are two A.P. such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals :

(1) 3200 (2) 1600 (3) 2650 (4) 2560

Ans. **(4)**

$x_1, x_2, x_3, \ldots\ldots x_n$ in AP.

$x_3 = 8$ & $x_8 = 20$ $\Rightarrow x_5 = \frac{64}{5}$

$h_1, h_2, h_3, \ldots\ldots h_n$, in HP

$h_2 = 8$, $h_7 = 20$ $\Rightarrow$ $h_{10} = 200$

$\Rightarrow x_5 h_{10} = \frac{64}{5}\times 200 = \frac{12800}{5} = 2560$

20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+2\vec{b}+2\vec{c}=0$, then $|\vec{a}\times\vec{c}|$ is equal to :

(1) $\frac{1}{4}$ (2) $\frac{15}{16}$ (3) $\frac{\sqrt{15}}{4}$ (4) $\frac{\sqrt{15}}{16}$

Ans. **(3)**

Sol. $\vec{a}+2\vec{b}+2\vec{c}=\vec{o}$

$\vec{a}+2\vec{c}=-2\vec{b}$

$|\vec{a}|^2+4|\vec{c}|^2+4\vec{a}.\vec{c}=4|\vec{b}|^2$

$1+4+4\cos\theta=-\frac{1}{4}$ $\therefore$ $\sin\theta=\frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\frac{1}{16}}}=\frac{\sqrt{15}}{4}$

Now $|\vec{a}\times\vec{c}| = |\vec{a}|\,|\vec{c}|\,\sin\theta = \frac{\sqrt{15}}{4}$

21. A variable plane passes through a fixed point (3,2,1) and meets x, y and z axes at A, B and C respectively . A plane is drawn parallel to yz - plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is :

(1) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (2) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (3) $x+y+z=6$ (4) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

Ans. **(1)**

Sol. Let plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

it passes through (3,2,1) $\therefore$ $\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$

Now A (a,0,0) , B (0, b, 0) , C (0,0,c)

$\therefore$ Locus of point of intersection of planes x = a

y = b, z = c is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

22. Let A be a matrix such that A . $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and |3A| = 108. Then A^2 equals :

(1) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ (2) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 36 & 0 \\ -32 & -32 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

Ans. **(2)**

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Now A. $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} a & 2a+3b \\ c & 2c+3d \end{bmatrix}$ is scalar

$\therefore$ c = 0, 2a + 3b = 0, a = 2c + 3d a = 3d $\therefore$ $a^2 = 9d^2 = 36$

|3A| = 108

$\therefore$ |A| = 12 = ad – bc = ad

$\therefore$ $d^2 = 4$

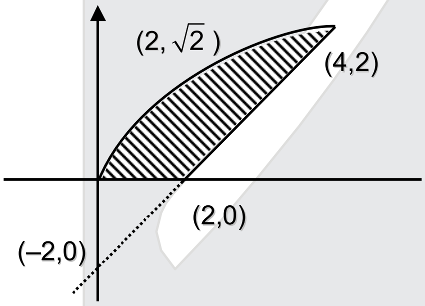
Now $A^2 = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix}$

23. The area (in sq. units) of the region $\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is :

(A) $\frac{13}{3}$ (2) $\frac{8}{3}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3}$

Ans. **(3)**

Sol.



$$\int_0^2 \sqrt{x}dx + \int_2^4 \left(\sqrt{x} - x + 2\right)dx$$

$$\int_0^4 \sqrt{x}dx + \int_2^4 (2 - x)dx$$

$$\left(\frac{2x^{\frac{3}{2}}}{3}\right)_0^4 + \left(2x - \frac{x^2}{2}\right)_2^4$$

$$= \frac{2}{3}(8) + -(4-2) = \frac{16}{3} - 2 = \frac{10}{3}$$

24. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point (–2, 0) is :

(1) – 34 (2) 4 (3) – 2 (4) – 32

Ans. **(1)**

Sol. $x^2 + y^2 + \sin y = 4 \Rightarrow \quad 2x + (2y + \cos y)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y+\cos y} \qquad$ at (–2, 0) $\qquad \Rightarrow \frac{dy}{dx} = \frac{4}{1} = 4$

and $(2y + \cos y)\frac{dy}{dx} + 2x = 0$

$(2y+\cos y)\frac{d^2y}{dx^2} + (2 - \sin y)\left(\frac{dy}{dx}\right)^2 + 2 = 0$

$\frac{d^2y}{dx^2} + (2-0)(4)^2 + 2 = 0$

$\Rightarrow \qquad \frac{d^2y}{dx^2} = -34$

25. An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is :

(1) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$ (2) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (3) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (4) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Ans. **(4)**

Sol. $\begin{vmatrix} i & j & k \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix}$

$i(0) - j\,(6-5) + k\,(24-20) = -\hat{j} + 4\hat{k}$

Angle = $\frac{\pi}{2} - \cos^{-1}\left(\frac{-1+4}{\sqrt{3}\sqrt{17}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{\sqrt{3}}{17}\right) = \sin^{-1}\sqrt{\frac{3}{17}}$

26. Let S = $\{(\lambda, \mu) \in R \times R : f(t) = (|\lambda|e^{|t|} - \mu) . \sin(2|t|), t \in R$, is a differentiable function$\}$. Then S is a subset of :

(1) $(-\infty, 0) \times R$ (2) $R \times [0, \infty)$ (3) $[0, \infty) \times R$ (4) $R \times (-\infty, 0)$

Ans. **(2)**

Sol. Let $s = \{(\lambda, \mu) \in R \times R\}$

$f(t) = \{|\lambda|\, e^{|t|} - \mu)\sin 2|t|$ $\quad 0^+ = 0$, $\quad 0^- = 0$

$\text{RHD} = \lim_{h\to 0}\frac{f(0+h)-0}{h} = \lim_{h\to 0}(|\lambda|e^h - \mu)\frac{\sin^2 h}{h} = 2(|\lambda|e^h - \mu) = 0$

$\text{LHD} = \lim_{h\to 0}\frac{f(0-h)-0}{-h} = \lim_{h\to 0}(\lambda^{e+h} - \mu)\frac{\sin^2 h}{-h} = -2(|\lambda|e^h - \mu)$

$|\lambda|e^h = \mu$

$|\lambda| = \mu$

$\Rightarrow \mu \ge 0 \;\&\; \lambda \in R$

27. Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$

$2x + y - 2 = 3$

$3x + 2y + kz = 4$

Has a unique solution. Then S is :

(1) equal to R – {0} (2) an empty set (3) equal to R (4) equal to {0}

Ans. **(4)**

Sol. $\Delta \neq 0$ for $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$1(k+2) - 1(2k+3) + 1(4-3) = 0 \Rightarrow k+2-2k-3+1 = 0 \Rightarrow -k = 0 \Rightarrow k = 0$

28. If n is the degree of the polynomial, $\left[\dfrac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\dfrac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, then the ordered pair (n,m) is equal to :

(1) $(8,5\,(10)^4)$ (2) $(12,\,8(10)^4)$ (3) $(12,(20)^4)$ (4) $(24,\,(10)^8)$

Ans. **(3)**

Sol. $\left[\dfrac{2\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)}{2}\right]^8 + \left[\dfrac{2\left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)}{2}\right]^8$

$$= 2\left[{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(5x^3-1\right) + {}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(5x^3-1\right)^2 + {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(5x^3-1\right)^3 + {}^8C_8\left(5x^3-1\right)^4\right]$$

$$= 2\left[\left(5x^3-1\right)^4 + 28\left(5x^3+1\right)^3\left(5x^3-1\right) + 70\left(5x^3+1\right)^2\left(5x^3-1\right)^2 + 28\left(5x^3+1\right)\left(5x^3-1\right)^3 + \left(5x^3-1\right)^4\right]$$

h = 12 & $m = 2(5^4 + 140.5^3 + 70.5^4 + 140.5^3 + 5^4)$

$= 160000 = (20)^4$

29. The mean of a set of 30 observations is 75. If each observations is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $\frac{10}{3}$ (4) $\frac{2}{3}$

Ans. **(1)**

Sol. $x_1 + x_2 + \ldots\ldots\ldots + x_{30} = 75 \times 30$

Now given $\lambda(x_1 + x_2 + x_3 + \ldots\ldots\ldots + x_{30}) - 25 \times 30 = 75 \times 30$

$\lambda(75 \times 30) = 100 \times 30$

$\lambda = \frac{4}{3}$

30. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are, respectively :

(1) T,T,T (2) F,T,F (3) T,F,T (4) F,F,F

Ans. **(3)**

Sol. $(p \wedge \sim q) \wedge (p \wedge \sim r) \to \sim p \vee q$

(1) $(T \wedge F) \wedge (T \wedge T) \to (F \vee T)$

$\equiv (F \wedge T) \to T$

$\equiv F \to T \equiv T$

(2) $(F \wedge F) \wedge (F \wedge F) \to T \vee T$

$F \to T \equiv T$

(3) $(T \wedge T) \wedge (T \wedge T) \to F \vee F$

$T \to F \equiv F$

(4) $(F \wedge T) \wedge (F \wedge F) \to T \vee F$

$F \wedge F \to T$

$F \to T \equiv T$