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JEE (ADVANCED) 2019

DATE: 27-05-2019

Questions & Solutions

PAPER – 1 SUBJECT:MATHEMATICS

MAX. MARKS: 186 **TIME: 3 HRS.**

PAPER-1 : INSTRUCTIONS TO CANDIDATES

- **Question Paper-1 has three (03) parts: Physics, Chemistry and Mathematics.**
- **Each part has a total eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3)**
- **Total number of questions in Question Paper-1 are Fifty Four (54) and Maximum Marks are One Hundred Eighty Six (186)**

Type of Questions and Marking Schemes

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

SECTION-2 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

SECTION-3 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered.

Zero Marks : **0** In all other cases.

Answering Questions :

- To select the option(s), use the mouse to click on the corresponding button(s) of the option(s).
- To deselect the chosen option for the questions of **SECTION-1**, click on the button of the chosen option again or click on the **Clear Response** button to clear the chosen option.
- To deselect the chosen option(s) for the questions of **SECTION-2**, click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.
- To change the option(s) of a previously answered question of **SECTION-1** and **SECTION-2** first deselect as given above and then select the new option(s).
- To answer questions of **SECTION-3**, use the mouse to click on numbers (and/or symbols) on the on–screen virtual numeric keypad to enter the numerical value in the space provided for answer.
- To change the answer of a question of **SECTION-3**, first click on the **Clear Response** button to clear the entered answer and then enter the new numerical value.
- To mark a question **ONLY** for review (i.e. without answering it), click on the **Mark for Review & Next** button.
- To mark a question for review (after answering it), click on **Mark for Review & Next** button – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next** button – the answered question will be evaluated.

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PART-III : MATHEMATICS

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–1** In all other cases.

खंड 1 (अधिकतम अंकः 12)

- इस खंड में चार **(04)** प्रश्न है।
- प्रत्येक प्रश्न के लिए सही चार विकल्प दिए गए हैं। इन चार विकल्पों में से **केवल एक** ही विकल्प सही उत्तर है।
- प्रत्येक प्रश्न के लिए दिए हुए विकल्पों में से सही उत्तर से संबधित विकल्प को चुनिए।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्न योजना के अनुसार होगा :

पूर्ण अंक : **+3** यदि सिर्फ सही विकल्प ही चुना गया है।
शून्य अंक : **0** यदि कोई भी विकल्प नहीं चुना गया है (अर्थात् प्रश्न अनुत्तरित है)।
ऋण अंक : **–1** अन्य सभी परिस्थितियों में।

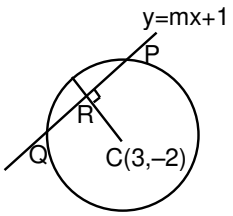
1. A line $y = mx + 1$ intersect the circle $(x – 3)^2 + (y + 2)^2 = 25$ at points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct

एक रेखा $y = mx + 1$ वृत्त $(x – 3)^2 + (y + 2)^2 = 25$ को बिन्दुओं P आरूैर Q पर प्रतिच्छेद करती है। अगर रेखाखण्ड (line segment) PQ के मध्यबिन्दु का x-निर्देशांक (x-corrdinate) $-\frac{3}{5}$ है, तब निम्नलिखित में से कौनसा एक विकल्प सही है ?

(1) $6 \le m < 8$ (2) $2 \le m < 4$ (3) $4 \le m < 6$ (4) $-3 \le m < -1$

Ans. **(2)**

Sol.



For point R, $x = -\frac{3}{5} \Rightarrow y = 1 - \frac{3m}{5}$ $\quad R\left(-\frac{3}{5}, 1-\frac{3m}{5}\right)$

slope of CR = $\frac{1-\frac{3m}{5}+2}{-\frac{3}{5}-3} = -\frac{1}{m} \Rightarrow \frac{15-3m}{-3-15} = -\frac{1}{m}$

$15m - 3m^2 = 18$

$m^2 - 5m + 6 = 0$

$m = 2, 3$

$2 \le m \le 4$

2. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers and I is the 2×2 identity matrix.

If α^* = is the minimum of the set $\{a(\theta) : \theta \in [0, 2\pi]\}$ and

β^* = is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$

Then the value of $\alpha^* + \beta^*$ is

माना कि $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

जहाँ $\alpha = \alpha(\theta)$ और $\beta = \beta(\theta)$ वास्तविक (real) संख्याएँ है, और I एक 2×2 तत्समक-आव्यूह (2×2 identity matrix) है। यदि

समुच्चय $\{a(\theta) : \theta \in [0, 2\pi]\}$ का निम्नतम (minimum) α^* है और

समुच्चय $\{\beta(\theta) : \theta \in [0, 2\pi)\}$ का निम्नतम (minimum) β^* है,

तो $\alpha^* + \beta^*$ का मान है -

(1) $\frac{-37}{16}$ (2) $\frac{-29}{16}$ (3) $\frac{-31}{16}$ (4) $\frac{-17}{16}$

Ans. **(2)**

Sol. $m = \sin^4\theta \cdot \cos^4\theta + (1 + \sin^2\theta)(1 + \cos^2\theta)$

$2 + \sin^4\cos^4\theta + \sin^2\theta\cos^2\theta$

$\begin{bmatrix} \sin^4\theta & -(1+\sin^2\theta) \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \beta = \frac{1}{|m|}\begin{bmatrix} \cos^4\theta & 1+\sin^2\theta \\ -1-\cos^2\theta & \sin^4\theta \end{bmatrix}$

$\sin^4\theta = \frac{\alpha+\beta}{|m|}\cos^4\theta$, $-1 - \sin^2\theta = \frac{\beta}{|m|}(1 + \sin^2\theta)$

$\beta = -|m|$

$\beta = -[\sin^4\theta\cos^4\theta + \sin^2\theta\cos^2\theta + 2] = -[t^2 + t + 2] \Rightarrow \beta_{min} = -\frac{37}{16}$

$\alpha = \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2}(\sin^2 2\theta) \quad \Rightarrow \quad \min\alpha = \frac{1}{2}$

$\alpha + \beta = -\frac{37}{16} + \frac{1}{2} = -\frac{37}{16} + \frac{8}{16} = -\frac{29}{16}$

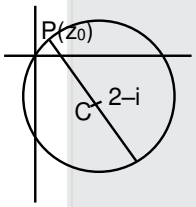
3. Let S be the set of all complex numbers z satisfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in s\right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is

माना कि S उन सभी सम्मिश्र संख्याओं (complex numbers) z का समुच्चय (set) है जो $|z - 2 + i| \geq \sqrt{5}$ को संतुष्ट करती है। यदि एक सम्मिश्र संख्या z_0 ऐसी है जिससे $\frac{1}{|z_0 - 1|}$ समुच्चय $\left\{\frac{1}{|z-1|} : z \in s\right\}$ का उच्चतम (maximum) है, तब $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ का मुख्य कोणांक (principal argument) है

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

Ans. **(3)**

Sol.



$|z - (2 - i)| \geq \sqrt{5}$

For $|z_0 - 1|$ to be minimum, $z_0 = x_0 + iy_0$ is at point P as shown in figure

$\arg\left(\frac{4 - (z_0 + \bar{z}_0)}{z_0 - \bar{z}_0 + 2i}\right) = \arg\left(\frac{4 - 2x}{2iy + 2i}\right) = \arg\left(\frac{-i(2-x)}{y+2}\right) = \arg(-i\lambda) = -\frac{\pi}{2} \quad (\because \lambda > 0)$

4. The area of the region $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is

क्षेत्र $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ का क्षेत्रफल (area) है -

(1) $16\log_e 2 - 6$ (2) $8\log_e 2 - \frac{7}{3}$ (3) $16\log_e 2 - \frac{14}{3}$ (4) $8\log_e 2 - \frac{14}{3}$

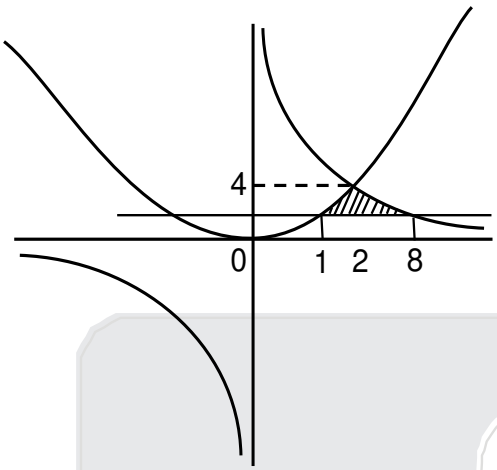
Ans. **(3)**

Sol. $xy \leq 8$

$1 \leq y \leq x^2$

$x^2 . x = 8$

$x = 2$



$$\text{Required Area} = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dy = \left[8\ell n\, y - \frac{y^{3/2}}{3/2}\right]_1^4 = 8\,\ell n4 - \frac{2}{3}\,.\,8 - 0 + \frac{2}{3} = 16\ell n2 - \frac{14}{3}$$

SECTION-2 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** In all other cases.

खंड 2 (अधिकतम अंक: 32)

- इस खंड में **आठ (08)** प्रश्न है।
- प्रत्येक प्रश्न के लिए चार विकल्प दिए गए हैं। इन चार विकल्पों में से **एक या एक से अधिक** विकल्प सही है(हैं)।
- प्रत्येक प्रश्न के लिए, दिए हुए विकल्पों में से सही उत्तर (उत्तरों) से संबधित विकल्प (विकल्पों) को चुनिए।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्न योजना के अनुसार होगा :

पूर्ण अंक	:	**+4** यदि केवल (सारे) सही विकल्प (विकल्पों) को चुना गया है।
आंशिक अंक	:	**+3** यदि चारों विकल्प सही हैं परन्तु केवल तीन विकल्पों को चुना गया है।
आंशिक अंक	:	**+2** यदि तीन या तीन से अधिक विकल्प सही है परन्तु केवल दो विकल्पों को चुना गया है और दोंनो चुने हुए विकल्प सही विकल्प हैं।
आंशिक अंक	:	**+1** यदि दो या दो से अधिक विकल्प सही हैं परन्तु केवल एक विकल्प को चुना गया है और चुना हुआ विकल्प सही विकल्प है।
शून्य अंक	:	**0** यदि किसी भी विकल्प को नहीं चुना गया है (अर्थात् प्रश्न अनुत्तरित है)।
ऋण अंक	:	**–1** अन्य सभी परिस्थितियों में।

1. Define the collectionss $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles of follows :

$E_1 : \frac{x^2}{9}+\frac{y^2}{4} = 1;$

R_1 : rectangle of largest area with sides parallel to the axes, inscribed in E_1:

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$

Then which of the following options is/are correct ?

(1) The eccentricities of E_{18} and E_{19} are NOT equal

(2) The length of latus rectum of E_9 is $\frac{1}{6}$

(3) $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N

(4) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

दीर्घवृत्तों (Ellipses) $\{E_1, E_2, E_3, ...\}$ और आयतों (rectangles) $\{R_1, R_2, R_3, ...\}$ के संग्रहों को निम्न प्रकार से परिभाषित करें :

$E_1 : \frac{x^2}{9}+\frac{y^2}{4} = 1;$

R_1 : अधिकतम क्षेत्र (largest area) का आयत, जिसकी भुजाएं अक्षों (axes) के समान्तर है, और जो E_1 में अंतर्स्थित (inscribed) है

E_n : अधिकतम क्षेत्र वाला दीर्घवृत्त $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ जो R_{n-1}, $n > 1$ में अंतरिर्स्थत है

R_n : अधिकतम क्षेत्र का आयत, जिसकी भुजाएं अक्षों के समान्तर है, और जो E_n, $n > 1$ में अंतर्स्थित है।

तब निम्न में से कौनसा (से) विकल्प सही है (है) ?

(1) E_{18} और E_{19} की उत्केन्द्रतायें (eccentricities) समान **नहीं** है

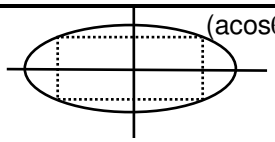
(2) E_9 के नाभिलम्ब (latus rectum) की लम्बाई $\frac{1}{6}$ है

(3) प्रत्येक पूर्णांक N के लिए $\sum_{n=1}^{N}$ (R_n का क्षेत्रफल) < 24 है

(4) E_9 में केन्द्र से एक नाभि (focus) की दूरी $\frac{\sqrt{5}}{32}$ है

Ans. **(2,3)**

Sol.



Area Max when $\theta = 45°$

	a	b
E_1	3	2
E_2	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
E_3	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{(\sqrt{2})^2}$
⋮	⋮	⋮
E_9	$\frac{3}{(\sqrt{2})^8}$	$\frac{2}{(\sqrt{2})^8}$

(A) $E_1 + E_2 + ... + E_m$

when $m \to \infty$ $\frac{2ab}{1-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}} = 4ab = 4.3.2 = 24$

(B) Length of LR is ellipse = $\frac{2b^2}{a} = 2.\frac{4.2^4}{2^8.3} = \frac{1}{6}$

(C) distance between focus and center of ellipse = $a_9 e_9 = \frac{3}{2^4}.\frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$

2. In a non-right-angled triangle ΔPQR, Let p, q, r denote the lengths of the sies opposite to the angles at P, Q, R respectively. The median form R meets the side PQ at S, the perpendicualr4 from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ?

(1) Length of RS = $\frac{\sqrt{7}}{2}$ (2) Area of ΔSOE = $\frac{\sqrt{3}}{12}$

(3) Radius of incircle of ΔPQR = $\frac{\sqrt{3}}{2}(2-\sqrt{3})$ (4) Length of OE = $\frac{1}{6}$

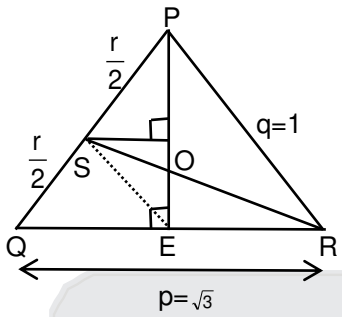
एक असमकोणीय त्रिभुज (non-right-angled) ΔPQR के लिए, माना कि p, q, r क्रमशः कोण P, Q, R के सामने वाली भुजाओं की लम्बाइयाँ दर्शायी है। R से खींची गयी माध्यिका (median) भुजा PQ से S पर मिलती है, P से खींचा गया अभिलम्ब (perpendicular) भुजा QR से E पर मिलता है तथा RS और PE एक दूसरे को O पर काटती है। यदि $p = \sqrt{3}$, $q = 1$ और ΔPQR के परिवृत्त (circumcircle) की त्रिज्या (radius) 1 है, तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) RS की लम्बाई = $\frac{\sqrt{7}}{2}$ (2) ΔSOE का क्षेत्रफल (area) = $\frac{\sqrt{3}}{12}$

(3) ΔPQR के अंतर्वृत (incircle) की त्रिज्या = $\frac{\sqrt{3}}{2}(2-\sqrt{3})$ (4) OE की लम्बाई = $\frac{1}{6}$

Ans. **(1,3,4)**

Sol.



$$\frac{p}{\sin P}=\frac{q}{\sin Q}=2(1) \quad \Rightarrow \quad \sin P=\frac{\sqrt{3}}{2},\ \sin Q=\frac{1}{2}$$

$\Rightarrow$ $\angle P = 60°$ or $120°$ and $\angle Q = 30°$ or $150°$

because $\angle P + \angle Q$ must be less than $180°$ but not equal to $90°$

$\angle P = 120°$ and $\angle Q = 30°$ and $\angle R = 30°$ $\qquad \frac{r}{\sin R}=2 \Rightarrow r=1$

Now length of median $RS=\frac{1}{2}\sqrt{2p^2+2q^2-r^2}=\frac{1}{2}\sqrt{6+2-1}=\frac{\sqrt{7}}{2}\Rightarrow$ option (A) is correct

$$\text{Inradius}=\frac{2\Delta}{p+q+r}=\frac{\frac{2pqr}{4\times(1)}}{p+q+r}=\frac{1}{2}\left(\frac{1\times1\times\sqrt{3}}{1+1+\sqrt{3}}\right)=\frac{\sqrt{3}}{2}\left(\frac{2-\sqrt{3}}{1}\right)\Rightarrow \text{option (C) is correct}$$

$$\Rightarrow \quad \frac{1}{2}\times\sqrt{3}\times PE=\frac{pqr}{4(1)}\ (\text{equal area of }\Delta) \quad \Rightarrow \quad PE=\frac{1\times1\times\sqrt{3}}{4}\times\frac{2}{\sqrt{3}}=\frac{1}{2}$$

$$\Rightarrow \quad OE=\frac{2(\text{Area of }\Delta OQR)}{QR}=\frac{2\times\frac{1}{3}\left(\frac{1}{2}.1.\sqrt{3}\sin30°\right)}{\sqrt{3}}=\frac{1}{6}$$

3. Let $f : R \to R$ be given by

$$f(x)=\begin{cases} x^5+5x^4+10x^3+10x^2+3x+1, & x<0 \\ x^2-x+1 & 0\le x<1 \\ (2/3)x^3-4x^2+7x-(8/3) & 1\le x<3 \\ (x-2)\ell n(x-2)-x+(10/3) & x\ge 3 \end{cases}$$

Then which of the following options is/are Correct ?

(1) f ′ is NOT differentiable at x = 1

(2) f is increasing on $(-\infty,0)$

(3) f is onto

(4) f ′ has a local maximum at x = 1

माना कि $f : R \to R$ निम्न प्रकार से दिया है

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ (2/3)x^3 - 4x^2 + 7x - (8/3) & 1 \le x < 3 \\ (x-2)\ell n(x-2) - x + (10/3) & x \ge 3 \end{cases}$$

तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) $x = 1$ पर f' अवकलनीय **नहीं** (NOT differentiable) है।

(2) f अंतराल $(-\infty,0)$ में वर्धमान (increasing) है

(3) f आच्छादक (onto) है।

(4) f' का एक स्थानीय उच्चतम (local maximum) $x = 1$ पर है

Ans. **(1,3,4)**

Sol. $$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1 & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & 1 \le x < 3 \\ (x-2)\ln(x-2) - x + \frac{10}{3} & x \ge 3 \end{cases}$$

$$f'(x) = \begin{cases} 5(x+1)^4 - 2 & x < 0 \\ 2x - 1 & 0 \le x < 1 \\ 2x^2 - 8x + 7 & 1 \le x < 3 \\ \ln(x-2) & x \ge 3 \end{cases}$$

$x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1$ takes value between $-\infty$ to 1

Also $(x-2)\ln(x-2) - x + \frac{10}{3}$ takes value between $\frac{1}{3}$ to ∞

So, range of f(x) is R. So option (A) is correct

$f''(1^-) = 2$ and $f''(1^+) = -4$

so $f'(x)$ is non-diff at $x = 1$ so option (B) is correct

$f'(x)$ has local maxima at $x = 1$ so option (C) is correct

4. Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$. For alll postive integers n. define

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \ge 1$

$b_1 = 1$ and $b_n = a_n = a_{n-1} + a_{n+1}$, $n \ge 2$

the which of the follwing options is/are correct ?

(1) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(2) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

(3) $a_1 + a_2 + \ldots a_n = a_{n+2} - 1$ for all $n \ge 1$

(4) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

माना कि $x^2 - x - 1 = 0$ के मूल (roots) α और β हैं, जहाँ $\alpha > \beta$ है। सभी धनात्मक पूर्णांकों n के लिए निम्न को परिभाषित किया गया है

$a_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$

$b_1 = 1$ और $b_n = a_n = a_{n-1} + a_{n+1}$, $n \geq 2$

तब निम्न में से कौनसा (से) विकल्प सही है (हैं)-

(1) $\sum_{n=1}^{\infty} \dfrac{a_n}{10^n} = \dfrac{10}{89}$ (2) प्रत्येक $n \geq 1$ के लिए, $b_n = \alpha^n + \beta^n$

(3) प्रत्येक $n \geq 1$ के लिए, $a_1 + a_2 + \ldots a_n = a_{n+2} - 1$ (4) $\sum_{n=1}^{\infty} \dfrac{b_n}{10^n} = \dfrac{8}{89}$

Ans. **(1,2,3)**

Sol. (1) $b_n = a_{n+1} + a_{n-1} = \dfrac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} + \dfrac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = \dfrac{\alpha^{n-1}(\alpha^2 + 1) - \beta^{n-1}(\beta^2 + 1)}{\alpha - \beta}$

$= \dfrac{\alpha^{n-1}(\alpha + 2) - \beta^{n-1}(\beta + 2)}{\alpha - \beta} = \dfrac{\alpha^{n-1}\left(\dfrac{5 + \sqrt{5}}{2}\right) - \beta^{n-1}\left(\dfrac{5 - \sqrt{5}}{2}\right)}{\alpha - \beta}$

$= \dfrac{\sqrt{5}\alpha^{n-1}\left(\dfrac{\sqrt{5} + 1}{2}\right) - \sqrt{5}\beta^{n-1}\left(\dfrac{\sqrt{5} - 1}{2}\right)}{\alpha - \beta} = \dfrac{\sqrt{5}(\alpha^n + \beta^n)}{\alpha - \beta} = \alpha^n + \beta^n$ $\because$ $\alpha - \beta = \sqrt{5}$

(2) $\sum_{n=1}^{\infty} \dfrac{b_n}{10^n} = \sum\left(\dfrac{\alpha}{10}\right)^n + \sum\left(\dfrac{\beta}{10}\right)^n = \dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} + \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \dfrac{\alpha}{10 - \alpha} + \dfrac{\beta}{10 - \beta}$

$= \dfrac{10(\alpha + \beta) - 2\alpha\beta}{100 - 10(\alpha + \beta) + \alpha\beta} = \dfrac{10 + 2}{89} = \dfrac{12}{89}$

(3) $\sum_{n=1}^{\infty} \dfrac{a_n}{10^n} = \sum \dfrac{\alpha^n - \beta^n}{(\alpha - \beta)10^n} = = \dfrac{1}{\alpha - \beta}\left(\dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}\right) \dfrac{1}{\alpha - \beta}\left(\dfrac{\alpha}{10 - \alpha} - \dfrac{\beta}{10 - \beta}\right)$

$= \dfrac{1}{\alpha - \beta} \cdot \dfrac{(10(\alpha - \beta) - \alpha\beta + \alpha\beta)}{100 - 10(\alpha + \beta) + \alpha\beta} = \dfrac{10}{89}$ Option (C) is correct.

(4) $a_1 + a_2 + \ldots . a_n = \Sigma a_i = \dfrac{\Sigma\alpha^i - \Sigma\beta^i}{\alpha - \beta} = \dfrac{\frac{\alpha(1 - \alpha^n)}{(1 - \alpha)} - \frac{\beta(1 - \beta^n)}{(1 - \beta)}}{\alpha - \beta}$

$= \dfrac{(\alpha + 1)(1 - \alpha^n) - (\beta + 1)(1 - \beta^n)}{(1 - \alpha)(1 - \beta)(\alpha - \beta)} = \dfrac{\alpha^2 - \alpha^{n+2} - \beta^2 + \beta^{n+2}}{(1 - \alpha)(1 - \beta)(\alpha - \beta)} = \dfrac{\sqrt{5} + \beta^{n+2} - \alpha^{n+2}}{\beta - \alpha} = -1 + a_{n+2}$

5. Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k})$, $\lambda \in R$ and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k})$, $\mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ?

माना कि L_1 और L_2 क्रमशः निम्न रेखाएं है : $\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k})$, $\lambda \in R$ और $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k})$, $\mu \in R$ यदि L_3 एक रेखा है जो L_1 और L_2 दोनों के लम्बवत है और दोनों को काटती है, तब निम्नलिखित विकलपों में से कौन सा (से) L_3 को निरूपित करता (करते) है (हैं) ?

(1) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(2) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(3) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(4) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

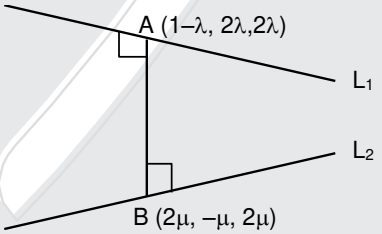
Ans. **(1,2,3)**

Sol. Both given lines are skew lines.

So direction ratios of any line perpendicular to these lines are $6\hat{i} + 6\hat{j} - 3\hat{k}$

<2, 2, –1>

Points at shortest distance between given lines are



$\overrightarrow{AB} \perp$ line L_1

$\overrightarrow{AB} \perp$ line L_2

So $A\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$

Now equation of required line $\vec{r} = \left(\frac{8}{9}\hat{i} + \frac{2}{9}\hat{j} + \frac{2}{9}\hat{k}\right) + \alpha\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

Now by option B, C, D are correct.

6. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

(1) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(2) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

(3) Probability that the chosen ball is green equals $\frac{39}{80}$

(4) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{3}{10}$

तीन थैले (bags) B_1, B_2 और B_3 है। B_1 थैले में 5 लाल (red) और 5 हरी (green) गेंदे हैं, B_2 में 3 लाल ओर 5 हरी गेंदे है, और B_3 में 5 लाल और 3 हरी गेंदे है। थैले B_1, B_2 और B_3 के चुने जाने की प्रायिकतायें क्रमशः 3/10, 3/10 और 4/10 हैं। एक थैला यादृच्छिक (at random) लिया जाता है और एक गेंद उस थैले में से यादृच्छिया चुनी जाती है। तब निम्न में से कौनसा (से) विकलप सही है (है) ?

(1) चुनी गयी गेंद के हरे होने की प्रायिकता $\frac{3}{8}$ है, जब यह ज्ञात है कि चुना हुआ थैला B_3 है

(2) चुने हुए थैले के B_3 होने की प्रायिकता $\frac{5}{13}$ है, जब यह ज्ञात है कि चुनी गयी गेंद हरी है

(3) चुनी गयी गेंद के हरे होने की प्रायिकता $\frac{39}{80}$ है

(4) चुने हुए थैले के B_3 होने के साथ साथ गेंद के हरे होने की प्रायिकता $\frac{3}{10}$ है

Ans. **(1,3)**

Sol.

	Bag_1	Bag_2	Bag_3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

(1) $P(\text{Ball is Green}) = P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3)$

$$= \frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}=\frac{39}{80}$$

(2) $P(\text{Ball chosen is Green/ Ball is from } 3^{rd} \text{ Bag}) = \frac{3}{8}$

(3,4) $P(\text{Ball is from } 3^{rd} \text{ Bag / Ball chosen is Green}) = \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3)}$

$P(B_1) = \frac{3}{10}$

$P(B_2) = \frac{3}{10}$

$$P(B_3) = \frac{4}{10} = \frac{\frac{4}{10}\times\frac{3}{8}}{\frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}}=\frac{4}{13}$$

7. Let Γ denote a curve y = f(x) which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y-axis at Y_P. If PY_p has length 1 for each point P on Γ. Then which of the following options is/are correct?

माना कि Γ एक वक्र y = f(x) है जो प्रथम चतुर्थांश (first quadrant) में है ओर माना कि बिन्दु (1, 0) उस पर स्थित है। माना कि Γ के बिन्दु P पर खींची गयी स्पर्श रेखा (tangent) y-अक्ष को Y_P. पर प्रतिच्छेद (intersect) करती है। यदि Γ के प्रत्येक बिन्दु P के लिए PY_p की लम्बाई 1 है, तब निम्न में से कौन सा (से) कथन सही है (है) ?

(1) $y = -\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(2) $xy' + \sqrt{1-x^2} = 0$

(3) $xy' - \sqrt{1-x^2} = 0$

(4) $y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

Ans. **(2,4)**

Sol. (a, f(a)) ≡ r

f'(x) be differentiation of f(x) equation of tangent

(y – f(a)) = f'(a)(x – a)

put x = 0

y – f(a) = –af'(a)

y = f(a) – af'(a)

y_p = (0, f(a) – af'(a))

$py_p = \sqrt{a^2 + (af'(a))^2} = 1$

$a^2 + a^2 (f'(a))^2 = 1$

$(f'(a))^2 = \frac{1-a^2}{a^2}$

$\int (f'(x)) = \pm\int \sqrt{\frac{1-x^2}{x^2}}$

put $\sqrt{1-x^2} = t$

$\Rightarrow \quad y = \pm\int \frac{-t^2 dt}{1-t^2} = \pm\left(t - \frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right|\right) + c = \pm\left(t - \frac{1}{2}\ell n\frac{(1+t)^2}{1-t^2}\right) + c = \pm\left(\sqrt{1-x^2} - \ell n\frac{1+\sqrt{1-x^2}}{x}\right) + c$

$\Rightarrow$ Put x = 1 and y = 0 $\Rightarrow$ c = 0

8. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

where a and b are real numbers. Which of the following options is/are correct ?

(1) $\det(\text{adj } M^2) = 81$ (2) $a + b = 3$

(3) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$ (4) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

माना कि $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ और adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

जहाँ a और b वास्तविक संख्याएँ (real numbers) है। निम्न में से कौन सा (से) विकल्प सही है (है) ?

(1) $\det(\text{adj } M^2) = 81$ (2) $a + b = 3$

(3) यदि $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, तब $\alpha - \beta + \gamma = 3$ (4) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

Ans. **(2,3,4)**

Sol. $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and और adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

$(\text{adjM})_{11} = 2 - 3b$, $(\text{adjM})_{22} = -3a$

$\Rightarrow 2 - 3b = -1$

$\Rightarrow b = 1$ & $-3a = -6 \Rightarrow a = 2$

$|\text{adjM}| = -1(6 - 6) - 1(-8 + 10) - 1(24 - 30) = 4$

$\det\{\text{adj}(M^2)\} = |\det(\text{adjM})|^2 = 16$

Now अब $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \beta + 2\gamma = 1,\ \alpha + 2\beta + 3\gamma = 2,\ 3\alpha + \beta + \gamma = 3$

On solving $\alpha = 1, \beta = -1, \gamma = 1$ so $\alpha - \beta + \gamma = 3$

Now $(\text{adj } M)^{-1} + (\text{adj } M)^{-1} = 2(\text{adj } M)^{-1} = \dfrac{2\text{adj}(\text{adjM})}{|\text{adj M}|} = \dfrac{1}{2}|M|^{3-2}M = -M$

SECTION-3 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered.

Zero Marks : **0** In all other cases.

खंड 3 (अधिकतम अंक: 18)

- इस खंड में **छः (06)** प्रश्न है। प्रत्येक प्रश्न का उत्तर एक संख्यात्मक मान **(NUMERICAL VALUE)** हैं।
- प्रत्येक प्रश्न के उत्तर के सही संख्यात्मक मान को माउज (mouse) और ऑन–स्क्रीन (on–screen) वर्चुअल नुमेरिक कीपेड (virtaul numeric keypad) के प्रयोग से उत्तर के लिए चिन्हित स्थान पर दर्ज करें। यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है , तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।
- प्रत्येक प्रश्न के उत्तर का मूल्यांकन निम्न योजना के अनुसार होगा :–

पूर्ण अंक : **+3** यदि दर्ज किया गया संख्यात्मक मान **(Numerical value)** ही सही उत्तर है।

शून्य अंक : **0** अन्य सभी परिस्थितियों में।

1. Three lines are given by

$$\vec{r} = \lambda\hat{i},\ \lambda \in R$$

$$\vec{r} = \mu(\hat{i}+\hat{j}),\ n \in R \text{ and } \vec{r} = \nu(\hat{i}+\hat{j}+\hat{k}),\ \nu \in R$$

Let the lines cut the plane x + y + z = 1 at he points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\underline{\Delta})^2$ equals .....

तीन रेखाएं क्रमशः

$$\vec{r} = \lambda\hat{i},\ \lambda \in R$$

$$\vec{r} = \mu(\hat{i}+\hat{j}),\ n \in R \text{ and } \vec{r} = \nu(\hat{i}+\hat{j}+\hat{k}),\ \nu \in R$$

द्वारा दी गयी हैं। माना कि रेखाएं समतल (plane) x + y + z = 1 को क्रमशः बिन्दुओं A, B और C पर काटती हैं। यदि त्रिभुज ABC का क्षेत्रफल Δ है तब $(6\underline{\Delta})^2$ का मान बराबर .....

Ans. **(0.75)**

Sol. Put $(\lambda, 0, 0)$ in $x + y + z = 1 \quad \Rightarrow \lambda = 1 \Rightarrow P(1, 0, 0)$

Put $(\mu, \mu, 0) \quad \Rightarrow 2\mu = 1 \quad \Rightarrow Q\left(\frac{1}{2},\frac{1}{2},0\right)$

Put $(\gamma,\gamma,\gamma) \quad \Rightarrow \gamma = \frac{1}{3} \quad \Rightarrow R\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Area of triangle PQR $= \frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \frac{1}{2}\left|\left(\frac{\hat{i}-\hat{j}}{2}\right)\times\left(\frac{2\hat{i}-\hat{j}-\hat{k}}{3}\right)\right| = \frac{1}{12}\left|\hat{i}+\hat{j}+\hat{k}\right| = \frac{\sqrt{3}}{12} \Rightarrow (6\Delta)^2 = 0.75$

2. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S :$ Sum of entries of A is 7$\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals _____

माना कि S ऐसे 3 × 3 आव्यूहों (matrices) का प्रतिदर्श समिष्ट (sample space) है जिनकी प्रविष्टियाँ (entries) समुच्चय {0, 1} से है। माना कि घटनाएँ E_1 एवं E_2 निम्न हैं

$E_1 = \{A \in S : \det A = 0\}$ और

$E_2 = \{A \in S :$ A की प्रतिष्टियों का कुल योग 7 है$\}$

यदि एक आव्यूह S से यादृच्छिक (randomly) चुना जाजा है तब सप्रतिबंध प्रायिकता (conditional probability) $P(E_1|E_2)$ बराबर _____

Ans. **(0.50)**

Sol. E_2 : Sum of elements of A = 7 $\Rightarrow$ These are 7 ones and 2 zeros

Number of such matrices = ${}^9C_2 = 36$.

Out of all such matrices ; E_1 will be those when both zeros lie in the same row or in the same colume

eg. $\begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 1 & 1\end{bmatrix}$

$n(E_1 \cap E_2) = 2 \times {}^3C_2 \times {}^3C_2 = 18$

$\uparrow \quad \uparrow$

So $n(E1/E2) = \frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2}$

3. That $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2$; a, b, c are distinct non zero integers$\}$ equals _____.

माना कि $\omega \neq 1$ एकक का एक घनमूल (a cube root of unity) है। तब समुच्चय (set) $\{|a + b\omega + c\omega^2|^2$; a, b, c भिन्न अशून्य पूर्णांक (distinct non zero integers)$\}$ का निम्नतम (minimum) बराबर _____.

Ans. **(3.00)**

Sol. $|a + b\omega + c\omega^2|^2 = a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

it will be minimum when a,b,c are consecutive integers

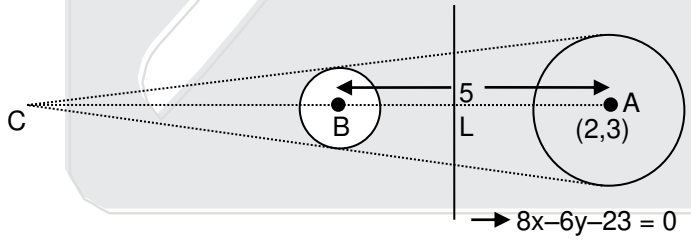
so minimum value is 3.

4. Let the point B be the reflection of the point A(2, 3) with respect to the line 8x – 6y – 23 = 0. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is…..

माना कि बिन्दु B रेखा 8x – 6y – 23 = 0 के सापेक्ष बिन्दु A(2, 3) का प्रतिबिम्ब (reflection) है। माना कि Γ_A और Γ_B क्रमशः त्रिज्याएं 2 और 1 वाले वृत्त हैं, जिनके केन्द्र क्रमशः A और B हैं। माना कि वृत्तों Γ_A और Γ_B की एक ऐसी उभयनिष्ठ स्पर्श (common tangent) रेखा T हैं, दोनो वृत्त जिसके एक ही तरफ हैं। यदि C बिन्दुओं A और B से जाने वाली रेखा और T का प्रतिच्छेद बिन्दु है, तब रेखाखण्ड (line segment) AC की लम्बाई है_____

Ans. **(10.00)**

Sol.



$$AL = \left|\frac{16 - 18 - 23}{10}\right| = \frac{5}{2}$$

$$\frac{CB}{CA} = \frac{1}{2}$$

$$\frac{CA - 5}{CA} = \frac{1}{2}$$

$CA = 10$

5. Let AP (a, d) denote the set of all the term of an infinite arithmetic progression with first term a and common difference d > 0. If

AP(1, 3) ∩ AP(2, 5) ∩ AP(3, 7) = AP(a, d)

then a + d equals……

माना कि AP (a, d) एक अनंत समान्तर श्रेणी (infinite arithmetic progression) के पदों का समुच्चय (set) है जिसका प्रथम पद a तथा सार्वअन्तर (common difference) d > 0 है। यदि

AP(1, 3) ∩ AP(2, 5) ∩ AP(3, 7) = AP(a, d)

है, तब a + d बराबर……

Ans. **(157.00)**

Sol. First series is {1, 4, 7, 10, 13,……..}

Second series is {2, 7, 12, 17, …….}

Third series is {3, 10, 17, 24, ……}

See the least number in the third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5.

⇒ 52 is the least number of third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5

Now, A = 52

D is L.C. M. of (3, 5, 7) = 105

⇒ A + D = 52 + 105 = 157

6. $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then find $27I^2$ equals …….

यदि $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ तब $27I^2$ बराबर है

Ans. **(4.00)**

Sol. $$I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)} \quad \ldots.(1)$$

by a + b – x property

$$I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{dx}{(1+e^{-\sin x})(2-\cos 2x)} = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{e^{\sin x}dx}{(1+e^{\sin x})(2-\cos 2x)}dx \quad \ldots..(2)$$

adding (1) and (2)

$$2I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{(1+e^{\sin x})}{(1+e^{\sin x})(2-\cos 2x)}dx \Rightarrow \quad I = \frac{1}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{1}{2-(2\cos^2 x-1)}dx = \frac{1}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\sec^2 x}{3\sec^2 x-2}dx$$

put tanx = t, $\sec^2 x dx = dt$

$$= \frac{2}{\pi}\int_0^1 \frac{dt}{3t^2+1} = \frac{2}{3\pi}\frac{1}{\left(\frac{1}{\sqrt{3}}\right)}\left(\tan^{-1}\left(\frac{t}{1/\sqrt{3}}\right)\right)_0^1 = \frac{2}{\sqrt{3}\,\pi}\left(\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}(0)\right) = \frac{2}{\sqrt{3}\,\pi}\left(\frac{\pi}{3}\right) = \frac{2}{3\sqrt{3}}$$

Now $27I^2 = 27 \times \frac{4}{27} = 4$

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