Resonance
Educating for better tomorrow

IIT-JEE 2012 : PAPER-2

Date : 08-04-2012 **Duration : 3 Hours** **Max. Marks : 198**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS :

A. General

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of the examination. The upper sheet is a rnachine-gradable Objective Response *Sheet* (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that and all the 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS

9. The ORS has **CODES** printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET**. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS (R4).

C. Question Paper Format

The question paper consists of 3 **parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. **Section I** contains **8 multiple choice questions**. Each question has four choices (A). (B), (C) and (D) out of which **ONLY ONE is correct.**
13. **Section II** contains **3 paragraphs** each describing theory, experiment, data etc. There are **6 multiple choice questions** relating to three paragraphs with **2 questions on each paragraph.** Each question of a particular paragraph has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
14. **Section III** contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY or MORE are correct.**

D. Marking Scheme

15. For each question in **Section I** and **Section II**, you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero (0) marks** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded in these sections.
16. For each question in **Section III**, you will be awarded **4 marks** If you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer(s) in this section.

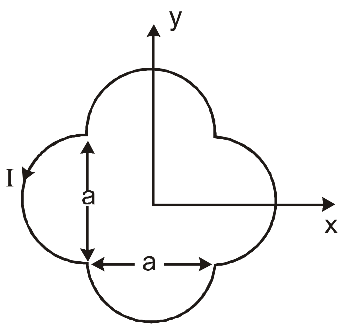
Write your Name, Registration Number and sign in the space provided on the back page of this booklet

PART - I : PHYSICS

SECTION - I : Single Correct Answer Type

This section contains 8 multiple choice questions, Each question has four choices, (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. A loop carrying current I lies in the x-y plane as shown in the figure. the unit vector $\hat{k}$ is coming out of the plane of the paper. the magnetic moment of the current loop is :



(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

Ans. **(B)**

Sol. Area = $a^2 + 4\times\frac{\pi\left(\frac{a}{2}\right)^2}{2}$

$= a^2 + \frac{\pi a^2}{2}$

$A = \left(1+\frac{\pi}{2}\right) a^2\ \hat{k}$

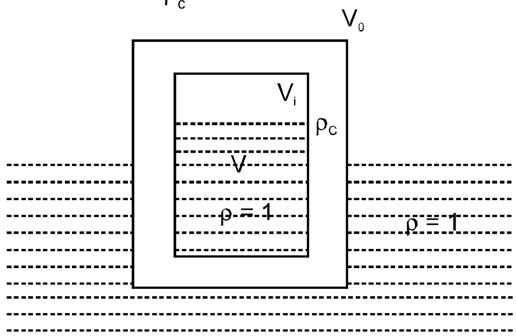
2. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half filled if ρ_c is less than 0.5 (B) more than half filled if ρ_c is less than 1.0

(C) half filled if ρ_c is less than 0.5 (D) less than half filled if ρ_c is less than 0.5

Ans. **(A)**

Sol.



Let outer volume of shell is V_0
Let inner volume of shell is V_i
Let volume of water inside the shall is v.

$\Rightarrow 1Vg + \rho_C (V_0 - V_i)g = \frac{1V_0}{2} g$ [Equlibrium]

$V + \rho_C (V_0 - V_i) = \frac{V_0}{2}$

$$\rho_C (V_0 - V_i) = \frac{V_0}{2} - V$$

$$\rho_C = \frac{\frac{V_0}{2} - V}{V_0 - V_i}$$

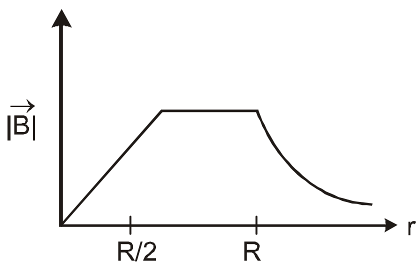
if $\quad \rho_C < \frac{1}{2} \quad \Rightarrow \quad \frac{\frac{V_0}{2} - V}{V_0 - V_i} < \frac{1}{2}$

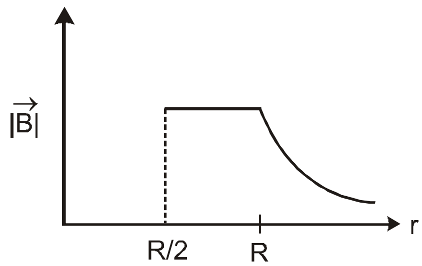
$$\frac{V_0}{2} - V < \frac{V_0}{2} - \frac{V_i}{2}$$

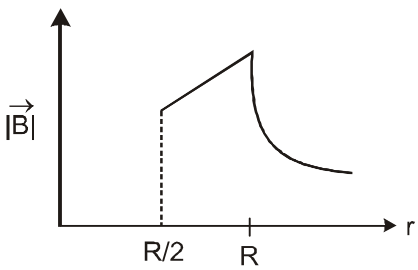
$$-V < -\frac{V_i}{2}$$

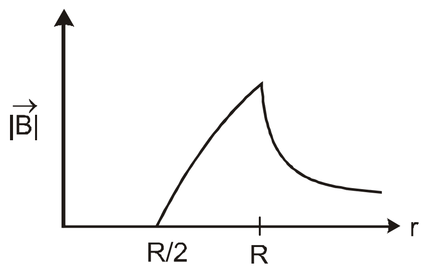
$V > \frac{V_i}{2}$ so (A)

3. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along is length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by :

(A)


(B)


(C)


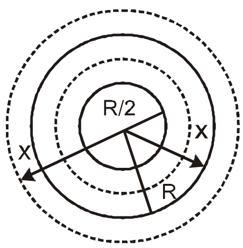
(D)


Ans. **(D)**

Sol. **Case-I** $x < \frac{R}{2}$

$|B| = 0$

Case-II $\frac{R}{2} \le x < R$



$$\int \vec{B}.d\vec{\ell} = \mu_0 I$$

$$|B|\ 2\pi x = \mu_0 \left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

$$|B| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right)$$

Case-III $x \geq R$

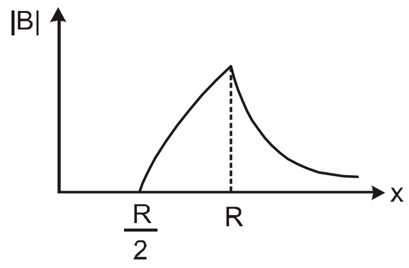
$\int \vec{B}.\, d\vec{\ell} = \mu_0 I$

$$|B|\, 2\pi x = \mu_0 \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

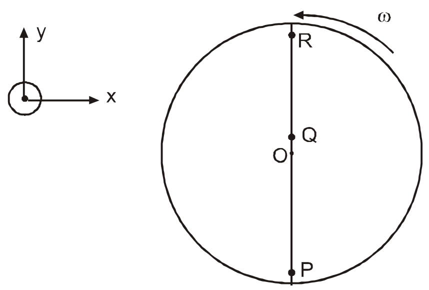
$$|B| = \frac{\mu_0 J}{2x}\ \frac{3}{2}\ R^2$$

$$|B| = \frac{3\mu_0 J R^2}{8x}$$

so



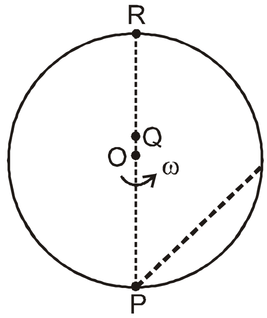
4. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc completed $\frac{1}{8}$ rotation. (ii) their range is less than half disc radius, and (iii) ω remains constant throughout . Then



(A) P lands in the shaded region and Q in the unshaded region
(B) P lands in the unshaded region and Q in the shaded region
(C) Both P and Q land in the unshaded region
(D) Both P and Q land in the shaded region

Ans. **(C)** **IIT answer C or D.**

Sol.



Since distance of particle P from point O is initially decreasing then increasing so, its angular velocity will initially increase then decrease. So, angle swept by P is more than angle swept by disc. So it will fall in unshaded portion.

Since distance of particle Q from O is continuously increasing so its ω is continuously decreasing. So angle swept by Q is less than angle swept by disc. So it will fall in unshaded portion.

5. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4cm . The distance frequency of the tuning for k is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is

(A) 14.0 (B) 15.2 (C) 16.4 (D) 17.6

Ans. **(B)**

Sol. $\dfrac{V}{4(\ell+e)} = f$

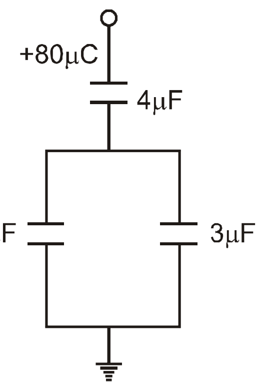
$\Rightarrow \ell + e = \dfrac{V}{4f}$

$\Rightarrow \ell = \dfrac{V}{4f} - e$

here $e = (0.6)r = (0.6)\ (2) = 1.2$ cm

so $\ell = \dfrac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2$ cm

6. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is :



(A) +32 μC (B) +40 μC (C) +48 μC (D) +80 μC

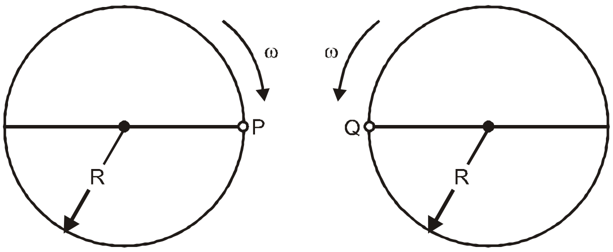
Ans. **(C)**

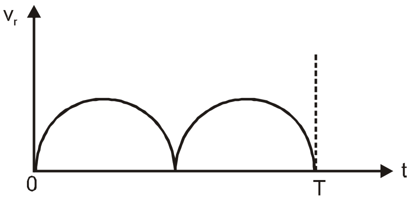
Sol. $q_3 = \dfrac{C_3}{C_2 + C_3}.Q$

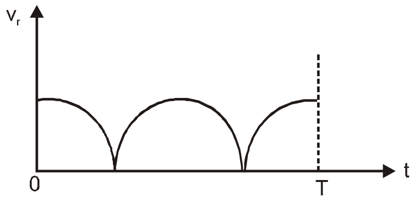
$q_3 = \dfrac{3}{3+2} \times 80 = \dfrac{3}{5} \times 80$

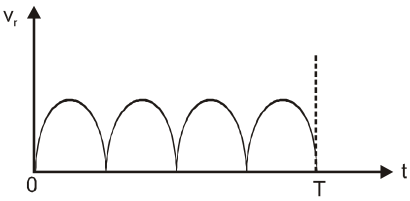
$= 48\ \mu C$

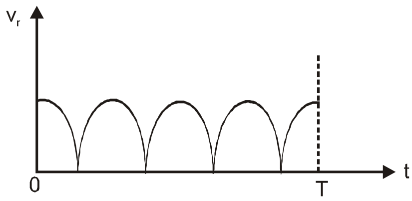
7. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0 , the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by



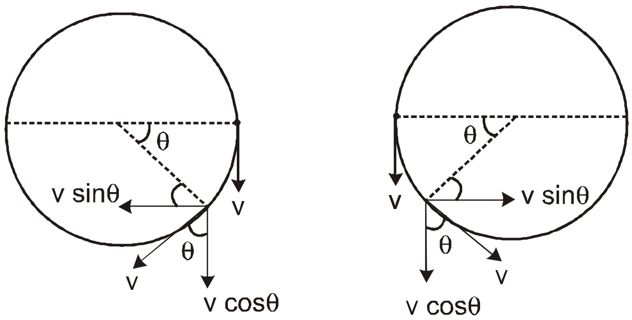
(A) 

(B) 

(C) 

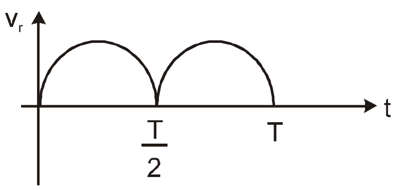
(D) 

Ans. **(A)**

Sol. 

$v_r = |2\ v \sin\theta)|$

$= |2v \sin \omega t)|$



8. Two moles of ideal helium gas are in a rubber balloon at 30° C .The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C.The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

(A) 62J (B) 104 J (C) 124 J (D) 208 J

Ans. **(D)**

Sol. $\Delta Q = nC_P\,\Delta T$

$= 2\left(\frac{f}{2}R + R\right)\Delta T$

$= 2\left[\frac{3}{2}R + R\right] \times 5$

$= 2 \times \frac{5}{2} \times 8.31 \times 5$

$= 208$ J

SECTION – II : Paragraph Type

This section contains **6 multiple choice questions** relating to **three paragraphs** with **two questions on each paragraph.** Each question has four choices (A), (B) (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 9 and 10

The β- decay process, discovered around 1900, is basically the decay of a neutron (n), In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. therefore, considering the decay of a neutron as a tro-body dcay process, it was predicted theoretically that thekinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the lectron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

9. What is the maximum energy of the anti-neutrino ?

(A) Zero (B) Much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV (D) Much larger than 0.8×10^6 eV

Ans. **(C)**

Sol. KE_{max} of β^-

$Q = 0.8 \times 10^6$ eV

$KE_P + KE_{\beta^-} + KE_{\bar{\nu}} = Q$

KE_P is almost zero

When $KE_{\beta^-} = 0$

then $KE_{\bar{\nu}} = Q - KE_P$

$\cong Q$

10. If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron ?

(A) $0 \le K \le 0.8 \times 10^6$ eV (B) 3.0 eV $\le K \le 0.8 \times 10^6$ eV

(C) 3.0 eV $\le K < 0.8 \times 10^6$ eV (D) $0 \le K < 0.8 \times 10^6$ eV

Ans. **(D)**

Sol. $0 \le KE_{\beta^-} \le Q - KE_P - KE_{\overline{\nu}}$

$0 \le KE_{\beta^-} < Q$

Paragraph for Question 11 and 12

Most materials have therefractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snells' law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

11. Choose the correct statement.

(A) The speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is v = c.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

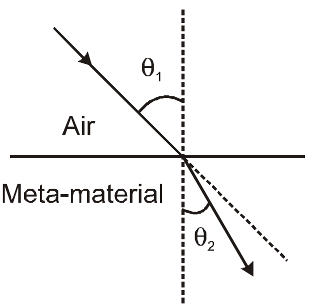
Ans. **(B)**

Sol. $n = \frac{c}{v}$

for metamaterials

$v = \frac{c}{|n|}$

12. For light incident from air on a meta-material, the appropriate ray diagram is :

(A)


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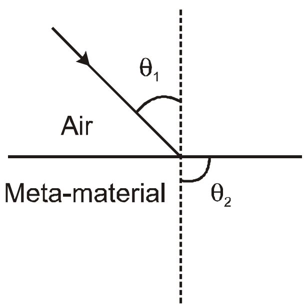
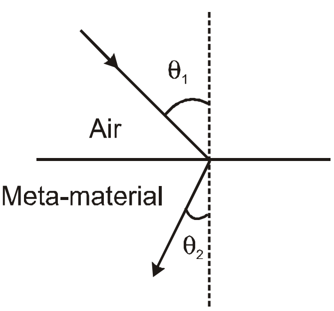


(C)


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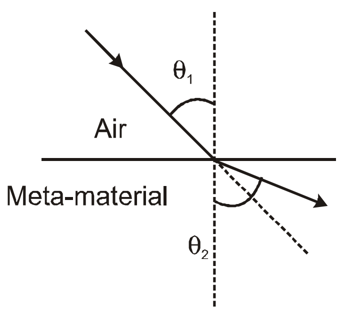


Ans. **(C)**

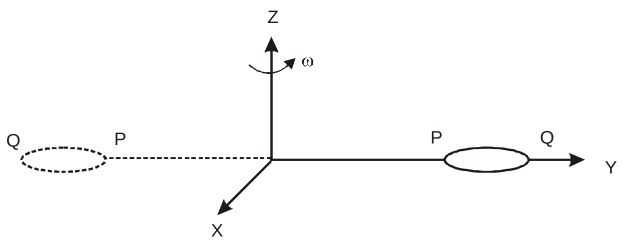
Sol. **(C)** Meta material has a negative refractive index

$\therefore$ (C) $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 \Rightarrow$ n_2 is negative

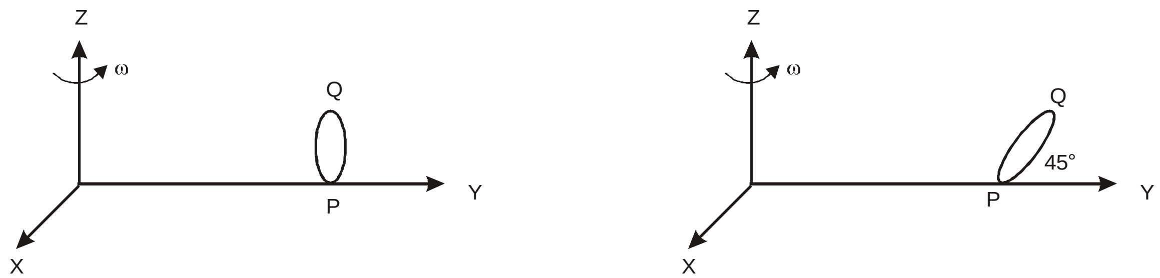
$\therefore$ θ_2 negative

Paragraph for Q. No. 13-14

The general motion of a rigid body can be considered to be a combination of (i) a motioon --- centre of mass about an axis, and (ii) its motion about an instantanneous axis passing through center of mass. These axes need not be stationary. Consider, for example, a thin uniform welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. Where disc-stick system is rotated about the origin ona horizontal frictionless plane with angular sp--- ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis pass through its centre of mass (as is seen from the changed orientation of points P and Q). Both the motions have the same angular speed ω in the case.



Now consider two similar systems as shown in the figure: case (a) the disc with its face ver--- and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane its horizontal diameter parallel to x-axis. In both the cases, the disc is weleded at point P, and systems are rotated with constant angular speed ω about the z-axis.



Case (a) Case (b)

13. Which of the following statement regarding the angular speed about the istantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2}\,\omega$ for boht the cases

(B) it is ω for case (a); and $\frac{w}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2}\,\omega$ for case (b)

(D) It is ω for both the cases

Ans. **(D)**

Sol. Angular Velocity of rigid body about any axes which are parallel to each other is same . So angular velocity is ω .

14. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is vertical for both the cases (a) and (b).

(B) It is verticle for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(C) It is horizontal ofr case (a); and is at 45° to the x - z plane and is normal to the plane of the disc for case (b).

(D) It is vertical of case (a); and is at 45° to the x - z plane and is normal to the plane of the disc for case (b).

Ans. **(A)**

Sol. Since z- coordinate of any particle is not changing with time so axis must be parellel to z axis.

SECTION – III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

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15. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane form the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement (s) is (are) correct?

(A) Both cylinders P and Q reach the ground at the same time

(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinder Q reaches the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

Ans. **(D)**

Sol. $I_P > I_Q$

$$a_P = \frac{g\sin\theta}{I_P + mR^2}$$

$$a_Q = \frac{g\sin\theta}{I_Q + mR^2}$$

$a_P < a_Q \Rightarrow V = u + at \Rightarrow t \propto \frac{1}{a}$

$t_P > t_Q$

$V^2 = u^2 + 2as \Rightarrow v \propto a \Rightarrow V_P < V_Q$

Translational K.E. = $\frac{1}{2}mV^2 \Rightarrow$ TR $KE_P <$ TR KE_Q

$V = \omega R \Rightarrow \omega \propto V \Rightarrow \omega_P < \omega_Q$

16. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are) :

(A) the emf induced in the loop is zero if the current is constant.

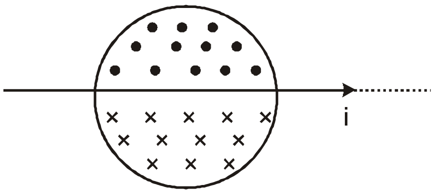
(B) The emf induced in the loop is finite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) Theemf induced in the loop is finite if the current decreases at a steady rate.

Ans. **(A,C)**

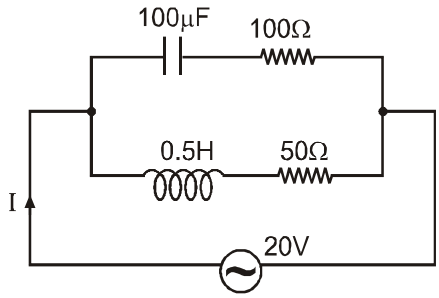
Sol.



$(\phi)_{loop} = 0$ for all cases

so induced emf $= 0$

17. In the given circuit, the AC source has $\omega = 100$ rad/s. considering the inductor and capacitor to be ideal, the correct choice (s) is(are)



(A) The current through the circuit, I is 0.3 A

(B) The current through the circuit, I is $0.3\sqrt{2}$ A.

(C) The voltage across 100Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50Ω resistor = 10V

Ans. **(A,C or C)**

Sol. C = 100 μF, $\frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})}$

$X_C = 100\ \Omega$, $X_L = \omega L = (100)\ (.5) = 50\ \Omega$

$Z_1 = \sqrt{x_C^2 + 100^2} = 100\ \sqrt{2}\Omega$

$Z_2 = \sqrt{x_L^2 + 50^2} = \sqrt{50^2 + 50^2}$

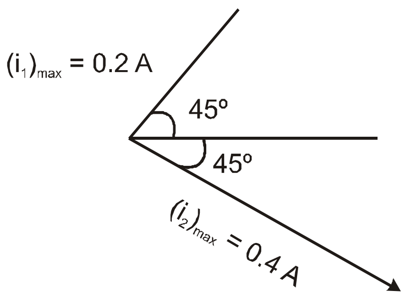
$= 50\ \sqrt{2}$

$\varepsilon = 20\sqrt{2}\sin\omega t$

$i_1 = \frac{20\sqrt{2}}{100\sqrt{2}}\sin(\omega t + \pi/4)$

$i_1 = \frac{1}{5}\sin(\omega t + \pi/4)$

$I_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin(\omega t - \pi/4)$



$I = \sqrt{(.2)^2 + (.4)^2}$

$= (.2)\ \sqrt{1+4}$

$= \frac{1}{5}\ \sqrt{5}\ = \frac{1}{\sqrt{5}}$

$(I)_{rms} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\approx 0.3\,A$

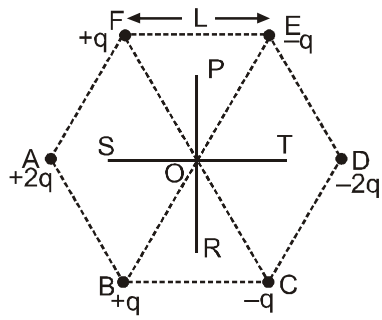
$(V_{100\Omega})_{rms} = (I_1)_{rms}) \times 100$

$= \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = \frac{20}{\sqrt{2}} = 10\ \sqrt{2}\ V$

$V_{50\Omega})_{rms} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ V$

Since $I_{rms} \approx$ 0.3 A so A may or may not be correct.

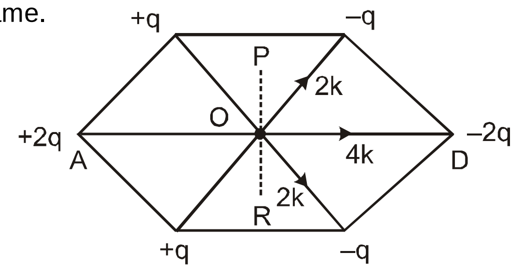
18. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct ?



(A) the elecric field at O is 6K along OD
(B) The potential at O is zero
(C) The potential at all points on the line PR is same
(D) The potential at all points on the line ST is same.

Ans. **(A,B,C)**

Sol. E_0 = 6 K (along OD)
$V_0 = 0$
Potential on line PR is zero
Ans. (A), (B), (C)



19. Two spherical planets P and Q have the same unfirom density ρ, masses M_P and M_Q, an surface areas A and 4A, respectively. A spherical planet R also has unfirom density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V respectivley. Then

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

Ans. **(B,D)**

Sol. $V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2.G\rho.\frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{4G\rho}{3}}\ R$

$V_{es} \propto R$
Sarface area of P = A = $4\pi R_P^2$
Surface area of Q = 4A = $4\pi\ R_Q^2$
$\Rightarrow R_Q = 2R_p$
mass R is $M_R = M_P + M_Q$

$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$

$\Rightarrow R_R^3 = R_P^3 + R_Q^3$

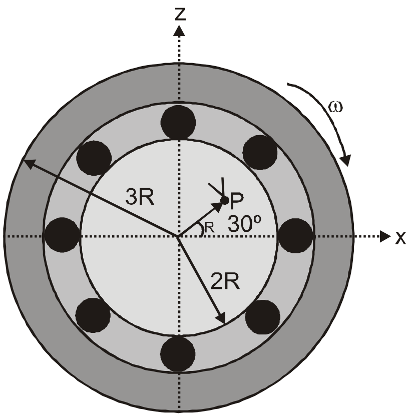
$= 9R_P^3$

$R_R = 9^{1/3} R_P \Rightarrow R_R > R_Q > R_P$

Therefore $V_R > V_Q > V_P$

$\frac{V_R}{V_P} = 9^{1/3}$ and $\frac{V_P}{V_Q} = \frac{1}{2}$

20. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated b frictionaless ball bearings. The system is in the x-z plane. The point P on the inner disc is at distance R from the origin, where OP makes an angle of 30º with the horizontal. Then with respect to the horizontal surface,



(A) the point O has linear velocity $3R\omega\,\hat{i}$.

(B) the point P has a linear velocity $\frac{11}{4}R\omega\,\hat{i} + \frac{\sqrt{3}}{4}R\omega\,\hat{k}$

(C) the point P has linear velocity $\frac{13}{4}Rw\,\hat{i} - \frac{\sqrt{3}}{4}R\omega\,\hat{k}$

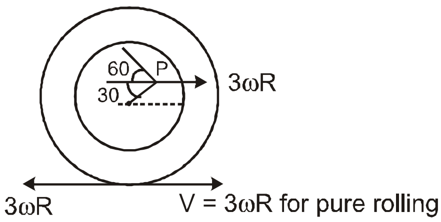
(D) The point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)Rw\,\hat{i} + \frac{1}{4}Rw\,\hat{k}$.

Ans. **(A,B)**

Sol. $V_0 = 3\omega R\,\hat{i}$

$V_P\ (3\omega R - \frac{\omega R}{2}\cos 60^\circ)\ \hat{i} + \frac{\omega R}{2}\sin 60\,\hat{j}$

$= \frac{11\omega R}{4}\hat{i} + \frac{\sqrt{3}\omega R}{4}\hat{i}$



PART - II : CHEMISTRY

SECTION - I : Single Correct Answer Type

This section contains 8 multiple choice questions, Each question has four choices, (A), (B), (C) and (D) out of which ONLY ONE is correct.

21. $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behaviour (paramagnetic/ diamagnetic) . the coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

(A) tetrahedral and tetrahedral

(B) square planar and square planar

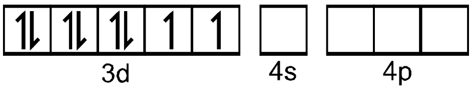
(C) tetrahedral and square planar

(D) square planar and tetrahedral

Ans. **(C)**

Sol. $[NiCl_2\{PEt_2Ph\}]$ contains Ni^{2+} with electronic configuration

Ni^{2+} = [Ar] $3d^8 4s^0$



In high spin state, it is paramagnetic, sp^3 hybridised, tetrahedral.

In low spin state, it is diamagnetic, dsp^2, square planar.

22. The reaction of white phosphorous with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type; the oxidation states of phosphorous in phosphine and the other product are respectively

(A) redox reaction; – 3 and –5

(B) redox reaction ; 3 and + 5

(C) disproportionation reaction ; – 3 and + 5

(D) disproportionation reaction; – 3 and + 3

Ans. **(C)**

Sol.
$$P_4(s) + NaOH \longrightarrow \underset{(-3)}{PH_3} + \underset{(+1)}{NaH_2PO_2}(aq)$$

$$NaH_2PO_2 \xrightarrow{\Delta} \underset{(+5)}{Na_3PO_4} + PH_3$$

Oxidation states of P in Na_3PO_4 & PH_3 are +5 & –3 respectively. It is a disproportionation reaction.

23. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agents used are

(A) O_2 and CO respectively

(B) O_2 and Zn dust respectively

(C) HNO_3 and Zn dust respectively.

(D) HNO_3 and CO respectively

Ans. **(B)**

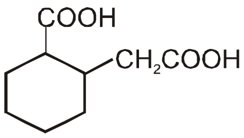
Sol. In extraction of silver, Ag_2S is leached with KCN in presence of air :

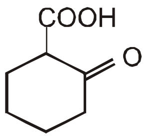
$$Ag_2S + NaCN + O_2 \rightleftharpoons Na[Ag(CN)_2] + Na_2S_2O_3$$

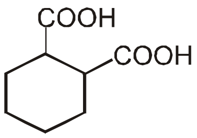
Thus, O_2 is oxidant.

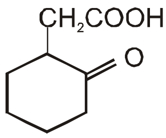
$$2Ag(CN)_2^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2Ag\downarrow$$

24. The compound that undergoes decarboxlylation most readily under mild condition is

(A) 

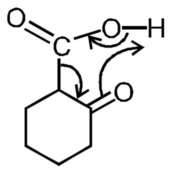
(B) 

(C) 

(D) 

Ans. **(B)**

Sol. In decarboxylation, β-carbon acquires $\delta-$ charge. Whenever $\delta-$ charge is stabilized, decarboxylation becomes simple. In (B), it is stabilized by –m & –I of C = O, which is best amongst the options offered,



25. Using the data provided, calculate the multiple bond energy (kJ mol^{-1}) of a C≡C bond C_2H_2. That energy is (take the bond energy of a C–H bond as 350 kJ mol^{-1})

$2C(s) + H_2(g) \longrightarrow C_2H_2(g)$ $\Delta H = 225$ kJ mol^{-1}

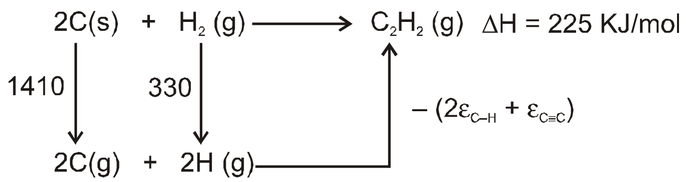
$2C(s) \longrightarrow 2C(g)$ $\Delta H = 1410$ kJ mol^{-1}

$H_2(g) \longrightarrow 2H(g)$ $\Delta H = 330$ kJ mol^{-1}

(A) 1165 (B) 837 (C) 865 (D) 815

Ans. **(D)**

Sol.



$\therefore$ $\Delta H = +1410 + 330 - (350 \times 2) - \varepsilon_{C\equiv C} = +225$

$\therefore$ $\varepsilon_{C\equiv C} = 1740 - 700 - 225 = +815$ KJ/mol.

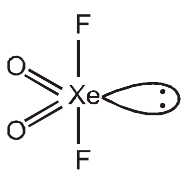
26. The shape of XeO_2F_2 molecule is

(A) trigonal bipyramidal (B) square plannar

(C) tetrahedral (D) see-saw

Ans. **(D)**

Sol. XeO_2F_2 has trigonal bipyramidal geometry. Due to presence of lone pair on equitorial position, the shape is see-saw.



27. The major product H in the given reaction sequence is

$$CH_3–CH_2–CO–CH_3 \xrightarrow{^{\ominus}CN} G \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} H$$

(A) $CH_3–CH=\underset{\substack{|\\CH_3}}{C}–COOH$

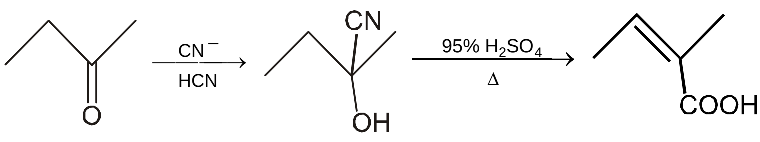
(B) $CH_3–CH=\underset{\substack{|\\CH_3}}{C}–CN$

(C) $CH_3–CH_2–\overset{\substack{OH\\|}}{\underset{\substack{|\\CH_3}}{C}}–COOH$

(D) $CH_3–CH=\underset{\substack{|\\CH_3}}{C}–CO–NH_2$

Ans. **(A)**

Sol.



(G)

28. For a dilute solution containing 2.5 g of a non- volatile non- electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2ºC. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take K_b = 0.76 K kg mol^{-1})

(A) 724 (B) 740 (C) 736 (D)718

Ans. **(A)**

Sol. $\Delta T_b = 2^oC$; $m_a = 2.5$ g

$m_{solvent} = 100$ g

$K_b = 0.76$ K. kg. mol^{-1}

$P_{solution} = ?$

$\Delta T_b = K_b \times m$

$2 = 0.76 \times m$ $\quad\therefore\ m = \dfrac{2}{0.76}$

$\dfrac{P^0 - P}{P} = m \times MM \times 10^{-3}$ $\quad\therefore\ \dfrac{760 - P}{P} = \dfrac{2}{0.76} \times 18 \times 10^{-3}$

$760 - P = \dfrac{36}{760}P$ $\quad\therefore\ 760 = \dfrac{796}{760}P$

$\therefore\ P = 760\left(\dfrac{796}{760}\right)$ torr = 725.6 torr $\approx$ 724 torr

SECTION – II : Paragraph Type

This section contains **6 multiple choice questions** relating to **three paragraphs** with **two questions on each paragraph.** Each question has four choices (A), (B) (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 29 to 30

The electrochemical cell shown below is a concentration cell.

M|M^{2+} (saturated solution of a sparingly soluble salt,MX_2)|| M^{2+} (0.001 mol dm^{-3}) |M

The emf of the cell depends on the difference in concetration of M^{2+} ions at the two electrodes. The emf of the cell at 298 is 0.059 V

29. The solubility product (K_{sp} ; mol^3 dm^{-9}) of MX_2 at 298 based on the information available the given concentration cell is (take 2.303× R × 298/F = 0.059 V)

(A) 1×10^{-15} (B) 4×10^{-15} (C) 1×10^{-12} (D) 4×10^{-12}

Ans. **(B)**

Sol. M|M^{2+} (aq) || M^{2+} (aq) | M

0.001 M

Anode : $M \longrightarrow M^{2+}(aq) + 2e^-$

Cathode : $M^{2+}(aq) + 2e^- \longrightarrow M$

$M^{2+}(aq)_c \rightleftharpoons M^{2+}(aq)_a$

$$E_{cell} = 0 - \frac{0.059}{2}\log\left\{\frac{M^{2+}(aq)_a}{10^{-3}}\right\}$$

$$0.059 = -\frac{0.059}{2}\log\left\{\frac{M^{2+}(aq)_a}{10^{-3}}\right\}$$

$$-2 = \log\left\{\frac{M^{2+}(aq)_a}{10^{-3}}\right\}$$

$10^{-2} \times 10^{-3} = M^{2+}(aq)_a$ = solubility = s

$K_{sp} = 4s^3 = 4 \times (10^{-5})^3 = 4 \times 10^{-15}$

30. The value of ΔG (kJ mol^{-1}) for the given cell is (take 1F = 96500 C mol^{-1})

(A) – 5.7 (B) 5.7 (C) 11.4 (D) –11.4

Ans. **(D)**

Sol. $\Delta G = -nFE_{cell} = -2 \times 96500 \times 0.059 \times 10^{-3}$ kJ/mole

$= -11.4$ kJ/mole

Paragraph for Questions Nos. 31 to 32

Bleaching powder and bleach solution are produced on a large scale and used in several house hold products. The effectiveness of bleach solution is often measured by iodometry.

31. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is

(A) 0.48 M (B) 0.96 M (C) 0.24 M (D) 0.024 M

Ans. **(C)**

Sol. milli mole of Hypo = 0.25 × 48

= 2 × milli mole of Cl_2

milli mole of Cl_2 = $\frac{0.25 \times 48}{2}$ = 6 milli mole

= milli mole of Cl_2 = milli mole of $CaOCl_2$

So, molarity = $\frac{6}{25}$M = 0.24 M

32. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

(A) Cl_2O (B) Cl_2O_7 (C) ClO_2 (D) Cl_2O_6

Ans. **(A)**

Sol. $CaOCl_2$ = Ca(OCl)Cl

OCl^- – Hypochlorite ion

which is anion of HOCl

Anhydride of HOCl = Cl_2O

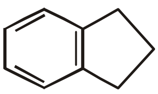
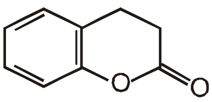
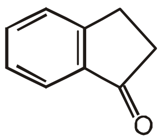
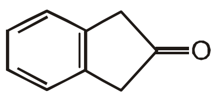
Paragraph for Questions Nos. 33 to 34

In the following reactions sequence, the compound J is an intermediate.

$$\mathrm{I}\ \xrightarrow[CH_3COONa]{(CH_3CO)_2O}\ \mathrm{J}\ \xrightarrow[\text{(ii) } SOCl_2 \text{ (ii) anhyd. } AlCl_3]{\text{(i) } H_2, Pd/C} \mathrm{K}$$

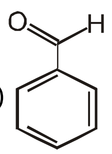
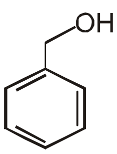
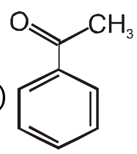
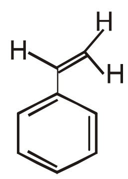
J ($C_9H_8O_2$) gives effervescence on treatment with $NaHCO_3$ and positive Baeyer's test

33. The compound K is

(A)  (B)  (C)  (D) 

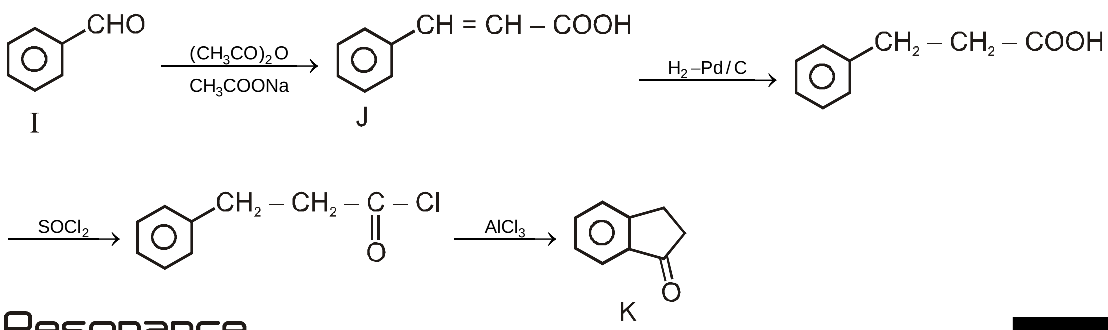
Ans. **(C)**

34. The compound I is

(A)  (B)  (C)  (D) 

Ans. **(A)**

Sol. **(33 to 34)**



SECTION – III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

35. With respect to graphite and diamond, which of the statement(s) given below is (are) correct ?

(A) Graphite is harder than diamond.

(B) Graphite has higher electrical conductivity than diamond

(C) Graphite has higher thermal conductivity than diamond

(D) Graphite has higher C–C bond order than diamond

Ans. **(BD)**

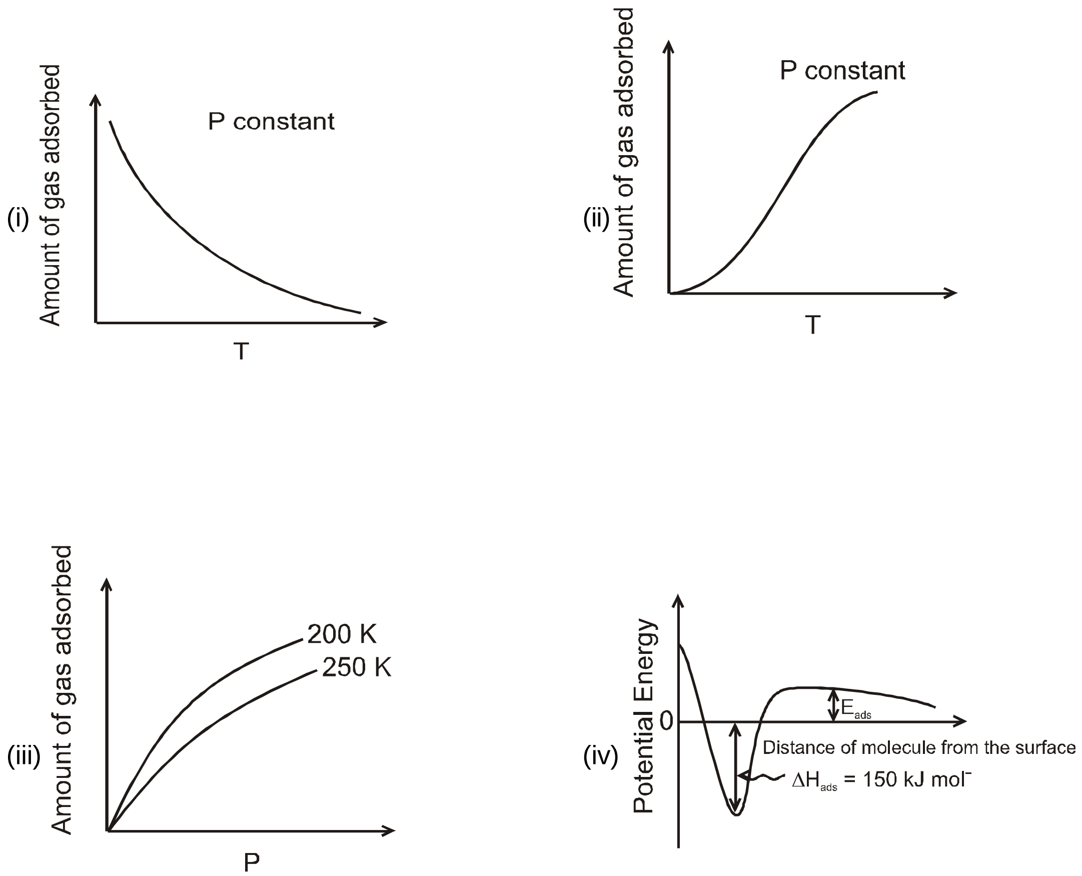
Sol. (A) Diamond is harder than graphite.

(B) Graphite is better conductor of electricity than diamond.

(C) Diamond is better conductor of heat than graphite.

(D) Bond order of graphite ($\simeq$ 1.5) > Bond order of diamond (= 1)

36. The given graph / data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice (s) about I, II, III and IV is (are) correct



(A) I is physisorption and II is chemisorption

(B) I is physisorption and III is chemisorption

(C) IV is chemisorption and II is chemisorption

(D) IV is chemisorption and III is chemisorption

Ans. **(AC)**

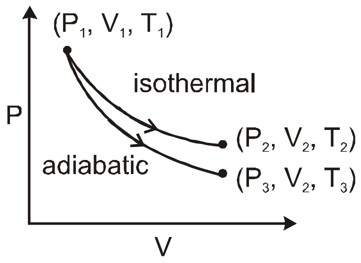
Sol. In physisorption on increasing temperature at constant pressure, adsorption decreases while in chemical adsorption on increasing temperature due to requirement of activation energy adsorption will increase at same pressure. So, I is physisorption while II is chemisorption.

III is physical adsorption as on increasing temperature, extent of adsorption is decreasing .

IV is representing enthalpy change (which is high) during chemical adsorption (due to bond formation) So, is valid for chemical adsorption.

So, answer is (A) and (C)

37. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct ?

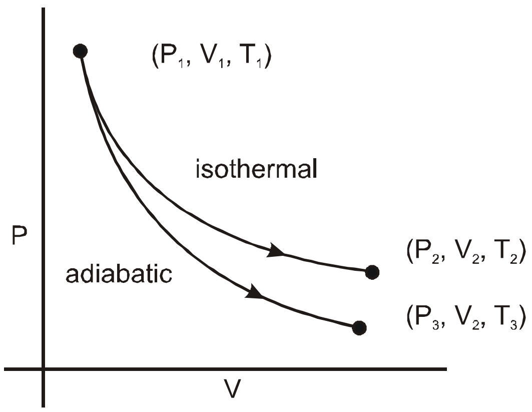


(A) $T_1 = T_2$ (B) $T_3 > T_1$

(C) $w_{isothermal} > w_{adiabatic}$ (D) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

Ans. **(AD)**

Sol.



(A) $T_1 = T_2$ (due to isothermal)

(B) $T_3 > T_1$ (incorrect) cooling will take place in adiabatic expansion)

(C) $W_{isothermal} > W_{adiabatic}$ { with sign, this is incorrect}

(D) $\Delta U_{isothermal} = 0 > \Delta U_{adiabatic} = -$ ve

So, answer is (A) and (D)

38. For the given aqueous reaction which of the statement(s) is (are) true ?

$$\text{excess KI} + K_3[Fe(CN)_6] \xrightarrow{\text{dilute } H_2SO_4} \text{brownish-yellow solution}$$

$$\downarrow ZnSO_4$$

(white precipitate + $\underbrace{\text{brownish- yellow filtrate}}$)

$$\downarrow Na_2S_2O_3$$

colourless solution

(A) The first reaction is a redox reaction

(B) White precipitate is $Zn_3[Fe(CN)_6]_2$

(C) Addition of filtrate to starch solution gives blue colour.

(D) White precipitate is soluble in NaOH solution

Ans. **(ACD)**

Sol. $\text{KI (aq)} + K_3[Fe(CN)_6](aq) \longrightarrow \underset{\text{Brownish - yellow}}{KI_3\ (aq)} + K_4[Fe(CN)_6]\ (aq)$

$$\downarrow + ZnSO_4\ (aq)$$

$\underset{\text{white ppt}}{K_2Zn_3[Fe(CN)_6]_2\downarrow}$ + KI_3 (aq)

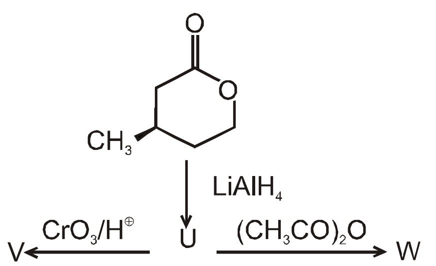
OR $\qquad \downarrow Na_2S_2O_3$

$\{K_2Zn[Fe(CN)_6]\downarrow\}$ $\qquad \underset{\text{Colourless}}{I^-\ (aq) + S_4O_6^{2-}\ (aq)}$

(D) with NaOH

$K_2Zn[Fe(CN)_6] + NaOH \longrightarrow [Zn(OH)_4]^{2-}(aq) + [Fe(CN)_6]^{4-}(aq)$

39. With reference to the scheme given, which of the given statments(s) about T, U, V and W is (are) correct?

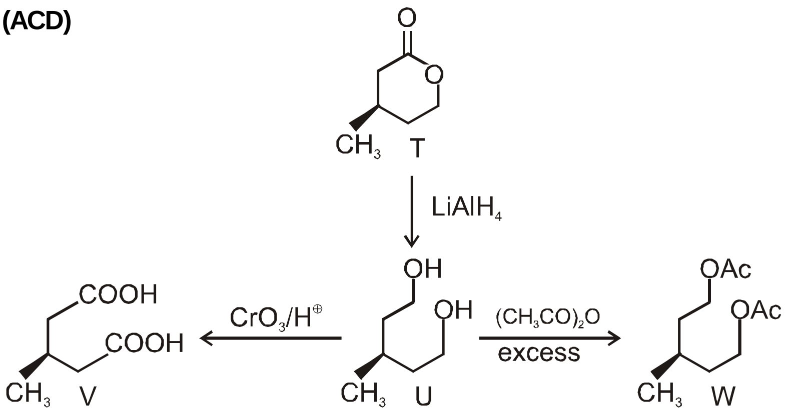


(A) T is soluble in hot aqueous NaOH (B) U is optically active

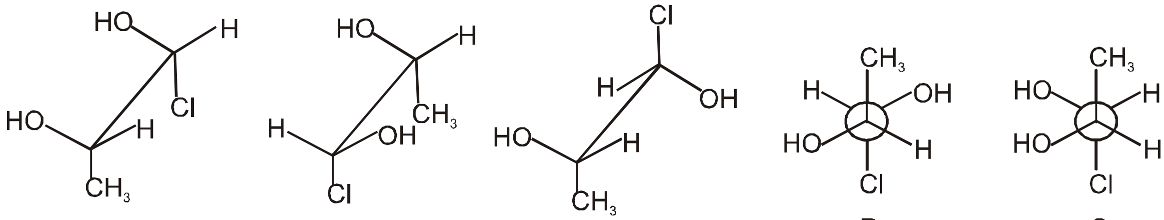
(C) Molecular formula of W is $C_{10}H_{18}O_4$ (D) V gives effervescence on treatment with aqueous $NaHCO_3$

Ans. **(ACD)**

Sol.



40. Which of the given statement(s) about N, O, P and Q with respect to M is (are) correct ?



M N O P Q

(A) M and N are non-mirror image stereoisomers

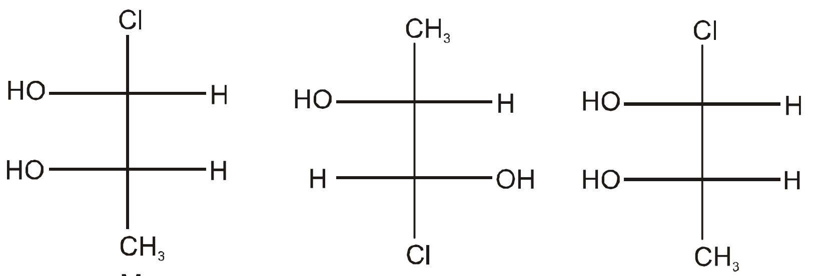
(B) M and O are identical

(C) M and P are enantiomers

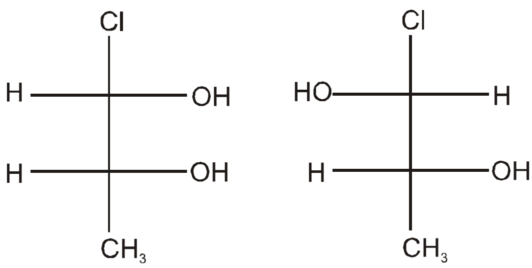
(D) M and Q are identical

Ans. **(ABC)**

Sol.



M N O



P Q

PART - III : MATHEMATICS

Section I : Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) x + y + z = $\sqrt{3}$ (D) x – $\sqrt{2}$y = 1 – $\sqrt{2}$

Sol. Ans. (A)

Equation of required plane

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow$ $(1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$

distance from point (3, 1, – 1)

$$= \left|\frac{3 + 3\lambda + 2 - \lambda - 3 - \lambda - 2 - 3\lambda}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2 + 4\lambda + 14}}\right| = \frac{2}{\sqrt{3}}$$

$\Rightarrow$ $3\lambda^2 = 3\lambda^2 + 4\lambda + 14$

$\Rightarrow$ $\lambda = -\frac{7}{2}$

equation of required plane

$5x - 11y + z - 17 = 0$

42. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

Sol. Ans. (C)

Let $\vec{c} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

$\Rightarrow$ $(\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$

$\Rightarrow$ $(\vec{a} + \vec{b}) \parallel \vec{c}$

Let $(\vec{a}+\vec{b}) = \lambda\vec{c}$

$\Rightarrow$ $|\vec{a}+\vec{b}| = |\lambda|\,|\vec{c}|$

$\Rightarrow$ $\sqrt{29} = |\lambda| .\ \sqrt{29}$

$\Rightarrow$ $\lambda = \pm 1$

$\therefore$ $\vec{a}+\vec{b} = \pm\ (2\hat{i}+3\hat{j}+4\hat{k})$

Now $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm\ (-14+6+12)$

$= \pm 4$

43. Let PQR be a triangle of area Δ with a = 2, b = $\frac{7}{2}$ and c = $\frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Sol. **Ans. (C)**

a = 2 = QR

b = $\frac{7}{2}$ = PR

c = $\frac{5}{2}$ = PQ

s = $\frac{a+b+c}{2} = \frac{8}{4} = 4$

$$\frac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \frac{2\sin P(1-\cos P)}{2\sin P(1+\cos P)} = \frac{1-\cos P}{1+\cos P} = \frac{2\sin^2\frac{P}{2}}{2\cos^2\frac{P}{2}} = \tan^2\frac{P}{2}$$

$$= \frac{(s-b)(s-c)}{s(s-a)} = \frac{(s-b)^2(s-c)^2}{\Delta^2} = \frac{\left(4-\frac{7}{2}\right)^2\left(4-\frac{5}{2}\right)^2}{\Delta^2} = \left(\frac{3}{4\Delta}\right)^2$$

44. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1,2,3,4,5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Sol. **Ans. (A)**

Favourable : D_4 shows a number and

only 1 of $D_1D_2D_3$ shows same number
or only 2 of $D_1D_2D_3$ shows same number
or all 3 of $D_1D_2D_3$ shows same number

Required Probability $= \dfrac{{}^6C_1({}^3C_1 \times 5 \times 5 + {}^3C_2 \times 5 + {}^3C_3)}{216 \times 6}$

$= \dfrac{6 \times (75 + 15 + 1)}{216 \times 6}$

$= \dfrac{6 \times 91}{216 \times 6}$

$= \dfrac{91}{216}$

45. The value of the integral $\displaystyle\int_{-\pi/2}^{\pi/2} \left(x^2 + \ell n \frac{\pi + x}{\pi - x}\right) \cos x \, dx$ is

(A) 0 (B) $\dfrac{\pi^2}{2} - 4$ (C) $\dfrac{\pi^2}{2} + 4$ (D) $\dfrac{\pi^2}{2}$

Sol. **Ans.** **(B)**

$$\int_{-\pi/2}^{\pi/2} \left(x^2 + \ell n\left(\frac{\pi + x}{\pi - x}\right)\right) \cos x \, dx = 2\int_0^{\pi/2} x^2 \cos x dx + 0 \qquad \left(\because \ell n\left(\frac{\pi + x}{\pi - x}\right) \text{is an odd function}\right)$$

$$= 2\left[\left(x^2 \sin x\right)_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x \, dx\right] = 2\left(\frac{\pi^2}{4} - 0\right) - 4\int_0^{\pi/2} x \sin x \, dx$$

$$= \frac{\pi^2}{2} - 4\left[\left(-x \cos x\right)_0^{\pi/2} + \int_0^{\pi/2} \cos x \, dx\right]$$

$$= \frac{\pi^2}{2} - 4$$

46. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$ (C) $PX = 2X$ (D) $PX = -X$

Sol. **Ans.** **(D)**

$P^T = 2P + I$

$\Rightarrow (P^T)^T = (2P + I)^T$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow 3P = -3I \quad \Rightarrow \quad P = -I$

$\Rightarrow PX = -IX = -X$

47. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

Sol. **Ans.** **(D)**

Corresponding A.P.

$\frac{1}{5}, \ldots\ldots\ldots\ldots \frac{1}{25}$ (20^{th} term)

$\frac{1}{25} = \frac{1}{5} + 19\,d \quad \Rightarrow \quad d = \frac{1}{19}\left(\frac{-4}{25}\right) = -\frac{4}{19 \times 25}$

$a_n < 0$

$\frac{1}{5} - \frac{4}{19 \times 25} \times (n-1) < 0$

$\frac{19 \times 5}{4} < n - 1$

$n > 24.75$

48. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$ where $a > -1$.

Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

Sol. **Ans.** **(B)**

$((1 + a)^{1/3} - 1)x^2 + ((a+1)^{1/2} - 1)x + ((a+1)^{1/6} - 1) = 0$

let $a + 1 = t^6$

$\therefore (t^2 - 1)x^2 + (t^3 - 1)x + (t - 1) = 0$

$(t + 1)x^2 + (t^2 + t + 1)x + 1 = 0$

As $a \to 0$, $t \to 1$

$2x^2 + 3x + 1 = 0 \Rightarrow x = -1$ and $x = -\frac{1}{2}$

Section II : Paragraph Type

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 49 to 50

Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ell nt\right) f(t)\, dt$ for all $x \in (1, \infty)$.

49. Which of the following is true ?

(A) g is increasing on $(1, \infty)$

(B) g is decreasing on $(1, \infty)$

(C) g is increasing on (1, 2) and decreasing on $(2, \infty)$

(D) g is decreasing on (1, 2) and increasing on $(2, \infty)$

Sol. **Ans.** **(B)**

$f(x) = (1 - x)^2 \sin^2 x + x^2 : x \in R$

$$g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)dt$$

$$\therefore\ g'(x) = \left(\frac{2(x-1)}{x+1} - \ln x\right) f(x)\ .\ 1$$

let $\phi(x) = \frac{2(x-1)}{x+1} - \ln x$

$$\phi'(x) = \frac{2[(x+1)-(x-1).1]}{(x+1)^2} - \frac{1}{x} = \frac{4}{(x+1)^2} - \frac{1}{x} = \frac{-x^2+2x-1}{x(x+1)^2} = \frac{-(x-1)^2}{x(x+1)^2}$$

$\therefore$ $\phi'(x) \le 0$

$\therefore$ for $x \in (1, \infty)$, $\phi(x) < 0$

$\therefore$ $g'(x) < 0$ for $x \in (1, \infty)$

50. Consider the statements :

P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) both P and Q are false

Sol. **Ans.** **(C)**

$f(x) + 2x = (1 - x)^2 \sin^2 x + x^2 + 2x$

$\because$ $f(x) + 2x = 2(1 + x^2)$

$\Rightarrow$ $(1 - x)^2 \sin^2 x + x^2 + 2x = 2 + 2x^2$

$(1 - x)^2 \sin^2 x = x^2 - 2x + 1 + 1$

$= (1 - x)^2 + 1$

$\Rightarrow \quad (1 - x)^2 \cos^2 x = -1$

which can never be possible

P is not true

$\Rightarrow$ Let $H(x) = 2f(x) + 1 - 2x(1 + x)$

$H(0) = 2f(0) + 1 - 0 = 1$

$H(1) = 2f(1) + 1 - 4 = -3$

$\Rightarrow$ so $H(x)$ has a solution

so Q is true

Paragraph for Question Nos. 51 to 52

Let a_n denote the number of all n-digit positive integers formed by the digits 0,1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

51. Which of the following is correct ?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Sol. **Ans. (A)**

$\underline{1}$----------------- $\underline{1}$ # a_{n-1}

----------------- $\underline{1}\,\underline{0}$ # a_{n-2}

So A choice is correct

consider B choice $c_{17} \neq c_{16} + c_{15}$

$c_{15} \neq c_{14} + c_{13}$ is not true

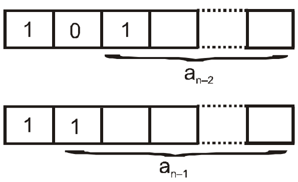
consider C choice $b_{17} \neq b_{16} + c_{16}$

$a_{16} \neq a_{15} + a_{14}$ is not true

consider D choice $a_{17} = c_{17} + b_{16}$

$a_{17} = a_{15} + a_{15}$ which is not true

Aliter



using the Recursion formula

$a_n = a_{n-1} + a_{n-2}$

Similarly $b_n = b_{n-1} + b_{n-2}$ and $c_n = c_{n-1} + c_{n-2}$ $\quad \forall\ n \geq 3$

and $\quad a_n = b_n + c_n \quad \forall\ n \geq 1$

so $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 5$, $a_5 = 8$..........

$b_1 = 1$, $b_2 = 1$, $b_3 = 2$, $b_4 = 3$, $b_5 = 5$, $b_6 = 8$ ....

$c_1 = 0$, $c_2 = 1$, $c_3 = 1$, $c_4 = 2$, $c_5 = 3$, $c_6 = 5$ ....

using this $b_{n-1} = c_n \ \forall\ n \geq 2$

52. The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

Sol. **Ans. (B)**

$b_6 = a_5$

$a_5 = \underline{1} - - - \underline{1} \qquad \underline{1} - - - \underline{0}$

${}^3C_0 + {}^3C_1 + 1 + {}^2C_1 + 1$

$1 + 3 + 1 + 2 + 1$

$4 + 4 = 8$

Paragraph for Question Nos. 53 to 54

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point P($\sqrt{3}$, 1). A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

53. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}\,y = 4$ (D) $x + 2\sqrt{2}\,y = 6$

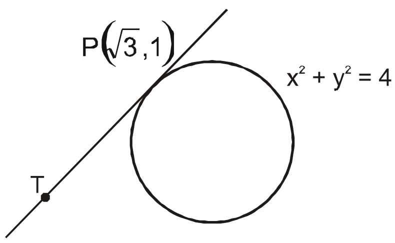
Ans. (D)

54. A possible equation of L is

(A) $x - \sqrt{3}\,y = 1$ (B) $x + \sqrt{3}\,y = 1$ (C) $x - \sqrt{3}\,y = -1$ (D) $x + \sqrt{3}\,y = 5$

Ans. (A)

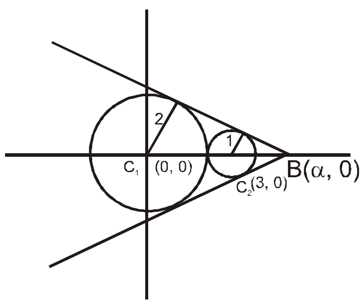
Sol. **Q.No. 53 to 54**



Equation of tangent at $(\sqrt{3}, 1)$

$\sqrt{3}x + y = 4$

53.



B divides C_1 C_2 in 2 : 1 externally

$\therefore$ B(6, 0)

Hence let equation of common tangent is

y – 0 = m(x – 6)

mx – y – 6m = 0

length of $\perp^r$ dropped from center (0, 0) = radius

$$\left|\frac{6m}{\sqrt{1+m^2}}\right| = 2 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$$

$\therefore$ equation is $x + 2\sqrt{2}\,y = 6$ or $x - 2\sqrt{2}\,y = 6$

54. Equation of L is

$x - y\sqrt{3} + c = 0$

length of perpendicular dropped from centre = radius of circle

$$\therefore \left|\frac{3+C}{2}\right| = 1 \quad \Rightarrow C = -1, -5$$

$\therefore$ $x - \sqrt{3}\,y = 1$ or $x - \sqrt{3}\,y = 5$

Section III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

55. Let X and Y be two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct ?

(A) $P(X \cup Y) = \frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^C \cap Y) = \frac{1}{3}$

Sol. Ans. (AB)

$P(X/Y) = \frac{1}{2}$

$$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$$

$P(Y/X) = \frac{1}{3}$

$$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$ A is correct

$P(X \cap Y) = P(X) \cdot P(X) \Rightarrow$ X and Y are independent B is correct

$P(X^c \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ D is not correct

56. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x \in (0, \infty)$, then

(A) f has a local maximum at x = 2

(B) f is decreasing on (2, 3)

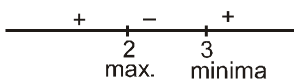
(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at x = 3

Sol. Ans. (ABCD)

$$f(x) = \int_0^x e^{t^2} \cdot (t-2)(t-3)\,dt$$

$$f'(x) = 1 \cdot e^{x^2} \cdot (x-2)(x-3)$$



(i) x = 2 is local maxima (ii) x = 3 is local minima (iii) It is decreasing in $x \in (2, 3)$

(iv) $f''(x) = e^{x^2} \cdot (x-2) + e^{x^2}(x-3) + 2x\,e^{x^2}(x-2)(x-3)$

$= e^{x^2} \cdot [x - 2 + x - 3 + 2x(x-2)(x-3)]$

$f''(x) = 0$

$f''(x) = e^{x^2}(2x^3 - 10x^2 + 14x - 5)$

$f''(0) < 0$ and $f''(1) > 0$

so $f''(c) = 0$ where $c \in (0, 1)$

57. For every integer n, let a_n and b_n be real numbers. Let function f : IR $\to$ IR be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers n.}$$

If f is continuous, then which of the following hold(s) for all n ?

(A) $a_{n-1} - b_{n-1} = 0$ (B) $a_n - b_n = 1$ (C) $a_n - b_{n+1} = 1$ (D) $a_{n-1} - b_n = -1$

Sol. Ans. (BD)

$$\left.\begin{aligned} f(2n) &= a_n \\ f(2n^+) &= a_n \\ f(2n^-) &= b_n + 1 \end{aligned}\right\} \quad \begin{aligned} a_n &= b_n + 1 \\ a_n - b_n &= 1 \\ &\text{So B is correct} \end{aligned}$$

$$\left.\begin{array}{r} f(2n+1) = a_n \\ f((2n+1)^-) = a_n \\ f((2n+1)^+) = b_{n+1} - 1 \end{array}\right\} \quad \begin{array}{l} a_n = b_{n+1} - 1 \\ a_n - b_{n+1} = -1 \\ a_{n-1} - b_n = -1 \end{array}$$

So D is correct

58. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) y + 2z = –1 (B) y + z = –1 (C) y – z = –1 (D) y – 2z = –1

Sol. **Ans.** **(BC)**

For co-planer lines $[\vec{a} - \vec{c}\ \vec{b}\ \vec{d}] = 0$

$\vec{a} \equiv (1, -1, 0)$, $\vec{c} = (-1, -1, 0)$

$\vec{b} = 2\hat{i} + k\hat{j} + 2\hat{k}$ $\vec{d} = 5\hat{i} + 2\hat{j} + k\hat{k}$

Now $\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \quad \Rightarrow \quad k = \pm 2$

$\vec{n}_1 = \vec{b}_1 \times \vec{d}_1 = 6\hat{j} - 6\hat{k}$ for k = 2

$\vec{n}_2 = \vec{b}_2 \times \vec{d}_2 = 14\hat{j} + 14\hat{k}$ for k = – 2

so the equation of planes are $(\vec{r} - \vec{a}).\vec{n}_1 = 0 \Rightarrow y - z = -1$ ...... (1)

$(\vec{r} - \vec{a}).\vec{n}_2 = 0 \Rightarrow y + z = -1$ ...... (2)

so answer is (B,C)

59. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

(A) –2 (B) –1 (C) 1 (D) 2

Sol. **Ans.** **(AD)**

Let A = $[a_{ij}]_{3\times 3}$

adj A = $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

|adj A| = 1(3 – 7) – 4(6 – 7) + 4(2 – 1) = 4

$\Rightarrow |A|^{3-1} = 4$

$\Rightarrow |A|^2 = 4$

$\Rightarrow |A| = \pm 2$

60. Let $f : (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1 - \sqrt{\frac{3}{2}}$ (B) $1 + \sqrt{\frac{3}{2}}$ (C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$

Sol. Ans. (AB)

$\cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

Now $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta} = \frac{1+\cos 2\theta}{\cos 2\theta} = 1 + \frac{1}{\cos 2\theta}$

$\Rightarrow f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

NOTE : Since a functional mapping can't have two images for pre-image 1/3, so this is ambiguity in this question perhaps the answer can be A or B or AB or marks to all.

Resonance
Educating for better tomorrow

ANSWER KEY

IIT-JEE -2012 : PAPER-2

CODE-0, 1, 2, 3, 4, 5, 6, 7, 8 & 9

Que	Paper-2 CODE									
	0	1	2	3	4	5	6	7	8	9
1	B	B	D	A	C	D	C or D	B	A	C or D
2	A	D	A	C	D	C or D	B	A	B	C
3	D	A	C	D	C or D	B	A	B	D	D
4	C or D	C	D	C or D	B	A	B	D	A	B
5	B	D	C or D	B	A	B	D	A	C	A
6	C	C or D	B	A	B	D	A	C	D	A
7	A	B	A	B	D	A	C	D	C or D	B
8	D	A	B	D	A	C	D	C or D	B	D
9	C	C	D	B	C	D	B	D	A	C
10	D	B	A	C	D	A	C	C	D	B
11	B	C	D	D	A	B	C	D	D	D
12	C	D	C	A	D	C	D	A	C	A
13	D	A	C	D	B	D	A	B	C	D
14	A	D	B	C	C	C	D	C	B	C
15	D	AC or C	AC	ABC	D	BD	AB	ABC	AC or C	AC
16	AC	AC	ABC	D	BD	AB	AC or C	BD	ABC	AC or C
17	AC or C	ABC	D	BD	AB	AC or C	AC	AB	BD	ABC
18	ABC	D	BD	AB	AC or C	AC	ABC	D	AB	BD
19	BD	BD	AB	AC or C	AC	ABC	D	AC	D	AB
20	AB	AB	AC or C	AC	ABC	D	BD	AC or C	AC	D
21	C	C	B	C	D	A	B	D	B	B
22	C	B	C	D	A	B	D	B	C	D
23	B	C	D	A	B	D	B	C	B	A
24	B	D	A	B	D	B	C	B	C	C
25	D	A	B	D	B	C	B	C	D	B
26	D	B	D	B	C	B	C	D	A	C
27	B	D	B	C	B	C	D	A	B	D
28	A	B	C	B	C	D	A	B	D	B
29	B	A	C	C	B	C	C	D	A	A
30	D	C	A	A	D	A	A	B	C	C
31	C	B	D	C	A	C	B	C	D	C
32	A	D	B	A	C	A	D	A	B	A
33	C	A	A	D	C	D	A	C	A	D
34	A	C	C	B	A	B	C	A	C	B
35	BD	AD	AC	ACD	BD	ACD	ABC	ACD	AD	AC
36	AC	AC	ACD	BD	ACD	ABC	AD	ACD	ACD	AD
37	AD	ACD	BD	ACD	ABC	AD	AC	ABC	ACD	ACD
38	ACD	BD	ACD	ABC	AD	AC	ACD	BD	ABC	ACD
39	ACD	ACD	ABC	AD	AC	ACD	BD	AC	BD	ABC
40	ABC	ABC	AD	AC	ACD	BD	ACD	AD	AC	BD
41	A	A	C	C	D	B	A	B	D	A
42	C	C	C	D	B	A	B	D	A	D
43	C	C	D	B	A	B	D	A	C	B
44	A	D	B	A	B	D	A	C	C	A
45	B	B	A	B	D	A	C	C	D	D
46	D	A	B	D	A	C	C	D	B	C
47	D	B	D	A	C	C	D	B	A	B
48	B	D	A	C	C	D	B	A	B	C
49	B	B	D	A	B	D	A	C	A	B
50	C	A	A	B	C	A	B	B	D	A
51	A	B	C	D	A	A	B	D	C	D
52	B	C	B	A	D	B	C	A	B	A
53	D	A	B	C	A	C	A	A	B	C
54	A	D	A	B	B	B	D	B	A	B
55	AB	BD	ABCD	BC	AB	AD	AB	BC	BD	ABCD
56	ABCD	ABCD	BC	AB	AD	AB	BD	AD	BC	BD
57	BD	BC	AB	AD	AB	BD	ABCD	AB	AD	BC
58	BC	AB	AD	AB	BD	ABCD	BC	AB	AB	AD
59	AD	AD	AB	BD	ABCD	BC	AB	ABCD	AB	AB