Questions & Solutions

PAPER-2 | SUBJECT : MATHEMATICS

PAPER-2 : INSTRUCTIONS TO CANDIDATES

- **Question Paper-2 has three (03) parts : Physics, Chemistry and Mathematics.**
- Each part has a total of **eighteen (18) questions** divided into **three (03) sections (Section-1, Section-2 and Section-3).**
- Total number of questions in **Question Paper-2** are **: Fifty Four (54)** and Maximum Marks are **One Hundred Ninety Eight (198).**

Type of Questions and Marking Schemes

SECTION-1 (Maximum Marks: 18)

- This section contains **SIX (06)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 to 9, BOTH INCLUSIVE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. .
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered.
Zero Marks : 0 If the question is unanswered.
Negative Marks : **–1** In all other cases.

SECTION 2 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–2** In all other cases.

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+4** If ONLY the correct numerical value is entered.
Zero Marks : **0** In all other cases.

PART : MATHEMATICS

SECTION-1 (Maximum Marks: 18)

- This section contains **SIX (06)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 to 9, BOTH INCLUSIVE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. .
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered.

Zero Marks : 0 If the question is unanswered.

Negative Marks : **–1** In all other cases.

1. For a complex number z, let Re(z) denote the real part of z. let S be the set of all complex numbers z satisfying $z^4 - |z|^4 = 4\,iz^2$, where I = $\sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$. where z_1, $z_2 \in S$ with $Re(z_1) > 0$ and $Re(z_2) < 0$, is ….

Ans. 8

Sol. $z^4 - |z|^4 = 4iz^2 \quad \Rightarrow \quad z^4 - \bar{z}^2 = 4iz^2$

$\Rightarrow \quad z^2(z^2 - \bar{z}^2) = 4iz^2 \quad \Rightarrow \quad z^2 - \bar{z}^2 = 4i$

$\Rightarrow \quad (z + \bar{z})(z - \bar{z}) = 4i$

$\Rightarrow \quad \left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right) = 1 \quad \Rightarrow \quad xy = 1$

for z_1 & z_2 $\Rightarrow$ $x_1\,y_1 = 1$ and $x_2y_2 = 1$

x_1 & x_2 are of opposite sign similarly

y_1 & y_2 are of opposite sign

$\Rightarrow \quad x_1 > 0,\ y_1 > 0,\ x_2 < 0,\ y_2 < 0$

Now $|z_1 - z_2|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= x_1^2 + x_2^2 + y_1^2 + y_2^2 - 2x_1x_2 - 2y_1y_2$

$= x_1^2 + x_2^2 + y_1^2 + y_2^2 + 2x_1(-x_2) + 2y_1(-y_2)$

$\geq 8(x_1^2.x_2^2.y_1^2.y_2^2.2x_1(-x_2).2y_1(-y_2)^{1/8}$

$\geq 8((x_1y_1)^3 (x_2y_2)^3)^{1/8}$

≥ 8

2. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is NOT less than 0.95, is …..

Ans. 6

Sol. P(of destroying the target) $\geq$ 0.95

$${}^nC_3\left(\frac{3}{4}\right)^3\cdot\left(\frac{1}{4}\right)^{n-3} + {}^nC_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^{n-4} + \ldots\ldots + {}^nC_r\left(\frac{3}{4}\right)^n \geq 0.95$$

$$1-\left\{{}^nC_0\left(\frac{3}{4}\right)^0\cdot\left(\frac{1}{4}\right)^n + {}^nC_1\left(\frac{3}{4}\right)^1\cdot\left(\frac{1}{4}\right)^{n-1} + {}^nC_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^{n-2}\right\} \geq 0.95$$

$$1-\frac{95}{100} \geq \frac{1}{4^n}+\frac{3n}{4^n}+\frac{n(n-1)}{2}\cdot\frac{9}{4^n}$$

$$\frac{4^n}{20} \geq \frac{2+6n+9n^2-9n}{2}$$

$$\frac{2^{2n-1}}{5} \geq 9n^2-3n+2$$

$2^{2n-1} \geq 5\ (9n^2 - 3n + 2)$

$n = 3 \Rightarrow 32 \geq 5 \times 74$

$n = 4 \Rightarrow 128 \geq 5 \times 132$

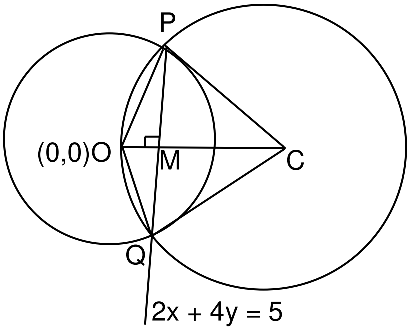
$n = 5 \Rightarrow 512 \geq 5 \times 212$

$n = 5 \Rightarrow 2048 \geq 5 \times 308$

3. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is …..

Ans. 2

Sol.



Circumcentre C of ΔOPQ lies on $x + 2y = 4$

Let $C \equiv (4 - 2\alpha, \alpha)$

$\because \quad OC \perp PQ \quad \Rightarrow \quad M_{OC}.M_{PQ} = -1$

$\Rightarrow \quad \left(\dfrac{\alpha}{4-2\alpha}\right)\left(-\dfrac{2}{4}\right) = -1 \quad \Rightarrow \quad \alpha = \dfrac{8}{5} \quad \Rightarrow \quad C\left(\dfrac{4}{5}, \dfrac{8}{5}\right)$

$$OM = \frac{|0+0-5|}{\sqrt{2^2+4^2}} = \frac{\sqrt{5}}{2}$$

$$CM = \frac{\left|2\left(\frac{4}{5}\right)+4\left(\frac{8}{5}\right)-5\right|}{\sqrt{2^2+4^2}} = \frac{3}{2\sqrt{5}}$$

$$OC = PC = \sqrt{\left(\frac{4}{5}\right)^2+\left(\frac{8}{5}\right)^2} = \frac{4}{\sqrt{5}}$$

Now $PM^2 = OP^2 - OM^2 = PC^2 - CM^2 \quad \Rightarrow \quad r^2 - \dfrac{5}{4} = \dfrac{16}{5} - \dfrac{9}{20} \quad \Rightarrow \quad r = 2$

4. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is ….

Ans. 5

Sol. $A = \begin{bmatrix} a & b \\ c & 3-a \end{bmatrix} \qquad |A| = 3a - a^2 - bc = 5$

$$A^3 = \begin{bmatrix} a & b \\ c & 3-a \end{bmatrix}\begin{bmatrix} a & b \\ c & 3-a \end{bmatrix}\begin{bmatrix} a & b \\ c & 3-a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+3b-ab \\ ac+3c-ac & cb+(3-a)^2 \end{bmatrix}\begin{bmatrix} a & b \\ c & 3-a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & 3b \\ 3c & cb+(3-a)^2 \end{bmatrix}\begin{bmatrix} a & b \\ c & 3-a \end{bmatrix}$$

$tr(A^3) = a^3 + abc + 3bc + 3bc + 3bc - abc + (3 - a)^3 = -18$

$a^3 + 9bc + (3 - a)^3 = -18$

$9bc + 27 - 3.3\, a(3 - a) = -18$

$bc + 3 - 3a + a^2 = -2$

$3a - a^2 - bc = 5 \quad \Rightarrow \quad |A| = 5$

5. Let the functions : $(-1, 1) \to \mathbb{R}$ and $g : (-1, 1) \to (-1, 1)$ be defined by

$f(x) = |2x - 1| + |2x + 1|$ and $g(x) = x - [x]$,

where [x] denotes the greatest integer less than or equal to x. Let $f \circ g$: $(-1,1) \to \mathbb{R}$ be the composite function defined by $(f \circ g)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1,1)$ at which $f \circ g$ is **NOT** continuous, and suppose d is the number of points in the interval $(-1,1)$ at which $f \circ g$ is **NOT** differentiable. Then the value of $c +$ is _____

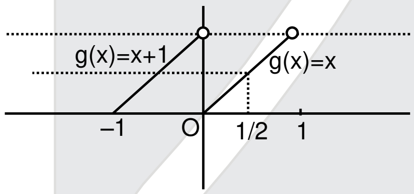
Ans. 4

Sol.
$$f(x) = |2x - 1| + |2x + 1| = \begin{cases} -4x, & x < -\dfrac{1}{2} \\ 2, & -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ 4x, & x > \dfrac{1}{2} \end{cases}$$

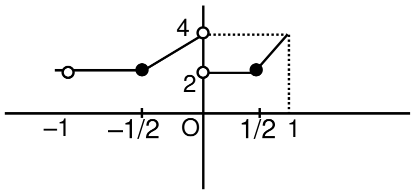
g(x) = {x}

$$f(g(x)) = \begin{cases} -4g(x), & g(x) < -\dfrac{1}{2} \\ 2, & -\dfrac{1}{2} \le g(x) \le \dfrac{1}{2} \\ 4g(x), & g(x) > \dfrac{1}{2} \end{cases}$$

graph of g(x)



$$f(g(x)) = \begin{cases} 2, & x \in \left(-1, -\dfrac{1}{2}\right) \cup \left(0, \dfrac{1}{2}\right) \\ 4(x+1), & x \in \left(-\dfrac{1}{2}, 0\right) \\ 4x, & x \in \left(\dfrac{1}{2}, 1\right) \end{cases}$$



c = 1, d = 3

$\therefore$ c + d = 4

6. The value of the limit

$$\lim_{x\to\frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x+\sin x)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$$

is _____

Ans. 8

Sol.
$$\lim_{x\to\frac{\pi}{2}} \frac{8\sqrt{2}\sin 2x.\cos x}{\cos\frac{x}{2}-\cos\frac{7x}{2}+\cos\frac{5x}{2}-\left(\sqrt{2}.2\cos^2 x+\cos\frac{3x}{2}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{4\sqrt{2}.2.2\sin x\cos x.\cos x}{2\sin x\sin\frac{x}{2}+2\sin 3x\sin\frac{x}{2}-2\sqrt{2}\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{2\sin\frac{x}{2}(2\sin 2x.\cos x)-2\sqrt{2}\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x}{2\sin\frac{x}{2}.4\sin x-2\sqrt{2}}$$

$$= \frac{16\sqrt{2}}{4\sqrt{2}-2\sqrt{2}} = \frac{16\sqrt{2}}{2\sqrt{2}} = 8$$

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- For each question, choose the option(s) corresponding to (all) the correct answer(s).
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7. Let b be a nonzero real number. Suppose f : R → R is a differentiable function such that f(0) = 1. If the derivative f' of f satisfies the equation

$$f'(x) = \frac{f(x)}{b^2 + x^2}$$

for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE?

(A) If $b>0$, then f is an increasing function

(B) If $b<0$, then f is a decreasing function

(C) $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$

(D) $(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

Ans. **(AC)**

Sol. $f'(x) = \frac{f(x)}{b^2 + x^2} \Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{b^2 + x^2}$

$\Rightarrow \ell nf(x) = \frac{1}{b}\tan^{-1}\frac{x}{b} + C$

$\therefore$ f(0) = 1 $\therefore$ 0 = C $\Rightarrow$ C = 0

$\therefore \ell nf(x) = \frac{1}{6}\tan^{-1}\frac{x}{b}$

$\Rightarrow f(x) = e^{\frac{1}{b}\tan^{-1}\frac{x}{b}}$

$\therefore f'(x) = e^{\frac{1}{b}\tan^{-1}\frac{x}{b}} \cdot \frac{1}{b^2 + x^2} > 0$ for $x \in R$

$\Rightarrow$ f(x) is increasing option (A)

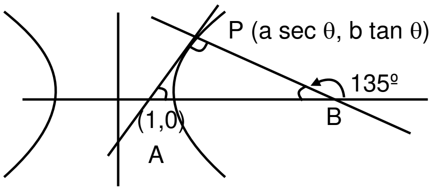
$f(x) \,.\, f(-x) = e^{\frac{1}{b}\tan^{-1}\frac{x}{b} - e^{-\frac{1}{b}\tan^{-1}\frac{x}{b}}} \neq 1$

8. Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let P be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Suppose the tangent to the hyperbola at P passes through the point (1, 0), and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes. Let Δ denote the area of the triangle formed by the tangent at P, the normal at P and the x-axis. If e denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE?

(A) $1 < e < \sqrt{2}$ (B) $\sqrt{2} < e < 2$ (C) $\Delta = a^4$ (D) $\Delta = b^4$

Ans. **(AD)**

Sol.



Tangent at P

$$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$$ Passes through (1, 0)

$\sec\theta = a$

Now slope of AP = 1

$$\frac{b\sec\theta}{a\tan\theta}=1 \Rightarrow \quad b=\tan\theta$$

Now $b^2=a^2\left(e^2-1\right)$

$$\tan^2\theta=\sec^2\theta\left(e^2-1\right) \Rightarrow e^2-1=\sin^2\theta$$

$$e^2-1\in[0,1] \because \theta\in\left(0,\frac{\pi}{2}\right)$$

$$e^2\in[1,\ 2]\Rightarrow 1<e<\sqrt{2}$$

Now $A(1,\ 0)$ and $P(a\sec\theta, b\tan\theta) = (\sec^2\theta,\ \tan^2\theta)$

$$AP=\sqrt{\tan^2\theta+\tan^2\theta} \Rightarrow AP=\sqrt{2}\tan\theta \qquad (\because \Delta APB \text{ is isosceles})$$

area = $\frac{1}{2}(AP)^2=\frac{1}{2}\times 2\tan^4\theta=b^4$

9. Let $f : R \to R$ and $g : R \to R$ be functions satisfying

$f(x + y) = f(x) + f(y) + f(x)f(y)$ and $f(x) = xg(x)$

for all $x, y \in R$. If $\lim_{x\to 0} g(x) = 1$, then which of the following statements is/are TRUE?

(A) f is differentiable at every $x \in R$

(B) If $g(0) = 1$, then g is differentiable at every $x \in R$

(C) The derivative $f'(1)$ is equal to 1

(D) The derivative $f'(0)$ is equal to 1

Ans. **(ABD)**

Sol. $\because$ Put x = y = 0 in given relation.

$\Rightarrow$ f(0) = f(0) + f(0) + $f^2(0)$

$\Rightarrow$ f(0) = 0 or –1

$\because$ f(x + y) = f(x) + f(y) + f(x).f(y)

$\Rightarrow \dfrac{f(x+y)-f(x)}{y} = \dfrac{f(y)(1+f(x))}{y}$

$\Rightarrow \lim\limits_{y\to 0}\left(\dfrac{f(x+y)-f(x)}{y}\right) = \lim\limits_{y\to 0}(1+f(x)).\dfrac{f(y)}{y}$

$\Rightarrow$ f′(x) = 1 + f(x)

$\Rightarrow$ f′(0) = 1 + f(0)

$\Rightarrow$ f′(0) = 1 + 0

$\Rightarrow$ f′(0) = 1

$\because \lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0}\dfrac{f(x)}{x} = 1$

Again $\dfrac{f'(x)}{1+f(x)} = 1 \Rightarrow \int \dfrac{f'(x)dx}{1+f(x)}dx = \int dx$

$\Rightarrow \ell n\ (1 + f(x)) = x + C$

$\Rightarrow \ell n\ [1 + f(x)] = x$

$\Rightarrow$ 1 + f(x)) = e^x

$\Rightarrow$ f(x) = e^x – 1 $\Rightarrow$ f′(x) = e^xs

$\Rightarrow$ f′(1) = e′

$g(x) = \dfrac{f(x)}{x} = \dfrac{e^x-1}{x}$ $y'(0^+) = \lim\limits_{h\to 0}\dfrac{g(0+h)-g(0)}{h}$

If g(0) = 1 then

$$g'(0^+) = \lim_{h\to 0}\frac{\frac{e^h-1}{h}-1}{h} = \lim_{h\to 0}\frac{e^h-1-h}{h^2} = \frac{1}{2}$$

$$g'(0^-) = \lim_{h\to 0}\frac{g(0-h)-g(0)}{-h} = \lim_{h\to 0}\frac{e^{-h}-1}{\frac{-h}{-h}}$$

$$\lim_{h\to 0}\frac{e^{-h}-1+h}{h^2} = \frac{1}{2}$$

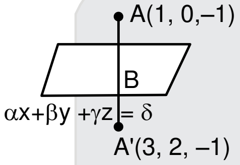
g(x) is differentiable

10. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\alpha+\gamma=1$. Suppose the point $(3, 2, -1)$ is the mirror image of the point $(1, 0, -1)$ with respect to the plane $\alpha x+\beta y+\gamma z=\delta$. Then which of the following statements is/are TRUE?

(A) $\alpha+\beta=2$ (B) $-\gamma=3$ (C) $\delta+\beta=4$ (D) $\alpha+\beta+\gamma=\delta$

Ans. **(ABC)**

Sol.



Find point at AA' = B(2, 1, –1)

$2\alpha+\beta-\gamma=\delta$ ... (1)

$\alpha+\gamma=1$ ....(2)

dr's of AA' (2, 2, 0)

$\frac{\alpha}{2}=\frac{\beta}{2}=\frac{\gamma}{0}=\lambda$

$\therefore\ 2\lambda+0=1 \quad \alpha=2\lambda,\ \beta=2\lambda,\ \gamma=0$

$\gamma=\frac{1}{2}$

$\therefore\ \alpha=1, \beta=1, \gamma=0, \delta=3$

Now (A) $\alpha+\beta+\gamma=1+1+0=2 \neq \delta$

(B) $\alpha+\beta+1+1+=2$

(C) $\gamma+\delta=0+3=3$

(D) $\delta+\gamma=-3-0=3$

11. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ}=a\hat{i}+b\hat{j}$ and $\overrightarrow{PS}=a\hat{i}-b\hat{j}$ are adjacent sides of a parallelogram PQRS. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w}=\hat{i}+\hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}|+|\vec{v}|=|\vec{w}|$ and if the area of the parallelogram PQRS is 8, then which of the following statements is/are TRUE?

(A) $a+b=4$

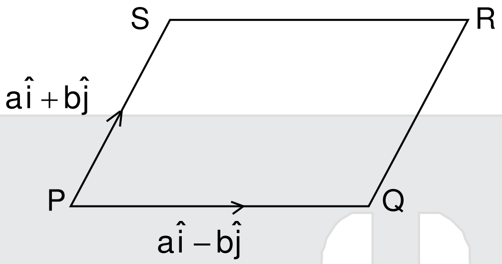
(B) $a-b=2$

(C) The length of the diagonal PR of the parallelogram $PQRS$ is 4

(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

Ans. **(AC)**

Sol.



$$\vec{x} = \left\{(\hat{i}+\hat{j}).\frac{a\hat{i}+b\hat{j}}{\sqrt{a^2+b^2}}\right\}\frac{a\hat{i}+b\hat{j}}{\sqrt{a^2+b^2}} = \frac{(a+b)}{\sqrt{a^2+b^2}}\cdot\frac{a\hat{i}+b\hat{j}}{\sqrt{a^2+b^2}}$$

$$\vec{v} = \left\{(\hat{i}+\hat{j}).\frac{a\hat{i}-b\hat{j}}{\sqrt{a^2+b^2}}\right\}\frac{a\hat{i}-b\hat{j}}{\sqrt{a^2+b^2}} = \frac{a-b}{\sqrt{a^2+b^2}}\times\frac{a\hat{i}-b\hat{j}}{\sqrt{a^2+b^2}}$$

given $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ a & -b & 0 \end{vmatrix} = 8$

$|\hat{k}\,(-ab - ab)| = 8$

$ab = 4$

Given $|\vec{x}| + |\vec{v}| = \vec{w}$

$$\frac{a+b}{\sqrt{a^2+b^2}}\ \frac{|a-b|}{\sqrt{a^2+B^2}} = \sqrt{2}$$

$a > b$

$2a = \sqrt{2}\sqrt{a^2+b^2}$

$2a^2 = a^2 + b^2$

$a + b = 4$

$a - b = 0$ diagonal $= \overrightarrow{PS} + \overrightarrow{PQ} = 2a\,\hat{i}$

length $= 2a = 4$

$a < b$

$2a = \sqrt{2}\ \sqrt{a^2+b^2}$

$a^2 = b^2 \Rightarrow \quad a = b = 2$

12. For nonnegative integers s and r, let

$$\binom{s}{r} = \begin{cases} \dfrac{s!}{r!(s-r)!} & \text{if } r \le s \\ 0 & \text{if } r > s \end{cases}$$

For positive integers m and n, let

$$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m, n, p)}{\binom{n+p}{p}}$$

where for any nonnegative integer p,

$$f(m, n, p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}$$

Then which of the following statements is/are TRUE?

(A) $g(m, n) = g(n, m)$ for all positive integers m, n

(B) $g(m, n+1) = g(m+1, n)$ for all positive integers m, n

(C) $g(2m, 2n) = 2\, g(m, n)$ for all positive integers m, n

(D) $(2m, 2n) = (g(m, n))^2$ for all positive integers m, n

Ans. **(ABD)**

Sol.
$$\frac{m!}{(m-1)!2!} \times \frac{n(+1)!}{(n+i-p)!} \times \frac{(p+n)!}{(p-1)!(n+1)!}$$

$$\frac{m!}{(m-1)!2!} \times \frac{(p+n)!}{p!n!} \times \frac{n!}{(n+1-p)!(p-1)!}$$

$$\frac{m!}{(m-1)!\,2!} \times {}^{p+n}C_p \,.\, {}^{n}C_{p-i}$$

$${}^{p+n}C_p . \sum_{i=0}^{p} {}^{m}C_1 . {}^{n}C_{p-i}$$

$${}^{p+n}C_p . \left[{}^{m}C_0 . {}^{n}C_p + {}^{m}C_1 . {}^{m}C_{p-1} + {}^{m}C_2 . {}^{n}C_{p-2} + ... {}^{m}C_p . {}^{n}C_0\right]$$

$$(1+x)^m . (1+x)^n$$

$$= \left({}^{m}C_0 + {}^{m}C_1 . x + {}^{m}C_2 x .....\right)\left({}^{n}C_0 + {}^{n}C_1 x + ... + {}^{n}C_p x^p + ... {}^{p+n}C_p . {}^{m+n}C_p\right)$$

$$g(m,n) = \sum_{p=0}^{m+n} {}^{m+n}C_p = 2^{m+n}$$

(A) $g(2m, 2n) = 2^{2m+2n}$

$2g(m, n) = 2.2^{m+n}$

(B) $g(m, n) = 2^{m+nH} = g\,(mH, n)$

(C) $g(2m, 2n) = 2^{2m+2n} = (2^{m+n})^2 = (g(m,n))^2$

(D) $g(m,n) = g(n,m)$

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+4** If ONLY the correct numerical value is entered.

Zero Marks : **0** In all other cases.

13. An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that **no** two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is _____

Ans. **495**

Sol. $^{15-4+1}C_4 = {}^{12}C_4 = \dfrac{12.11.10.9}{4.3.2.1} = 495$

14. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____

Ans. **1080**

Sol. $6 \rightarrow 1,1.2.2 \quad \dfrac{6!}{(1!1!2!2!)(2!2!)} \times 4!$

$= \dfrac{180 \times 24}{16} = 1080$

15. Two fair dice, each with faces numbered 1, 2, 3,4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If is the probability that this perfect square is an odd number, then the value of $14p$ is _____

Ans. **8**

Sol. Square $\rightarrow$ 4 or 9 $\rightarrow$ (1,3), (3,1) (2,2) , (6,3) (3,6), (5,4), (4,5)

Prime $\rightarrow$ 2 or 3 or 5 or 7 or 11

$\rightarrow$ (1,1) , (1,2), (2,1), (1,4), (4,1), (2,3) , (3,2), (1,6), (6,1), (5,2), (2,5), (3,4), (4,3), (6,5), (5,6)

P (square or prime) = $\frac{22}{36}$

$$P\left(\frac{\text{Perfect squre is odd}}{\text{perfect square before prime}}\right) = \frac{\frac{4}{36}+\frac{14}{36}\cdot\frac{4}{36}+\frac{14}{36}\cdot\frac{14}{36}\cdot\frac{14}{36}\cdot\frac{4}{36}+\ldots\ldots}{\frac{7}{36}+\frac{14}{36}\cdot\frac{7}{36}+\frac{14}{36}\cdot\frac{14}{36}\cdot\frac{7}{36}+\ldots\ldots}$$

$$\frac{\left(=\frac{\frac{4}{36}}{1-\frac{14}{36}}\right)}{\left(\frac{\frac{7}{36}}{1-\frac{14}{36}}\right)} = \frac{4}{7} = P$$

$\Rightarrow$ 14 P = 8

16. Let the function f: [0, 1] $\rightarrow$ R be defined by

$$f(x) = \frac{4^x}{4^x+2}$$

Then the value of

$$f\left(\frac{1}{40}\right)+f\left(\frac{2}{40}\right)+f\left(\frac{3}{40}\right)+\ldots\ldots+f\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$$

is _____

Ans. 19

Sol. $f(x)=\frac{4x}{4x+2}$, $f(1-x)=\frac{4^{1-x}}{4^{1-x}+2}=\frac{2}{4^x+2}$

$\Rightarrow$ f(x) + f(1 –x) = 1

$$f\left(\frac{1}{40}\right)+f\left(\frac{2}{40}\right)+\ldots..+f\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$$

$$=\left(f\left(\frac{1}{40}\right)+f\left(\frac{39}{40}\right)+\left(f\left(\frac{2}{40}\right)+f\left(\frac{38}{40}\right)+\ldots.+f\left(\frac{19}{40}\right)+f\left(\frac{21}{40}\right)+f\left(\frac{20}{40}\right)-f\left(\frac{1}{2}\right)\right)\right)$$

$$\Rightarrow \underbrace{1+1+\ldots\ldots\ldots+1}_{19\text{ times}} + f\left(\frac{1}{2}\right) - f\left(\frac{1}{2}\right) = 19$$

17. Let : R → R be a differentiable function such that its derivative f' is continuous and $f(\pi) = -6$.

If F : [0, π] → R is defined by $F(x) = \int_0^x f(t)\,dt$, and if $\int_0^\pi (f'(x)+F(x))\cos x\,dx = 2$

then the value of $f(0)$ is _____

Ans. 4

Sol. $I = \int_0^\pi (f'(x).\cos x + f(x).\cos x)dx = 2$

$= \int_0^\pi f'(x).\cos x.dx + \int_0^\pi f(x)\cos x.dx = 2$

$= (\cos x.f(x))_0^\pi - \int_0^\pi (-\sin x).f(x)dx + \int_0^\pi f(x).\cos x.dx = 2$

$\Rightarrow \quad (\cos\pi.f(\pi) - \cos) + \int_0^\pi \sin x.f'(x)dx + \int_0^\pi f(x).\cos x dx = 2$

$\Rightarrow \quad (-1).(-6) - f(0)) + + (\sin x.f(x))_0^\pi - \int_0^\pi \cos x.f(x)dx + \int_0^\pi f(x).\cos x.dx = 2$

$\Rightarrow \quad 6 - f(0) + (\sin\pi - f(p) - \sin 0.f(0)) = 2$

$\Rightarrow \quad f(0) = 4$

18. Let the function $f : (0, \pi) \to \mathbb{R}$ be defined by $f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$.
Suppose the function f has a local minimum at θ precisely when $\theta \in \{\lambda_1\pi, \ldots, \lambda_r\pi\}$, where $0 < \lambda_1 < \cdots < \lambda_r < 1$. Then the value of $\lambda_1 + \cdots + \lambda_r$ is _____

Ans. 0.5

Sol. $f(\theta) = 1 + \sin 2\theta + (1 - \sin 2\theta)^2$

$= \sin^2 2\theta - \sin 2\theta + 2$

$= \left(\sin 2\theta - \frac{1}{2}\right)^2 + \frac{7}{4}$

minima when $\sin 2\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\theta(10, \pi)$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$

$\lambda_1 = \frac{1}{12}, \lambda_2 = \frac{5}{12}$

sum = $\frac{1}{2}$

JEE (MAIN) 2020 RESULT

Few things remain unchanged forever...
So, is our tradition of delivering

Self-Earned & Owned Result

58.68%
Success % of Eligibility to appearance

14717
Classroom: 11047 | Distance: 3670

STUDENTS ELIGIBLE FOR JEE (ADVANCED) 2020

ALL INDIA RANKS (AIRs) IN TOP-200 FROM CLASSROOM

AIR-37
AVI KUMAR
With us since Class 11th

AIR-74
SANKALP PARASHAR
With us since Class 11th

AIR-87
ACHINTYA NATH
With us since Class 11th

AIR-124
UTKARSH P. SINGH
With us since Class 10th

AIR-129
AARYAN K. GUPTA
With us since Class 9th

STATE TOPPER WEST BENGAL

AIR-131
SREEMANTI DEY
With us since Class 11th

AIR-143
RAHUL KANOONGO
With us since Class 13th
हिन्दी माध्यम विद्यार्थी

AIR-148
DHANANJAY KEJRIWAL
With us since Class 9th

AIR-150
PRAKHAR BANSAL
With us since Class 11th

AIR-155
HARSH TRIVEDI
With us since Class 8th

AIR-161
DARSHAN RAKHEWAR
With us since Class 11th

AIR-167
JATIN K. GUPTA
With us since Class 11th

AIR-190
MIR MOHAMMAD ASIF
With us since Class 11th

AIR-198
KUSHAGRA GUPTA
With us since Class 11th

RESULT HIGHLIGHTS

7 AIRs
(AIR-53, 81, 91, 106, 113, 132, 142)
in TOP-200
Distance (DLP)

41 Students
(Classroom: 25 | Distance: 16)
with 100 %tile
in P/C/M

50 AIRs in TOP-500
(Classroom: 36 | Distance: 14)

109 AIRs in TOP-1000
(Classroom: 77 | Distance: 32)

4 AIRs
(SC: AIR-4, 8 & 10 | ST: AIR-9)
in TOP-10
Classroom